PHYS 132 HW #1

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Problem 11:

(a): The initial force experienced by the proton is $F = kq_1q_2/r^2 = k(q^2)/(0.0025^2)$ where q is the charge of a proton. By $F = ma$, we have

$$a = F/m = \frac{kq^2}{0.0025^2 \cdot m} = \frac{(9 \cdot 10^9)(1.6 \cdot 10^{-19})^2}{(2.5 \cdot 10^{-3})^2 \cdot 1.67 \cdot 10^{-27}} = 2.2 \cdot 10^4 m/s^2.$$

(b): The force will decrease over time as the protons get further apart, so acceleration will decrease as well. But it will always be positive, so velocity will always increase, though at a decreasing rate. I hope this description is sufficient.

Problem 16: q_3 must be to the left of q_1 to make q_1 accelerate to the left, so we can let the position of q_3 be $-x$ where $x > 0$. Then the total force on q_1 can be calculated by Coulomb's Law as

$$F = \frac{kq_1q_2}{0.2^2} - \frac{kq_1q_3}{x^2}$$

using the known values for F, q_1, q_2, q_3 gives

$$\begin{aligned}
-7 &= (9 \cdot 10^{-9})(3 \cdot 10^{-6})\left(\frac{(5 \cdot 10^{-6})}{0.2^2} - \frac{(8 \cdot 10^{-6})}{x^2}\right) \\
-7 &= 2.7 \cdot 10^{-14}\Big((1.25 \cdot 10^{-4}) - (8 \cdot 10^{-6})/x^2\Big) \\
-2.6 \cdot 10^{14} &= (1.25 \cdot 10^{-4}) - (8 \cdot 10^{-6})/x^2 \\
-(8 \cdot 10^{-6})/x^2 &= 1.25 \cdot 10^{-4} - 2.6 \cdot 10^{14} \\
x^2 &= -\frac{8 \cdot 10^{-6}}{1.25 \cdot 10^{-4} - 2.6 \cdot 10^{14}} \\
x^2 &= 3.1 \cdot 10^{-20} \\
x &= 1.8 \cdot 10^{-10}.
\end{aligned}$$

Problem 20:

(a): Let q, m be the charge and mass of a proton and E the electric field. The force on the proton is Eq and its acceleration is Eq/m. Since it begins at rest, its average speed over time t is $t(Eq/m)/2$, and thus its displacement is $t^2(Eq/m)/2$, giving

$$1.60 \cdot 10^{-2} = E \cdot (3.20 \cdot 10^{-6})(q/2m) \implies E = 1.04 \cdot 10^{-4} N.$$

(b): The final speed is twice the average speed, which is also $2d/t$, giving

$$v = 2(1.60 \cdot 10^{-2})/(3.20 \cdot 10^{-6}) = 10^4 m/s$$

Problem 31:

The force locally at any given point $(x, 0)$ on the rod is given by Coulomb's Law as

$$F = \frac{k(-2 \cdot 10^{-6})(4.8 \cdot 10^{-9})}{x^2 + 0.05^2}$$

from which the y-component is

$$F_y = \frac{k(-2\cdot 10^{-6})(4.8\cdot 10^{-9})(0.05)}{(x^2+0.05^2)^{3/2}}$$

By symmetry over the y axis, the x components of force cancel, so we only need to integrate F_y over the rod, giving

$$\int_{-0.1}^{0.1} \frac{k(-2\cdot 10^{-6})(4.8\cdot 10^{-9})(0.05)}{(x^2+0.05^2)^{3/2}}\,\mathrm{d}x = -3.1\cdot 10^{-3}N.$$

This force is pulling the rod upward along the $+y$ axis.

Problem 44:

In the y direction, the q_1 charge contributes no force and the q_2 charge contributes

$$\frac{k(-1)(-4\cdot 10^{-9})}{1^2}\cdot\frac{0.6}{1} = (9\cdot 10^9)(4\cdot 10^{-9})(0.6) = 21.6N$$

in the positive y direction. In the x direction, there are contributions from q_1 and q_2, giving

$$\frac{k(-1)(-4\cdot 10^{-9})}{1^2}\cdot\frac{0.8}{1} + \frac{k(-1)(6\cdot 10^{-9})}{0.6^2} = -121.2N$$

in the negative x direction.

Problem 54:

Using the same method as example 21.14, we can approximate the electric field as $2pk/x^3$, and $F = Eq$, which gives

$$F = \frac{2pkq}{x^3} = \frac{2\cdot(6.17\cdot 10^{-30})\cdot(9\cdot 10^9)\cdot(-1.6\cdot 10^{-19})}{(3\cdot 10^{-9})^3} = -6.6\cdot 10^{-13}$$

and the charge will be pulled in the $-x$ direction.

Problem 59:

(a): In the x component, the charge q_1 contributes

$$F = \frac{kq_1q_2}{0.05^2}\cdot\frac{0.04}{0.05} = \frac{(9\cdot 10^9)(5\cdot 10^{-9})(6\cdot 10^{-9})}{0.05^2}\cdot\frac{0.04}{0.05} = 8.6\cdot 10^{-5}$$

in the $+x$ direction. In the y component, both charges contribute, giving

$$F = \frac{k(5\cdot 10^{-9})(6\cdot 10^{-9})}{0.05^2}\cdot\frac{0.03}{0.05} + \frac{k(-2\cdot 10^{-9})(6\cdot 10^{-9})}{0.03^2} = -5.5\cdot 10^{-5}$$

in the $-y$ direction.

(b): The total magnitude of this force is

$$\sqrt{(8.6\cdot 10^{-5})^2 + (-5.5\cdot 10^{-5})^2} = 1.0\cdot 10^{-4}$$

and its direction is $\arctan(-5.5/8.6) = -32.6$ degrees (with the positive x axis being 0 degrees).

Problem 79:

(a): The electric field can be computed by the integral

$$\int_0^a \frac{k(Q/a)}{(x-t)^2}\,\mathrm{d}t = \frac{kQ}{a}\Big[-\frac{1}{3}(x-t)^3\Big]_0^a = \frac{kQ}{3a}\Big(x^3-(x-a)^3\Big).$$

(b): The force on q is

$$F = Eq = \frac{kQq}{3a}\Big(x^3 - r^3\Big).$$

and in the positive x direction.

(c): When r is large, $(r+a)^3 - r^3 = 3r^2a + 3ra^2 + a^3$ is asymptotically close to $3r^2a$ (since $r \gg a$, making $3r^2a$ the dominant term), which gives

$$F \approx \frac{kQq}{3a}\left(3r^2a\right) = \frac{kQq}{r^2}.$$

This makes sense because when we zoom out the rod looks like a point charge Q, giving back Coulomb's Law.