Parameterizing Play: Exploring Game Dynamics in Snakes and Ladders

Jai Bakshi
Symbiosis School for Liberal Arts
Symbiosis International (Deemed University)

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Research Project submitted in partial fulfillment
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by
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Approved By

First Chair:____

Signature:________________________ Date:____________________

Second Chair:____

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- Link: Learn LaTeX in 30 minutes
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Chapter 1

Introduction

From the simple delight of childhood board games to the intricate strategies of modern tabletop experiences, games hold a fundamental appeal for humanity. As Peter Gray (2017) argues, play is not merely frivolous pastime but a powerful vehicle for learning and development, deeply ingrained in our nature. Games, in their essence, are structured systems that invite players to engage in artificial conflicts governed by predefined rules, ultimately leading to quantifiable outcomes (Puentedura, n.d.). This act of play, this engagement within a rule-bound system, is where the potential for enjoyment resides. Understanding and quantifying this 'enjoyment' becomes a complex undertaking. It is influenced by a myriad of factors, ranging from individual preferences and social dynamics to the inherent design and mechanics of the game itself. In the context of game studies and design, it becomes crucial to move beyond subjective impressions and explore methods to systematically analyse and potentially quantify the elements that contribute to a game's enjoyability.

This dissertation focuses on the specific domain of table-top games – a rich and tangible space for exploring game mechanics and player interaction. Tabletop games, encompassing board games, card games, and dice games, offer a unique lens through which to examine the relationship between game design and player experience. Their tangible nature, the direct manipulation of components, and the face-to-face social interaction create a particularly fertile ground for investigating the sources of game enjoyment. Within this exploration, one must consider the philosophical underpinnings of what constitutes a game. Bernard Suits, in his seminal work The Grasshopper: Games, Life and Utopia (1978), provides one such valuable framework. Suits introduces the concept of the "lusory attitude", the willing acceptance of constitutive rules to engage in activity aimed at achieving a specific state of affairs (the lusory goal), where such rules prohibit the most efficient means of achieving that state. This “lusory attitude” is central to understanding games as distinct from ordinary life, operating within what Johan Huizinga (1938) termed the “magic circle”—a bounded space where different rules and expectations apply. By embracing this perspective, one can begin to dissect the intricate relationship between game mechanics, player engagement, and the elusive quality of enjoyability. This research, will distinguish between two key aspects of game enjoyment: mechanical enjoyability – the enjoyment derived from the inherent design and mechanics of the game system itself, and experiential enjoyability – the fluctuating enjoyment stemming from player decisions, social interactions, and the unfolding narrative of a particular game session.

1.1 Moving Beyond Subjectivity

While the allure of games is universally acknowledged, the nature of enjoyment itself remains inherently subjective. As Nicole Lazzaro (2004) emphasizes in her examination of “Four Keys to More Emotion Without Story,” factors like social interaction (“People Factor”) and individual player preferences significantly shape the overall gaming experience. Consequently, assessing game enjoyability is not as straightforward as evaluating objective features. Existing game review systems, as analysed by Yang and Mei (2010), often grapple with this subjectivity. These systems, while providing valuable consumer feedback, are inherently limited by their reliance on subjective opinions and the tendency to focus on “search attributes” – features readily apparent before playing – rather than “experiential attributes” – those felt only through gameplay. Furthermore, Yang and Mei’s research reveals that negative reviews can disproportionately influence perceptions, and the network effect, where shared experiences amplify enjoyment (or dissatisfaction), further complicates objective assessment.

This inherent subjectivity does not negate the need for a more systematic and potentially quantifiable approach to understanding game enjoyment. Indeed, to advance game design and analysis, one must strive to bridge the gap between subjective experience and objective analysis. This research affirms that while experiential enjoyability remains inherently variable, mechanical enjoyability, rooted in the game’s core mechanics, can be approached as a more quantifiable construct. By focusing on the design elements and rule systems that structure gameplay, this project aims to develop methods for objectively assessing and potentially predicting the level of enjoyment a game's mechanics might elicit.

1.2 Snakes and Ladders: A timeless classic

To ground our exploration of game enjoyability in a tangible example, this dissertation will reference the game of Snakes and Ladders. As Marcus du Sautoy (2023) illuminates by exploring the mathematical underpinnings of games, Snakes and Ladders is far from being a trivial childhood pastime; it boasts a rich history and a remarkable universality that makes it an ideal case study for understanding fundamental game mechanics and player engagement. Its origins can be traced back to ancient India, where it was known as *Moksha Patam* or *Gyan Chaupar*. Reflecting insights often found in Sautoy's discussions on the history of games and the mathematics surrounding them, this game, believed to have emerged as early as the 2nd century BC, was not merely entertainment but served as a moral and didactic tool. The ladders represented virtues like generosity, faith, and humility, while the snakes symbolized vices such as lust, anger, theft, and pride. The ascent and descent on the board mirrored the karmic cycle of life, illustrating the consequences of good and bad actions in a visually compelling and accessible way.

Over centuries, *Moksha Patam* travelled beyond India, evolving and adapting as it spread across cultures, a journey that resonates with Sautoy's (2023) broader narrative of how ideas and concepts traverse geographical and cultural boundaries. By the late 19th century, a Westernised version, "Snakes and Ladders," emerged in England and quickly gained popularity worldwide. While the overt moralistic undertones diminished in its global iteration, the core mechanics of chance, progression, setbacks, and the simple pursuit of a defined goal remained intact. This appeal across diverse cultures and time periods, echoing themes of in mathematical and historical contexts, underscores the game's ability to tap into fundamental aspects of human engagement and enjoyment.

