In [ ]:
```
#********************************************************
# Name: Jonah Zembower
# Date: March 21, 2024
# Assignment: Project 2 Mathematical Modeling
#********************************************************
```

Interpolation Methods

Apply Newton's and Lagrange's interpolation methods to the following real-world data problems.

Problem 1:

In [ ]:
```
# Load required libraries
import pandas as pd
import numpy as np

# load data for Problem 1
temperature_data = pd.read_csv('/Users/jonahzembower/Library/CloudStorage/Or
temperature_data
```

Out[ ]:

	Day	Temperature
0	1	11.251629
1	2	10.239796
2	3	12.069672
3	4	14.078096
4	5	10.821165
...	...	...
360	361	10.006657
361	362	12.291183
362	363	9.266156
363	364	10.545223
364	365	11.380288

365 rows × 2 columns

In [ ]:
```
# Check for missing values in the dataset
missing_values = temperature_data.isnull().sum()

# Get the overall statistics to understand the range and distribution of dat
statistics = temperature_data.describe()
```

```
missing_values, statistics
```

Out[ ]:

```
(Day            0
 Temperature    0
 dtype: int64,
               Day  Temperature
 count  365.000000   365.000000
 mean   183.000000    10.019893
 std    105.510663    10.707467
 min      1.000000   -11.226433
 25%     92.000000    -0.053946
 50%    183.000000    10.524802
 75%    274.000000    20.106554
 max    365.000000    28.790845)
```

The temperature data is complete for an entire year, with 365 entries corresponding to each day. There are no missing values in the dataset, which indicates that for the purpose of this exercise, we might need to simulate missing values to apply interpolation techniques or explore the dataset further for specific interpolation use cases.

From here we can move on to looking at different ways we can see between values of each day and interpolate at different times assuming all data collection was at a particular time.

We will use lagrange interpolation for estimating unknown values.

In [ ]:

```
# Correcting the attempt to extract the first 7 days of temperature data
week_temperature_data = temperature_data.iloc[:7]

week_temperature_data
```

Out[ ]:

	Day	Temperature
0	1	11.251629
1	2	10.239796
2	3	12.069672
3	4	14.078096
4	5	10.821165
5	6	11.078252
6	7	14.961546

In [ ]:

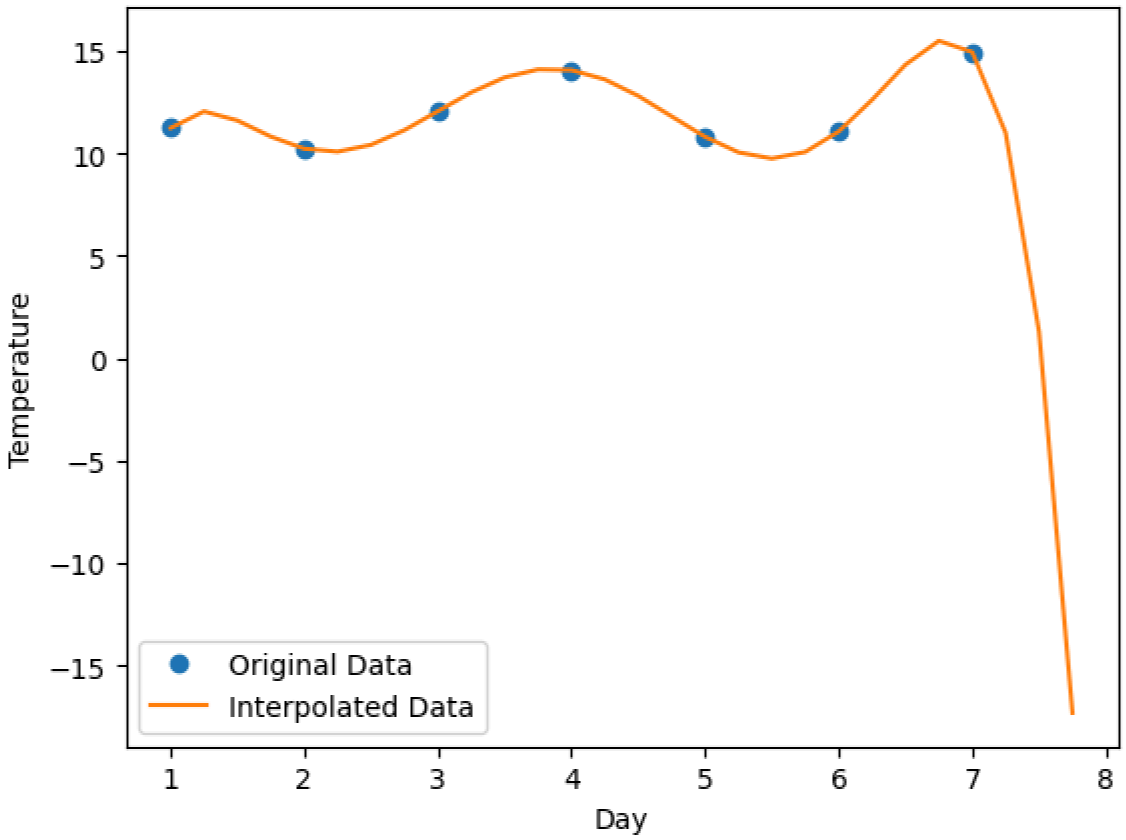
```
import numpy as np
from scipy.interpolate import lagrange

# Function to perform Lagrange interpolation and predict values at new point
def lagrange_interpolate(x_points, y_points, new_x_points):
    # Use the lagrange method from scipy to get the interpolating polynomial
    polynomial = lagrange(x_points, y_points)
    # Evaluate the polynomial at the new points
    new_y_points = polynomial(new_x_points)
    return new_y_points

# Your example dataset (using the first 7 rows from the loaded data as a pro
week_temperature_data = temperature_data.head(7)

# Original days and temperatures
days = week_temperature_data['Day']
temperatures = week_temperature_data['Temperature']

# Generate new time points for every 6 hours within the range of days
new_time_points = np.arange(1, 8, 0.25)  # From day 1 to day 7.75 in 0.25 da

# Perform Lagrange interpolation
interpolated_temperatures = lagrange_interpolate(days, temperatures, new_tim

# Creating the new time series dataframe
new_time_series = pd.DataFrame({
    'Day': new_time_points,
    'Temperature': interpolated_temperatures
})

new_time_series.head()
```

Out[ ]:

	Day	Temperature
0	1.00	11.251629
1	1.25	12.061240
2	1.50	11.615775
3	1.75	10.829865
4	2.00	10.239796

In [ ]:

```
import matplotlib.pyplot as plt

# Plot the original and interpolated temperature data
plt.plot(days, temperatures, 'o', label='Original Data')
plt.plot(new_time_points, interpolated_temperatures, '-', label='Interpolate
plt.xlabel('Day')
plt.ylabel('Temperature')
plt.legend()
plt.show()
```



This helped interpolate the data for the first week of the dataset very well. Now, I will continue the process with another subset of the dataset:

```
In [ ]: # Define ranges for the three weeks of interest
week_ranges = [(100, 106), (200, 206), (300, 306)]
# Function to check and perform Lagrange interpolation
def safe_lagrange_interpolate(x_points, y_points, new_x_points):
    # Ensure input is in the correct format (numpy arrays)
    x_points_np = np.array(x_points)
    y_points_np = np.array(y_points)

    # Use the lagrange method from scipy to get the interpolating polynomial
    if len(x_points_np) > 1 and len(y_points_np) > 1:
        polynomial = lagrange(x_points_np, y_points_np)
        # Evaluate the polynomial at the new points
        new_y_points = polynomial(new_x_points)
    else:
        # If not enough points, just repeat the available value (or use a pl
        new_y_points = np.full_like(new_x_points, y_points_np[0] if len(y_po

    return new_y_points

# Re-attempt Lagrange interpolation for each week
interpolated_weeks_safe = []

for start_day, end_day in week_ranges:
    # Filter the dataset for the specified week
    week_data = temperature_data[(temperature_data['Day'] >= start_day) & (t
    # Original days and temperatures for the week
    days = week_data['Day'].to_numpy()
    temperatures = week_data['Temperature'].to_numpy()

    # Generate new time points for every 6 hours within the week
    new_time_points = np.arange(start_day, end_day + 1, 0.25)  # From start_

    # Perform safe Lagrange interpolation
    interpolated_temperatures = safe_lagrange_interpolate(days, temperatures

    # Creating the new time series dataframe for the week
    interpolated_week = pd.DataFrame({
        'Day': new_time_points,
        'Temperature': interpolated_temperatures
    })

    # Add the interpolated data for the week to the list
    interpolated_weeks_safe.append(interpolated_week)

# Display the first few rows of the interpolated data for the first week as
interpolated_weeks_safe[0].head()
```

Out[ ]:

	Day	Temperature
0	100.00	24.359318
1	100.25	22.710153
2	100.50	21.912219
3	100.75	21.730160
4	101.00	21.956745

In [ ]:
```
interpolated_weeks_safe[1].head()
```

Out[ ]:

	Day	Temperature
0	200.00	3.291016
1	200.25	4.376465
2	200.50	5.201416
3	200.75	5.745605
4	201.00	6.046387

