# Parameterising Play:
# Exploring Game Dynamics in Snakes and Ladders

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Symbiosis School for Liberal Arts

Symbiosis International (Deemed University)

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Research Project submitted in partial fulfillment

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by

Jai Bakshi

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# Contents

# Acknowledgements

This dissertation was a journey, filled to the brim with countless late nights, and error filled programs but as I come to the end of this journey, I would be amiss if I did not extend my gratitude to the support and guidance to firstly, my parents, for encouragement and support throughout this academic endeavour. To Dr. Sumithra Surendralal, I offer my deepest appreciation for her guidance, mentorship and unwavering support, one could say it served as a *ladder* taking me through this research project. I am also grateful to my friends for their support, willingness to engage with my explorations, and hyping me up by telling me how interesting this topic is to them. A special acknowledgement is due to Simba, my cat, for his *consistent* involvement in the writing process, although it wasn't always conducive to productivity. Lastly, I would like to particularly thank Sheuli, for her invaluable role as a sounding board, and for always supporting me throughout this process. This dissertation is a testament to the collective support I have received, and I am deeply appreciative of each contribution.

# Abstract

In contrast to the idea of "experiential enjoyment" of games, a qualitative attribute that reflects a player's subjective immersion and emotional response, this dissertation investigates "mechanical enjoyment"—defined as pleasure derived from the game's design and mechanics—through parametric analysis, this study conducts a methodical investigation of interactions between several game parameters in the classic board game of Snakes and Ladders. This study makes use of a mixed-methods approach, combining agent-based simulations and Markov Chain modelling, to examine game dynamics in Snakes and Ladders. The study investigates the effects of entity characteristics such as the numbers of snakes and ladders on the board, or the lengths of snakes and ladders and related metrics, as well as the board size on game duration and win probabilities, going beyond subjective interpretations of game enjoyment. Several kinds of relationships are found, including some surprising results that suggest that larger boards increase the likelihood of faster wins even though they lead to longer game durations on average. For game designers looking to identify how to balance difficulty and enjoyment through the means of setting up

the pacing in tabletop games, the Markov Chain model provides to be an effective tool for analysing mechanical enjoyment while minimising computational costs required in agent-based simulations.

*Keywords*: *snakes and ladders, tabletop games, markov chain, agent-based simulations, game design, mechanical enjoyment, game duration, win probability*

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# Chapter 1

# Introduction

From the simple delight of childhood board games to the intricate strategies of modern tabletop experiences, games hold a fundamental appeal for humanity. Play itself, as Peter Gray (2017) argues, is not merely a frivolous pastime but a powerful vehicle for learning and development, deeply ingrained in our nature. Games, in their essence, are structured systems that invite players to engage in artificial conflicts constrained by predefined rules, ultimately leading to quantifiable outcomes (Puentedura, n.d.). This act of play, this engagement within a rule-bound system, is where the potential for enjoyment resides. However, understanding and quantifying this 'enjoyment' becomes a complex undertaking. As Nicole Lazzaro (2004) emphasises, enjoyment is inherently subjective, influenced by myriad factors ranging from individual preferences and social dynamics to the inherent design and mechanics of the game itself.

Given this subjectivity, a challenge arises - how could we systematically analyse game enjoyment? Such a question necessitates a multidisciplinary approach, combining insights from ludology, psychology, mathematics, and game design to examine game enjoyment comprehensively. Tabletop games, encompassing board games, card games, and dice games, offer a vibrant and tangible space for this exploration. The direct manipulation of components and face-to-face social interaction creates a fertile ground for investigating the sources of game enjoyment. Within this exploration, one must consider the philosophical underpinnings of what constitutes a game. Bernard Suits, in his seminal work "The Grasshopper: Games, Life and Utopia" Suits (1978), provides one such valuable framework that positions enjoyment within the structure of gameplay itself. Suits introduces the concept of the "lusory attitude", the willing acceptance of constitutive rules to engage in activity aimed at achieving a specific state of affairs (the lusory goal), where such rules prohibit the most efficient means of achieving that state. This "lusory attitude" is central to understanding games as distinct from ordinary life, operating within what Johan Huizinga (1998) termed the "magic circle"—a bounded space where different rules and expectations apply. Both Suits and Huizinga highlight the structured nature of play, which is crucial in understanding enjoyment.

## 1.1 Game Typologies

Games can be categorised in various ways, reflecting different modes of play and player motivations. The ways to engage with games can range anywhere from competitive, cooperative, solo, casual, exploratory, and role-playing, each incentivising a distinct playstyle.

Caillois and Barash Caillois and Barash (2001) propose four main groups into which different game styles fall: Agon is a group that is constituted of games that allow for competition amongst the players elicit “Hard Fun” (Lazzaro, 2004) like Chess and such games revolve around mastery and challenge. While, games that utilise luck and uncertainty as one of their core mechanics fall into the category of games called Alea, such games may elicit “Easy Fun” (Lazzaro, 2004), for example, Snakes and Ladders. Engaging in role-play or make-believe scenarios, where the player dons a new persona to immerse themselves in fictional universes, such as Dungeons and Dragons come under the category of games called Mimicry. These games offer an “altered state” (Lazzaro, 2004) of enjoyment proving to be escapist and transformational experiences. Lastly, Ilinx is a category of games that temporarily disrupts the stability of perception and create a sense of panic in an otherwise clear mind, examples of the same include Bungee Jumping.

### 1.1.1 Deterministic vs. Stochastic Games

Mathematically speaking, games can be broadly classified based on the predictability of their outcomes and the role of chance: deterministic and stochastic games. Deterministic games have entirely predictable outcomes determined by player actions and game rules. In games like Chess, assuming perfect play, the optimal move can be determined from any board configuration, making the game deterministic in its outcome. Stochastic games, conversely, incorporate elements of uncertainty or randomness, leading to less predictable outcomes. Snakes and Ladders is a prime example of a stochastic game. Gameplay is infused with randomness through die rolls, making the game's progression probabilistic rather than predetermined.

## 1.2 Moving Beyond Subjectivity

While the allure of games is universally acknowledged, the nature of enjoyment itself remains inherently subjective (Lazzaro, 2004). Existing game review systems, as analysed by Yang and Mai (2010), often grapple with this subjectivity, relying on consumer feedback inherently limited by subjective opinions and the tendency to focus on "search attributes"—features readily apparent before playing—rather than "experiential attributes"—those felt only through gameplay.

However, this inherent subjectivity does not negate the need for a more systematic and potentially quantifiable approach to understanding game enjoyment. Indeed, to advance game design and analysis, one must strive to bridge the gap between subjective experience and objective analysis. While experiential enjoyment remains inherently variable, enjoyment, rooted in the game design's core mechanics (or the rules), can be approached as a more quantifiable construct. With this theoretical foundation in place, this dissertation focuses on the specific domain of tabletop games, chosen for their tangible nature and the direct player interaction they allow. By focusing on the design elements and rule systems that structure gameplay, this project aims to develop methods for objectively assessing and potentially predicting the level of enjoyment a game's mechanics might elicit, focusing on what can be termed "mechanical enjoyment"—the enjoyment derived from the inherent design and mechanics of the game system itself rather than reflecting upon a player's subjective immersion and emotional response. This "mechanical enjoyment," attributed to the various components of a game's design refers to a quantitative measure that can potentially be assigned to a game to indicate the level of enjoyment or the utility a player derives from the game.

The academic study of games is a diverse and interdisciplinary field, spanning user experience design, social context analysis, mathematical game theory, and more (Vlachopoulos & Makri, 2017). Key perspectives within game studies include narratology, focusing on games as narrative experiences, and ludology, emphasising game rules and structures (McManus & Feinstein, 2006). While narratology examines the story and narrative elements within games, ludology prioritises the systematic analysis of game mechanics and player interaction with these systems. The framework used in "Four Keys to More Emotion", derived from user experience research, further categorises game enjoyment into "Hard Fun", "Easy Fun", "Altered State", and the "People Factor", highlighting the diverse sources of player engagement and providing a user-centric perspective (Lazzaro, 2004).

Mathematical analysis provides another crucial lens for examining games. Game theory, a branch of applied mathematics, offers tools to study strategic decision-making in competitive situations (von Neumann et al., 1944). Furthermore, methods like combinatorial analysis and, significantly for this research, Markov Chains, have been applied to analyse game mechanics and dynamics. For instance, Raposo and Lamont (2023) employed mathematical analysis to investigate the Royal Game of Ur. These diverse analytical approaches, ranging from theoretical frameworks to mathematical modelling, provide a rich toolkit for objectively investigating game mechanics and their impact on player experience. Specifically, within this domain, we will specifically examine the game of Snakes and Ladders.

## 1.3 Snakes and Ladders: A timeless classic

Snakes and Ladders, far from being a trivial childhood pastime boasts a rich history and a remarkable universality that makes it an ideal case study for understanding fundamental game mechanics and player engagement. Its origins can be traced back to ancient India, where it was known as *Moksha Patam* or *Gyan Chaupar* (Du Sautoy, 2024). The ladders represented virtues like generosity, faith, and humility, while the snakes symbolized vices such as lust, anger, theft, and pride. The ascent and descent on the board mirrored the karmic cycle of life, illustrating the consequences of good and bad actions in a visually compelling and accessible way. In the earliest board layouts from India, the snakes handedly outnumbered the number of ladders; The nine snakes to four ladders made achieving *moksha* quite hard (Du Sautoy, 2024). These boards usually were arranged in a rectangular fashion, with the most common boards being $8 \times 9$ with 72 tiles.

Over centuries, *Moksha Patam* travelled beyond India, evolving and adapting as it spread across cultures, a journey that resonates with Sautoy's (2024) broader narrative of how ideas and concepts traverse geographical and cultural boundaries. By the late 19th century, a Westernised version, "Snakes and Ladders," emerged in England and quickly gained popularity worldwide. While the moralistic undertones were done away with in its global iteration, the core mechanics of chance, progression, setbacks, and the simple pursuit of a defined goal remained intact.

While often perceived as a simple children's game, Snakes and Ladders has also proven to be a rich subject for academic inquiry, particularly within the field of mathematical game analysis. By as early as 1967, we saw that the game constitutes an "interesting example of a Markov chain," amenable to rigorous mathematical modelling (Daykin et al., 1967). Researchers have since employed Markov chains to analyse various aspects of Snakes and Ladders, including the probability distributions of game length (Tun, 2021) and the calculation of expected playing time (Althoen et al., 1993; Daykin et al., 1967). Althoen et al. (1993) estimated the average game length to be approximately 39 moves for a $10 \times 10$ board, through both analytical calculations and computer simulations, providing a benchmark figure for understanding the typical duration of a game of Snakes and Ladders.

Snakes and Ladders, in its simplicity, offers insight into the broader challenges of quantifying game enjoyment. Its mechanics are easily grasped—the roll of a die dictates movement, and predetermined snakes and ladders introduce elements of both fortune and misfortune. However, even within this seemingly straightforward system, players experience various emotions: anticipation with each dice roll, frustration upon encountering a snake, elation when climbing a ladder, and the ultimate satisfaction of reaching the final square. The game's accessibility and widespread familiarity make it an excellent lens to examine how even basic game mechanics, governed by chance and simple rules, can generate engaging and emotionally resonant player experiences. By analysing Snakes and Ladders through the framework of mechanical enjoyment, one can isolate and understand the core design elements that contribute to the enduring appeal of tabletop games and, potentially, games more broadly. Therefore, this dissertation will use Snakes and Ladders as a central case study to explore and quantify mechanical enjoyment.

## 1.4 Dissertation Structure

Beginning with the establishment of essential frameworks and a review of existing approaches to game analysis and enjoyability in Chapter 1 of which this section is a part, the research moves towards an empirical investigation of game dynamics. In Chapter 2 we employ agent-based simulations to quantify the influence of specific game parameters on how the game duration is affected, thereby altering the experience of playing Snakes and Ladders. Building upon these empirical findings in Chapter 3, the investigation then advances to explore the impact of another key design element—the size of the game board—through further simulation-based analysis. In Chapter 4, we develop a Markov model to analytically derive key game metrics and provide a comparative validation against the empirical results from Chapters 2 and 3. Finally, the dissertation concludes in Chapter 5 by aggregating the insights gained throughout this exploration, discussing their implications for game design and game enjoyment, also suggesting potential avenues for future research and inquiry.