Snakes and Ladders, in its simplicity, offers a microcosm of the broader challenges in quantifying game enjoyability. Its mechanics are easily grasped – the roll of a die dictates movement, and predetermined snakes and ladders introduce elements of both fortune and misfortune. Yet, even within this seemingly straightforward system, players experience a range of emotions: anticipation with each dice roll, frustration upon encountering a snake, elation when climbing a ladder, and the ultimate satisfaction of reaching the final square. The game's accessibility and widespread familiarity make it an excellent lens through which to examine how even basic game mechanics, governed by chance and simple rules, can generate engaging and emotionally resonant player experiences. By analyzing Snakes and Ladders through the framework of mechanical and experiential enjoyability, one can begin to isolate and understand the core design elements that contribute to the enduring appeal of tabletop games, and potentially, games more broadly.

Chapter 2

The Dynamics of Snakes and Ladders

The objectivity of a set of rules provides a strong foundation to set up the notion of mechanical enjoyability when it comes to various kinds of systems, especially those like table-top games. Within the clearly defined structure of a game's rules, we can begin to analyze and potentially quantify the sources of enjoyment that arise purely from the system's design, independent of individual player preferences or social contexts. This concept resonates deeply with the notion of the "Magic Circle" (Huazinga, 1938), which describes games as existing within a bounded space governed by self-contained rules and conventions. It is within this "circle" of rules that mechanical enjoyability takes shape – an inherent quality of the game system itself, derived from its internal logic and the interactions it engenders through its mechanics.

This chapter delves into the mechanics of the classic game *Snakes and Ladders*, aiming to quantify the impact of various game parameters on the overall game dynamics. This is achieved by simulating numerous games while systematically varying parameters. In this chapter, we will investigate two key aspects: firstly, the impact of the *number* of snakes and ladders on the board, and secondly, the effects of varying the *lengths* of these entities. To simplify the analysis and isolate the effects of these parameters, the model reduces the game to its essential elements. This allows us to systematically examine how changes in these parameters affect the distribution of game duration – specifically, the number of moves needed to reach the end state. This distinction in parameter variability helps separate the core game mechanics from the broader gameplay experience.

The average game time is chosen as a primary metric for quantifying these mechanical aspects. Game time, defined as the number of moves required to reach the end state, serves as a readily measurable and intuitively understandable indicator of game dynamics. It directly reflects the efficiency and predictability of the game system in guiding a player towards its objective. For a game like Snakes and Ladders, where the goal is simply to reach the final tile, the number of turns taken to achieve this outcome becomes a crucial measure of the game's mechanical properties. Variations in average game time, as we will explore, can reveal how different configurations of snakes and ladders, governed by the game's rules, alter the overall pace and challenge of the experience.

2.1 Setting up the board

In our pilot study, the game board is modelled as a 10x10 grid, comprising 100 squares, akin to the classic Snakes and Ladders game. The player, represented by an Agent in our model, starts at tile 1, with no requirement to roll a specific number to begin (i.e., no starting condition). The goal state is to reach or exceed tile 100. The Agent's movement is determined by a fair six-sided die roll.

To facilitate a systematic investigation of game dynamics, this research introduces several controllable parameters that define the entities on the board:

1. **Board Size** ($BoardSize$): The maximum size of the board in terms of the number of tiles. The board is of the form $m * m$ and there are a total of m^2 tiles on the board.

2. **Number of Snakes** (N_s): The total number of snakes on the board.

3. **Number of Ladders** (N_l): The total number of ladders on the board.

4. **Length of Snakes** (L_s^i): This parameter determines the length of the i^{th} snake on the board for $i = 1, 2, ...N_s$. It dictates the number of tiles the agent is set back when landing on a snake's head.

5. **Length of Ladders** (L_l^i): This parameter determines the length of i^{th} ladder on the board for $i = 1, 2, ...N_l$. It dictates the number of tiles the agent climbs when encountering a ladder's base.

6. **Ladder Position** ($Ladder^i_{base/top}$): The position of the i^{th} ladder's terminal ends.

7. **Snake Position** ($Snake^i_{head/tail}$): The position of the i^{th} snake's terminal ends.

To ensure the board configuration remains valid and avoids conflicts—such as positioning snakes or ladders at invalid tiles where they might extend beyond the board's boundaries—certain constraints are implemented:

1. **Ladder Constraint:** Ladders cannot begin within the L_l^i tiles of the board to prevent them from extending beyond the game's end. The ladder's starting position therefore becomes:

$$Ladder_{start}^i \leq BoardSize - L_l$$

2. **Snake Constraint:** Snakes cannot begin within the first L_s^i tiles to avoid their tails going below the starting position. The snake's end therefore becomes:

$$Snake_{start}^i \geq 1 + L_s$$

3. **Overlap Constraint:** To maintain game integrity, no terminal ends of a snake or ladder (start or end) can overlap with any part of another snake or ladder. The paths of snakes and ladders can coincide at various points so long as they don't have overlaps at the ends of the entities. If an overlap occurs, the simulation set-up randomly decides whether to remove the overlapping snake or ladder based on a probability of 0.5.

