

MODULE <i>AWSet</i>	
EXTENDS <i>Naturals, Sequences, SEC</i>	
CONSTANTS	
<i>Data</i>	the set of data
VARIABLES	
<i>aSet</i> , <i>tSet</i> , <i>seq</i> , <i>incoming</i> , <i>msg</i> , <i>messageSet</i>	<i>aSet</i> [<i>r</i>]: set of active <i>Instance</i> (<i>s</i>) maintained by <i>r</i> ∈ <i>Replica</i> <i>tSet</i> [<i>r</i>]: set of tombstone <i>Instance</i> (<i>s</i>) maintained by <i>r</i> ∈ <i>Replica</i> <i>seq</i> [<i>r</i>]: local sequence number at replica <i>r</i> ∈ <i>Replica</i> <i>incoming</i> [<i>r</i>]: incoming messages at replica <i>r</i> ∈ <i>Replica</i>
$\text{vars} \triangleq \langle aSet, tSet, seq, incoming, msg, messageSet, SECvars \rangle$	
$Instance \triangleq [d : Data, r : Replica, k : Nat]$ $Msg \triangleq [r : Replica, seq : Nat, update : \text{SUBSET } Update, A : \text{SUBSET } Instance, T : \text{SUBSET } Instance]$	
$Network \triangleq \text{INSTANCE } Network$	
$TypeOK \triangleq$ $\wedge \quad aSet \in [Replica \rightarrow \text{SUBSET } Instance]$ $\wedge \quad tSet \in [Replica \rightarrow \text{SUBSET } Instance]$ $\wedge \quad seq \in [Replica \rightarrow Nat]$	
$Init \triangleq$ $\wedge Network!NInit$ $\wedge SECInit$ $\wedge seq = [r \in Replica \mapsto 0]$ $\wedge aSet = [r \in Replica \mapsto \{\}]$ $\wedge tSet = [r \in Replica \mapsto \{\}]$	
$Add(d, r) \triangleq$ $\wedge seq' = [seq \text{ EXCEPT } ![r] = @ + 1]$ $\wedge SECUpdate(r, seq[r])$ $\wedge aSet' = [aSet \text{ EXCEPT } ![r] = @ \cup \{[d \mapsto d, r \mapsto r, k \mapsto seq'[r]]\}]$ $\wedge \text{UNCHANGED } \langle tSet, incoming, msg, messageSet \rangle$	
$Remove(d, r) \triangleq$ $\wedge seq' = [seq \text{ EXCEPT } ![r] = @ + 1]$ $\wedge SECUpdate(r, seq[r])$ $\wedge \text{LET } D \triangleq \{ins \in aSet[r] : ins.d = d\}$ $\quad \text{IN } \wedge aSet' = [aSet \text{ EXCEPT } ![r] = @ \setminus D]$ $\quad \wedge tSet' = [tSet \text{ EXCEPT } ![r] = @ \cup D]$ $\wedge \text{UNCHANGED } \langle incoming, msg, messageSet \rangle$	

$Broadcast(s, m) \triangleq$
 $\quad [r \in Replica \mapsto \text{IF } s = r \text{ THEN } incoming[s]$
 $\quad \quad \quad \text{ELSE } incoming[r] \circ \langle m \rangle]$

$Send(r) \triangleq$
 $\quad \wedge Network!NBroadcast(r, [r \mapsto r, seq \mapsto seq[r], update \mapsto StateUpdate(r), A \mapsto aSet[r], T \mapsto tSet[r]])$
 $\quad \wedge SECSend(r)$
 $\quad \wedge \text{UNCHANGED } \langle aSet, tSet, seq \rangle$

$Deliver(r) \triangleq$
 $\quad \wedge Network!NDeliver(r)$
 $\quad \wedge SECDeliver(r, msg'[r])$
 $\quad \wedge seq' = [seq \text{ EXCEPT } ![r] = @ + 1]$
 $\quad \wedge tSet' = [tSet \text{ EXCEPT } ![r] = @ \cup msg'[r].T]$
 $\quad \wedge aSet' = [aSet \text{ EXCEPT } ![r] = (@ \cup msg'[r].A) \setminus tSet'[r]]$
 $\quad \wedge \text{UNCHANGED } \langle \rangle$

$Next \triangleq$
 $\quad \vee \exists r \in Replica : \exists a \in Data :$
 $\quad \quad \quad Add(a, r) \vee Remove(a, r)$
 $\quad \vee \exists r \in Replica :$
 $\quad \quad \quad Send(r) \vee Deliver(r)$

$Spec \triangleq Init \wedge \square[Next]_{vars}$

$Read(r) \triangleq \{ins.d : ins \in aSet[r]\}$

$QC: \text{ Quiescent Consistency}$
 $Quiescence \triangleq$
 $\quad \forall r \in Replica : incoming[r] = \langle \rangle$

$Convergence \triangleq$
 $\quad \forall r, s \in Replica : Read(r) = Read(s)$

$QC \triangleq Quiescence \Rightarrow Convergence$

$SEC: \text{ Strong Eventual Consistency}$
 $SEC \triangleq \forall r1, r2 \in Replica :$
 $\quad Sameupdate(r1, r2) \Rightarrow Read(r1) = Read(r2)$

\backslash * Modification History
 \backslash * Last modified Tue May 14 11:24:42 CST 2019 by zfwang
 \backslash * Last modified Sun Apr 28 15:09:12 CST 2019 by jywellin
 \backslash * Created Sun Apr 28 14:02:54 CST 2019 by jywellin