# Chapter 2

# The Dynamics of Snakes and Ladders

Within the clearly defined structure provided by a game's rules, we can begin to analyse and potentially quantify the sources of enjoyment that arise purely from the system's design, independent of individual player preferences or social contexts. The notion of the "Magic Circle" (Huizinga, 1998), describes games as existing within a bounded space governed by self-contained rules and conventions. It is within this "circle" of rules that mechanical enjoyment takes shape—an inherent quality of the game system itself, derived from its internal logic and the interactions it engenders through its mechanics. The objectivity of a set of rules provides a strong foundation to set up the notion of mechanical enjoyment when it comes to various kinds of systems, especially those like table-top games.

The average game duration is a critical metric for quantifying these mechanical aspects of enjoyment. Game duration, defined as the number of moves required to reach the end state, is a readily measurable and intuitively understandable indicator of game dynamics. It directly reflects the efficiency and predictability of the game system in guiding a player towards its objective. For a game like Snakes and Ladders, where the goal is to reach the final tile, the number of turns taken to achieve this outcome becomes a crucial measure of the game's mechanical properties. A longer game would detract from the playing experience and negatively impact the enjoyment since the time commitment required to go through a round would grow, while, achieving a swift victory in fewer number of turns although could feel satisfactory at first, a game that finishes too quickly may also leave the player with no sense of achievement. As we will explore, variations in average game time can reveal how different configurations of snakes and ladders, governed by the game's rules, alter the overall pace and challenge of the experience.

By simulating numerous games while systematically varying parameters we study the impacts on game duration. In this chapter, we investigate two key aspects: firstly, the impact of the *number* of snakes and ladders on the board, and secondly, the effects of varying the *lengths* of these entities. To simplify the analysis and isolate the effects of these parameters, the model reduces the game to its essential elements. This allows us to systematically examine how changes in these parameters affect the distribution of game duration—specifically, the number of moves needed to reach the end state. This distinction in parameter variability helps separate the core game mechanics from the broader gameplay experience.

## 2.1 Setting up the board

The classic game board as mentioned in Chapter 1 has 8 rows and 9 columns to form a $8 \times 9$ grid. In this dissertation, the game board is modelled as a 1-dimensional board of $n^2$ tiles for ease of computation, where $n$ depicts the number of tiles along the side of the board. In this Chapter, we keep the $n = 10$, i.e. the board comprises of a 100 tiles, akin to the modern Snakes and Ladders game. The player, represented by an Agent in our model, starts at tile 1, with no requirement to roll a specific number to begin (i.e., no starting condition). The goal is to reach the tile 100. The Agent's movement is determined by a fair six-sided die roll. Each roll of a fair six-sided die produces an outcome $k \in \{1, 2, 3, 4, 5, 6\}$, each with an equal probability of $\frac{1}{6}$.

To facilitate a systematic investigation of game dynamics, this research introduces several controllable parameters that define the entities on the board:

1. **Board Size** (BoardSize): The maximum size of the board in terms of the number of tiles. The board is of the form $n \times n$ and there are a total of $n^2$ tiles on the board.

2. **Number of Snakes** ($N_s$): The total number of snakes on the board. Kept to be $BoardSize = 100$ across this chapter.

3. **Number of Ladders** ($N_l$): The total number of ladders on the board.

4. **Length of Snakes** ($L_i^s$): This parameter determines the length

of the $i^{th}$ snake on the board for $i = 1, 2, ...N_s$. It dictates the number of tiles the agent is set back when landing on a snake's head.

5. **Length of Ladders** ($L_i^l$): This parameter determines the length of $i^{th}$ ladder on the board for $i = 1, 2, ...N_l$. It dictates the number of tiles the agent climbs when encountering a ladder's base.

6. **Ladder Position** ($\text{Ladder}_i^{base/top}$): The position of the $i^{th}$ ladder's terminal ends.

7. **Snake Position** ($\text{Snake}_i^{head/tail}$): The position of the $i^{th}$ snake's terminal ends.

When a board is set up computationally [1], to ensure the board configuration remains valid and avoids conflicts—such as positioning snakes or ladders at invalid tiles where they might extend beyond the board's boundaries, or an overlap is at the terminal positions of entities — certain constraints are implemented:

1. **Ladder Constraint:** Ladders cannot begin within the $L_i^l$ tiles of the board to prevent them from extending beyond the game's end. The ladder's starting position therefore becomes:

$$\text{Ladder}_i^{base} \leq \text{BoardSize} - L_i^l$$

[1] The computations were implemented in Python 3.13.2 (Python Software Foundation, 2025)

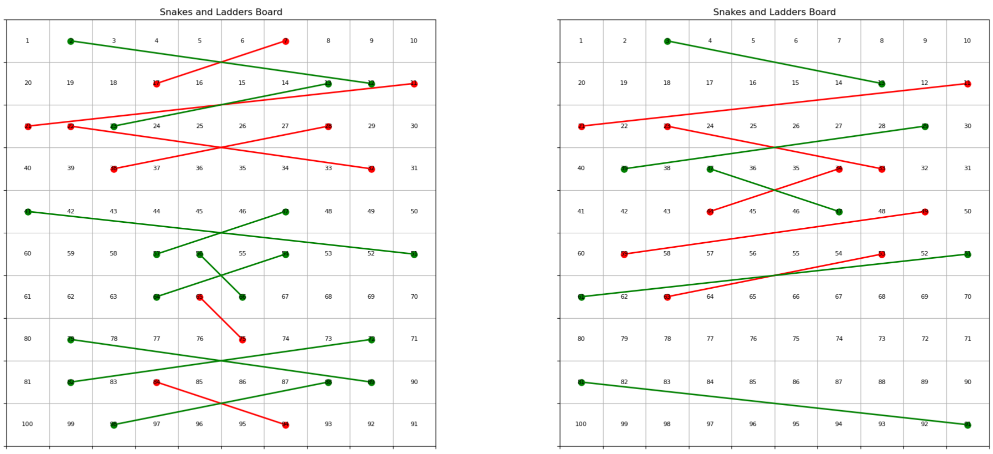
2. **Snake Constraint:** Snakes cannot begin within the first $L_i^s$ tiles to avoid their tails going below the starting position. The snake's end therefore becomes:

$$\text{Snake}_i^{head} \geq 1 + L_s^i$$

3. **Overlap Constraint:** No terminal ends of a snake or ladder (start or end) can overlap with the terminal ends of another snake or ladder. The paths of snakes and ladders can coincide at various points so long as they don't have overlaps at the ends of the entities. If an overlap occurs in the simulation set-up, we randomly decide whether to remove the overlapping snake or ladder based on a probability of 0.5.

The board generation process involves randomly selecting starting positions for snakes and ladders within these permissible ranges. This is followed by a validation step to resolve any overlaps. This iterative process continues until a valid board configuration is achieved. Figures 2.1 (a) and (b) show examples of boards generated under the constraints. The number of iterations required to generate a valid board is recorded and can be analysed to understand the complexity of board creation under different parameter settings.

This structured approach to board generation allows us to systematically vary the parameters and study their individual and combined effects on the game dynamics. By analysing the resulting distributions of game durations, we would like to understand the trends and patterns that reveal the interplay of these factors in shaping the player's experience of the game.




(a) (b)

Figure 2.1: **Sample Board Layouts:** (a) $N_s = 10, N_l = 10$ (b) $N_s = 5, N_l = 5$ The lines in green represent ladders while, the lines in red represent snakes

## 2.2 Approaches to Assign Entity Parameters

We employ different approaches for assigning the key parameters: the $N_s$ and $N_l$, and in particular, the $L_i^s$ and $L_i^l$.

### 2.2.1 Varying the Number of Snakes and Ladders

During the preliminary exploration, the primary focus is on how the $N_S$ and $N_L$ affects the average game time, while keeping the lengths of these entities consistent across simulations. Using simulated data, we try to explore the relationship between different counts of snakes and ladders on the duration of the game. This is done while keeping $L_i^s$ and $L_i^l$ constant to 10 tiles. A set of 10 distinct board configurations are generated, varying the $N_S$ and $N_L$ independently and finally 10,000 games are simulated to study their isolated and combined impacts.

In this attempt to capture the effects of varying the $N_s$ and $N_l$, the simulations were conducted in various different configurations. These were of a form where the $N_s$ (starting at $N_s = 5$)was kept constant for a varying $N_l$ between $[5, 10]$. After generating 10 board layouts for each particular set of $N_s$ and $N_l$, 10,000 games were simulated assuming a 1-player game, and the $N_s$ was increased by 1. This method was repeated until $N_s = 10$. This gave us a total of 25 different set of parameters, with 10 board layouts for each configuration. The average game duration were collected, which represent the mean number of turns taken across 10,000 simulated games for each board size, providing a measure of typical game duration.

**Distribution of Average Game Duration: Varying Number of Entities**

Figure 2.2 presents a comparative overview of game duration across varying $\frac{N_s}{N_l}$. The box plot effectively visualises how the average number of turns taken to complete a single game, effectively changes as the balance between obstacles (Snakes) and shortcuts (Ladders) shifts. Notably, configurations categorised by a significantly large or small ratio, especially ones with large $N_s$ for example, when $N_s = 9$ and $N_l = 5$ (i.e. the ratio is 1.8) we are able to see great fluctuation in the game durations, as denoted by the large inter-quartile gap and similarly when $N_s = 10$ and $N_l = 5$ (i.e. the ratio is 2) the median value of average game duration is much higher than those where the situation is flipped. A clear trend emerges indicating that board configurations with a scarcity of ladders tend to prolong gameplay.

Conversely, as the number of ladders in a configuration increases, a general reduction in average game durations is observed. This is logically consistent as ladders expedite progress towards the final square, thus decreasing the total moves needed. Adding to this, the fact that configurations with more ladders, lead to shorter average game durations, these generally display more compact distributions with fewer outliers, suggesting a more predictable and consistent game duration.

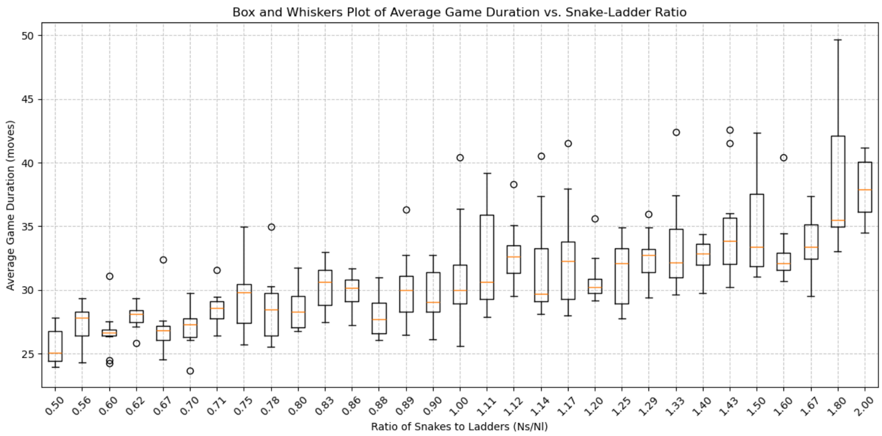
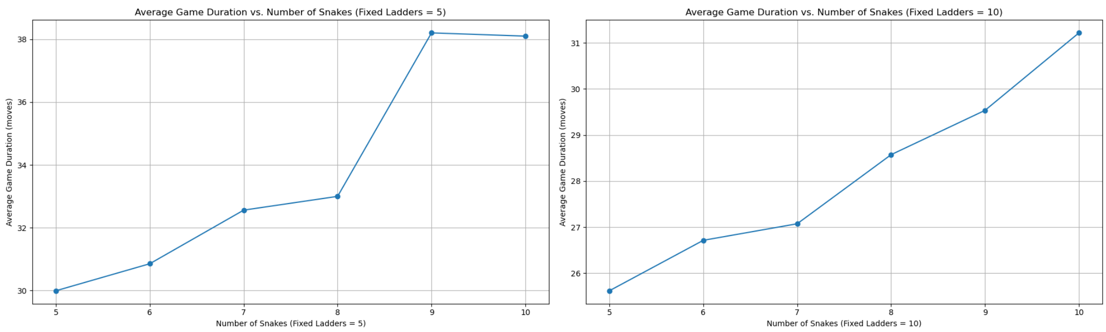



Figure 2.2: **Distribution of Average Game Durations Grouped by $N_S$ and $N_L$:** Box plot shows higher variability in average game duration for extreme $N_S/N_L$ differences. Fewer ladders correlate with longer game durations; outliers indicate luck-dependent game lengths.