The board generation process involves randomly selecting starting positions for snakes and ladders within these permissible ranges. This is followed by a validation step to resolve any overlaps. This iterative process continues until a valid board configuration is achieved. The number of iterations required to generate a valid board is recorded and can be analysed to understand the complexity of board creation under different parameter settings.

This structured approach to board generation allows us to systematically vary the parameters and study their individual and combined effects on the game dynamics. By analysing the resulting distributions of game durations, this research aims to uncover trends and patterns that reveal the interplay of these factors in shaping the player's experience.

2.2 Approaches to Assign Entity Parameters

To comprehensively explore the dynamics of *Snakes and Ladders*, this research employ different approaches for assigning the key parameters: the number of entities and, in particular, the lengths of snakes and ladders.

2.2.1 Varying the Number of Snakes and Ladders

During the preliminary exploration, the primary focus is on how the *number* of snakes (N_S) and ladders (N_L) affects the average game time, while keeping the lengths of these entities consistent across simulations. Using simulated data, the project explores the relationship between different counts of snakes and ladders, maintaining their lengths as variables influenced by board constraints but not systematically varied in this phase. The game is simulated 1000 times for each of the 10 distinct board configurations, varying the number of snakes and ladders independently of each other to observe their isolated and combined impacts.

2.2.2 Varying the Lengths of Snakes and Ladders

To investigate the impact of entity lengths, three distinct approaches for assigning lengths (L_i^S and L_i^L) on the game board are deployed. Each approach allows for unique characteristics of the board configuration to facilitate a comparative analysis of game time under varying assumptions:

1. **Fixed Unequal Lengths:** This deterministic approach assigns fixed but unequal lengths to all snakes and ladders. For instance, one might set $L_1^S = 10$, $L_1^L = 5$, $L_2^S = 20$, $L_2^L = 10$ and so on, ensuring that $L_i^S \neq L_i^L$ for all $i \in [1, N]$, while maintaining consistency in the lengths assigned to entities of the same type across different simulations within this approach. This method provides a baseline for analyzing gameplay outcomes under deterministic length conditions and ensures uniformity across experiments.

2. **Sampling from Distributions:** To introduce variability in lengths, this approach employs three distinct probability distributions—uniform, normal, and exponential—to sample lengths for each snake and ladder (L_i^S & L_i^L $\forall i \in [1, N]$).

   (a) **Uniform distribution:** All valid lengths between 1 and L_max are equally likely, this ensures an unbiased selection across the entire range of lengths, providing a uniform probability for shorter and longer lengths.

   $$P(L = x) = \frac{1}{L_{max}}, \forall x \in \{1, 2, \dots, L_{max}\}$$

   (b) **Normal distribution:** A Normal/Gaussian distribution is characterized by a mean μ and a standard deviation σ, for the lengths of snakes and ladders:

- μ is set to $\frac{L_{max}}{2}$, placing the most likely lengths near the midpoint of the range
- σ is set to $\frac{L_{max}}{6}$, which suggests that most lengths fall within the range $[\mu - 3\sigma, \mu + 3\sigma]$

$$P(L = x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{(x-\mu)^2}{2\sigma^2}}, \forall x \in [1, L_{max}]$$

(c) **Exponential distribution:** This emphasizes on the shorter lengths, with the probability of longer lengths decreasing exponentially. The scaling parameter λ is set to $\frac{L_{max}}{3}$ ensuring a reasonable spread of values.

$$P(L = x) = \lambda e^{-\lambda x}, \forall x \in [1, L_{max}]$$

3. **Fixed Start and End Points:** This approach diverges from directly controlling the L_s and L_l. Instead, it involves assigning randomized $Ladders^i_{base/top}$ and $Snakes^i_{head/tail}$. This approach to the problem introduces another layer of variability by purely focusing on their placement rather than predetermined or sampled lengths. For each snake, the $Snake^i_{head}$ is chosen from the range $[2, BoardSize - 1]$ abiding by the snake constraint. While, the $Snake^i_{tail}$ is determined by randomly selecting tile below its starting position, i.e.

$$1 \leq Snake^i_{tail} < Snake^i_{head}$$

For ladders, the $Ladder^i_{base}$ is chosen randomly from $[2, BoardSize - 1]$ keeping the ladder constraint in check, whilst its $Ladder^i_{top}$ is assigned randomly above its starting position, i.e.

$$Ladder^i_{top} > Ladder^i_{base}$$

By decoupling length from predetermined distributions, the method accommodates a wider variety of configurations, making it suitable for exploring edge cases in gameplay. The method allows for a high degree of randomness in gameplay and will be used to test the robustness of the study, offering insights into how random placement and implicit lengths impact game duration, difficulty, and variability.

2.3 Presenting Findings: Impact of Entity Number and Length on Game Time

The simulations represent the inherently probabilistic nature of *Snakes and Ladders*, where outcomes are largely determined by the interplay between board design, entity parameters, and the randomness of dice rolls. The analysis of simulation results is presented through distinct visualizations: box plots, trend lines, and heatmaps, offering insights into distribution, trends, and overall patterns, respectively.