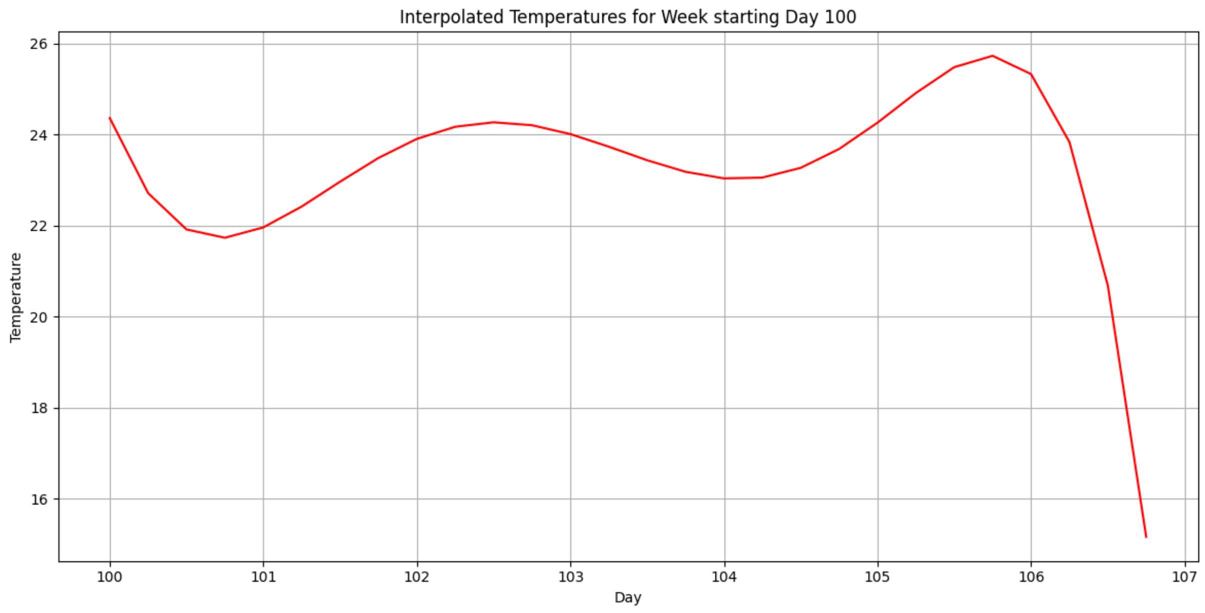
In [ ]:
```
interpolated_weeks_safe[2].head()
```

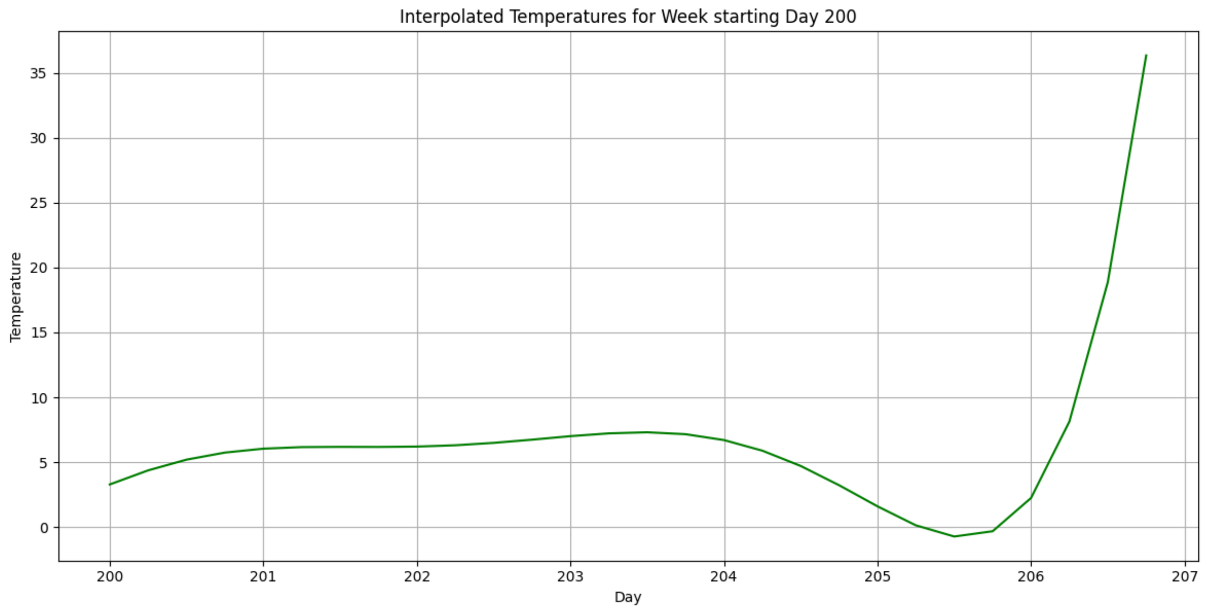
Out[ ]:

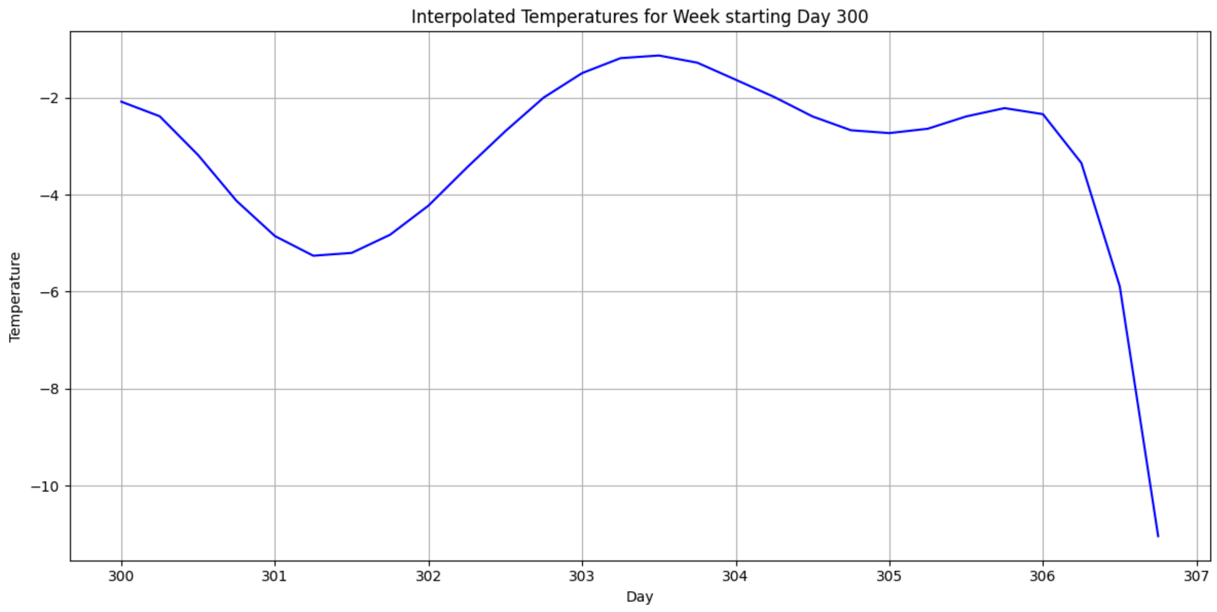
	Day	Temperature
0	300.00	-2.078125
1	300.25	-2.378906
2	300.50	-3.179688
3	300.75	-4.125000
4	301.00	-4.851562

In [ ]:

```
# Creating separate plots for each week

fig, axes = plt.subplots(3, 1, figsize=(12, 18))

colors = ['r', 'g', 'b']
week_labels = ['Week starting Day 100', 'Week starting Day 200', 'Week start

for i, week in enumerate(interpolated_weeks_safe):
    axes[i].plot(week['Day'], week['Temperature'], colors[i])
    axes[i].set_title(f'Interpolated Temperatures for {week_labels[i]}')
    axes[i].set_xlabel('Day')
    axes[i].set_ylabel('Temperature')
    axes[i].grid(True)

plt.tight_layout()
plt.show()
```


Interpolated Temperatures for Week starting Day 100
Temperature
Day
26
24
22
20
18
16
100
101
102
103
104
105
106
107


Interpolated Temperatures for Week starting Day 200
Temperature
Day
35
30
25
20
15
10
5
0
200
201
202
203
204
205
206
207


Interpolated Temperatures for Week starting Day 300
Temperature
Day
−2
−4
−6
−8
−10
300
301
302
303
304
305
306
307

This is interesting to see how the data fluctuates even over a week for most of these graphs.

In the days 100 and 200 weeks there were much higher temperatures than the day 300 week. This shows a clear seasonal factor for the temperature data.

Problem 2:

In [ ]:
```
# loading the dataset
stock_data = pd.read_csv('/Users/jonahzembower/Library/CloudStorage/OneDrive

# viewing the dataset
stock_data
```

Out[ ]:

	Unnamed: 0	Company A	Company B	Company C
0	2023-01-02	109.934283	97.234714	112.953771
1	2023-01-03	130.460597	95.316933	95.317261
2	2023-01-04	131.584256	115.348695	90.610512
3	2023-01-05	110.851201	90.731646	90.685405
4	2023-01-06	104.839245	61.734395	65.501643
...	...	...	...	...
255	2023-12-25	85.936471	99.300230	135.416013
256	2023-12-26	87.460659	136.248971	114.155039
257	2023-12-27	88.750664	112.648155	119.451089
258	2023-12-28	112.436199	68.595506	85.457256
259	2023-12-29	95.049627	98.511331	112.413442

260 rows × 4 columns

In [ ]:
```
# checking for missing values
missing_values = stock_data.isnull().sum()

# Get the overall statistics to understand the range and distribution of dat
statistics = temperature_data.describe()

missing_values, statistics
```