#### Trend in Average Game Durations for Different Configurations

Figure 2.3 presents line graphs illustrating the average game duration across different board layouts for configurations with $N_l = 5, 10$ and an increasing number of snakes. The line charts reveal a clear increasing trend related to the $N_s$ , i.e. the average duration of a game increases. Configurations with a smaller $\frac{N_s}{N_l}$ also tend to have shorter durations in general. This suggests that a higher $N_l$ can effectively buffer against the negative impact of snakes, contributing to a predictable game dynamic overall. Additionally, in Figure 2.3 (a), we observe a sudden spike in the average duration of a game, while, this can be attributed to the randomness of a die roll, or to the board layout itself, it is apparent that these spikes indicate that strategic snake placement can substantially impede player progress, likely due to increased instances of players landing on snakes and being forced to regress.




(a) (b)

Figure 2.3: **Trend in Average Game Duration:** For configurations with a fixed number of ladders and increasing number of snakes, an increasing trend can be observed w.r.t average game duration, the average game duration for a larger number of ladders is also lower consistently

#### Interaction Between Number of Snakes and Ladders

The heatmap (Fig. 2.4) provides a visual overview of the interaction between $N_S$ and $N_L$ on average game duration. Colours range from dark shades (indicating shorter game durations) to bright shades (indicating longer game durations). A clear pattern emerges: as the number of ladders increases, the average game duration decreases. This trend is most pronounced for higher snake counts ($N_S \geq 9$) and the $\frac{N_s}{N_l}$ is high, where additional ladders significantly reduce game durations. The bottom-right corner of the heatmap ($N_l \geq 9, N_s = 5$) shows the shortest game durations, reinforcing the idea that ladders effectively mitigate the delays caused by snakes.

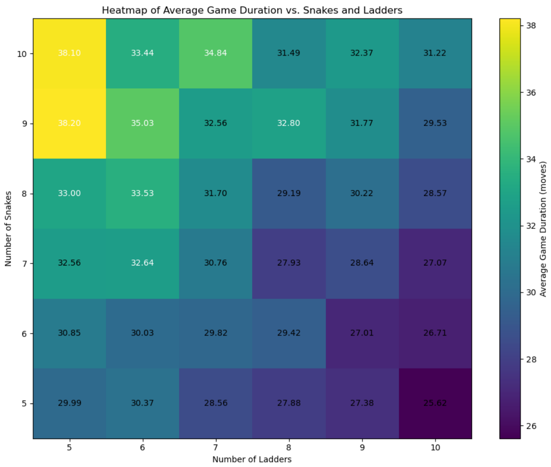



Figure 2.4: **Heatmap of Average Game Durations:** Heatmap shows decreasing average game duration with increasing $N_L$, especially at higher $N_S$. $N_S$ effect is non-linear; game duration significantly increases only beyond a certain $N_S$ threshold.

The effect of snakes on game duration, however, is not strictly linear. For example, increasing the number of snakes from 5 to 6 does not dramatically alter the game duration. Yet, when the number of snakes is increased to 9 or 10, game durations rise substantially. This suggests a threshold beyond which additional snakes significantly increase the likelihood of players encountering them, thus extending game duration considerably. This indicates that while snakes introduce challenges, their negative impact on game duration can be mitigated by providing players with ample ladders to climb back up.

### 2.2.2 Varying the Lengths of Snakes and Ladders

To investigate the impact of entity lengths, three distinct approaches for assigning lengths ($L_i^s$ and $L_i^l$) on the game board are deployed.

The maximum length of any entity (Denoted by $L_{max}$) is kept to be 40 tiles. Each approach allows for unique characteristics of the board configuration to facilitate a comparative analysis of game duration:

1. **Fixed Unequal Lengths:** This approach assigns fixed lengths to all entities within the same group (i.e. $\forall i \in [1, N_{s/l}]$ all $L_i^l$ are equal to each other and the same for $L_i^s$) but unequal across the two groups. For instance, one pair of values is $L_i^s = 10$ and $L_i^l = 5$, another would be $L_i^s = 20$ and $L_2^l = 10$ and so on. This method provides a baseline for analysing gameplay outcomes under deterministic length conditions and ensures uniformity across experiments.

2. **Sampling from Distributions:** To introduce variability in lengths, this approach employs three distinct probability distributions: uniform, normal, and exponential—to sample lengths for each snake and ladder.

   (a) **Uniform distribution:** All valid lengths between 1 and $L_{max}$ are equally likely, this ensures an unbiased selection across the entire range of lengths, providing a uniform probability for shorter and longer lengths.

   $$P(L = x) = \frac{1}{L_{max}}; x \in \{1, 2, \ldots, L_{max}\}$$

   (b) **Normal distribution:** A Normal/Gaussian distribution is characterized by a mean $\mu$ and a standard deviation $\sigma$, for the lengths of snakes and ladders:

- $\mu$ is set to $\frac{L_{max}}{2}$, placing the most likely lengths near the midpoint of the range
- $\sigma$ is set to $\frac{L_{max}}{6}$, which suggests that most lengths fall within the range $[\mu - 3\sigma, \mu + 3\sigma]$

$$P(L = x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{(x-\mu)^2}{2\sigma^2}}; \forall x \in [1, L_{max}]$$

(c) **Exponential distribution:** Using a decaying exponential distribution emphasises on the shorter lengths, with the probability of longer lengths decreasing exponentially. The scaling parameter $\lambda$ is set to $\frac{L_{max}}{3}$ ensuring a reasonable spread of values.

$$P(L = x) = \lambda e^{-\lambda x}, \forall x \in [1, L_{max}]$$

3. **Fixed Start and End Points:** This approach diverges from directly controlling the $L_s$ and $L_l$. Instead, it involves assigning randomized $\text{Ladders}_i^{base/top}$ and $\text{Snakes}_i^{head/tail}$. This approach to the problem introduces another layer of variability by purely focusing on their placement rather than predetermined or sampled lengths. For each snake, the $\text{Snake}_i^{head}$ is chosen from the range $[2, \text{BoardSize} - 1]$ abiding by the snake constraint. While, the $\text{Snake}_i^{tail}$ is determined by randomly selecting tile below its starting position, i.e.

$$1 \leq \text{Snake}_i^{tail} < Snake_i^{head}$$

For ladders, the $\text{Ladder}_i^{base}$ is chosen randomly from $[2, \text{BoardSize}-1]$ keeping the ladder constraint in check, whilst its $\text{Ladder}_i^{top}$ is assigned randomly above its starting position, i.e.

$$\text{BoardSize} > \text{Ladder}_i^{top} > \text{Ladder}_i^{base}$$

By decoupling length from predetermined distributions, the method accommodates a wider variety of configurations, making it suitable for exploring edge cases in gameplay. In the following section, we observe the evidence collected by running the aforementioned simulations while varying the said parameters.

#### Controlled Approach: Unequal Snake and Ladder Lengths

This findings from conducting simulations using fixed, unequal lengths pairs for snakes and ladders across 10 different board configurations, with 10000 simulations per configuration. $L^S$ and $L^L$ were systematically varied in pairs, ensuring $L^S$ and $L^L$ were consistently unequal within each pair type. The bar plot (Fig. 2.5) illustrates the average game duration for various length pairs. It is evident that average game duration generally increases as the difference between snake length and ladder length ($L^S - L^L$) widens. This suggests that when snakes are significantly longer than ladders, players experience more setbacks, contributing to longer average game durations. Conversely, no significant impact on game duration is observed across pairs where ladder lengths exceed snake lengths ($L^L > L^S$).

The frequency distribution plot (Fig. 2.6) provides a view of one such game duration distribution for the configuration with one of the more extreme length disparities ($L^S = 40$ & $L^L = 20$). The distribution is notably right-skewed, indicating that while most games

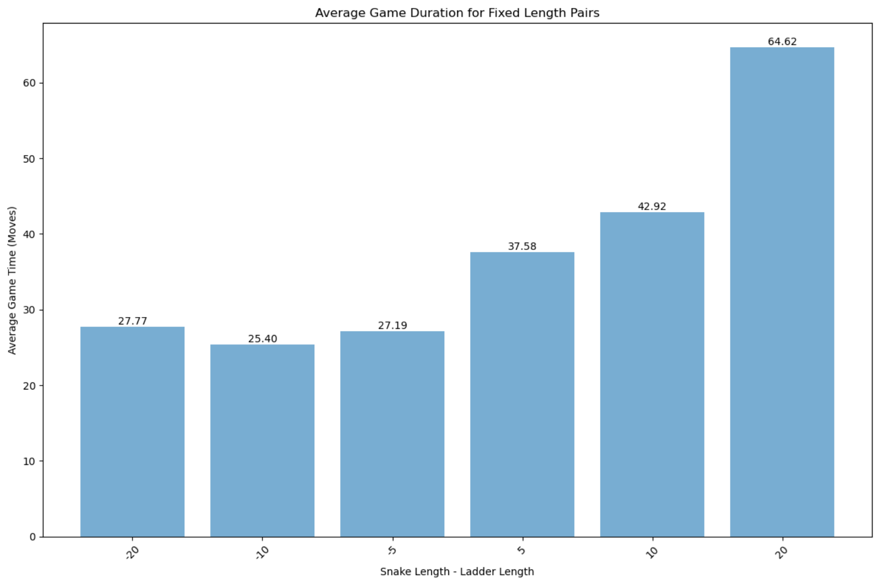



Figure 2.5: **Average Game Durations for Fixed Unequal Lengths:** Bar plot shows game duration increases with widening snake-ladder length difference ($L_S - L_L$), particularly when snakes are longer. $L_L > L_S$ pairs show minimal game duration impact.

conclude within a moderate number of moves, occasional runs extend significantly longer. This skew is likely attributable to the inherent randomness of dice rolls and the frequency of snakes encountered, even with relatively long ladders present. Comparing configurations with $L^L > L^S$ reveals that those with smaller differences ($L^L - L^S < 20$) exhibit tighter distributions with fewer outliers. The average game durations in these configurations also cluster more closely together, unlike the pronounced spikes observed in configurations with larger length disparities.