2.3.1 Distribution of Average Game Times: Varying Number of Entities

The box plot (Fig. 2.1) illustrates the distribution of average game times across various combinations of N_S and N_L. Game configurations with extreme differences in the number of snakes and ladders (e.g., $N_S = 10$ and $N_L = 5$) exhibit the highest variability in average game time, observable through wider interquartile ranges and more frequent outliers. A key observation is that configurations with fewer ladders tend to result in longer game times. For instance, configurations with 10 snakes and 5 ladders frequently show average game times above 33 moves, with some extreme outliers nearing 37 moves. Conversely, average game time tends to decrease as the number of ladders increases. This is intuitive, as ladders facilitate faster progress towards the goal state, reducing the number of turns required to complete the game. Lower average game times, especially in configurations with more ladders, often show tighter distributions with fewer outliers, suggesting a more consistent game time. The plot concludes that outliers are more frequent in configurations with more snakes and fewer ladders. These outliers could represent scenarios where luck significantly impacts the game, either by allowing a player to avoid major snakes (resulting in an unusually short game) or by repeatedly encountering snakes (leading to exceptionally long games).

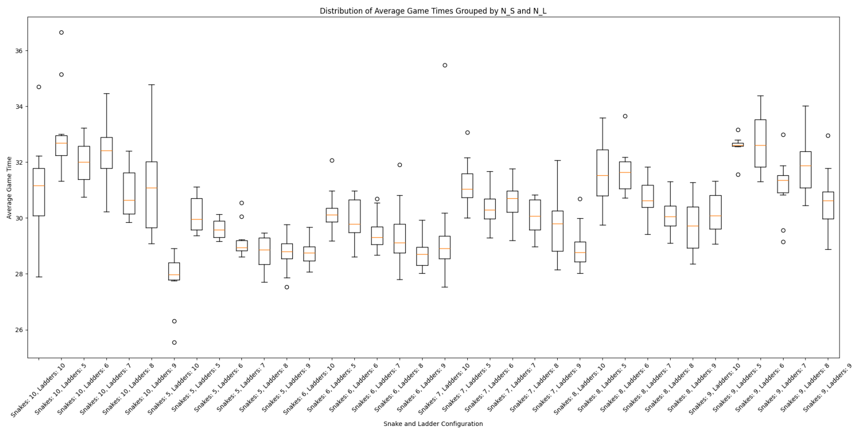


Figure 2.1: **Distribution of Average Game Times Grouped by N_S and N_L:** Box plot shows higher variability in average game time for extreme N_S/N_L differences. Fewer ladders correlate with longer game times; outliers indicate luck-dependent game lengths.

2.3.2 Interaction Between Number of Snakes and Ladders

The heatmap (Fig. 2.2) provides a visual overview of the interaction between N_S and N_L on average game time. Colors range from dark shades (indicating shorter game times) to bright shades (indicating longer game times). A clear pattern emerges: as the number of ladders increases, the average game time decreases. This trend is most pronounced for higher snake counts ($N_S \geq 9$), where additional ladders significantly reduce game times. The bottom-right corner of the heatmap ($N_S \geq 9, N_L = 10$) shows the shortest game times, reinforcing the idea that ladders effectively mitigate the delays caused by snakes.

The effect of snakes on game time, however, is not strictly linear. For example, increasing the number of snakes from 5 to 6 does not dramatically alter the game time. Yet, when the number of snakes is increased to 9 or 10, game times rise substantially. This suggests a threshold beyond which additional snakes significantly increase the likelihood of players encountering them, thus extending game time considerably. This indicates that while snakes introduce challenges, their negative impact on game duration can be mitigated by providing players with ample ladders to climb back up.

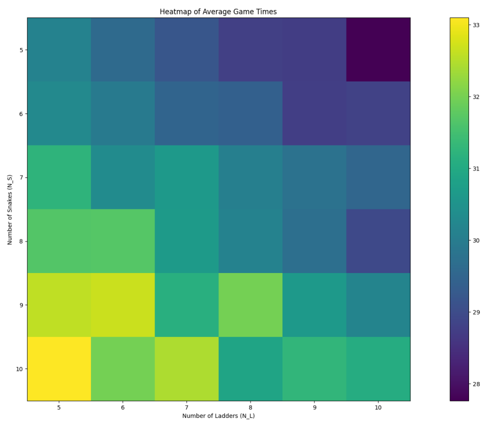


Figure 2.2: **Heatmap of Average Game Times:** Heatmap shows decreasing average game time with increasing N_L, especially at higher N_S. N_S effect is non-linear; game time significantly increases only beyond a certain N_S threshold.

2.3.3 Controlled Approach: Unequal Snake and Ladder Lengths

This section presents findings from simulations using fixed, unequal lengths for snakes and ladders across 10 different board configurations, with 1000 simulations per configuration. L^S and L^L were systematically varied in pairs, ensuring L^S and L^L were consistently unequal within each pair type. The bar plot (Fig. 2.3) illustrates the average game times for various length pairs. It is evident that average game time generally increases as the difference between snake length and ladder length ($L^S - L^L$) widens. This suggests that when snakes are significantly longer than ladders, players experience more setbacks, contributing to longer average game times. Conversely, no significant impact on game time is observed across pairs where ladder lengths exceed snake lengths ($L^L > L^S$).