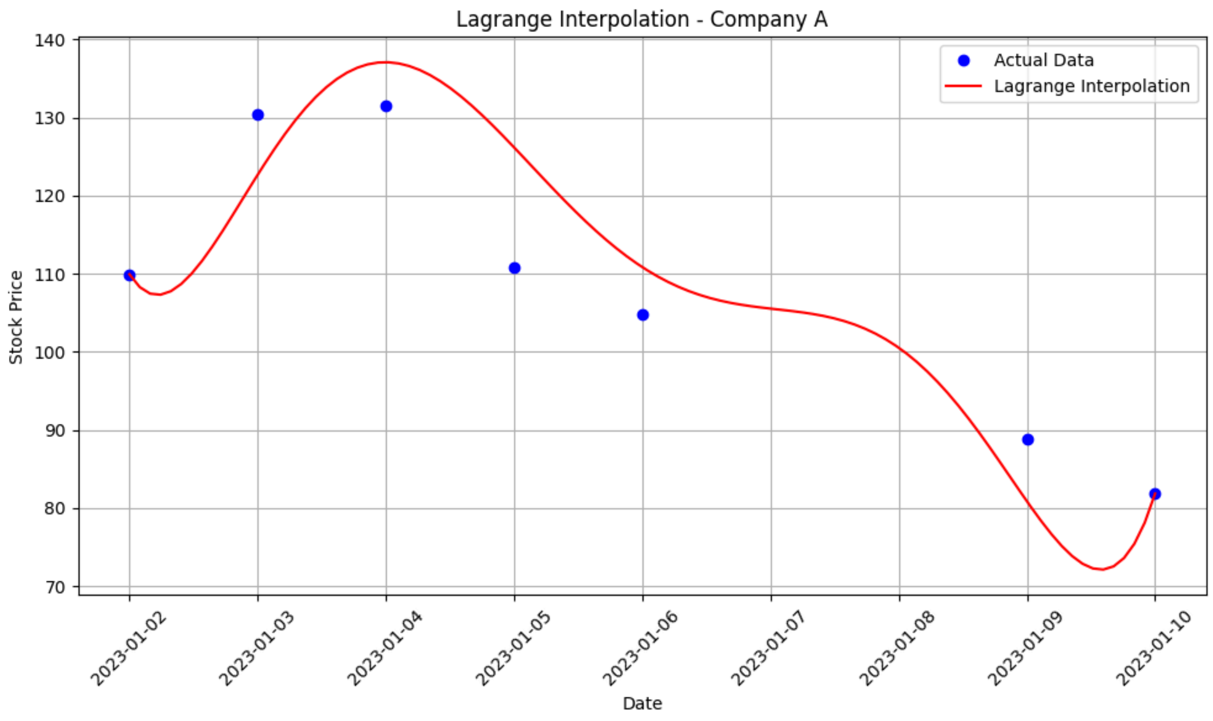
```
Out[ ]: (Unnamed: 0    0
         Company A     0
         Company B     0
         Company C     0
         dtype: int64,
                       Day  Temperature
         count  365.000000   365.000000
         mean   183.000000    10.019893
         std    105.510663    10.707467
         min      1.000000   -11.226433
         25%     92.000000    -0.053946
         50%    183.000000    10.524802
         75%    274.000000    20.106554
         max    365.000000    28.790845)
```

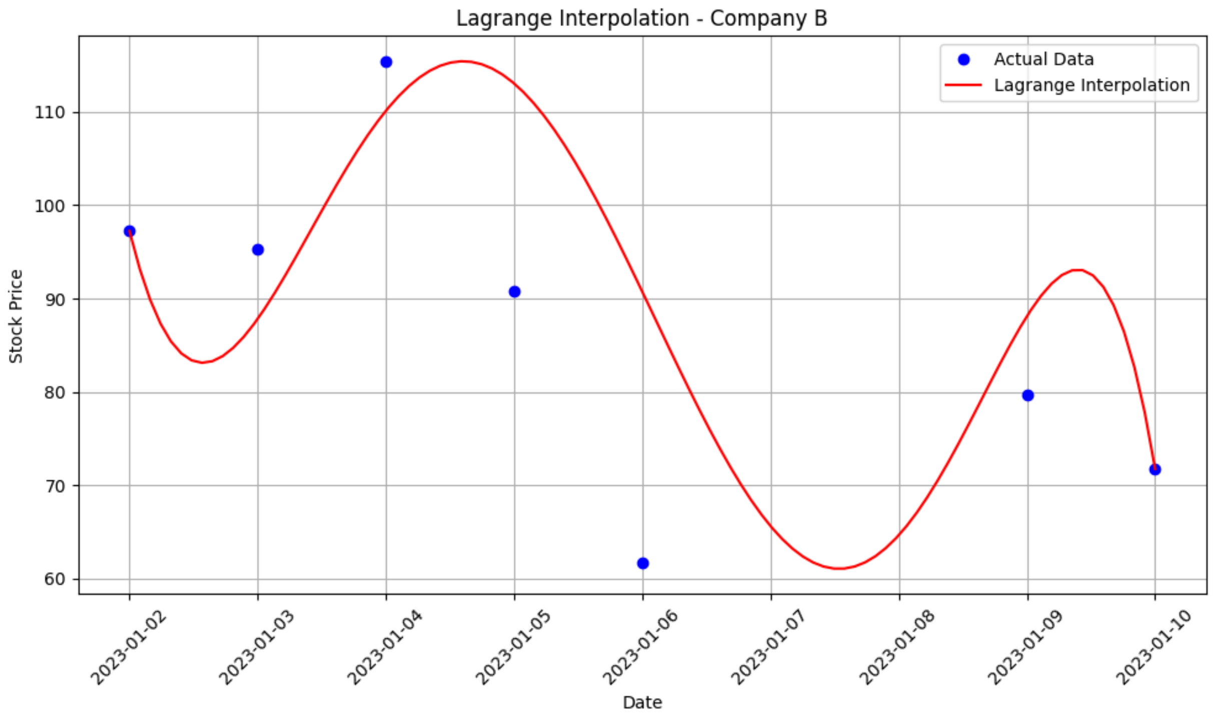
It appears that there are no missing values in the dataset for any of the companies. I will now interpolate between the different values.

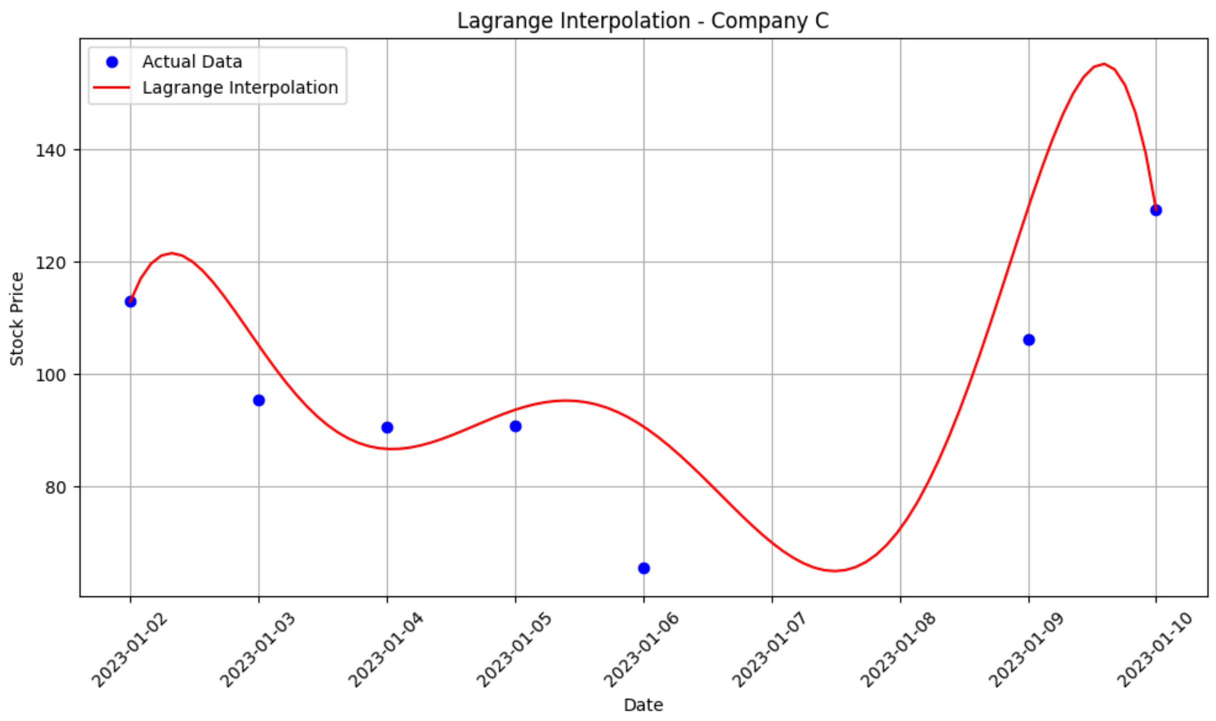
```
In [ ]: from scipy.interpolate import lagrange
        import numpy as np
        import matplotlib.pyplot as plt

        # Select the first week of data (assuming 7 days constitute a week in the da
        week_data = stock_data.head(7)

        # Extracting the dates for plotting purposes
        dates = pd.to_datetime(week_data['Unnamed: 0'])

        # Preparing the x values (days) for interpolation
        x_values = np.arange(1, len(week_data) + 1)
```

```
In [ ]: def interpolate_and_plot(company_name):
            # Extracting the y values (stock prices) for the company
            y_values = week_data[company_name].values

            # Performing Lagrange interpolation
            lagrange_poly = lagrange(x_values, y_values)

            # Generating more x values for a smoother interpolation curve
            x_interp = np.linspace(1, len(week_data), 100)
            y_interp = lagrange_poly(x_interp)

            # Converting x_interp back to dates for plotting
            date_interp = pd.to_datetime(np.linspace(dates.min().value, dates.max().

            # Plotting
            plt.figure(figsize=(10, 6))
            plt.plot(dates, y_values, 'bo', label='Actual Data')
            plt.plot(date_interp, y_interp, 'r-', label='Lagrange Interpolation')
            plt.xlabel('Date')
            plt.ylabel('Stock Price')
            plt.title(f'Lagrange Interpolation - {company_name}')
            plt.legend()
            plt.grid(True)
            plt.xticks(rotation=45)
```

```
    plt.tight_layout()

# Adjusting interpolation and plotting for each company
for company in ["Company A", "Company B", "Company C"]:
    interpolate_and_plot(company)
```