#### Using Sampling Distributions for Lengths

This section investigates the effects of sampling $L^S$ and $L^L$ from various probability distributions. We explore three such distributions—uniform, normal, and exponential—each with $N_S, N_L = 10$,

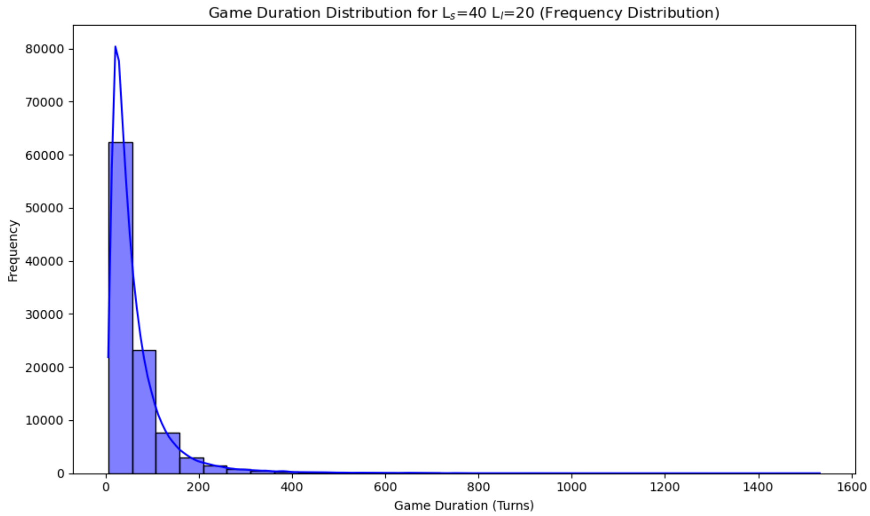



Figure 2.6: **Game Duration Distributions for Configurations with $L_S - L_L = 20$** Histogram of such configurations show right-skewness, indicating longer games.

from which $L^S$ and $L^L$ are sampled. For each distribution, 10000 games were simulated across 10 different boards. Figure 2.7 shows the aggregated average game durations across these distributions. The highest average game duration stems from the exponential sampling method, followed by the normal distribution, with the uniform distribution yielding the lowest average game duration. Exponential sampling tends to produce more shorter lengths, with a lower probability of longer entities, suggesting that a higher density of smaller snakes and ladders extends game duration. In contrast, normal distribution, which clusters lengths around a midpoint, serves as a middle-ground with lower average game durations due to the $L^S$ and $L^L$ not being extremely small.

Frequency distributions of game durations for a fixed board layout under each sampling method (Fig. 2.8) reveal right-skewness across all three distributions, indicating that while most games finish within

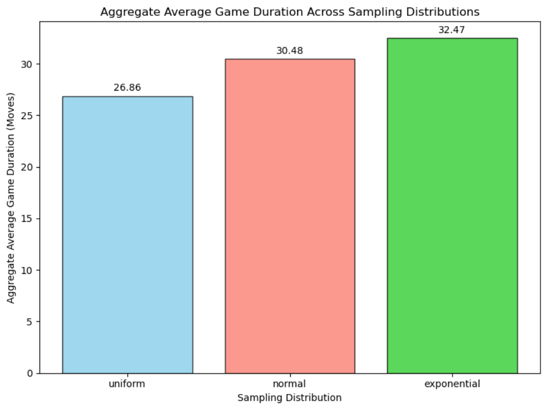
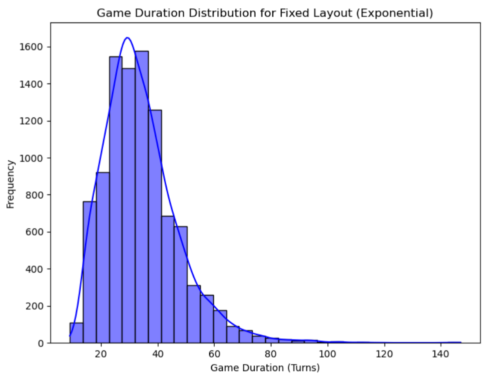



Figure 2.7: **Aggregated Averages of Game Duration across the sampling distributions:** Bar plot compares average game durations for uniform, normal, and exponential length distributions. Exponential distribution yields highest, Uniform distribution lowest average game duration, suggesting shorter lengths extend game duration.

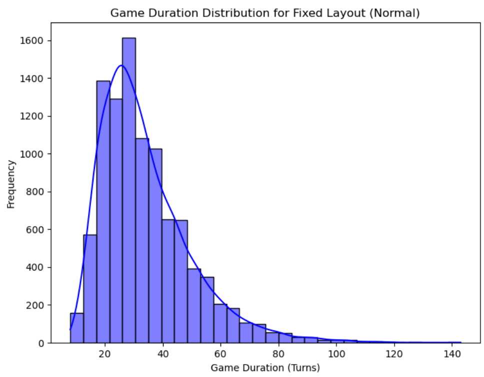
a moderate set of moves, outliers leading to longer games are possible. Exponential sampling exhibits the largest outliers, due to the prevalence of smaller entities and their random placement. Figure 2.9 (a) indicates that boards generated using exponential sampling consistently result in higher average game durations compared to other methods. Figure 2.9 (b) shows that the normal distribution results in more consistently lower average game durations across different board layouts, albeit with some boards exhibiting higher averages. The Uniform distribution 2.9 (c) displays the most variability across the boards played.

#### Fixed Start and End Points

This section examines the effect of $L^s$ and $L^l$ being assigned randomly based on a fixed start and end position. Positions are generated while adhering to length constraints (i.e. the Snake and Ladder Constraints mentioned before and $L_{\max} = 40$). This approach in-




(a)

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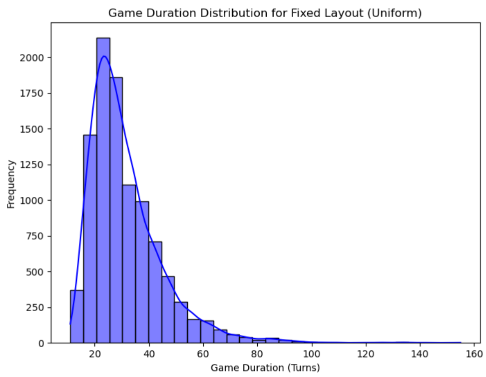
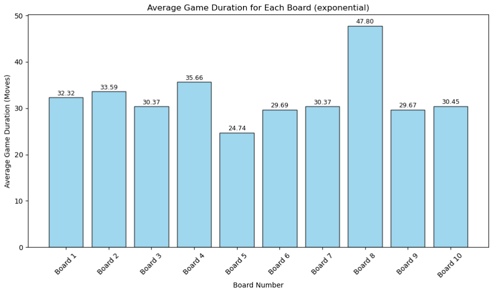


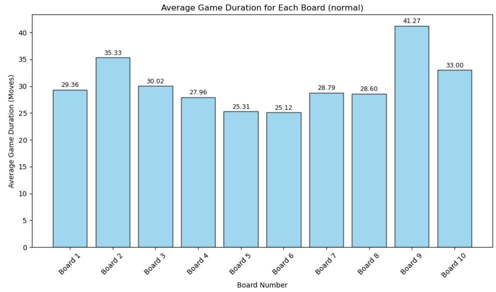

(c)

Figure 2.8: **Game Durations Distributions for a Fixed Layout, by Sampling Method:** Histograms for a fixed layout show right-skewness across all sampling methods. Exponential sampling exhibits highest variability, possibly due to smaller entities and placement.

troduces greater variability compared to methods where lengths are predetermined because the lengths were being picked at random and can allow for certain board configurations to only have small $L^s$ and $L^l$ or conversely, it could lead to the average game duration falling down considerably due to larger entities. Figure 2.10 (b) illustrates average game durations across all 10 boards that were generated. Each bar represents the average game duration for a specific board, revealing significant variability in average game durations across different board layouts, ranging from approximately 22 to 55 moves.




(a)

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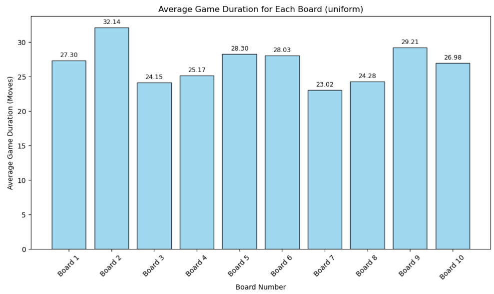


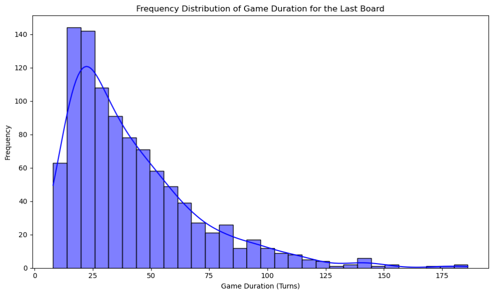

(c)

Figure 2.9: **Average Game Duration for Each Board, by Sampling Distribution:** The plots show average game duration for 10 boards, by sampling distribution. Uniform distribution (c) yields consistently lower averages. Normal distribution (b) shows highest board variability, but low average times. Exponential sampling (a) generally results in higher average times.

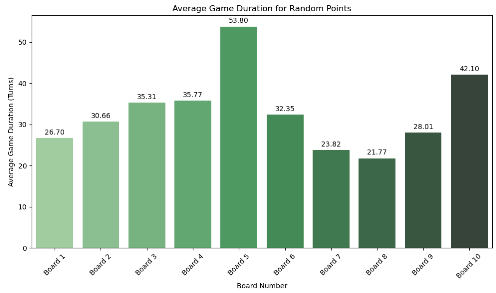
Figure 2.10 (a) displays the frequency distribution of game durations across all simulated games, indicating the right skewness of the approach to assigning lengths. Certain boards, due to their specific configurations, present varying challenges and opportunities for the agent, leading to a wide range of game durations.

#### Insights from varying the lengths systematically

This chapter explores the impacts of both entity $N_s$ and $N_l$ and, $L^S$ and $L^L$ on game duration in *Snakes and Ladders*. By simulating games under systematically varied parameters through different approaches—static length assignment, statistical length sampling, and randomised start/end points, we obtained several insights.




(a)




(b)

Figure 2.10: **Analysis of Randomly Generated Boards:** (a) Game Duration distribution for the last board shows variability within a board. (b) Average game duration across 10 random boards varies significantly (22-55 moves), highlighting random placement impact.

Firstly, variations in both the *number* and *lengths* of snakes and ladders significantly influence game duration and its variability. Increased numbers of snakes generally prolong games, while more ladders tend to shorten them. However, the growth/decay in game duration with entity lengths is not always linear, with thresholds and interactions between entity types playing a crucial role. For example, while simply increasing snake count does not always linearly increase game duration, exceeding a certain density of snakes dramatically extends game duration. Conversely, the positive impact of ladders is more pronounced in mitigating the negative effects of high snake counts.

Secondly, the method of assigning entity lengths introduces another layer of complexity. Deterministic length assignments provide a baseline for understanding game mechanics, while statistical sampling reveals how different length distributions affect game variability and average duration. Exponential distributions, favouring shorter lengths, tend to increase game duration and variability, whereas normal distributions offer a more balanced outcome. Uniform distributions, in contrast, result in shorter, more predictable games. Randomly generated boards based on fixed start and end points introduce the highest degree of variability, highlighting the significant impact of entity placement on overall game dynamics.

Thirdly, the presence of outliers and fluctuating trend lines across different board layouts underscores the significant influence of board layout itself. Strategic placement of snakes and ladders can create "traps" or "shortcuts," leading to substantial variations in game duration even within the same parameter configurations.

These insights demonstrate how adjusting the number and lengths of snakes and ladders can fine-tune game difficulty and duration. The choice of length assignment method—deterministic, sampled, or position-based—further allows designers to control the level of variability and unpredictability in gameplay. For games with mechanics similar to *Snakes and Ladders*, these insights can inform the creation of engaging experiences with carefully calibrated challenge and playtime.

Additionally, this chapter's exploration of entity number and length, while illuminating, represents just one facet of game design parametrisation. The observed sensitivity of game dynamics to these entity-level adjustments naturally prompts further investigation into other fundamental game design elements. In the subsequent chapter, we pivot our focus to another core parameter: the *scale* of the game board itself. By systematically varying board dimensions, while holding entity characteristics constant, we aim to unravel how board size, as a determinant of game space and traversal distance, independently shapes game hardness/difficulty, through game duration, and overall player experience. This shift in focus, from entity-level parameters to board-level dimensions, represents the next logical step in our systematic parametrisation of Snakes and Ladders gameplay, allowing for a more holistic understanding of the game's design space.

# Chapter 3

# Scaling the Game Board: Impact on Game Duration

## 3.1 Board Size as a Determinant of Game Difficulty

The preceding chapter systematically examined the influence of snake and ladder *lengths* and the *number* of these entities on the board. How varying entity lengths and numbers alter the average game duration was observed. Having established the sensitivity of game dynamics to entity lengths and numbers, this chapter now turns its attention to another fundamental parameter: the *size* of the game board itself.

