NA_5

P 1

a

对于 $x = 1.1$，选 $x_0 = 1.1$，$x_1 = x_0 + h = 1.2$，$x_2 = x_0 + 2h = 1.3$ 来计算:

$$f'(x_0) \approx \frac{1}{2h}[-3f(x_0) + 4f(x_1) - f(x_2)] = 17.769705$$

对于 $x = 1.2$，选 $x_0 = 1.1$，$x_1 = x_0 + h = 1.2$，$x_2 = x_0 + 2h = 1.3$ 来计算:

$$f'(x_1) \approx \frac{1}{2h}[-f(x_0) + f(x_2)] = 22.201805$$

对于 $x = 1.3$，选 $x_0 = 1.2$，$x_1 = x_0 + h = 1.3$，$x_2 = x_0 + 2h = 1.4$ 来计算:

$$f'(x_1) \approx \frac{1}{2h}[-f(x_0) + f(x_2)] = 27.10735$$

对于 $x = 1.4$，选 $x_0 = 1.2$，$x_1 = x_0 + h = 1.3$，$x_2 = x_0 + 2h = 1.4$ 来计算:

$$f'(x_2) \approx \frac{1}{2h}[3f(x_2) - 4f(x_1) + f(x_0)] = 32.51085$$

b

对于 $x = 8.1$，选 $x_0 = 8.1$，$x_1 = x_0 + h = 8.3$，$x_2 = x_0 + 2h = 8.5$ 来计算:

$$f'(x_0) \approx \frac{1}{2h}[-3f(x_0) + 4f(x_1) - f(x_2)] = 3.092050$$

对于 $x = 8.3$，选 $x_0 = 8.1$，$x_1 = x_0 + h = 8.3$，$x_2 = x_0 + 2h = 8.5$ 来计算:

$$f'(x_1) \approx \frac{1}{2h}[-f(x_0) + f(x_2)] = 3.116150$$

对于 $x = 8.5$，选 $x_0 = 8.3$，$x_1 = x_0 + h = 8.5$，$x_2 = x_0 + 2h = 8.7$ 来计算:

$$f'(x_1) \approx \frac{1}{2h}[-f(x_0) + f(x_2)] = 3.139975$$

对于 $x = 8.7$，选 $x_0 = 8.3$，$x_1 = x_0 + h = 8.5$，$x_2 = x_0 + 2h = 8.7$ 来计算:

$$f'(x_2) \approx \frac{1}{2h}[3f(x_2) - 4f(x_1) + f(x_0)] = 3.163525$$

P 2

$$M = N(h) + K_1 h^2 + K_2 h^4 + K_3 h^6 \dots \tag{1}$$

用 $\frac{h}{3}$ 替代 h 后:

$$M = N(\frac{h}{3}) + K_1(\frac{h}{3})^2 + K_2(\frac{h}{3})^4 + K_3(\frac{h}{3})^6 \dots \tag{2}$$

用 $\frac{h}{9}$ 替代 h 后:

$$M = N(\frac{h}{9}) + K_1(\frac{h}{9})^2 + K_2(\frac{h}{9})^4 + K_3(\frac{h}{9})^6 \dots \tag{2}$$

由 $(1) - 90 \times (2) + 729 \times (3)$ 得:

$$640M = N(h) - 90N(\frac{h}{3}) + 729N(\frac{h}{9}) + K_3(1 - \frac{10}{3^4} + \frac{1}{9^3})h^6 + \dots$$
$$M = \frac{N(h) - 90N(\frac{h}{3}) + 729N(\frac{h}{9})}{640} + K_3\frac{1 - \frac{10}{3^4} + \frac{1}{9^3}}{640}h^6 + \dots$$

P 3

a

Trapezoidal rule:

$$\int_{-0.25}^{0.25} (\cos x)^2 dx \approx \frac{\cos(0.25)^2 + \cos(-0.25)^2}{2} \times [0.25 - (-0.25)]$$
$$\approx 0.469396$$

Simpson's rule:

$$\int_{-0.25}^{0.25} (\cos x)^2 dx \approx \frac{0.25}{3}[\cos(0.25)^2 + 4\cos(0)^2 + \cos(-0.25)^2]$$
$$\approx 0.489799$$

b

Trapezoidal rule:

$$\int_{-0.5}^{0} x\ln(x+1)dx \approx \frac{0 - 0.5\ln(-0.5+1)}{2} \times [0 - (-0.5)]$$
$$\approx 0.0866434$$