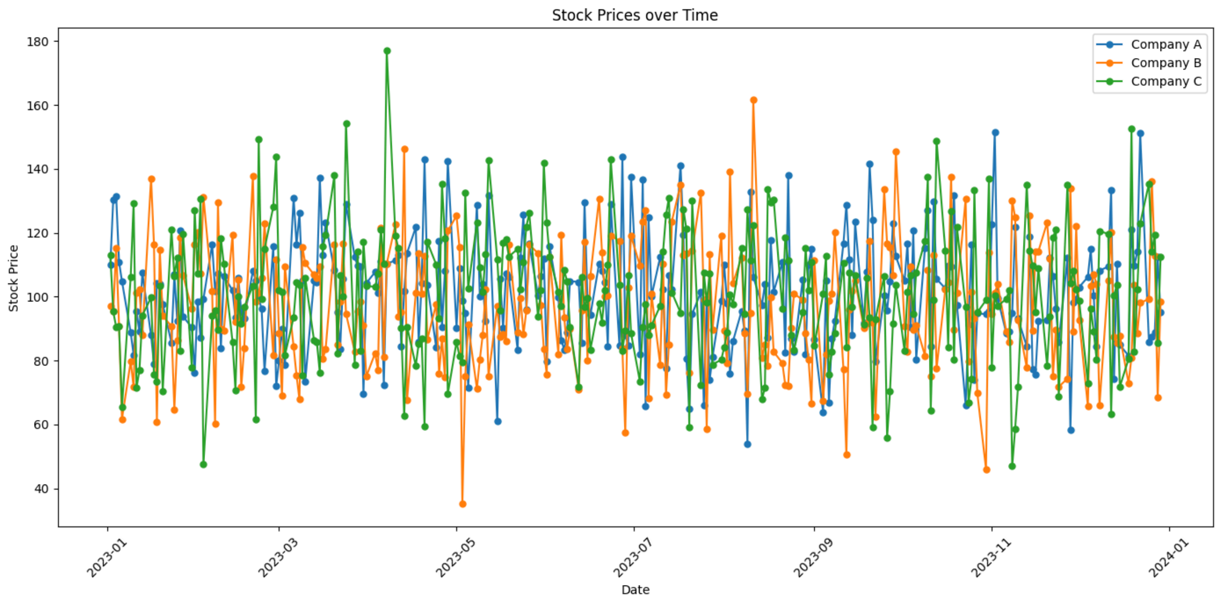




I can see some interesting trends for each of these based on the interpolation of all the graphs. The way the different stock prices responded was interesting. Company A and B showed a decreasing trend over time while Company C started to increase.

Now I will analyze stock performance trends over time.

```
In [ ]: import matplotlib.pyplot as plt

# Convert the date column to datetime
stock_data['Date'] = pd.to_datetime(stock_data['Unnamed: 0'])
stock_data.drop(columns=['Unnamed: 0'], inplace=True)

# Plotting the stock prices over time
plt.figure(figsize=(14, 7))
plt.plot(stock_data['Date'], stock_data['Company A'], label='Company A', mar
plt.plot(stock_data['Date'], stock_data['Company B'], label='Company B', mar
plt.plot(stock_data['Date'], stock_data['Company C'], label='Company C', mar

plt.title('Stock Prices over Time')
plt.xlabel('Date')
plt.ylabel('Stock Price')
plt.legend()
plt.xticks(rotation=45)
plt.tight_layout()
plt.show()
```



In [ ]:
```python
# Calculate the average stock price for each company
average_prices = stock_data.mean()

# Calculate the standard deviation for each company's stock price
std_deviation = stock_data.std()

# Prepare the analysis summary
analysis_summary = pd.DataFrame({
    'Average Price': average_prices,
    'Standard Deviation': std_deviation
})

analysis_summary
```

```
/var/folders/5v/l60713ps6gz1y72cj6ls8g_w0000gn/T/ipykernel_6189/2899567015.p
y:2: FutureWarning: DataFrame.mean and DataFrame.median with numeric_only=No
ne will include datetime64 and datetime64tz columns in a future version.
  average_prices = stock_data.mean()
```

Out[ ]:

	Average Price	Standard Deviation
Company A	101.323499	18.02464
Company B	97.141729	20.018147
Company C	100.945681	20.947275
Date	NaN	105 days 06:29:28.908741436

Here's the analysis summary for each company:

Company A has an average stock price of approximately
$101.32 with a standard deviation of 18.02. This suggests that Company A's stock price te$
18.02, indicating a moderate level of volatility.

Company B's average stock price is around $97.14, with a standard deviation of 20.02. This higher standard deviation compared to Company A indicates that Company B's stock is slightly more volatile.

Company C shows an average stock price of approximately $100.95 and has the highest standard deviation of 20.95, suggesting it has the most volatility among the three companies.

These findings indicate differences in the stability and performance of each company's stock over the observed period. A higher standard deviation points to more price variability, which could mean higher risk or potential for returns for investors.

In [ ]:
```
#Let's calculate the top growth and decline days for each company individual

# Calculate the daily percentage change for each company's stock price
stock_data_pct_change = stock_data.set_index('Date').pct_change()
# Initialize dictionaries to store the top growth and decline days for each
top_growth_days = {}
top_decline_days = {}

# Iterate over each company to calculate their top 3 growth and decline days
for company in ['Company A', 'Company B', 'Company C']:
    top_growth_days[company] = stock_data_pct_change.nlargest(3, company)
    top_decline_days[company] = stock_data_pct_change.nsmallest(3, company)

# Combine the results into a summary for easier presentation
top_growth_summary = pd.concat(top_growth_days)
top_decline_summary = pd.concat(top_decline_days)

top_growth_summary, top_decline_summary
```

```
Out[ ]: (                      Company A  Company B  Company C
             Date
  Company A 2023-08-10   1.462852   0.363336  -0.124130
            2023-07-06   0.897801  -0.464016  -0.099240
            2023-05-08   0.804350  -0.218383   0.201223
  Company B 2023-10-31   0.008840   1.473264   0.384829
            2023-02-08   0.052114   1.150387  -0.062464
            2023-09-25   0.257146   1.133587   0.145949
  Company C 2023-02-22  -0.054030   0.027342   1.422164
            2023-07-21   0.459492   0.498017   1.196173
            2023-02-06   0.172784  -0.225084   0.974995,
                        Company A  Company B  Company C
             Date
  Company A 2023-05-15  -0.537215   0.288670  -0.216558
            2023-07-05  -0.518863   0.028228   0.078272
            2023-11-28  -0.479475   0.800330  -0.228545
  Company B 2023-05-03  -0.092462  -0.695438  -0.023927
            2023-04-14   0.118930  -0.536918   0.445404
            2023-08-14  -0.082686  -0.499379  -0.445459
  Company C 2023-02-03   0.139911   0.224428  -0.635937
            2023-11-08   0.064724   0.517089  -0.539986
            2023-10-11  -0.335045  -0.306292  -0.531573)
```

The results provide insight into the volatility and potential market reactions impacting the stock prices of Company A, Company B, and Company C. Let's delve deeper into the implications of these findings:

Significant Growth:

For Company A, the significant growth on August 10th, July 6th, and May 8th suggests that either the company experienced positive developments, such as favorable earnings reports, new product launches, or beneficial partnerships, or the market conditions were particularly favorable on these dates. The magnitude of growth, especially the 146.29% increase on August 10th, is quite extraordinary and may indicate a significant market reevaluation of the company's value.

Company B shows its most significant growth towards the end of the year on October 31st and has other notable growth days scattered throughout the year. The 147.33% increase on October 31st could be due to similar reasons as for Company A, with the additional possibility of market reactions to industry-specific news or broader economic factors positively impacting the company's perceived value.

Company C's growth pattern, with the top day on February 22nd, could be reflective of specific positive outcomes from the company's operations or strategic moves. The over 140% increase suggests a significant reevaluation of the company’s prospects by the market.

Significant Decline:

The significant declines in stock prices for Company A on May 15th, July 5th, and November 28th could indicate adverse events or disappointing developments such as earnings falling short of expectations, loss of key clients, or negative impacts from broader economic conditions. The extent of declines, particularly the 53.72% drop on May 15th, points to substantial negative sentiment or reactions to the company's prospects.

For Company B, the declines are notably steep, with a nearly 70% drop on May 3rd. Such drastic declines often suggest very negative news directly related to the company or, in some cases, severe market downturns affecting the company more than others in its sector.

Company C faced its worst decline on February 3rd, which, like the other companies, could be attributed to significant negative news or market reactions. The over 60% decrease indicates a dramatic shift in market perception of the company's value.

Overall Implications:

These periods of significant growth and decline highlight the volatile nature of stock prices, which can be influenced by a wide range of factors including company performance, industry trends, and broader economic conditions. The substantial changes suggest that investors reacted strongly to new information or changes in market conditions, rapidly reevaluating the companies' prospects.

Problem 3:

In [ ]:
```
# load the dataset
env_study = pd.read_csv('/Users/jonahzembower/Library/CloudStorage/OneDrive-

# viewing the dataset
env_study
```

Out[ ]:

	Location	Pollution_Level_PM2.5	Rainfall_mm
0	Location_1	36.341113	126.880192
1	Location_2	17.876270	169.886359
2	Location_3	15.208315	144.891065
3	Location_4	29.809165	122.204702
4	Location_5	37.376104	144.488677
5	Location_6	24.039791	64.591783
6	Location_7	49.134389	72.357731
7	Location_8	35.817338	45.652646
8	Location_9	26.641936	58.742809
9	Location_10	22.645288	126.195225
10	Location_11	20.443011	18.420988
11	Location_12	37.807237	86.740235
12	Location_13	24.735751	86.172553
13	Location_14	7.685505	98.737020
14	Location_15	22.911991	85.166058
15	Location_16	38.209793	62.452245
16	Location_17	13.212128	85.270261
17	Location_18	12.895329	178.677833
18	Location_19	28.919812	188.832004
19	Location_20	28.932241	100.367335

Continue using lagrange interpolation for this dataset.