This chapter investigates how scaling the dimensions of the game board while maintaining a constant density of snakes and ladders, and then using this notion of density constant while alternating between the $N_s$ and $N_l$, then experimenting with the $\frac{N_s}{N_l}$ ratio, keeping their individual lengths fixed, impacts game duration. The primary focus was on understanding how board size influences the *difficulty* of the game. In this context, game hardness is operationalised through two readily quantifiable measures: average game duration, representing the typical duration of a play session, and probability of winning within a specified number of turns, reflecting the likelihood of achieving a relatively quick victory. These metrics provide complementary perspectives on the game's challenge and player experience, with average game duration indicating the overall time investment required and win probability offering insight into the game's pace and potential for swift success.

By systematically varying the board size and analysing the resulting changes in average game duration and win probabilities, this chapter aims to elucidate how board dimensions, in conjunction with fixed entity characteristics, shape the game mechanics and, by extension, the mechanical enjoyment of Snakes and Ladders. We hypothesise that increasing board size, even with constant entity density and lengths, will lead to longer average game durations, reflecting the greater distance to traverse to reach the goal. In the following sections, we test the hypothesis by conducting simulations, keeping the constraints on the board design intact from the prior chapter.

## 3.2 Simulation Setup for Board Size Scaling

To investigate the impact of board size, a simulation experiment was designed, varying the linear dimension, $n$, of a square Snakes and Ladders board, resulting in board sizes $\text{BoardSize} = n^2$. The following board sizes were systematically explored: $8 \times 8$ (64 tiles), $10 \times 10$ (100 tiles), $12 \times 12$ (144 tiles), $14 \times 14$ (196 tiles), $16 \times 16$

(256 tiles), $18 \times 18$ (324 tiles), and $20 \times 20$ (400 tiles). For each board size, a constant density of snakes and ladders was maintained, set at 0.1 entities per tile, meaning the number of snakes and the number of ladders were both calculated as $0.1 \times \text{BoardSize}$. Crucially, the individual lengths of all snakes and ladders were also kept *fixed* at 10 tiles, irrespective of board size, to isolate the effect of board dimensions on smaller boards. For each board size configuration, 10,000 game simulations[1] were conducted . Mentioned below are the metrics that were collected from the aforementioned simulations:

1. **Average Game Duration (Simulation):** The mean number of turns taken across 10,000 simulated games for each board size, providing a measure of typical game duration.

2. **Probability of Winning within $\frac{\textbf{BoardSize}}{2}$ Turns:** Calculated as the proportion of games completed within $\frac{\text{BoardSize}}{2}$ turns, representing the probability of a relatively quick win (within a half the number of tiles).

3. **Probability of Winning within $\frac{\textbf{BoardSize}}{3}$ Turns:** Calculated as the proportion of games completed within $\frac{\text{BoardSize}}{3}$ turns, representing the probability of a swift win (within a third of the number of tiles).

4. **Probability of Winning within $\frac{\textbf{BoardSize}}{4}$ Turns:** Calculated as the proportion of games completed within $\frac{\text{BoardSize}}{4}$ turns, representing the probability of a very swift win (within a quarter of the number of tiles).

[1]The computations were implemented in Python 3.13.2 (Python Software Foundation, 2025)

### 3.2.1 Keeping $\frac{N_S}{N_L}$ and Density of Snakes and Ladders as 0.1

In order to set up a control study of the effects of varying the board size on the average game duration and the probabilities of achieving a quick win, we keep $\frac{N_s}{N_l} = 1$ and the density of both Snakes and Ladders as 0.1. The following section explores the observations from conducting the simulations.

#### Findings: Impact of Board Size on Game Duration and Winning Probability

While keeping $\frac{N_s}{N_l} = 1$ and the density of both Snakes and Ladders as 0.1 gives us configurations with an equal, but proportionally growing $N_s$ and $N_l$ on the board. The effects of increasing the board size on average game duration can be seen in Figure 3.1. The findings fall in line with our initial hypothesis which suggested that game duration should increase as the board scales, and we see a near linear relationship between the board size and average duration. Another insight from conducting this simulation suggests that the probability of winning in both $\frac{\text{BoardSize}}{2}$ and $\frac{\text{BoardSize}}{3}$ turns increases as the board size increases.

Now, we move towards varying the other parameters systematically to isolate the effects of individual parameters on these metrics.

### 3.2.2 Varying $\frac{N_s}{N_l}$ and Keeping either Density of Snakes or Ladders as 0.1

To further explore the interplay between board size and the relative number of snakes and ladders, simulations were conducted across

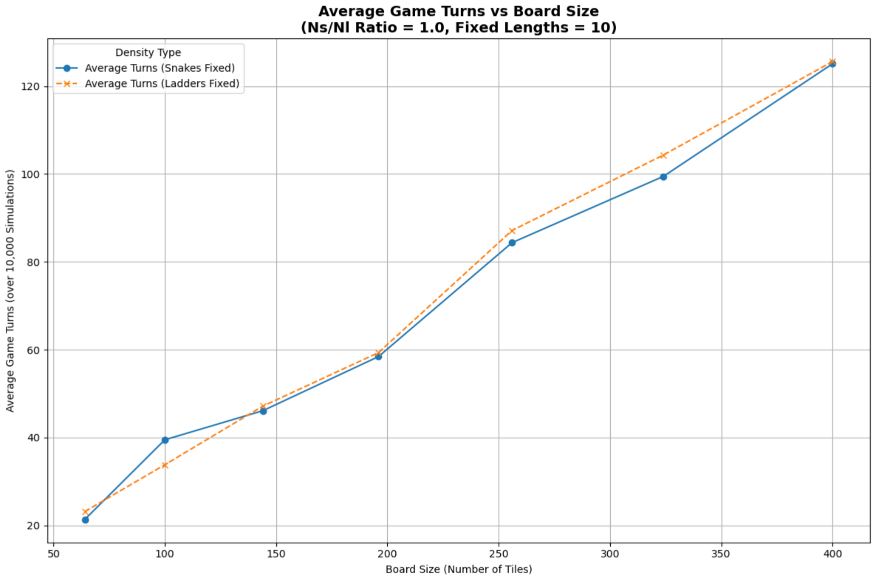



Figure 3.1: **Average Game Duration against Growing Board Size:** Placing an equal but proportionally growing $N_s$ and $N_l$ on the boards as they scale show that average game duration increases as board size increases

the aforementioned board sizes for varying ratios of snakes to ladders ($\frac{N_s}{N_l}$). It becomes crucial to vary $\frac{N_s}{N_l}$ while keeping either of the densities fixed to understand the impact having asymmetrical $N_s$ and $N_l$ have on the board, it allows us to distinguish whether the over abundance or shortage of any of the two lead to any kind of emergent behaviour. For this two sets of simulations were performed:

1. **Fixed Snake Density, Varying Ladder Density:** The number of snakes ($N_s$) was fixed at $0.1\times$BoardSize for each board size. The number of ladders ($N_l$) was then varied to achieve $\frac{N_s}{N_l}$ ratios of 0.5, 1.0, 1.5, and 2.0.

2. **Fixed Ladder Density, Varying Snake Density:** The number of ladders ($N_l$) was fixed at $0.1\times$BoardSize for each board size. The number of snakes ($N_s$) was varied to achieve $\frac{N_s}{N_l}$ ratios

of 0.5, 1.0, 1.5, and 2.0.

These simulations, systematically varying board size and $\frac{N_s}{N_l}$ ratio, aim to provide a comprehensive understanding of how these parameters influence game dynamics, hardness, and duration in Snakes and Ladders.

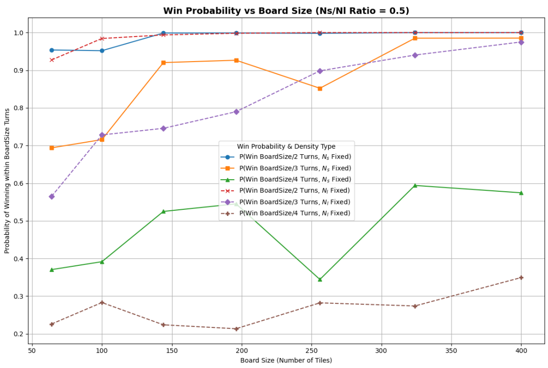
#### Findings: Impact of Board Size on Game Duration and Winning Probability

This section presents an analysis of the simulation findings, focusing on how board size and the ratio of snakes to ladders ($\frac{N_s}{N_l}$) impact the probability of winning within a specified number of turns, and the average game duration. This analysis aims to provide a nuanced understanding of how board size scaling and entity balance may shape the perceived player experience in Snakes and Ladders.

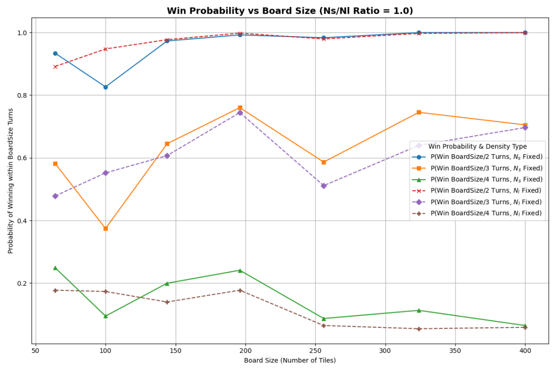
#### Win Probability vs. Board Size for Varying Ns/Nl Ratios

Figure 3.2 visually represent the relationship between board size and win probability across different $\frac{N_s}{N_l}$ (0.5, 1.0, 1.5, and 2.0, respectively). Each figure distinctly portrays data derived from both fixed snake density and fixed ladder density simulations, facilitating a comparative examination of how these density configurations modulate the observed gameplay dynamics.

**Overall Trend: Increasing Win Probability with Board Size**
A consistent trend, robust across all $\frac{N_s}{N_l}$ ratios and density configurations, is the tendency for the probability of achieving a win within a limited number of turns (specifically, within $\frac{\text{BoardSize}}{2}$ and $\frac{\text{BoardSize}}{4}$ turns) to exhibit an *increase as the board size expands.* This trend




(a)

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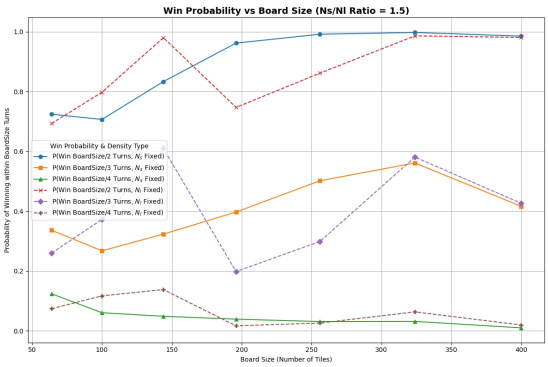


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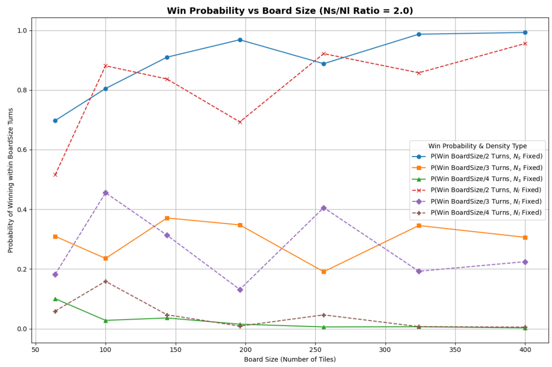



(d)

Figure 3.2: **Win Probability vs Board Size:** For $\frac{N_s}{N_l} \in [0.5, 2]$

robustly confirms the initial hypothesis positing that larger game boards, despite inherently leading to extended average game durations, may counter intuitively enhance the likelihood of a player securing a swift victory. For instance, examining Figure 3.2 (b) for a balanced $\frac{N_s}{N_l}$ ratio of 1.0, the probability of winning within $\frac{\text{BoardSize}}{4}$ turns increases from approximately 10% on an 8x8 board to over 25% on a 20x20 board in the fixed snake density configuration. This seemingly paradoxical effect can be intuitively explained by the proportionally greater number of pathways and tile options available on larger boards. The increased tile count provides players with more avenues to circumvent snake encounters and capitalise on ladder climbs, thereby statistically improving the chances of a quicker, luck-favoured game resolution, even when the density of entities remains constant.