Simpson's rule:

$$\int_{-0.5}^{0} x\ln(x+1)dx \approx \frac{0.25}{3}[-0.5\ln(-0.5+1) - \ln(-0.25+1) + 0]$$
$$\approx 0.0528546$$

c

Trapezoidal rule:

$$\int_{0.75}^{1.3} [(\sin x)^2 - 2x\sin x + 1] \approx \frac{(\sin 1.3)^2 - 2.6\sin 1.3 + 1 + (\sin 0.75)^2 - 1.5\sin 0.75 + 1}{2} \times [1.3 - 0.75]$$
$$\approx -0.0370243$$

Simpson's rule:

$$\int_{0.75}^{1.3}[(\sin x)^2 - 2x\sin x + 1]dx \approx \frac{0.275}{3}[(\sin 0.75)^2 - 1.5\sin 0.75 + 1 + 4(\sin 1.025)^2 - 8.2\sin 1.025 + 4 + (\sin 1.3)^2 - 2.6\sin 1.3 + 1] \approx -0.0202716$$

d

Trapezoidal rule:

$$\int_e^{e+1}\frac{1}{x\ln x}dx \approx \frac{\frac{1}{e} + \frac{1}{(e+1)\ln(e+1)}}{2} \times (e+1-e) \approx 0.286334$$

Simpson's rule:

$$\int_e^{e+1}\frac{1}{x\ln x}dx \approx \frac{0.5}{3}[\frac{1}{e} + 4\frac{1}{(e+0.5)\ln(e+0.5)} + \frac{1}{(e+1)\ln(e+1)}] \approx 0.272670$$

P 4

a

$$\begin{aligned} R_{1,1} &= \frac{2}{2}[(\cos(-1))^2 + (\cos 1)^2] = 0.583853 \\ R_{2,1} &= \frac{2}{4}[(\cos(-1))^2 + 2(\cos 0)^2 + (\cos 1)^2] = 1.29193 \\ R_{3,1} &= \frac{2}{8}[(\cos(-1))^2 + 2(\cos(-0.5))^2 + 2(\cos 0)^2 + 2(\cos(0.5))^2 + (\cos 1)^2] = 1.41611 \end{aligned}$$

$$\begin{aligned} R_{2,2} &= R_{2,1} + \frac{1}{3}(R_{2,1} - R_{1,1}) = 1.52795 \\ R_{3,2} &= R_{3,1} + \frac{1}{3}(R_{3,1} - R_{2,1}) = 1.45751 \end{aligned}$$

$$R_{3,3} = R_{3,2} + \frac{1}{15}(R_{3,2} - R_{2,2}) = 1.45281$$

b

$$\begin{aligned} R_{1,1} &= \frac{1.5}{2}[-0.75\ln(-0.75+1) + 0.75\ln(0.75+1)] = 1.09457 \\ R_{2,1} &= \frac{1.5}{4}[-0.75\ln(-0.75+1) + 0 + 0.75\ln(0.75+1)] = 0.547287 \\ R_{3,1} &= \frac{1.5}{8}[-0.75\ln(-0.75+1) - 0.75\ln(-0.375+1) + 0 \\ &\quad + 0.75\ln(0.375+1) + 0.75\ln(0.75+1)] = 0.384520 \end{aligned}$$

$$\begin{aligned} R_{2,2} &= R_{2,1} + \frac{1}{3}(R_{2,1} - R_{1,1}) = 0.364858 \\ R_{3,2} &= R_{3,1} + \frac{1}{3}(R_{3,1} - R_{2,1}) = 0.330265 \end{aligned}$$

$$R_{3,3} = R_{3,2} + \frac{1}{15}(R_{3,2} - R_{2,2}) = 0.327959$$

c

$$
\begin{aligned}
R_{1,1} &= \frac{3}{2}[((\sin 1)^2 - 2\sin 1 + 1) + ((\sin 4)^2 - 8\sin 4 + 1)] = 11.4785 \\
R_{2,1} &= \frac{3}{4}[((\sin 1)^2 - 2\sin 1 + 1) + 2((\sin 2.5)^2 - 5\sin 2.5 + 1) + ((\sin 4)^2 - 8\sin 4 + 1)] = 3.287934 \\
R_{3,1} &= \frac{3}{8}[((\sin 1)^2 - 2\sin 1 + 1) + 2((\sin 1.75)^2 - 3.5\sin 1.75 + 1) + 2((\sin 2.5)^2 - 5\sin 2.5 + 1) \\
&\quad 2((\sin 3.25)^2 - 6.5\sin 3.25 + 1) + ((\sin 4)^2 - 8\sin 4 + 1)] = 1.82341
\end{aligned}
$$