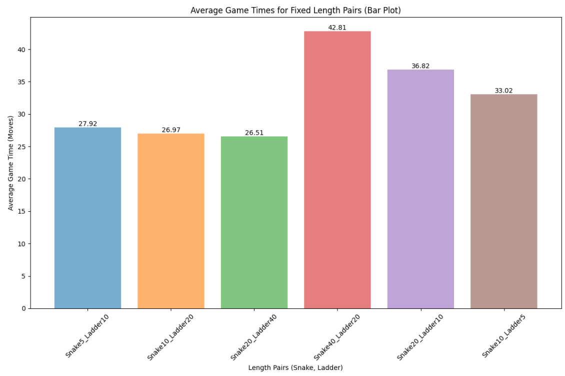
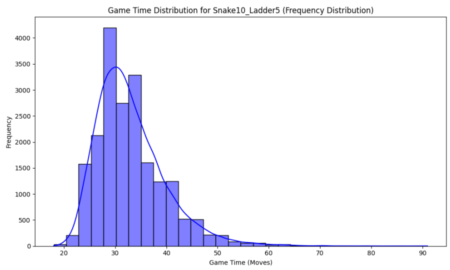


Figure 2.3: **Average Game Times for Fixed Unequal Lengths:**Bar plot shows game time increases with widening snake-ladder length difference ($L_S - L_L$), particularly when snakes are longer. $L_L > L_S$pairs show minimal game time impact.

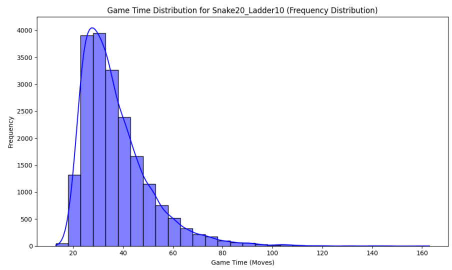
The frequency distribution plot (Fig. 2.4 (c)) provides a detailed view of game time distribution for the configuration with the most extreme length disparity ($L^S = 40$ & $L^L = 20$). The distribution is notably right-skewed, indicating that while most games conclude within a moderate number of moves, occasional runs extend significantly longer. This skew is likely attributable to the inherent randomness of dice rolls and the frequency of agent-snake encounters, even with relatively long ladders present. Comparing configurations with $L^L > L^S$ reveals that those with smaller differences ($L^L - L^S < 20$) exhibit tighter distributions resembling a normal distribution with few outliers. The average game times in these configurations also cluster more closely together, unlike the pronounced spikes observed in configurations with larger length disparities.

2.3.4 Using Sampling Distributions for Lengths

This section investigates the effects of different sampling distributions on snake and ladder lengths (L^S and L^L). We explore three statistical distributions—uniform, normal, and exponential—each with N_S, $N_L = 10$, from which L^S and L^L are sampled. For each distribution, 1000 games were simulated across 10 different boards. Figure 2.5 shows the aggregated average game times across these distributions. The highest average game time results from the exponential



(a)



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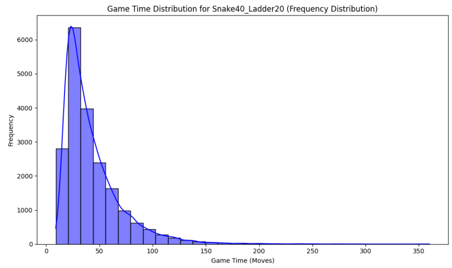


(c)

Figure 2.4: **Game Time Distributions for Configurations with $L_S > L_L$:**Histograms of these configurations show right-skewness, indicating occasional longer games. Skew is attributed to dice roll randomness and snake encounters, despite ladders.

sampling method, followed by the normal distribution, with the uniform distribution yielding the lowest average game time. Exponential sampling tends to produce more shorter lengths, with a lower probability of longer entities, suggesting that a higher density of smaller snakes and ladders extends game duration. In contrast, normal distribution, which clusters lengths around a midpoint, results in more stable configurations with lower average game times.

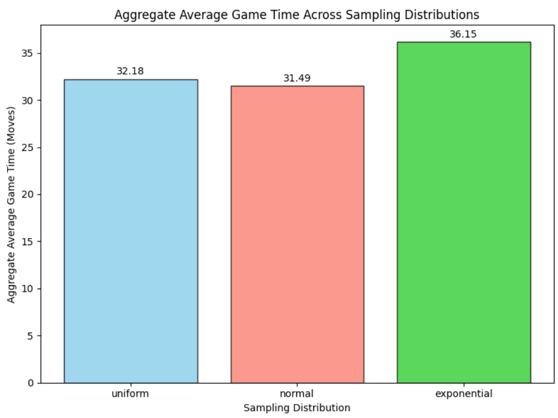
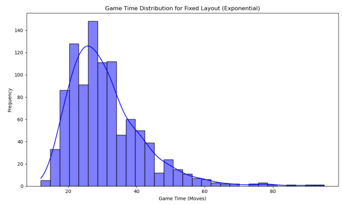


Figure 2.5: **Aggregated Averages of Game Time across the sampling distributions:** Bar plot compares average game times for uniform, normal, and exponential length distributions. Exponential distribution yields highest, normal lowest average game time, suggesting shorter lengths extend game duration.