**Impact of $\frac{N_s}{N_l}$ Ratio on Win Probability:** The $\frac{N_s}{N_l}$ ratio emerges as a significant modulator, substantially influencing the baseline win probabilities and the scaling relationship between board size and win likelihood, as detailed in the preceding subsections.

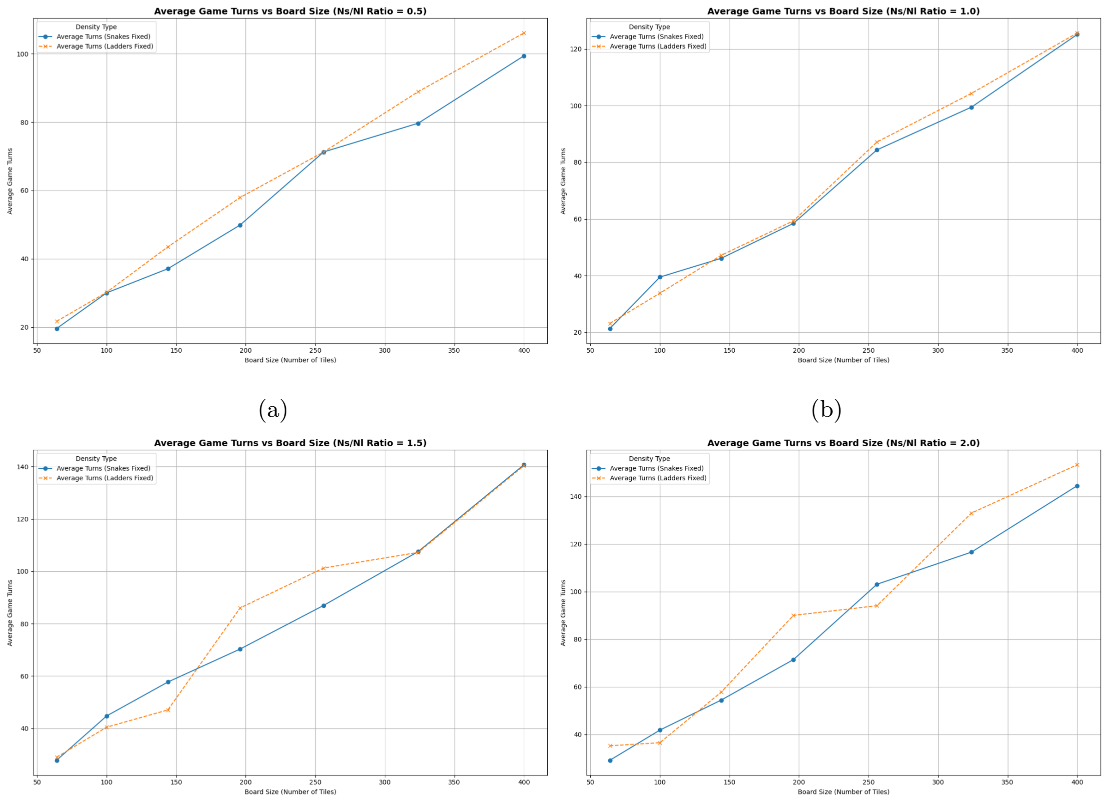
- **Low $\frac{N_s}{N_l}$ Ratio (0.5) - High Baseline Win Probability:** Figure 3.2, representing a low $\frac{N_s}{N_l}$ ratio indicative of ladder abundance relative to snakes, demonstrates consistently elevated win probabilities across the spectrum of board sizes. Notably, the incremental increase in win probability associated with board size expansion is less pronounced in this configuration.

- **Balanced $\frac{N_S}{N_L}$ Ratio (1.0) - Moderate and Scaling Probabilities:** In contrast, Figure 3.2 (b), depicting a balanced configuration with an equal number of snakes and ladders, portrays a more graduated and discernible increase in win probability as board size scales.

- **Elevated $\frac{N_S}{N_L}$ Ratios (1.5 and 2.0) - Reduced and Fluctuating Probabilities:** Figures 3.2 (c) and 3.2 (d), characterising higher $\frac{N_s}{N_l}$ ratios where snakes outnumber ladders, illustrate a departure from strictly linear scaling patterns and reveal suppressed win probabilities, particularly for swift victories.

- **Density Configuration - Minor Influence:** Comparative analysis within each figure, contrasting fixed snake density and fixed ladder density lines, reveals that the specific choice between fixing snake or ladder density exerts a comparatively subordinate influence on win probability distributions, with the $\frac{N_s}{N_l}$ ratio being the dominant factor.

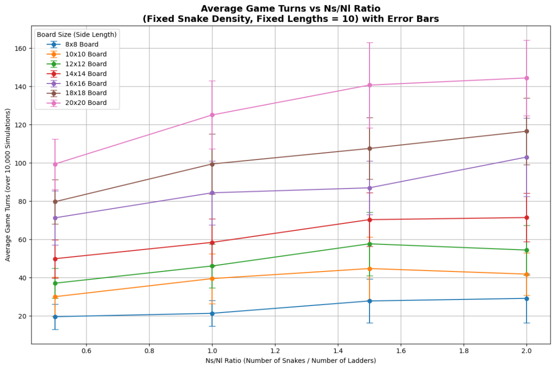
**Average Game Duration vs. Board Size for Varying Ns/Nl Ratios**



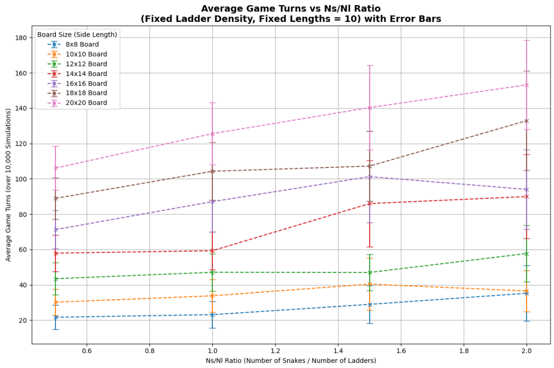

(a) (b) (c) (d)

Figure 3.3: **Average Game Turns vs Board Size** for varying $\frac{N_s}{N_l}$ ratios, regardless of which parameter was fixed, the probabilities follow a similar trend across all ratios.

Complementing the win probability analysis, Figure 3.3, depicts the relationship between board size and average game duration for the same varying $\frac{N_s}{N_l}$ ratios (0.5, 1.0, 1.5, and 2.0). These figures provide a direct measure of how board dimensions and entity balance




(a)




(b)

Figure 3.4: **Average Game Turns vs Board Size** for varying $\frac{N_s}{N_l}$ ratios tends to increase as the ratio increases, due to the increased $N_s$ on the board.

influence the typical duration of a *Snakes and Ladders* game.

#### Consistent Increase in Average Game Duration with Board Size

A highly consistent and pronounced trend across all $\frac{N_s}{N_l}$ ratios and density configurations is the increase in average game duration as the board size scales upward. This observation directly validates the initial hypothesis that larger board dimensions, even with constant entity density and lengths, inherently lead to longer gameplay durations.

#### Influence of $\frac{N_s}{N_l}$ Ratio on Average Game Duration

While board size dictates the overall scaling of game duration , the $\frac{N_s}{N_l}$ ratio exerts a substantial influence on the *absolute magnitude* of average game durations across different board dimensions, as detailed below:

- **Low $\frac{N_s}{N_l}$ Ratio (0.5) - Shorter Game Duration:** Figure 3.3 (a) illustrates that at a low $\frac{N_s}{N_l}$ ratio, average game durations are consistently lower across all board sizes compared to higher ratios, indicating quicker game completion due to ladder abundance.

- **Balanced $\frac{N_s}{N_l}$ Ratio (1.0) - Moderate Game Durations:** Figure 3.3 (b) demonstrates moderately increased average game durations compared to the low $\frac{N_s}{N_l}$ ratio scenario, with game durations scaling more visibly with board size, offering a conventionally paced gameplay experience.

- **Elevated $\frac{N_s}{N_l}$ Ratios (1.5 and 2.0) - Prolonged Game Durations:** Figures 3.3 (c) and 3.3 (d) reveal a marked elongation of average game durations, particularly at higher $\frac{N_s}{N_l}$ ratios, reflecting the impeding effect of a higher density of snakes and leading to protracted gameplay.

Similar to the win probability analysis, the density configuration (fixed snake vs. fixed ladder density) exhibits a comparatively negligible influence on the scaling of average game durations, with the overall game duration primarily governed by board size and the $\frac{N_s}{N_l}$ ratio. Also, Figure 3.4 reaffirms our findings from the prior chapter, which suggested an increase in the average duration of games as the $N_s$ on the board increases.

### 3.3 Game Duration and Player Engagement

The observed scaling of average game duration with board size has direct implications for player engagement and game experience. The simulations quantitatively demonstrate that board controls the typical time investment required to play Snakes and Ladders. Designers can leverage this predictable scaling to tailor game sessions to different player preferences and contexts. Furthermore, the modulatory effect of the $\frac{N_s}{N_l}$ ratio on average game duration offers an additional layer of control over game pacing to the already predictable impacts that stem from changing BoardSize.

Finally, this chapter lays the groundwork for considering ways in which the game can be modelled, since it can be repeated as a stochastic process. In the next Chapter, we will be modelling the game of *Snakes and Ladders* using a Markov Chain. We hope to gain further insight into analytical approaches as opposed to sticking to empirical observations.

# Chapter 4

# Modelling Game Dynamics with Markov Chains

Building upon the empirical insights into Snakes and Ladders game dynamics from agent-based simulations in Chapters 2 and 3, this chapter introduces a complementary analytical approach: a Markov Chain model. This chapter details construction and validation of this model to predict expected game length, win probabilities, and come to a steady-state distributions. Crucially, this Markovian framework serves to analytically verify the empirical findings from our simulations.

## 4.1 What are Markov Chains

A Markov chain is a mathematical construct describing a system transitioning between discrete states in a sequential, chain-like manner (Du Sautoy, 2024). The defining characteristic, and analytical power, of a Markov chain lies in its 'memoryless' property: future state transitions depend solely on the current state, irrespective of the sequence of states that preceded it. In the context of Snakes and Ladders, we conceptualise each tile on the board as a distinct state. The probabilistic transition from one tile to the next is governed by the outcome of a dice roll, rendering the game dynamics amenable to Markovian analysis.

By adopting this framework, we can mathematically represent the inherent randomness of dice rolls and the deterministic rules of snake and ladder placements. This allows us to move beyond empirical observation and offers a powerful means of validating and extending our simulation-based research.

#### 4.1.1 Fundamental Concepts

To ensure clarity and precision in our model description, it is essential to define the core concepts underpinning our Markovian approach:

1. **States:** Each tile on the Snakes and Ladders board, indexed from 1 to $BoardSize$, is defined as a unique state within our Markov model. State 1 represents the starting position, and state $BoardSize$ corresponds to the final tile, the absorbing goal state.

2. **Transitions:** Game progression is modelled as probabilistic transitions between these states. Transitions occur in discrete steps, driven by dice rolls. Each roll of a fair six-sided die produces an outcome $k \in \{1, 2, 3, 4, 5, 6\}$, each with an equal probability of $\frac{1}{6}$. Adding this outcome to the current tile number

determines a tentative next position, subject to board boundaries and entity adjustments.

3. **Absorbing State:** The final tile, state $BoardSize$, is designated as an absorbing state. Once a player reaches or surpasses tile $BoardSize$, the game concludes, and the player remains in this state indefinitely. This is mathematically represented by a self-loop in the transition matrix, with a probability of 1.

4. **Memorylessness (Markov Property):** The defining 'memoryless' property dictates that the probability of transitioning to any future state depends exclusively on the current state. The history of previous moves or states is irrelevant. This assumption is valid for Snakes and Ladders, as each dice roll and subsequent move are probabilistically independent of prior game events.