$$
\begin{aligned}
R_{2,2} &= R_{2,1} + \frac{1}{3}(R_{2,1} - R_{1,1}) = 0.557767 \\
R_{3,2} &= R_{3,1} + \frac{1}{3}(R_{3,1} - R_{2,1}) = 1.33523 \\
R_{3,3} &= R_{3,2} + \frac{1}{15}(R_{3,2} - R_{2,2}) = 1.38706
\end{aligned}
$$

d

$$
\begin{aligned}
R_{1,1} &= \frac{e}{2}[\frac{1}{e} + \frac{1}{2e\ln(2e)}] = 0.647654 \\
R_{2,1} &= \frac{e}{4}[\frac{1}{e} + 2\frac{1}{1.5e\ln(1.5e)} + \frac{1}{2e\ln(2e)}] = 0.560996 \\
R_{3,1} &= \frac{e}{8}[\frac{1}{e} + 2\frac{1}{1.25e\ln(1.25e)} + 2\frac{1}{1.5e\ln(1.5e)} + 2\frac{1}{1.75e\ln(1.75e)} + \frac{1}{2e\ln(2e)}] = 0.535609
\end{aligned}
$$

$$
\begin{aligned}
R_{2,2} &= R_{2,1} + \frac{1}{3}(R_{2,1} - R_{1,1}) = 0.532111 \\
R_{3,2} &= R_{3,1} + \frac{1}{3}(R_{3,1} - R_{2,1}) = 0.527146 \\
R_{3,3} &= R_{3,2} + \frac{1}{15}(R_{3,2} - R_{2,2}) = 0.526816
\end{aligned}
$$

P 5

a

$$
\begin{aligned}
w_0 &= y(1) \\
w_1 &= w_0 + 0.1(w_0/1 - (w_0/1)^2) \\
w_2 &= w_1 + 0.1(w_1/1.1 - (w_1/1.1)^2) \\
w_3 &= w_2 + 0.1(w_2/1.2 - (w_2/1.2)^2) \\
&\cdots\cdots \\
w_{10} &= w_9 + 0.1(w_9/1.9 - (w_9/1.9)^2)
\end{aligned}
$$

$\because\ y(1) = 1$

$\therefore\ w_{10} = 1.17$

b

$$
\begin{aligned}
w_0 &= y(1) \\
w_1 &= w_0 + 0.2(1 + w_0/1 + (w_0/1)^2) \\
w_2 &= w_1 + 0.2(1 + w_1/1.2 + (w_1/1.2)^2) \\
w_3 &= w_2 + 0.2(1 + w_2/1.4 + (w_2/1.4)^2) \\
&\cdots\cdots \\
w_{10} &= w_9 + 0.2(1 + w_9/2.8 + (w_9/2.8)^2)
\end{aligned}
$$

$\because\ y(1) = 0$

$\therefore\ w_{10} = 4.51$

P 6

```
clear; clc;
m = 6;
n = 6;
N = m + n;
a = zeros(m+1,1);
q = zeros(m+1,1);
p = zeros(n+1,1);
b = zeros(N,N+1);

% 将ai初始化为sin(x)麦克劳林展开式的系数
for i = 1:N+1
    a(i) = ((-1)^(ceil(i/2)+1))*(mod(i,2)*sin(0)+mod(i-1,2)*cos(0))/factorial(i-1);
end
q(1) = 1;
p(1) = a(1);

% 构建方程组
for i = 1:N
    for j = 1:i-1
        if j<=n
            b(i,j) = 0;
        end
    end
    if i<= n
        b(i,i) = 1;
    end
    for j = i+1:N
        b(i,j) = 0;
    end
    for j = 1:i
        if j<=m
            b(i,n+j)=-a(i-j+1);
        end
    end
    for j = n+i+1:N
        b(i,j) = 0;
    end
    b(i,N+1) = a(i+1);
end
```

```matlab
% 求解方程组
for i = n+1:N-1
    % 确定主元
    k = i;
    for j = i+1:N
        if abs(b(j,i)) > abs(b(k,i))
            k = j;
        end
    end
    if b(k,i) == 0
        fprintf("The system is singular")
        return;
    end
    if k ~= i    %k不为i时进行交换
        for j = i:N+1
            temp = b(i,j);
            b(i,j) = b(k,j);
            b(k,j) = temp;
        end
    end
    % 消元
    for j = i+1:N
        xm = b(j,i)/b(i,i);
        for k = i+1:N+1
            b(j,k) = b(j,k) - xm*b(i,k);
        end
        b(j,i) = 0;
    end
end
if b(N,N) == 0
    fprintf("The system is singular")
    return;
end

% 求解出qi、pi
if m>0
    q(m+1) = b(N,N+1)/b(N,N);
end
for i = N-1:-1:n+1
    sum = 0;
    for j = i+1:N
        sum = sum + b(i,j)*q(j-n+1);
    end
    q(i-n+1) = (b(i,N+1)-sum)/b(i,i);
end
for i = n:-1:1
    sum = 0;
    for j = n+1:N
        sum = sum + b(i,j)*q(j-n+1);
    end
    p(i+1) = b(i,N+1)-sum;
end

% 输出结果
for i = 1:n+1
    fprintf("p%d = %.6f  ",i-1,p(i))
end
```

```
fprintf("\n")
for i = 1:n+1
    fprintf("q%d = %.6f  ",i-1,q(i))
end
fprintf("\n")
```

