NA_4

P 1

a

$$L_0=\frac{(x-0.3)(x-0.6)}{(0-0.3)(0-0.6)}=\frac{50}{9}(x-0.3)(x-0.6),\quad L_1=\frac{(x-0)(x-0.6)}{(0.3-0)(0.3-0.6)}=-\frac{100}{9}x(x-0.6),$$
$$L_2=\frac{(x-0)(x-0.3)}{(0.6-0)(0.6-0.3)}=\frac{50}{9}x(x-0.3)$$
$$\Rightarrow P(x)_2=\sum_{k=0}^{2}f(x_k)L_k(x)=-11.22x^2+3.81x+1$$

$f(x)=e^{2x}\cos 3x\Rightarrow f'''(x)=-46e^{2x}\cos(3x)-9e^{2x}\sin(3x)$

对于 $x\in[0,0.6]$，$|f'''(x)|_{\max}=|f'''(0.260)|=65.7$

设 $g(x)=(x-0)(x-0.3)(x-0.6)=x^3-\frac{9}{10}x^2+\frac{9}{50}x$

则 $D_x(x^3-\frac{9}{10}x^2+\frac{9}{50}x)=3x^2-\frac{9}{5}x+\frac{9}{50}=\frac{3}{50}(50x^2-30x+3)$

对于 $x\in[0,0.6]$，$|g(x)|_{\max}=0.0103923$

$|\frac{f'''(\xi(x))}{3!}(x-0)(x-0.3)(x-0.6)|<\frac{0.0103923\times 65.7}{6}\approx 0.11$

b

$$L_0=\frac{(x-2.4)(x-2.6)}{(2-2.4)(2-2.6)}=\frac{25}{6}(x-2.4)(x-2.6),\quad L_1=\frac{(x-2)(x-2.6)}{(2.4-2)(2.4-2.6)}=\frac{25}{2}(x-2)(x-2.6),$$
$$L_2=\frac{(x-2)(x-2.4)}{(2.6-2)(2.6-2.4)}=\frac{25}{3}(x-2)(x-2.4)$$
$$P(x)_2=\sum_{k=0}^{2}f(x_k)L_k(x)=-0.13x^2+0.90x-0.63$$

$f(x)=\sin(\ln x)\Rightarrow f'''(x)=\frac{3\sin(\ln x)+\cos(\ln x)}{x^3}$

对于 $x\in[2,2.6]$，$|f'''(x)|_{\max}=0.335765$

设 $g(x)=(x-2)(x-2.4)(x-2.6)=x^3-7x^2+16.24x-12.48$

由计算得，当$x\in[2,2.6]$ 时，$|g(x)|_{\max}=0.0169009$

$|\frac{f'''(\xi(x))}{3!}(x-0)(x-0.3)(x-0.6)|<|\frac{0.335765\times 0.0169009}{6}|\approx 0.00095$

P 2

$$
\begin{aligned}
L_0 &= \frac{(x-0.5)(x-1)(x-2)}{(0-0.5)(0-1)(0-2)} = -(x-0.5)(x-1)(x-2), \\
L_1 &= \frac{(x-0)(x-1)(x-2)}{(0.5-0)(0.5-1)(0.5-2)} = \frac{8}{3}x(x-1)(x-2), \\
L_2 &= \frac{(x-0)(x-0.5)(x-2)}{(1-0)(1-0.5)(1-2)} = -2(x-0)(x-0.5)(x-2), \\
L_3 &= \frac{(x-0)(x-0.5)(x-1)}{(2-0)(2-0.5)(2-1)} = \frac{1}{3}(x-0)(x-0.5)(x-1) \\
& f(0)=0, \quad f(0.5)=y, \quad f(1)=3, \quad f(2)=2
\end{aligned}
$$

由 $P(x)_3 = \sum_{k=0}^{3} f(x_k)L_k(x)$ 得 x^3 前得系数为 $\frac{8y-16}{3}$

$\because \ \frac{8y-16}{3} = 6$

$\therefore \ y = \frac{17}{4}$

P 3

a

由 Neville's method 得:

$$
\begin{aligned}
-3.2 + 1.4P_2 &= 2.4 \\
P_2 &= 4
\end{aligned}
$$

$\therefore \ P_2 = f(0.5) = 4$

b

$$
\begin{aligned}
P_{0,1,2}(2.5) &= \frac{(2.5-1)\times P_{0,2}(2.5) - (2.5-2)\times P_{0,1}(2.5)}{2-1} \\
&= 2.25 \\
P_{0,1,2,3}(2.5) &= \frac{(2.5-0)\times P_{1,2,3}(2.5) - (2.5-3)\times P_{0,1,2}(2.5)}{3-0} \\
&= 2.875
\end{aligned}
$$

P 4

$$
\begin{cases}
10 = \dfrac{6 - f[x_1]}{0.7-0.4} \\
f[x_0,x_1] = \dfrac{f[x_1]-f[x_0]}{0.4-0} \\
\dfrac{50}{7} = \dfrac{10 - f[x_0,x_1]}{0.7-0}
\end{cases}
$$

$$
\Rightarrow \begin{cases}
f[x_0] = 1 \\
f[x_1] = 3 \\
f[x_0,x_1] = 5
\end{cases}
$$

P 5

设 $x_0=0,\ x_1=1,\ x_2=2$，且

$$S(x)=S_j(x)=a_j+b_j(x-x_j)+c_j(x-x_j)^2+d_j(x-x_j)^3$$

$\because\ h_j=x_{j+1}-x_j,\ a_j=f(x_j),\ j=0,1,2$

$\therefore\ h_0=h_1=1,\ a_0=0,\ a_1=1,\ a_2=2$

$$A\mathbf{x}=\mathbf{b}$$
$$A=\begin{bmatrix}1\,0\,0\\1\,4\,1\\0\,0\,1\end{bmatrix}$$

$$\mathbf{b}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\mathbf{x}=\begin{bmatrix}c_0\\c_1\\c_2\end{bmatrix}$$

$$\Rightarrow \mathbf{x}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{aligned}b_0&=\frac{1}{h_0}(a_1-a_0)-\frac{h_0}{3}(c_1+2c_0)=1\\b_1&=\frac{1}{h_1}(a_2-a_1)-\frac{h_1}{3}(c_2+2c_1)=1\\d_0&=\frac{1}{3h_0}(c_1-c_0)=0\\d_1&=\frac{1}{3h_1}(c_2-c_1)=0\\&\Rightarrow S(x)=x\quad\text{for } x\in[0,2]\end{aligned}$$

P 6

反证法：

假设 $\mathbf{A}$ 为对角线严格主导矩阵，且满足 $det(A)=0$

$\therefore \mathbf{Ax}=0$ 存在解 $\mathbf{x}=(x_1,x_2,\cdots,x_n)^T$

记 $|x_k|=\max\{|x_1|,|x_2|.\cdots,|x_n|\}$

$\because \sum\limits_{j=1}^{n}a_{kj}x_j=0$

$\therefore |\sum\limits_{j=1,j\neq k}^{n}a_{kj}x_j|=|-a_{kk}x_k|=|a_{kk}|\ |x_k|$

又$\because$ $\mathbf{A}$ 为对角线严格主导矩阵

$|a_{kk}|\ |x_k|>|\sum\limits_{j=1,j\neq k}^{n}a_{kj}||x_k|\geq|\sum\limits_{j=1,j\neq k}^{n}a_{kj}x_j|$

与 $|\sum\limits_{j=1,j\neq k}^{n} a_{kj}x_j| = |a_{kk}|\,|x_k|$，原假设不成立

即 $det(A) \neq 0$, A 为可逆矩阵