

---

MODULE *AWSet*

---

EXTENDS *Naturals, Sequences, SEC*

---

CONSTANTS

*Data*            the set of data

VARIABLES

*aSet*,            *aSet*[*r*]: set of active *Instance*(*s*) maintained by *r* ∈ *Replica*

*tSet*,            *tSet*[*r*]: set of tombstone *Instance*(*s*) maintained by *r* ∈ *Replica*

*seq*,            *seq*[*r*]: local sequence number at replica *r* ∈ *Replica*

*incoming*,    *incoming*[*r*]: incoming messages at replica *r* ∈ *Replica*

*msg*,

*messageset*

*vars*  $\triangleq$   $\langle aSet, tSet, seq, incoming, msg, messageset, SECvars \rangle$

---

*Instance*  $\triangleq$  [*d* : *Data*, *r* : *Replica*, *k* : *Nat*]

*Msg*  $\triangleq$  [*r* : *Replica*, *seq* : *Nat*, *update* : SUBSET *Update*, *A* : SUBSET *Instance*, *T* : SUBSET *Instance*]

---

*Network*  $\triangleq$  INSTANCE *Network*

---

*TypeOK*  $\triangleq$

$\wedge$  *aSet* ∈ [*Replica* → SUBSET *Instance*]

$\wedge$  *tSet* ∈ [*Replica* → SUBSET *Instance*]

$\wedge$  *seq* ∈ [*Replica* → *Nat*]

---

*Init*  $\triangleq$

$\wedge$  *Network*! *NInit*

$\wedge$  *SECInit*

$\wedge$  *seq* = [*r* ∈ *Replica* ↦ 0]

$\wedge$  *aSet* = [*r* ∈ *Replica* ↦ {}]

$\wedge$  *tSet* = [*r* ∈ *Replica* ↦ {}]

---

*Add*(*d*, *r*)  $\triangleq$

$\wedge$  *seq*' = [*seq* EXCEPT ![*r*] = @ + 1]

$\wedge$  *SECUpdate*(*r*, *seq*[*r*])

$\wedge$  *aSet*' = [*aSet* EXCEPT ![*r*] = @ ∪ {[*d* ↦ *d*, *r* ↦ *r*, *k* ↦ *seq*'[*r*]}]

$\wedge$  UNCHANGED  $\langle tSet, incoming, msg, messageset \rangle$

*Remove*(*d*, *r*)  $\triangleq$

$\wedge$  *seq*' = [*seq* EXCEPT ![*r*] = @ + 1]

$\wedge$  *SECUpdate*(*r*, *seq*[*r*])

$\wedge$  LET *D*  $\triangleq$  {*ins* ∈ *aSet*[*r*] : *ins.d* = *d*}

IN  $\wedge$  *aSet*' = [*aSet* EXCEPT ![*r*] = @ \ *D*]

$\wedge$  *tSet*' = [*tSet* EXCEPT ![*r*] = @ ∪ *D*]

$\wedge$  UNCHANGED  $\langle incoming, msg, messageset \rangle$

---


$$Broadcast(s, m) \triangleq$$

$$[r \in Replica \mapsto \text{IF } s = r \text{ THEN } incoming[s] \\ \text{ELSE } incoming[r] \circ \langle m \rangle]$$

$$Send(r) \triangleq$$

$$\wedge Network!NBroadcast(r, [r \mapsto r, seq \mapsto seq[r], update \mapsto StateUpdate(r), A \mapsto aSet[r], T \mapsto tSet[r]]) \\ \wedge SECSend(r) \\ \wedge \text{UNCHANGED } \langle aSet, tSet, seq \rangle$$

$$Deliver(r) \triangleq$$

$$\wedge Network!NDeliver(r) \\ \wedge SECDeliver(r, msg'[r]) \\ \wedge seq' = [seq \text{ EXCEPT } ![r] = @ + 1] \\ \wedge tSet' = [tSet \text{ EXCEPT } ![r] = @ \cup msg'[r].T] \\ \wedge aSet' = [aSet \text{ EXCEPT } ![r] = (@ \cup msg'[r].A) \setminus tSet'[r]] \\ \wedge \text{UNCHANGED } \langle \rangle$$


---


$$Next \triangleq$$

$$\vee \exists r \in Replica : \exists a \in Data : \\ Add(a, r) \vee Remove(a, r) \\ \vee \exists r \in Replica : \\ Send(r) \vee Deliver(r)$$

$$Spec \triangleq Init \wedge \square[Next]_{vars}$$


---


$$Read(r) \triangleq \{ins.d : ins \in aSet[r]\}$$

**QC: Quiescent Consistency**

$$Quiescence \triangleq$$

$$\forall r \in Replica : incoming[r] = \langle \rangle$$

$$Convergence \triangleq$$

$$\forall r, s \in Replica : Read(r) = Read(s)$$

$$QC \triangleq Quiescence \Rightarrow Convergence$$

**SEC: Strong Eventual Consistency**

$$SEC \triangleq \forall r1, r2 \in Replica :$$

$$Sameupdate(r1, r2) \Rightarrow Convergence$$


---

\ \* Modification History  
 \ \* Last modified *Wed May 15 20:55:42 CST 2019* by *xhdn*  
 \ \* Last modified *Mon May 13 13:28:35 CST 2019* by *zfwang*  
 \ \* Last modified *Sun Apr 28 15:09:12 CST 2019* by *jywellin*  
 \ \* Created *Sun Apr 28 14:02:54 CST 2019* by *jywellin*