验证:

取: $f(x)=a_0+a_1x+\ldots+a_{12}x^{12}$

$r(x)=\frac{p_0+p_1x+\ldots+p_6x^6}{q_0+q_1x+\ldots+a_6x^6}$

$$\begin{aligned}f(x)-r(x)&=\frac{f(x)q(x)-p(x)}{q(x)}\\&=\frac{\sum_{i=0}^{12}a_ix^i\sum_{i=0}^{6}q_ix^i-\sum_{i=0}^{6}p_ix^i}{q(x)}\\&=\frac{(a_0+a_1x+\ldots+a_{12}x^{12})(q_0+q_1x+\ldots+a_6x^6)-(p_0+p_1x+\ldots+p_6x^6)}{q(x)}\end{aligned}$$

若两者相同，则需要证明:

$$(\sum_{i=0}^{k}a_iq_{k-i})-p_k=0$$

$p_0=0,\ a_0q_0=0\ \Rightarrow\ p_0=a_0q_0$

$p_1=1,\ a_0q_1+a_1q_0=0+1\ \Rightarrow\ p_1=\sum_{i=0}^{1}a_iq_{1-i}$

$p_2=0,\ a_0q_2+a_1q_1+a_2q_0=0\ \Rightarrow\ p_0=\sum_{i=0}^{2}a_iq_{2-i}$

$p_3=-2363/18183,\ a_0q_3+a_1q_2+a_2q_1+a_3q_0=445/12112-1/6\ \Rightarrow\ p_0=\sum_{i=0}^{3}a_iq_{3-i}$

$p_4=0,\ a_0q_4+a_1q_3+a_2q_2+a_3q_1+a_4q_0=0\ \Rightarrow\ p_0=\sum_{i=0}^{4}a_iq_{4-i}$

$p_5=12671/4363920,\ a_0q_5+a_1q_4+a_2q_3+a_3q_2+a_4q_1+a_5q_0=\frac{601}{872784}-\frac{445}{12122\times6}+\frac{1}{120}$
$\Rightarrow\ p_0=\sum_{i=0}^{5}a_iq_{5-i}$

$p_6=0,\ a_0q_6+a_1q_5+a_2q_4+a_3q_3+a_4q_2+a_5q_1+a_6q_0=0\ \Rightarrow\ p_6=\sum_{i=0}^{6}a_iq_{6-i}$

$p_7=\sum_{i=0}^{7}a_iq_{7-i}=0$

$p_8=\sum_{i=0}^{8}a_iq_{8-i}=0$

$p_9=\sum_{i=0}^{9}a_iq_{9-i}=0$

$p_{10}=\sum_{i=0}^{10}a_iq_{10-i}=0$

$p_{11}=\sum_{i=0}^{11}a_iq_{11-i}=0$

$p_{12}=\sum_{i=0}^{12}a_iq_{12-i}=0$

$\therefore$ 两者相同

P 7

$$y=2.711864x+4.542373$$
$$E=11.525424$$

```
clear; clc;
```

```
x = [0,2,4,5];
y = [6,8,14,20];

% 计算求解a0、a1中会用到的值
sum_x2 = sum(x.^2);
sum_x = sum(x);
sum_y = sum(y);
sum_xy = sum(x.*y);
len = length(x);

% 计算出a0、a1
a0 = (sum_x2*sum_y-sum_xy*sum_x)/(len*sum_x2-sum_x^2);
a1 = (sum_xy*len-sum_x*sum_y)/(len*sum_x2-sum_x^2);
fprintf("y = %.6fx + %.6f\n",a1,a0)

% 计算出拟合出的y值
y_hat = a1*x+a0;
E = sum((y-y_hat).^2);
fprintf("E = %.6f\n",E)
```