#### 4.1.2 Constructing the Transition Matrix

The core of our Markov model is the transition matrix, $P$, a square matrix of size $(\text{BoardSize} - (N_s + N_l)) \times (\text{BoardSize} - (N_s + N_l))$. Each entry $P(i, j)$ quantifies the probability of transitioning from state $i$ to state $j$ in a single turn. To maintain the state space and focus on board position transitions, we do not treat snake heads or ladder bases as distinct states (Du Sautoy, 2024). An BoardSize $\times$ BoardSize matrix may be used to study the same phenomena, but since the probability of staying on tiles with entity triggers is going to remain the same as landing on the other terminal tile, therefore, we reject these tiles. This aids the computation as well, since Matrix

Multiplication using the Naive Algorithm has a time complexity of $O(N^3)$ and would scale tremendously as we scale board sizes. For baseline analysis within this chapter, we configure the board with 10 snakes ($N_S$) and 10 ladders ($N_L$), with their lengths assigned randomly using the fixed start and end points methodology detailed in Chapter 2.

For each non-absorbing state $i < N$:

1. **Dice Roll Simulation:** We simulate the roll of a fair six-sided die, generating an outcome $k \in \{1, 2, 3, 4, 5, 6\}$, each with a probability of $\frac{1}{6}$.

2. **Tentative Movement:** A tentative next state is calculated by adding the dice roll outcome $k$ to the current state $i$ (Constrained to Chapter 2's set up).

3. **Entity-Based Adjustment:** We then check if the tentative next state coincides with the start of a snake or the base of a ladder, based on the pre-defined board layout. If so, the state is immediately updated to the corresponding snake tail or ladder top, respectively.

4. **Probability Assignment:** For each dice outcome $k$, the transition probability ($\frac{1}{6}$) is added to the matrix entry $P(i, j)$, where $j$ represents the final state reached after all movement and entity-based adjustments.

For the absorbing state BoardSize, the transition matrix row is configured to represent absorption: $P(\text{BoardSize}, \text{BoardSize}) = 1$, and $P(\text{BoardSize}, j) = 0$ for all $j \neq \text{BoardSize}$. This ensures that once the final state is reached, the probability of transitioning to any other state is zero. For all other (non-absorbing states) $i < \text{BoardSize}$, the transition probabilities are determined directly based on the board layout. For each state $i$ and each possible dice roll outcome, we directly determine the next state $j$ based on the board layout. The $P_{ij}$ is then simply the count of dice rolls that lead to $j$ divided by 6.

Notationally, for non-absorbing states $i < N$, the transition probability is expressed as:

$$P(i, j) = \sum_{k=1}^{6} \frac{1}{6} \cdot \mathbf{1}\{f(i, k) = j\}$$

where $f(i, k)$ is a function encapsulating the game's movement rules: it computes the next state $j$ reached from state $i$ after rolling $k$, incorporating boundary reflections and snake/ladder adjustments. The indicator function $\mathbf{1}\{\cdot\}$ evaluates to 1 if the condition is true, and 0 otherwise.

For the absorbing state $N$, the transition probabilities are deterministic:

$$P(N, N) = 1, \quad P(N, j) = 0 \quad \forall j \neq N.$$

This formulation guarantees that each row of the transition matrix sums to unity, a fundamental property of stochastic matrices and Markov chains (Du Sautoy, 2024).

## 4.2 Analytical Derivation of Game Metrics

With the transition matrix constructed, we can analytically derive key game metrics using matrix-based methods. Specifically, we leverage the concept of the fundamental matrix to calculate expected game turns and estimate win probabilities.

### 4.2.1 Fundamental Matrix and Expected Turns

To calculate the expected number of turns to reach the absorbing state, we first partition the transition matrix $P$ into submatrices corresponding to transient (non-absorbing) and absorbing states. Let $Q$ be the submatrix representing transitions between transient states, and $R$ be the submatrix for transitions from transient to absorbing states. The fundamental matrix $N$, central to our analysis, is then computed as:

$$N = (I - Q)^{-1}$$

where $I$ is the identity matrix of the same dimension as $Q$. The entry $N(i, j)$ of the fundamental matrix provides the expected number of times the system visits transient state $j$ before absorption, starting from transient state $i$.

The expected number of turns, $t$, to reach the absorbing state from a starting state $i$ is derived by summing the entries in the $i^{th}$ row of the fundamental matrix $N$ and multiplying by a column vector of ones, $\mathbf{1}$:

$$t_i = (N \cdot \mathbf{1})_i = \sum_j N(i, j)$$

For a standard Snakes and Ladders game starting at state 0, the expected number of turns to reach the final tile is:

$$t = t_0 = \sum_j N(0, j)$$

This sum represents the total expected number of steps spent in transient states before absorption, effectively quantifying the average game duration in turns (Du Sautoy, 2024).

For example, for a 2x2 board, with 4 tiles labelled 1 through 4 and using a 2-sided die (i.e. only move 1 or 2 tiles ahead). The board has a snake going from tile 3 back to tile 1, and a ladder going from tile 2 to tile 4. The probability of going from tile 1 to tile 2 or 3 becomes 1/2, but since tiles 1 and 2 are entity triggers, they are removed from the transition matrix. The transition matrix thus for the same is of the form:

$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & 1 \end{pmatrix}$$

Here, the first row, represents the probability of going from tiles 1 to 1 and 1 to 4, and the second row shows the probability of going from tile 4 to tile 1 and finally the absorbing state which goes from tile 4 to 4 with absolute certainty.

The sub-matrix Q for the transition matrix is:

$$Q = \begin{pmatrix} \frac{1}{2} \end{pmatrix}$$

After performing all the necessary operations to Q, i.e. $(I - Q)^{-1}$, we get a matrix N:

$$N = \begin{pmatrix} 2 \end{pmatrix}$$

The sum of all elements in the first row, in the case of our current board and transition matrix is 2. Therefore, the average duration of a game on a $2 \times 2$ matrix in the specified configuration is 2 turns.

### 4.2.2 Win Probabilities within Turn Limits

Beyond expected game duration, we extend the Markov model to estimate win probabilities within specified a specified number of turns, providing a measure of game difficulty as defined in Chapter 3. Agent-based simulations empirically estimate win probabilities within $\frac{\text{BoardSize}}{2}$, $\frac{\text{BoardSize}}{3}$, and $\frac{\text{BoardSize}}{4}$ turns by tracking game completion rates within these thresholds over numerous trials. The Markov model, through iterative state transitions, provides a computationally derived estimate of these win probabilities. While a direct analytical derivation of win probability distributions from the fundamental matrix is mathematically complex, our iterative approach offers estimates, allowing for a direct comparison with simulation results and further validation of the model's predictive power.

## 4.3 Validation: Markov Model vs. Agent-Based Simulations

To validate our Markov Chain model, we compared its analytical predictions against empirical results from the agent-based simulations detailed in Chapters 2 and 3. This section presents a comparative analysis of expected game turns, win probabilities, and game turn distributions.

### 4.3.1 Comparative Analysis of Expected Turns

The comparison of analytically calculated expected game turns with the average game turns observed in agent-based simulations shows that for a representative $10 \times 10$ board configuration (Figure 4.1) with $N_s = N_l = 10$ and $L_i^l = L_i^s = 10$, simulations across 10,000 games yielded an average game duration of 33.99 turns. Strikingly, the expected number of turns derived from our Markov model's fundamental matrix is 33.72 turns. This provides strong empirical validation for the Markov chain model's accuracy in predicting average game length in Snakes and Ladders. This analytical verification also enhances the credibility of the simulation-based findings presented in Section 3.2, demonstrating convergent validity across methodological approaches.

### 4.3.2 Comparative Analysis of Win Probabilities

Further validation is achieved by comparing win probabilities within a specified number of turns, calculated using both agent-based simulations and the Markov model. Figure 4.2 presents a comparison of win probabilities for varying board sizes at a fixed $\frac{N_s}{N_l}$ ratio of 0.5 (Ns fixed). The plot juxtaposes win probabilities derived from agent-based simulations against those predicted by the Markov model for turn limits of $\frac{\text{BoardSize}}{2}$, $\frac{\text{BoardSize}}{3}$, and $\frac{\text{BoardSize}}{4}$.

As illustrated in Figure 4.2, the Markov model's predictions for win probabilities closely align with the empirical results from agent-based simulations. Both methodologies demonstrate a similar trend:

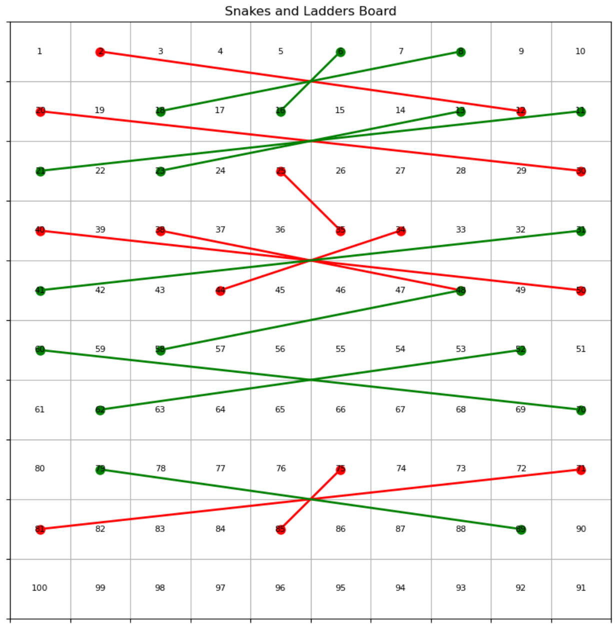



Figure 4.1: **Representative Board Layout:** Snakes and Ladders configuration used to construct the Markov Model—Red lines represent snakes, Green lines represent ladders

win probabilities within limited turns generally *increase* as board size expands, despite the longer average game durations associated with larger boards, as discussed in Chapter 3. This counter-intuitive finding, validated by both simulation and analytical methods, underscores the impact of board size on game dynamics. The close agreement between the predicted and simulated probabilities extends the confidence in our implementation of the Markov model; Its ability to capture essential aspects of game difficulty impacting beyond just the average game duration and by extension the enjoyment. Further comparisons across other $\frac{N_s}{N_l}$ ratios and density configurations would similarly strengthen this validation.

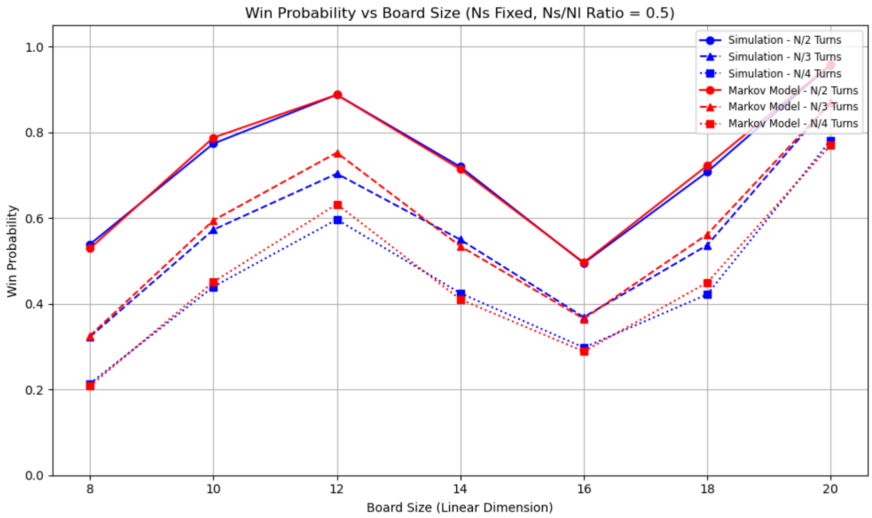



Figure 4.2: **Win Probability vs Board Size** Comparison of win probabilities within $\frac{\text{BoardSize}}{2}$, $\frac{\text{BoardSize}}{3}$, and $\frac{\text{BoardSize}}{4}$ turns, derived from agent-based simulations and Markov model predictions, demonstrating a high degree of agreement.