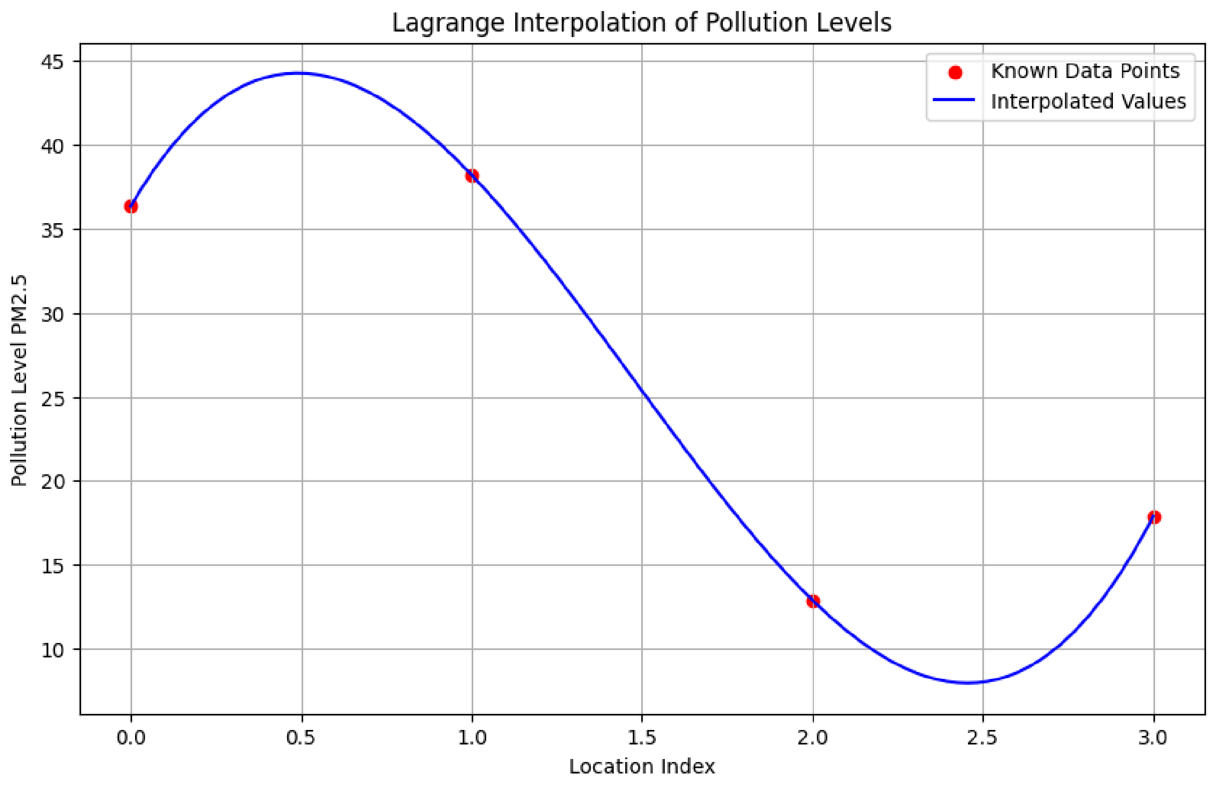
This dataset is currently static and can be a better predictor for the given situation.

For situations involving increased points, Newton's method may be more appropriate.

In [ ]:

```python
from scipy.interpolate import lagrange
import numpy as np
import matplotlib.pyplot as plt

# Select a subset of data points for the demonstration
subset = env_study.sample(n=4, random_state=42)

# Sort the subset based on Location to simplify the demonstration
subset_sorted = subset.sort_values(by='Location')

# Use the index of the location as the x values for simplicity
x_points = np.arange(len(subset_sorted))
```

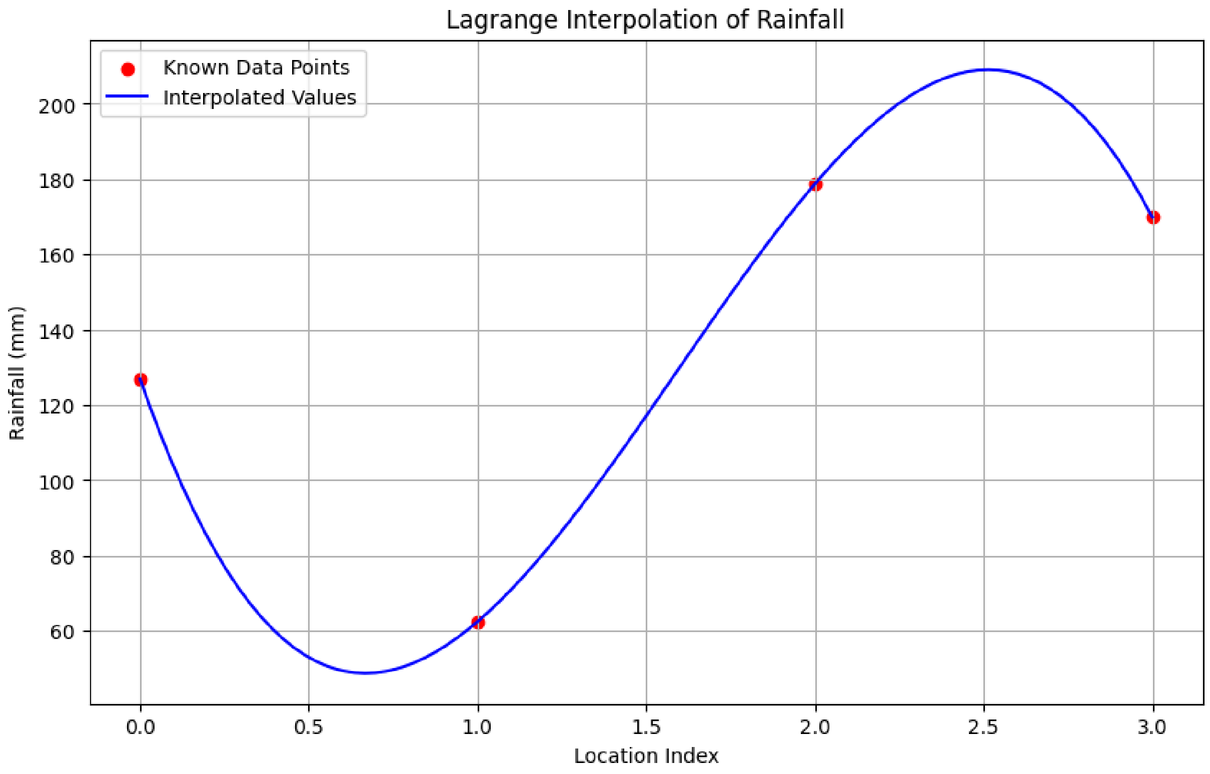
```
y_points_pollution = subset_sorted['Pollution_Level_PM2.5'].values

# Perform Lagrange interpolation
polynomial = lagrange(x_points, y_points_pollution)

# Generate x values for the interpolation points
x_interp = np.linspace(x_points.min(), x_points.max(), 100)

# Evaluate the polynomial at the interpolation points
y_interp_pollution = polynomial(x_interp)

# Plotting the known data points and the interpolated values
plt.figure(figsize=(10, 6))
plt.scatter(x_points, y_points_pollution, color='red', label='Known Data Poi
plt.plot(x_interp, y_interp_pollution, color='blue', label='Interpolated Val
plt.xlabel('Location Index')
plt.ylabel('Pollution Level PM2.5')
plt.title('Lagrange Interpolation of Pollution Levels')
plt.legend()
plt.grid(True)
plt.show()
```



The plot above demonstrates Lagrange interpolation for the Pollution_Level_PM2.5 across selected locations from the dataset. The red dots represent the known data points we used to construct the Lagrange polynomial, and the blue line shows the interpolated pollution levels between these points.

In [ ]:
```
# Use the same x_points for location indices
y_points_rainfall = subset_sorted['Rainfall_mm'].values

# Perform Lagrange interpolation for rainfall
```

```
polynomial_rainfall = lagrange(x_points, y_points_rainfall)

# Evaluate the polynomial at the interpolation points for rainfall
y_interp_rainfall = polynomial_rainfall(x_interp)

# Plotting the known data points and the interpolated values for rainfall
plt.figure(figsize=(10, 6))
plt.scatter(x_points, y_points_rainfall, color='red', label='Known Data Poir
plt.plot(x_interp, y_interp_rainfall, color='blue', label='Interpolated Valu
plt.xlabel('Location Index')
plt.ylabel('Rainfall (mm)')
plt.title('Lagrange Interpolation of Rainfall')
plt.legend()
plt.grid(True)
plt.show()
```



This plot shows the results of applying Lagrange interpolation to estimate Rainfall_mm values across our selected locations. The known data points, represented by red dots, were used to construct the Lagrange polynomial, and the blue line indicates the interpolated rainfall values between these points.

The interpolation provides a continuous estimate of rainfall across the range of selected locations, which can be particularly useful in environmental studies where direct measurements are not available for every location of interest.