Frequency distributions of game times for a fixed board layout under each sampling method (Fig. 2.6) reveal right-skewness across all three distributions, indicating that while most games finish within a moderate set of moves, outliers leading to longer games are possible. Exponential sampling exhibits the highest variability, possibly due to the prevalence of smaller entities and their random placement. Figure 2.7 (b) indicates that boards generated using exponential sampling consistently result in higher average game times compared to other methods. Figure 2.7 (a) shows that uniform distribution results in more consistently lower average game times across different board layouts, albeit with some boards exhibiting higher averages. Normal distribution displays the most variability across the boards played but with consistently low average game times and fewer outliers pushing game times upward.



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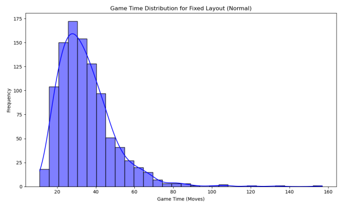

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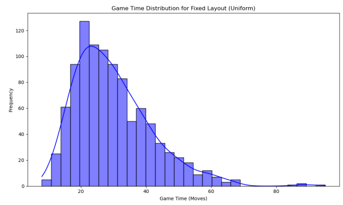

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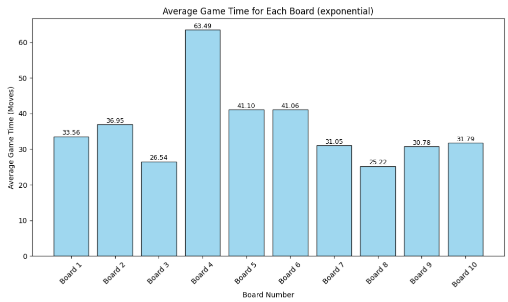
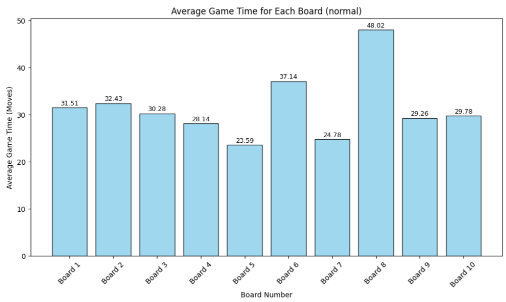
Figure 2.6: **Game Time Distributions for a Fixed Layout, by Sampling Method:** Histograms for a fixed layout show right-skewness across all sampling methods. Exponential sampling exhibits highest variability, possibly due to smaller entities and placement.



(a)



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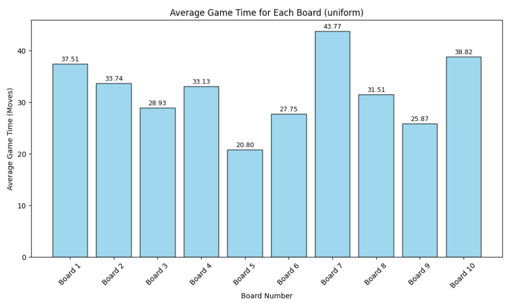
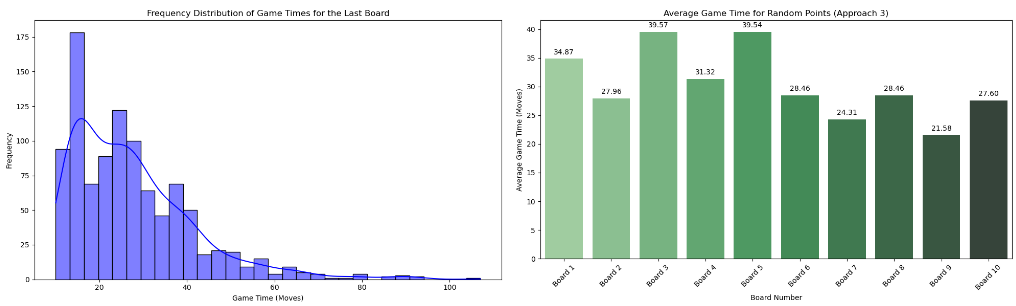


(c)

Figure 2.7: **Average Game Time for Each Board, by Sampling Distribution:** The plots show average game time for 10 boards, by sampling distribution. Uniform distribution yields consistently lower averages. Normal distribution shows highest board variability, but low average times. Exponential sampling generally results in higher average times.

2.3.5 Randomly Generated Boards: Fixed Start and End Points

This section examines the effect of randomly generated snakes and ladders based on fixed start and end positions. Positions are generated while adhering to length constraints. This approach introduces greater variability compared to methods where lengths are predetermined. Figure 2.8 (a) illustrates average game times across 10 randomly generated boards. Each bar represents the average game time for a specific board, revealing significant variability in average game times across different board layouts, ranging from approximately 22 to 40 moves. Figure 2.8 (b) displays the frequency distribution of game times across all simulated games, indicating similar variability, as evidenced by random spikes in certain sections and the kernel density line. Certain boards, due to their specific configurations, present varying challenges and opportunities for the agent, leading to a wide range of game durations.



(a) (b)

Figure 2.8: **Analysis of Randomly Generated Boards:** (a) Game time distribution for a random board shows variability within a board. (b) Average game time across 10 random boards varies significantly (22-40 moves), highlighting random placement impact.

2.4 Analysis

This chapter explores the impacts of both entity number and length (L^S and L^L) on game dynamics in *Snakes and Ladders*. By simulating games under systematically varied parameters across different approaches—static length assignment, statistical length sampling, and randomized start/end points, revealing several key insights.