### 4.3.3 Comparative Distribution Analysis: Game Turns

A more granular validation of the Markov model is achieved by comparing the full distribution of game turns. Figure 4.3 presents a comparative view of a particular configuration's turn distributions. Here, the turn distribution displays the probabilities of winning in certain number of turns by juxtaposing empirical distributions from 10,000 agent-based simulations against the distribution derived from the Markov model. The Markov-derived distribution is generated by probabilistically simulating[1] game progression through the transition matrix for 10,000 iterations, mirroring the simulation approach.

Visual inspection of Figure 4.3 indicates a strong qualitative agreement between the two distributions. Both exhibit a characteristic right-skewed pattern, peaking in the 10-20 turn range and displaying a long tail extending towards higher turn counts. This near

[1]The computations were implemented in Python 3.13.2 (Python Software Foundation, 2025)

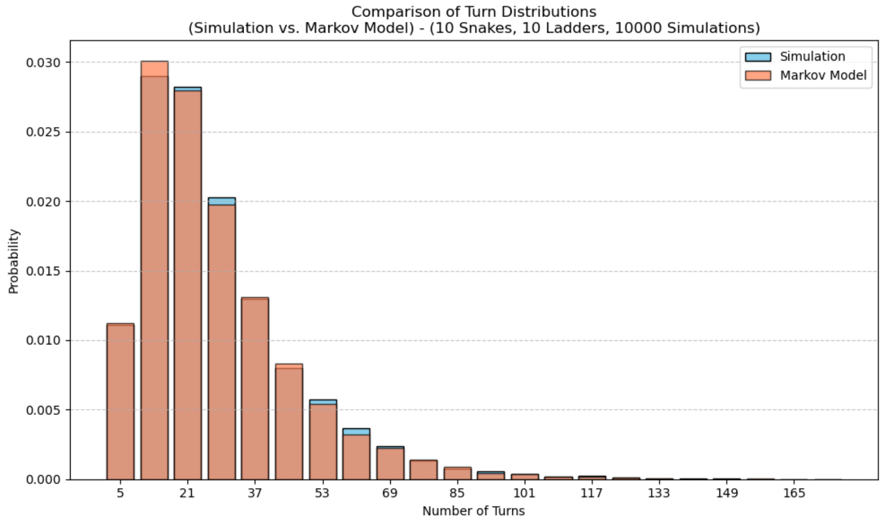



Figure 4.3: **Distribution of Game Turns: Simulation vs. Markov Model (10 Snakes, 10 Ladders, 10,000 Games):** Probability distributions of game turns from agent-based simulations and Markov model-based predictions exhibit a high degree of congruence, demonstrating model validity.

identical mirroring of the turns taken to achieve a victory indicates the efficacy of the model at being able to replicate the results from empirical observations. It shows, that the model is able to accurately capture the game's characteristics and is flexible enough to accommodate changes and effectively see their effects on the metrics such as Average duration of a game (Althoen et al., 1993).

This close mirroring of distributional shapes and central tendencies provides compelling evidence that the Markov model accurately captures the stochasticity and overall dynamics of game progression in Snakes and Ladders. The histogram derived from agent-based simulations closely aligns with the distribution predicted by the Markov model, further solidifying the model's capacity to represent the game's probabilistic nature.

### 4.3.4 Steady-State Distribution and Gameplay Hotspots

A steady-state distribution, is a probability distribution that remains unchanged over time, i.e. the probability of being in each state converges to a fixed value, regardless of the initial state. A Markov chain has a unique steady-state distribution if every state is reachable from every other state and is aperiodic (i.e. it doesn't go through a set of states with only a fixed period of validity) (McGregor, n.d.).

The steady-state distribution, derived analytically from our Markov model, offers insights into the long-term probabilities of tile occupation, representing the equilibrium state of the game over infinite plays. Figure 4.4 visualises this steady-state distribution as a heatmap.

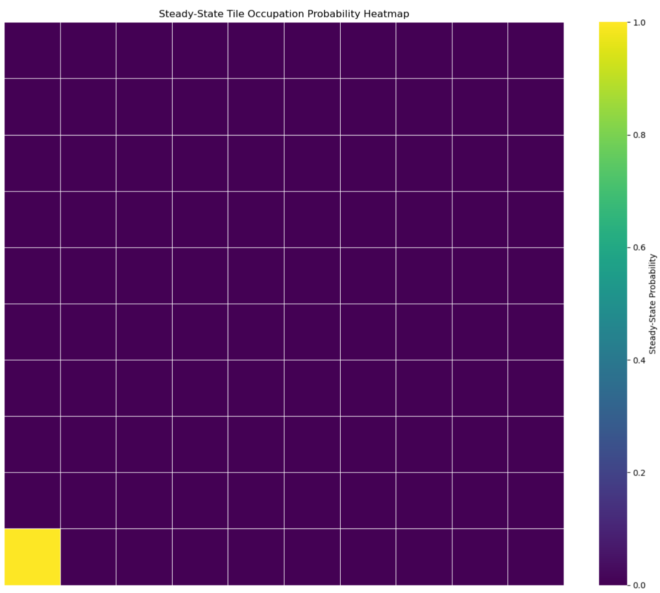



Figure 4.4: **Steady-State Heatmap:** From Markov Model, illustrating long-term equilibrium.

As theoretically expected for a well-formulated absorbing Markov chain, the Steady-State Heatmap (Figure 4.4 demonstrates a probability of 1.0 concentrated on tile 100 (bottom-right corner), with probabilities for all other tiles approaching zero. This analytically confirms that, in the long run, the game predictably terminates at the absorbing winning state.

### 4.3.5 Analytical Efficiency in Game Duration Derivation

A significant advantage of the Markov model is its analytical efficiency in deriving expected game duration. Unlike agent-based simulations, which require computationally intensive repeated trials to approximate average game durations, the Markov model offers a direct, deterministic calculation through fundamental matrix analysis. This analytical solution provides a rigorous and computationally efficient alternative for determining average game durations across diverse board configurations and parameter settings. The demonstrated close agreement between the analytical expected turns and simulation-based average turns validates the Markovian approach as not only accurate but also a computationally advantageous tool for analysing game duration in Snakes and Ladders.

## 4.4 Conclusion: Validation and Utility of the Markov Model

This chapter has successfully constructed a Markov Chain model for Snakes and Ladders. By comparing our findings across the prior chapters, encompassing expected game duration, win probabilities, game turn distributions, and steady-state behaviour, we have observed a clear convergence between the predictions of the Markov model and the empirical findings from agent-based simulations. And finally, in the next chapter, we conclude this dissertation by assessing the collective insights we have gained throughout this dissertation.

# Chapter 5

# Conclusion

This dissertation systematically explores the game of *Snakes and Ladders*, driven by the ambition to look beyond the subjective interpretations of game enjoyment and establishing a more quantifiable understanding of what we define as “mechanical enjoyment”. Our understanding of “mechanical enjoyment” refers to the enjoyment derived by simply the game design and mechanics. By embracing a mixed approach, combining agent-based simulations to build a base and then Markov Chain modelling, we tried to explore the impact of diverse game parameters and design elements on the emergent game dynamics. While the findings mostly agree with our hypotheses, but at times they proved to be counter-intuitive. The interplay between the strategic board design, and inherent probabilistic outcomes was studied, not only within the specific context of *Snakes and Ladders* but also offering broader insights applicable to the wider realm of tabletop games.

Looking at the empirical approach to gaining insight, Chapter 2’s agent-based study investigated the effects of systematically varying the number and lengths of snakes and ladders. This chapter unveiled a crucial bit of information: while mere jumps in the number of snakes and ladders did not linearly correlate with game duration, extreme disparities in their counts significantly affected a game's duration. Specifically, configurations with a disproportionately high number of snakes coupled with fewer ladders were demonstrably prone to producing larger outlier game sessions of extended length, indicative of heightened 'luck dependency' and potential player frustration. Conversely, increasing $N_l$ was shown to act as a counter-balance, tending to lower game durations and diminish variability, thus promoting a more consistent and predictable player experience. Furthermore, our investigation into entity lengths revealed that the relationship between $L_s$ and $L_l$, rather than absolute lengths themselves, acted as a bigger modulator of game duration. Wider disparities, particularly scenarios where snakes were designed to be markedly longer than ladders, consistently translated to shorter game durations, shedding light on the intuitive impact of setbacks outweighing the short-cuts on perceived game "difficulty".

Chapter 3 extended the investigation to examine the impact of scaling the game board, maintaining consistent entity density. Counter-intuitively, and perhaps most strikingly, this section of the research revealed that while larger boards predictably *increase* average game time—a finding consistent with the notion of more tiles to traverse—they simultaneously, and paradoxically, *enhance* the probability of achieving quicker victories within defined turn limits. This effect was attributed to the proportionally greater number of pathways and expanded tile options inherent in larger board configurations. Increased board size, in essence, provides players with a more "diffused" game space, offering a statistically higher likelihood of circumventing clusters of snakes and capitalising on strategically advantageous ladder placements, even within the context of overall prolonged game sessions. Across both parameter variation and board scaling experiments, the ratio of snakes to ladders consistently emerged as a dominant and readily tunable factor, demonstrably influencing both mean game duration and the probability of achieving a quick win, thereby solidifying its crucial role as a primary modulator of overall game difficulty and player-perceived pacing.

Chapter 4 we moved on from agent-based simulations and focused on constructing a Markov Chain model. This model, constructed to capture the transitions by dice rolls and entity interactions, proved effective in analytically predicting several game metrics. Upon comparing our insights from the prior chapters, and, juxtaposing model-derived predictions against empirically observed data from agent-based simulations, we saw a compelling similarity across expected game durations, win probabilities within a certain number of turns, and entire game turn distributions. Beyond just the predictive accuracy, the Markov model distinguished itself through its analytical efficiency, offering a direct and computational method to derive these crucial game metrics.

The "ideal" Snakes and Ladders board layout discussed in this study does not mean reducing the overall duration of games, but a carefully balanced set of several design objectives, which may be in conflict with one another. The $\frac{N_s}{N_l}$ ratio stands out for its simplicity as a good design parameter, in that it enables a good balancing of the challenge and the pacing, whereas the board size appears to be a much more complicated parameter: it impacts the overall length of the game, as well as the subtle changes in the statistical probability of the player's victory speed, which is faster but not that often consistent. In the scope of this dissertation, we propose a clear and measurable way of making such critical decisions regarding the design that will lead to further design attempts toward the more appealing, more balanced, and more commercially successful tabletop game iterations.

It is, however, important to acknowledge the limitations both in the methodology and scope of this research. Firstly, the stochasticity that characterises *Snakes and Ladders* and which is suitably captured by probabilistic models also brings along an element of randomness that bounds the predictability. Even though Markov modelling and simulation analyses generate estimates for average game behaviour across many iterations, every single game session will inherently show some kind of variance because that's what the probability of a die roll does. Secondly, the exploration of board layout design space, while systematic in its parameter variations and board scaling experiments, necessarily remained constrained to a relatively limited subset of all conceivable board configurations. The research primarily focused on changing the entity counts, lengths, and overall board size, but did not exhaustively explore the potentially significant impact of *specific placements* of snakes and ladders, etc.

In conclusion, this dissertation has tried to establish that, there is great value in having a systematic and quantifiable approach to understanding the complex dynamics behind simple games like *Snakes and Ladders*. By effectively combining empirical simulation with the analytical power of Markov Chain modelling, the research not only provides practical and actionable insights for game designers seeking to create more engaging and balanced tabletop experiences but also contributes a methodological framework and a more objective, data-driven foundation for future research within the ever-evolving field of game studies, bridging the gap between game mechanics and player enjoyment within tabletop games. Future research should try to study all kinds of interactions that relative features such as $\frac{N_s}{N_l}$ and $L_i^s - L_i^l$ have with other possible metrics which directly have impacts on enjoyment, such as getting stuck between a section of the board due to the nature of the configuration itself. Apart from just looking at the mechanical enjoyment that this dissertation has already looked at, future research should aim to close the distance between qualitative tools to assess experiential enjoyment, and the notion of mechanical enjoyment.

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