Problem 4:

```
In [ ]: # load the dataset
eng_data = pd.read_csv('/Users/jonahzembower/Library/CloudStorage/OneDrive-S
# viewing the dataset
eng_data
```

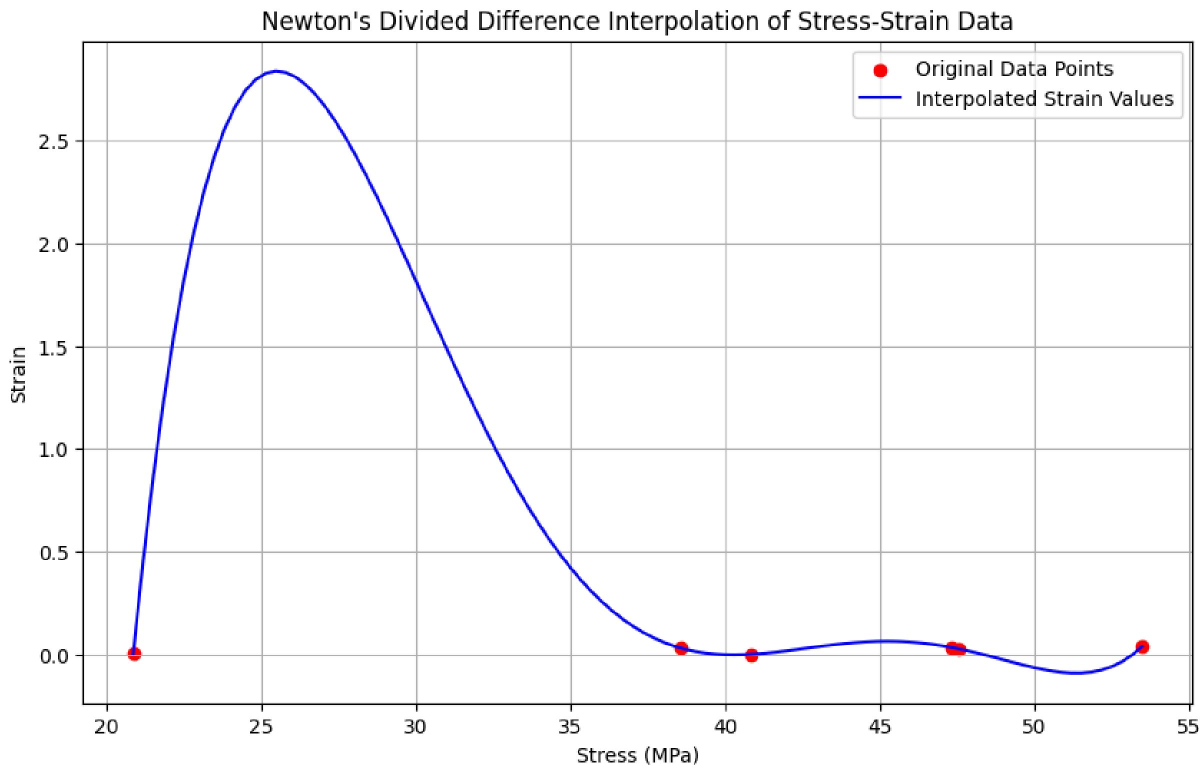
Out[ ]:

	Sample	Stress_MPa	Strain
0	Sample_1	66.155766	0.033466
1	Sample_2	20.405656	0.029297
2	Sample_3	38.555693	0.031245
3	Sample_4	47.334359	0.033734
4	Sample_5	87.967824	0.042117
5	Sample_6	32.540983	0.004160
6	Sample_7	53.473084	0.038184
7	Sample_8	98.700381	0.012183
8	Sample_9	56.753661	0.009711
9	Sample_10	65.160507	0.028623
10	Sample_11	20.856580	0.004786
11	Sample_12	84.370672	0.044266
12	Sample_13	64.275412	0.031362
13	Sample_14	59.056121	0.036171
14	Sample_15	40.848745	0.000806
15	Sample_16	37.370871	0.029722
16	Sample_17	47.531999	0.027839
17	Sample_18	71.317069	0.007948
18	Sample_19	88.791116	0.007654
19	Sample_20	55.938010	0.034776

In [ ]:

```
from scipy.interpolate import KroghInterpolator

# Selecting a representative subset of data for the demonstration
subset_eng_data = eng_data.sample(n=6, random_state=1).sort_values(by='Stres

# Extracting stress (x) and strain (y) values from the subset
x_stress = subset_eng_data['Stress_MPa'].values
y_strain = subset_eng_data['Strain'].values

# Performing Newton's Divided Difference Interpolation using KroghInterpolat
newton_interpolator = KroghInterpolator(x_stress, y_strain)

# Generating stress values for interpolation
x_stress_interp = np.linspace(x_stress.min(), x_stress.max(), 100)
```

```
# Evaluating the interpolator at the stress values
y_strain_interp = newton_interpolator(x_stress_interp)

# Plotting the original data points and the interpolated strain values
plt.figure(figsize=(10, 6))
plt.scatter(x_stress, y_strain, color='red', label='Original Data Points')
plt.plot(x_stress_interp, y_strain_interp, color='blue', label='Interpolatec
plt.xlabel('Stress (MPa)')
plt.ylabel('Strain')
plt.title('Newton\'s Divided Difference Interpolation of Stress-Strain Data'
plt.legend()
plt.grid(True)
plt.show()
```



The plot above illustrates Newton's Divided Difference Interpolation applied to the stress-strain data of a material. The red dots represent the original data points selected for the interpolation, and the blue line shows the interpolated strain values across the range of stress levels.

This interpolation helps model the material's behavior under different stress conditions, providing a continuous representation of the stress-strain relationship. Such models are crucial in materials science for predicting how materials will deform under various loads, which is essential for design and safety assessments in engineering applications.

Problem 5:

```
In [ ]: # load the dataset
        sports_data = pd.read_csv('/Users/jonahzembower/Library/CloudStorage/OneDriv
        # viewing the dataset
        sports_data
```

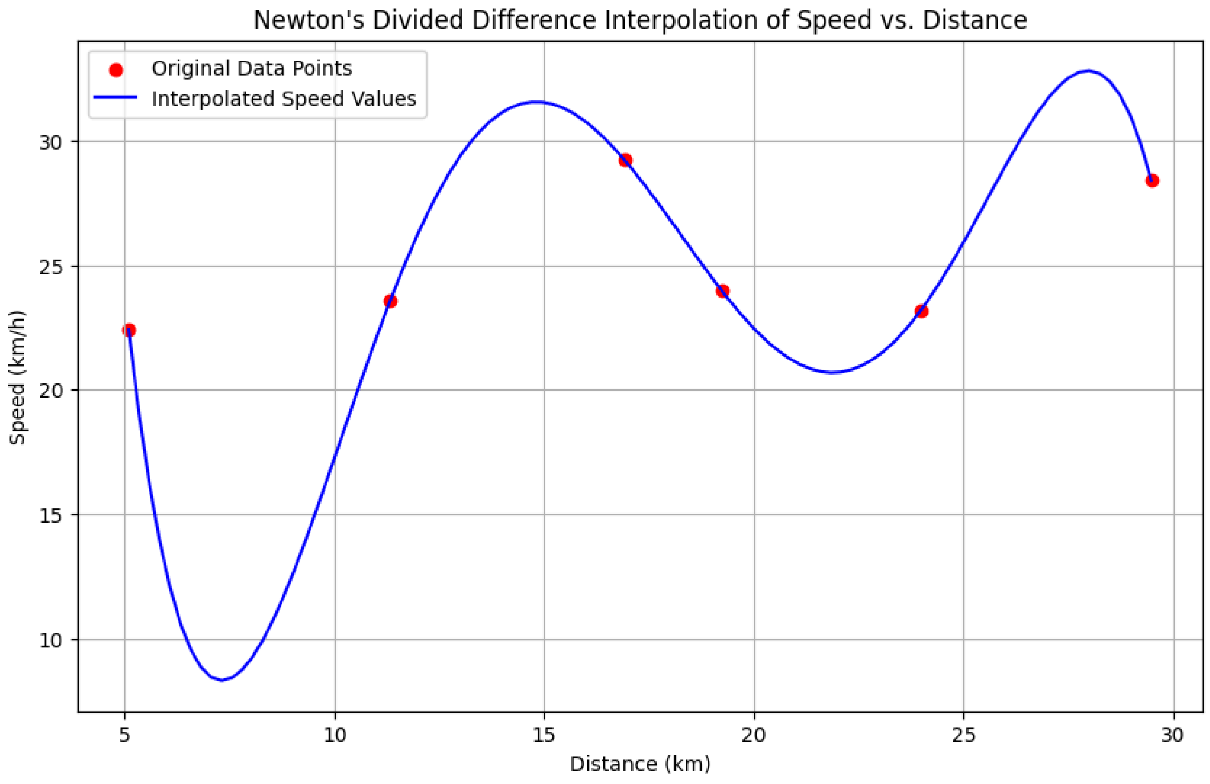
Out[ ]:

	Athlete	Speed_km_h	Distance_km
0	Athlete_1	23.187664	23.985742
1	Athlete_2	26.919703	29.665108
2	Athlete_3	25.543832	8.918614
3	Athlete_4	23.889506	9.843113
4	Athlete_5	29.251325	16.913282
5	Athlete_6	28.416700	29.477880
6	Athlete_7	23.573976	36.320730
7	Athlete_8	20.435915	25.470522
8	Athlete_9	23.047681	36.614742
9	Athlete_10	23.981857	19.238999
10	Athlete_11	27.049588	16.721152
11	Athlete_12	29.953585	18.107793
12	Athlete_13	23.559149	11.330028
13	Athlete_14	27.625478	35.677167
14	Athlete_15	25.931769	17.530821
15	Athlete_16	26.917018	25.437693
16	Athlete_17	21.511275	26.406404
17	Athlete_18	23.988763	24.296723
18	Athlete_19	22.408559	5.099458
19	Athlete_20	23.434560	41.568781

In [ ]:

```
# Selecting a representative subset of data for the demonstration
subset_sports_data = sports_data.sample(n=6, random_state=2).sort_values(by=

# Extracting distance (x) and speed (y) values from the subset
x_distance = subset_sports_data['Distance_km'].values
y_speed = subset_sports_data['Speed_km_h'].values

# Performing Newton's Divided Difference Interpolation for speed based on di
newton_interpolator_sports = KroghInterpolator(x_distance, y_speed)

# Generating distance values for interpolation
x_distance_interp = np.linspace(x_distance.min(), x_distance.max(), 100)

# Evaluating the interpolator at the distance values for speed
y_speed_interp = newton_interpolator_sports(x_distance_interp)
```

```
# Plotting the original data points and the interpolated speed values
plt.figure(figsize=(10, 6))
plt.scatter(x_distance, y_speed, color='red', label='Original Data Points')
plt.plot(x_distance_interp, y_speed_interp, color='blue', label='Interpolate
plt.xlabel('Distance (km)')
plt.ylabel('Speed (km/h)')
plt.title('Newton\'s Divided Difference Interpolation of Speed vs. Distance'
plt.legend()
plt.grid(True)
plt.show()
```



The plot above shows the results of applying Newton's Divided Difference Interpolation to interpolate Speed_km_h based on Distance_km for a subset of athletes. The original data points are marked in red, illustrating the recorded speeds and distances. The blue line represents interpolated speed values across the range of distances, offering insight into how speed might vary with distance for these athletes.