Firstly, variations in both the *number* and *lengths* of snakes and ladders significantly influence game duration and its variability. Increased numbers of snakes generally prolong games, while more ladders tend to shorten them. However, the relationship is not always linear, with thresholds and interactions between entity types playing a crucial role. For example, while simply increasing snake count does not always linearly increase game time, exceeding a certain density of snakes dramatically extends game duration. Conversely, the positive impact of ladders is more pronounced in mitigating the negative effects of high snake counts.

Secondly, the method of assigning entity lengths introduces another layer of complexity. Deterministic length assignments provide a baseline for understanding game mechanics, while statistical sampling reveals how different length distributions affect game variability and average duration. Exponential distributions, favoring shorter lengths, tend to increase game time and variability, whereas normal distributions offer a more balanced outcome. Uniform distributions, in contrast, result in shorter, more predictable games. Randomly generated boards based on fixed start and end points introduce the highest degree of variability, highlighting the significant impact of entity placement on overall game dynamics.

Thirdly, the presence of outliers and fluctuating trend lines across different board layouts underscores the significant influence of board layout itself. Strategic placement of snakes and ladders can create "traps" or "shortcuts," leading to substantial variations in game duration even within the same parameter configurations.

These findings have valuable implications for game design and balancing. They demonstrate how adjusting the number and lengths of snakes and ladders can fine-tune game difficulty and duration. The choice of length assignment method—deterministic, sampled, or position-based—further allows designers to control the level of variability and unpredictability in gameplay. For games with mechanics similar to *Snakes and Ladders*, these insights can inform the creation of engaging experiences with carefully calibrated challenge and playtime.

Additionally, this chapter's exploration of entity number and length, while illuminating, represents just one facet of game design parameterisation. The observed sensitivity of game dynamics to these entity-level adjustments naturally prompts further investigation into other fundamental game design elements. In the subsequent chapter, we pivot our focus to another core parameter: the *scale* of the game board itself. By systematically varying board dimensions, while holding entity characteristics constant, we aim to unravel how board size, as a determinant of game space and traversal distance, independently shapes game hardness, duration, and overall player experience. This shift in focus, from entity-level parameters to board-level dimensions, represents the next logical step in our systematic parameterisation of Snakes and Ladders gameplay, allowing for a more holistic understanding of the game's design space.

Chapter 3

Scaling the Game Board: Impact on Game Hardness and Duration

3.1 Introduction: Board Size as a Determinant of Game Hardness

In the preceding chapter, the influence of snake and ladder *lengths* and the *number* of these entities on the board was systematically examined. It was observed how varying entity lengths, through different distributional approaches, alters the average game time and the overall shape of the game's turn distribution. Having established the sensitivity of game dynamics to entity lengths, this chapter now turns its attention to another fundamental parameter: the *size* of the game board itself.

This chapter investigates how scaling the dimensions of the game board, while maintaining a consistent density of snakes and ladders and keeping their individual lengths fixed, impacts key gameplay metrics. The primary focus was on understanding how board size influences the *hardness* of the game. In this context, game hardness is operationally defined through two readily quantifiable measures: **average game time**, representing the typical duration of a play session, and **probability of winning within a specified number of turns**, reflecting the likelihood of achieving a relatively quick victory. These metrics provide complementary perspectives on game challenge and player experience, with average game time indicating the overall time investment required and win probability offering insight into the game's pace and potential for swift success.

By systematically varying the board size and analysing the resulting changes in average game time and win probabilities, this chapter aims to elucidate how board dimensions, in conjunction with fixed entity characteristics, shape the mechanical properties and, by extension, the mechanical enjoyability of Snakes and Ladders. The hypothesis is that increasing board size, even with constant entity density and lengths, will lead to longer average game times, reflecting the greater distance to traverse to reach the goal.

3.2 Methodology: Simulation Setup for Board Size Scaling

To investigate the impact of board size, a simulation experiment was designed, varying the linear dimension, n, of a square Snakes and Ladders board, resulting in board sizes $N = n^2$. The following board sizes were systematically explored: 8×8 (64 tiles), 10×10 (100 tiles), 12×12 (144 tiles), 14×14 (196 tiles), 16×16 (256 tiles), 18×18 (324 tiles), and 20×20 (400 tiles). For each board size, a constant density of snakes and ladders was maintained, set at 0.1 entities per tile, meaning the number of snakes and the number of ladders were both calculated as $0.1 \times N$. Crucially, the individual lengths of all snakes and ladders were also kept *fixed* at 10 tiles, irrespective of board size, to isolate the effect of board dimensions. For each board size configuration, 10,000 game simulations were conducted.

1. **Average Game Time (Simulation):** The mean number of turns taken across 10,000 simulated games for each board size, providing a measure of typical game duration.
2. **Probability of Winning within N/2 Turns:** Calculated as the proportion of games completed within $N/2$ turns, where N is the board size, representing the probability of a relatively quick win (within half the number of tiles).
3. **Probability of Winning within N/3 Turns:** Calculated as the proportion of games completed within $N/4$ turns, representing the probability of a swift win (within a third of the number of tiles).
4. **Probability of Winning within N/4 Turns:** Calculated as the proportion of games completed within $N/4$ turns, representing the probability of a very swift win (within a quarter of the number of tiles).

