MATH 317 HW 3

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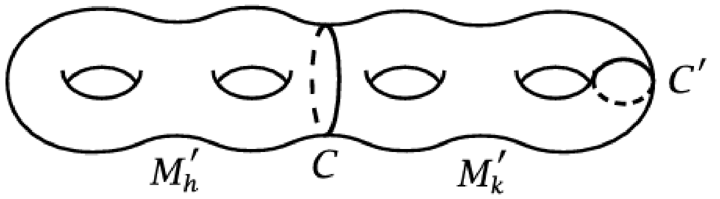
Problem 1 (Hatcher 1.2:9): In the surface M_g of genus g, let C be a circle that separates M_g into two compact surfaces M'_h and M'_k obtained from the closed surfaces M_h and M_k by deleting an open disk from each. Show that M'_h does not retract onto C, and hence M_g does not retract onto C. But show that M_g does retract onto the non-separating circle C' in the figure.



Problem 2 (Hatcher 1.2:11): The mapping torus T_f of a map $f : X \to X$ is the quotient of $X \times I$ obtained by identifying each point $(x, 0)$ with $(f(x), 1)$. In the case $X = S^1 \vee S^1$ with f preserving basepoints, compute a presentation for $\pi_1(T_f)$ in terms of the induced map $f_* : \pi_1(X) \to \pi_1(X)$. Do the same when $X = S^1 \times S^1$.

Problem 3 (Hatcher 1.2:14): Consider the quotient space X of a cube I^3 obtained by identifying each square face with the opposite square face via a clockwise quarter-rotation and translation by one unit. Show that X is a cell complex with two 0-cells, four 1-cells, three 2-cells, and one 3-cell. Using this structure, show that $\pi_1(X)$ is the quaternion group.

Proof. $\square$

Problem 4 (Hatcher 1.2:16): Show that the fundamental group of the surface of infinite genus is free on an infinite number of generators.

Problem 5 (Hatcher 1.2:22): In this exercise, we describe an algorithm for computing the *Wirtinger presentation* of the fundamental group of the complement of a knot in $\mathbb{R}^3$. We begin with the knot lying almost flat on a table T so that K consists of finitely-many disjoint arcs α_i contained within T and finitely-many β_ℓ which leave T to cross over another part of K. We build a 2-dimensional complex X that is a deformation retract of $\mathbb{R}^3 - K$ in the following steps:

For each α_i, let R_i be a curved rectangle so that its long edges are on T and it has α_i underneath it, and let and β_ℓ crossing over α_i lie along the curve of R_i. For each β_ℓ, let S_ℓ be the square-shaped piece which covers the crossing. Let X be the union of T, R_i, and S_ℓ for all i, ℓ. Lift K off the table slightly so that it does not intersect X. Then we can retract $R^3 - K$ to X.

(a) Assuming that this retraction is possible, show that $\pi_1(\mathbb{R}^3 - K)$ has a presentation with one generator x_i for each R_i and one relation $x_i x_j x_i^{-1} = x_k$ whenever α_j, α_k are two pieces which cross over α_i via some β_ℓ.
(b) Use this presentation to show that the Abelianization of $\pi_1(\mathbb{R}^3 - K)$ is $\mathbb{Z}$.

Problem 6 (Hatcher 1.B:5): Consider the graph of groups Γ having one vertex $\mathbb{Z}$ and one edge $n \mapsto 2n$. Show that $\pi_1(K\Gamma)$ has presentation $\langle a, b | bab^{-1}a^{-2}\rangle$ and describe the universal cover of $K\Gamma$ explicitly as a product $T \times \mathbb{R}$ with T a tree.

Problem 7 (Hatcher 2.1:5): Compute the simplicial homology groups of the Klein bottle using the Δ-complex structure.

Problem 8 (Hatcher 2.1:8): Construct a 3-dimensional Δ-complex X from n tetrahedra $T_1, \ldots, T_n$, with all sharing a common edge and each sharing a face with its two neighbors. Show that the simplicial homology groups of X in dimensions 0,1,2,3 are $\mathbb{Z}, \mathbb{Z}_n, 0, \mathbb{Z}$ respectively.

Problem 9 (Hatcher 2.1:11): Show that if X is a retract of X then the map $H_n(A) \to H_n(X)$ induced by the inclusion $A \subset X$ is injective.