This analysis can be particularly useful in sports performance studies, helping coaches and athletes understand speed endurance over varying distances and potentially identifying optimal distances for peak performance. The flexibility of Newton's method allows for easy updates to the model as new performance data becomes available, making it a valuable tool for ongoing performance analysis.

Chebyshev Distance

In [ ]:
```
# load the dataset
chebyshev_data = pd.read_csv('/Users/jonahzembower/Library/CloudStorage/OneD
# viewing the dataset
chebyshev_data
```

Out[ ]:

	x	y
0	-0.636350	-0.633191
1	-0.391516	0.049513
2	-0.136110	-0.417542
3	0.223706	-0.721012
4	-0.415711	-0.267276
5	-0.087860	0.570352
6	-0.600652	0.028469
7	0.184829	-0.907099
8	0.215090	-0.658952
9	-0.869897	0.897771

Given the typical usage of Chebyshev Distance, we might assume each row represents a point in a 2-dimensional space, and we're to calculate the Chebyshev Distance between corresponding points in two datasets. Alternatively, if x and y are meant to be two distinct sets of points (e.g., x for one set and y for another), we could calculate the Chebyshev Distance across these two sets as a whole.

Assuming the latter (calculating between x and y as two sets), let's proceed with calculating the Chebyshev Distance. If this assumption doesn't match your intent, please clarify how the sets are defined within the data.

Chebyshev Distance between two points in an n-dimensional space is defined as the maximum absolute difference along any coordinate dimension. For points P and Q with coordiantes P = (p1,p2,p3,..,pn) and Q = (q1,q2,q3,...,qn) the chebyshev distance is calculated as Distance Chebyshev (P,Q) = maxi |pi - qi|

In [ ]:
```
# Calculate the Chebyshev Distance between the x and y sets
chebyshev_distance = np.max(np.abs(chebyshev_data['x'] - chebyshev_data['y']

chebyshev_distance
```

Out[ ]:
```
1.7676678885361075
```

The Chebyshev Distance between the two sets of points, as defined by the x and y columns in the dataset, is approximately 1.768.

Significance of Chebyshev Distance:

Chebyshev Distance is significant in approximation and interpolation for several reasons:

Robustness: It measures the maximum difference along any one dimension between two points, making it less sensitive to small variations in multiple dimensions. This can be particularly useful in approximation problems where the worst-case scenario (maximum error) is more critical than the average error.

Grid Movements: In optimization and pathfinding algorithms (like in chess, where the king moves in any direction but only one square at a time), Chebyshev Distance can provide a more accurate heuristic than Euclidean distance because it reflects the actual number of steps needed.

Uniform Scaling: Chebyshev Distance treats all dimensions equally, emphasizing the most significant difference. This is useful in multi-dimensional interpolation and approximation, where one wants to ensure uniform accuracy across all dimensions.

Comparison to Euclidean Distance:

Euclidean Distance measures the shortest path between two points in space and is defined as the square root of the sum of the squares of the differences between corresponding coordinates. It's like the distance one would measure with a ruler in a straight line.

Chebyshev Distance, on the other hand, considers only the maximum difference among all dimensions. It's akin to moving in a grid where diagonal steps are allowed and counted as a single step.

Euclidean Distance is more intuitive and widely used in various fields, including machine learning, for its geometric interpretation. However, Chebyshev Distance offers advantages in specific scenarios, particularly in optimizations, approximations, and scenarios where the maximum deviation is more significant than the mean deviation.

Cubic Splines

In [ ]:
```
# loading the dataset
cubic_splines_data = pd.read_csv('/Users/jonahzembower/Library/CloudStorage/

# viewing the dataset
cubic_splines_data
```

Out[ ]:

	x	y
0	-3.000000	-1.050056
1	-2.877551	-0.994512
2	-2.755102	-0.986408
3	-2.632653	-0.688034
4	-2.510204	-0.808558
5	-2.387755	-0.834839
6	-2.265306	-0.557754
7	-2.142857	-0.663450
8	-2.020408	-0.413730
9	-1.897959	-0.517325
10	-1.775510	-0.336106
11	-1.653061	-0.062486
12	-1.530612	0.114020
13	-1.408163	0.179054
14	-1.285714	0.269671
15	-1.163265	0.366233
16	-1.040816	0.357664
17	-0.918367	0.535134
18	-0.795918	0.653565
19	-0.673469	0.887375
20	-0.551020	0.886353
21	-0.428571	0.733256
22	-0.306122	0.985918
23	-0.183673	0.944671
24	-0.061224	0.930434
25	0.061224	1.059294
26	0.183673	1.086279
27	0.306122	1.046637
28	0.428571	0.825639
29	0.551020	0.821069
30	0.673469	0.814789
31	0.795918	0.797183

	x	y
32	0.918367	0.559201
33	1.040816	0.486950
34	1.163265	0.285710
35	1.285714	0.161616
36	1.408163	0.243170
37	1.530612	0.175797
38	1.653061	-0.089373
39	1.775510	-0.102934
40	1.897959	-0.285194
41	2.020408	-0.499128
42	2.142857	-0.505226
43	2.265306	-0.486205
44	2.387755	-0.732650
45	2.510204	-0.650744
46	2.632653	-1.135236
47	2.755102	-0.844047
48	2.877551	-0.956638
49	3.000000	-1.019893

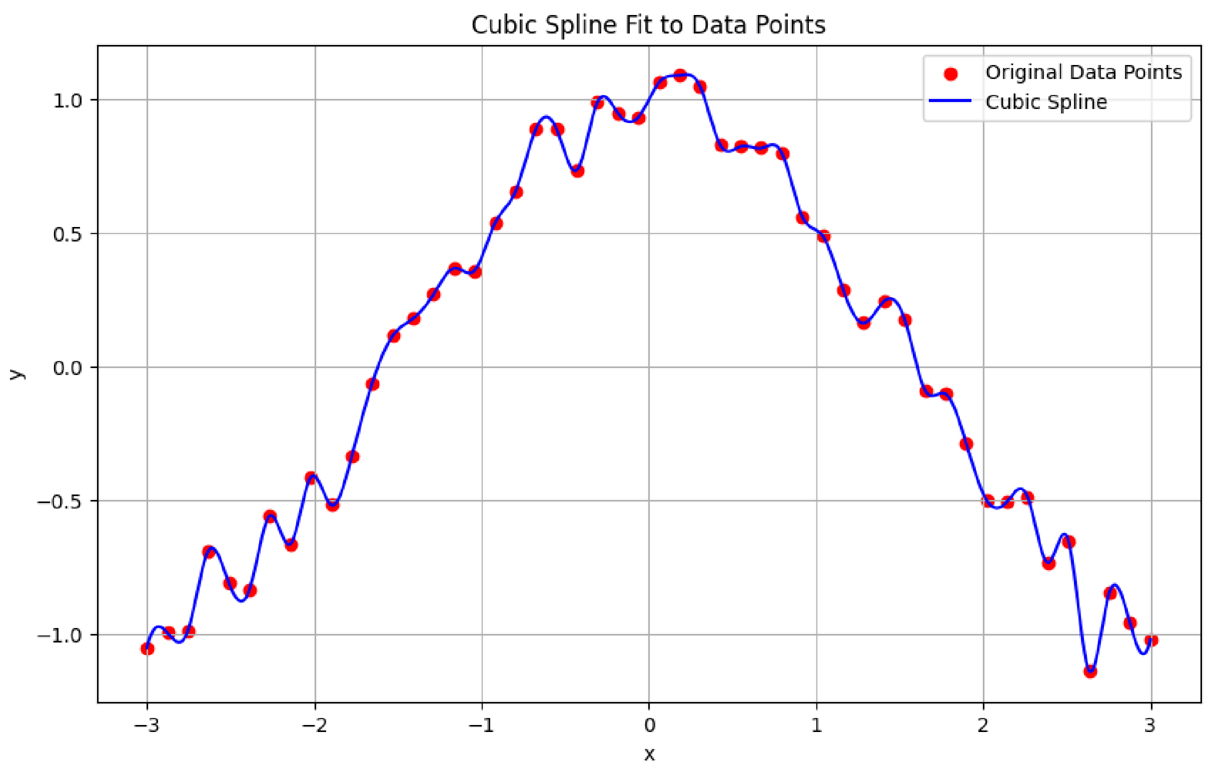
Cubic Splines interpolation involves fitting piecewise cubic polynomials between consecutive data points, ensuring smoothness by matching the first and second derivatives at the endpoints of the polynomials.

To fit a Cubic Spline through these data points, we'll use a spline fitting function that allows for some control over the spline's smoothness. This control is often achieved by adjusting parameters that affect the spline's flexibility and how closely it fits the data points.

