MATH 318 HW 2

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Problem 1.4.4: Let M, T be manifolds, $Q \subset N$ a submanifold and $F : T \times M \to N$ a smooth map that is transversal to Q, so that $F^{-1}Q$ is a submanifold of $T \times M$. For $t \in T$, let $f_t : M \to N$ be the map defined by $f_t(p) = F(t, p)$. Show that f_t is transversal to Q iff t is a regular value of $F^{-1}Q \xrightarrow{\pi_T} T$.

Proof. If t is a regular value of π_T, then $D_p\pi_T$ is surjective on $t \times Q$. $\square$

Problem 2.1.2: Prove that the tangent bundle of P^m can be obtained from TS^m by identifying the pairs (p, v) and $(-p, -v)$.

Problem 2.1.4: Let G be a Lie group with unit e. For $g \in G$ let L_g be the map $L_g : h \mapsto gh$.

(a) Prove that the map $G \times T_eG \to TG$, $(g, v) \mapsto D_eL_g(v)$ is a diffeomorphism. Conclude that G has a basis of vector fields.

(b) Prove that for every $v \in T_eG$ there exists a unique vector field V on G that is left-invariant.