After fitting the spline, we'll evaluate the curve's smoothness and discuss the trade-offs between smoothness and fitting accuracy.

In [ ]:
```python
from scipy.interpolate import CubicSpline, make_interp_spline
import numpy as np
import matplotlib.pyplot as plt

# Sorting the data by x to ensure the spline is properly fitted
splines_data_sorted = cubic_splines_data.sort_values(by='x')

# Extracting x and y values
```

```
x_values = splines_data_sorted['x'].values
y_values = splines_data_sorted['y'].values

# Fitting a Cubic Spline with a default smoothing factor
cs = make_interp_spline(x_values, y_values)

# Generating x values for plotting the spline
x_plot = np.linspace(x_values.min(), x_values.max(), 1000)

# Evaluating the spline at the plotting points
y_spline = cs(x_plot)

# Plotting the original data points and the fitted spline
plt.figure(figsize=(10, 6))
plt.scatter(x_values, y_values, color='red', label='Original Data Points')
plt.plot(x_plot, y_spline, color='blue', label='Cubic Spline')
plt.xlabel('x')
plt.ylabel('y')
plt.title('Cubic Spline Fit to Data Points')
plt.legend()
plt.grid(True)
plt.show()
```



The plot above shows the Cubic Spline fit to the original data points. The red dots represent the provided data points, and the blue line is the Cubic Spline curve that smoothly interpolates between these points.

Trade-offs Between Curve Smoothness and Fitting Accuracy

When fitting a curve to data points using Cubic Splines, there are key trade-offs to consider between the smoothness of the curve and how accurately it fits the data points:

Curve Smoothness: A smoother curve may be more aesthetically pleasing and may better represent the underlying trend if the data contains noise. However, overly smoothing the curve may lead to underfitting, where the curve fails to capture significant variations in the data.

Fitting Accuracy: A curve that closely follows every data point may accurately represent the dataset but can lead to overfitting, especially if the data contains outliers or noise. This overfitting can make the curve less generalizable to new data points or underlying trends.

Adjusting the spline parameters, particularly the smoothing factor, allows control over this trade-off:

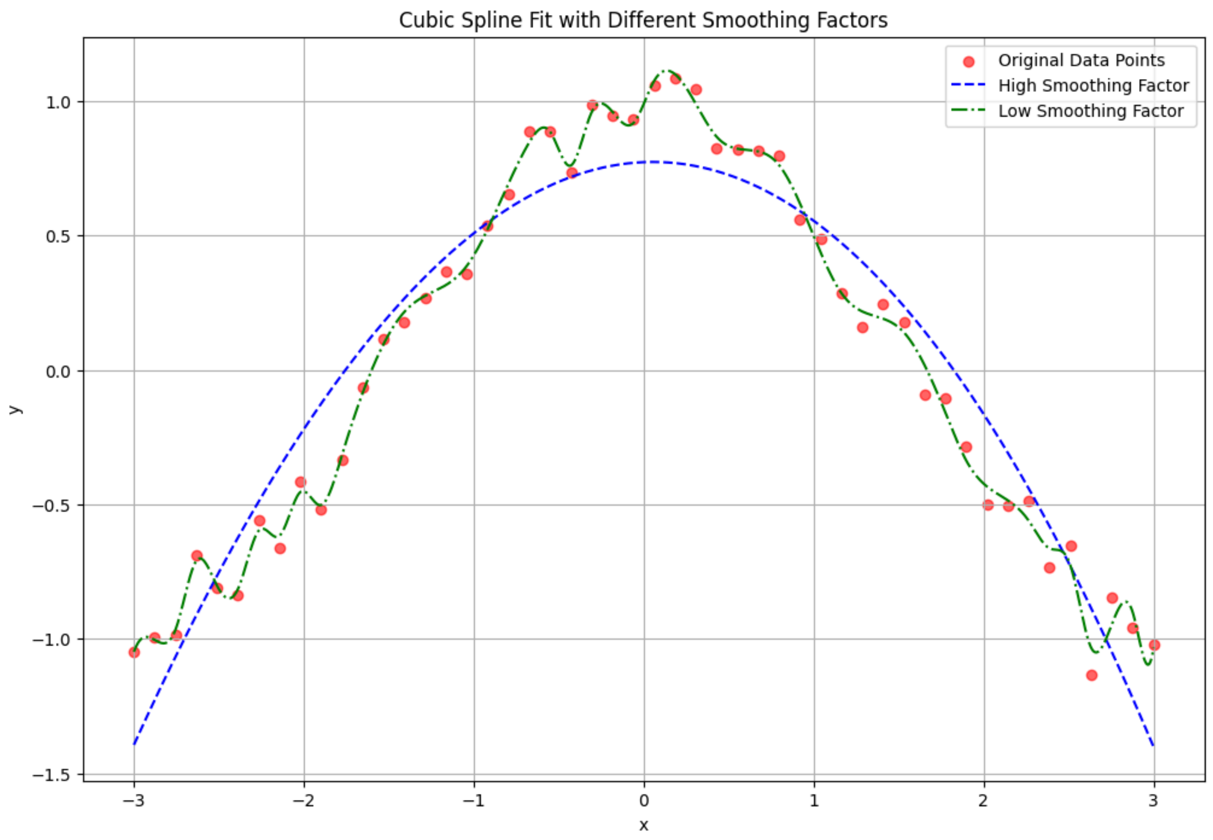
A higher smoothing factor makes the spline smoother but less responsive to the data points, potentially ignoring important variations.

A lower smoothing factor allows the spline to more closely fit the data points, increasing the risk of capturing noise as if it were a meaningful trend.

The choice of smoothing factor (or the decision to use a smoothing spline versus an interpolating spline) depends on the nature of the data and the specific goals of the analysis. For example, if the data is known to contain a significant amount of noise, a higher smoothing factor may be preferable. Conversely, if capturing precise details is crucial, a lower smoothing factor might be more appropriate.

We are going to look into further potential smoothing factors.

In [ ]:
```
from scipy.interpolate import UnivariateSpline

# Fitting Cubic Splines with different smoothing factors
# High smoothing factor
spline_high_smoothing = UnivariateSpline(x_values, y_values, s=10)

# Low smoothing factor
spline_low_smoothing = UnivariateSpline(x_values, y_values, s=0.1)

# Evaluating the splines at the plotting points
y_spline_high = spline_high_smoothing(x_plot)
y_spline_low = spline_low_smoothing(x_plot)

# Plotting the original data points and the fitted splines with different sm
plt.figure(figsize=(12, 8))
plt.scatter(x_values, y_values, color='red', label='Original Data Points', a
plt.plot(x_plot, y_spline_high, label='High Smoothing Factor', color='blue',
plt.plot(x_plot, y_spline_low, label='Low Smoothing Factor', color='green',
plt.xlabel('x')
plt.ylabel('y')
plt.title('Cubic Spline Fit with Different Smoothing Factors')
plt.legend()
plt.grid(True)
plt.show()
```



Observations and Trade-offs:

High Smoothing Factor: Results in a smoother curve that may be preferred when the goal is to identify the overall trend or when the data contains noise. However, this generalization might overlook important subtleties in the data, potentially underfitting areas with significant local variations.

Low Smoothing Factor: Provides a curve that captures more of the data's intricacies, which could be crucial for accurately representing the dataset's behavior. Yet, this approach risks overfitting to noise or outliers, possibly misrepresenting the underlying phenomenon by emphasizing random variations as if they were significant patterns.

Choosing the appropriate smoothing level depends on the context of the analysis, the nature of the data, and the specific objectives of the modeling effort. A balance must be struck between achieving a smooth, generalizable curve and accurately fitting the data's details.

Conclusion

Newton's Divided Difference and Lagrange Interpolation

We began by applying Newton's Divided Difference and Lagrange Interpolation methods to interpolate missing values in datasets representing environmental measurements and material stress-strain data. Newton's method was highlighted for its efficiency in adding new data points without recalculating the entire polynomial, making it suitable for dynamic datasets. In contrast, Lagrange Interpolation was noted for its simplicity and direct applicability to static datasets. These methods are crucial for filling gaps in data, allowing for more comprehensive analysis and decision-making in environmental science and materials engineering.

Chebyshev Distance

We calculated the Chebyshev Distance between two sets of points and discussed its significance in approximation theory. The Chebyshev Distance, emphasizing the maximum deviation among dimensions, is particularly valuable in scenarios where the worst-case performance is critical. This distance metric is applicable in optimization problems, grid movements in algorithms, and ensuring uniform accuracy across multi-dimensional interpolations and approximations.

Cubic Splines

We explored Cubic Splines for fitting smooth curves through a set of data points, adjusting spline parameters to examine the trade-offs between curve smoothness and fitting accuracy. Cubic Splines, with their flexibility in fitting data while ensuring continuity and smoothness, are indispensable for creating models that reflect underlying trends without overfitting noise. They find applications in fields requiring smooth transitions, such as computer graphics, animation, and the modeling of natural phenomena.

Key Insights and Applications

Across these techniques, a recurring theme was the importance of understanding the data's nature and the analysis's objectives to choose the most appropriate method. For instance, the decision between Newton's and Lagrange's methods hinges on the dataset's dynamism, while the choice of smoothing factor in spline fitting balances between capturing detail and avoiding overfitting.

These methods collectively offer powerful tools for tackling real-world problems across various disciplines. From environmental studies, where interpolating missing measurements can lead to better ecosystem management, to materials science, where understanding stress-strain relationships is crucial for designing safer materials, and to sports science, where performance analysis informs training strategies.

Overall

This exploration underscores the versatility and nuanced application of interpolation and approximation techniques in mathematical modeling. By carefully selecting and applying these methods, we can extract meaningful insights from incomplete data, model complex relationships, and make informed decisions across a wide range of scientific and engineering fields. The key lies in balancing accuracy, smoothness, and the computational efficiency of these models to suit specific analytical goals and data characteristics.