To further explore the interplay between board size and the relative number of snakes and ladders, simulations were conducted across the aforementioned board sizes for varying ratios of snakes to ladders (N_S/N_L). Two sets of simulations were performed:

1. **Fixed Snake Density, Varying Ladder Denexperiencesity:** The number of snakes (N_S) was fixed at $0.1 \times N$ for each board

size N. The number of ladders (N_L) was then varied to achieve N_S/N_L ratios of 0.5, 1.0, 1.5, and 2.0.

2. **Fixed Ladder Density, Varying Snake Density:** The number of ladders (N_L) was fixed at $0.1 \times N$ for each board size N. The number of snakes (N_S) was varied to achieve N_S/N_L ratios of 0.5, 1.0, 1.5, and 2.0.

These simulations, systematically varying board size and Ns/Nl ratio, aim to provide a comprehensive understanding of how these parameters influence game dynamics, hardness, and duration in Snakes and Ladders.

3.3 Findings: Impact of Board Size on Game Hardness and Duration

This section presents a comprehensive analysis of the simulation findings, focusing on how board size and the ratio of snakes to ladders (N_S/N_L) impact key indicators of game hardness and duration: the probability of winning within a specified number of turns, and the average game time. This analysis aims to provide a nuanced understanding of how board size scaling and entity balance shape the perceived player experience in Snakes and Ladders.

3.3.1 Win Probability vs. Board Size for Varying Ns/Nl Ratios

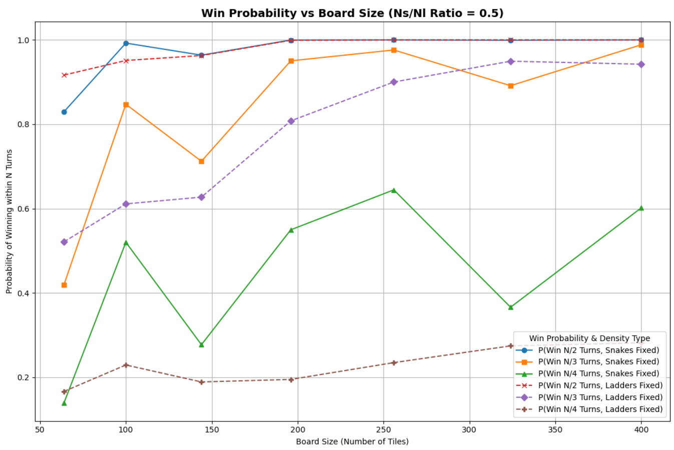


Figure 3.1: Win Probability vs Board Size for Ns/Nl Ratio = 0.5

Figures 3.1, 3.2, 3.3, and 3.4 visually represent the intricate relationship between board size and win probability across different N_S/N_L ratios (0.5, 1.0, 1.5, and 2.0, respectively). Each figure distinctly portrays data derived from both fixed snake density and fixed ladder den-

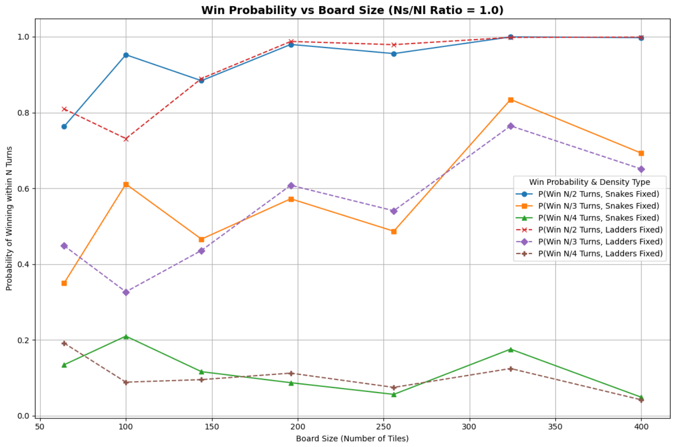


Figure 3.2: Win Probability vs Board Size for Ns/Nl Ratio = 1.0

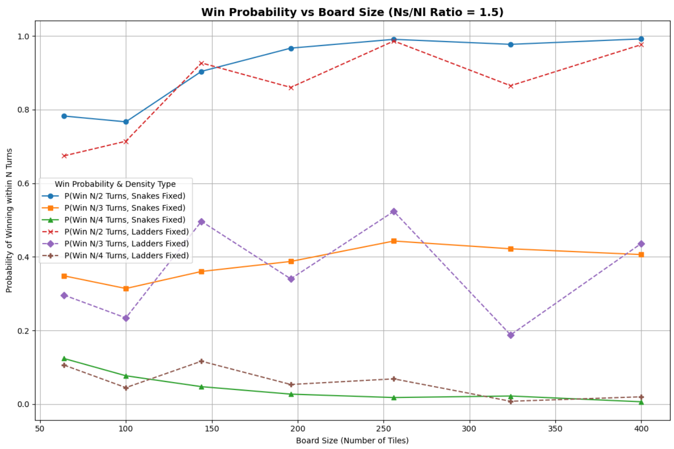


Figure 3.3: Win Probability vs Board Size for Ns/Nl Ratio = 1.5

sity simulations, facilitating a comparative examination of how these density configurations modulate the observed gameplay dynamics.

Overall Trend: Increasing Win Probability with Board Size A consistent trend, robust across all N_S/N_L ratios and density configurations, is the tendency for the probability of achieving a win within a limited number of turns (specifically, within N/2 and N/4 turns) to exhibit an *increase as the board size expands*. This trend robustly confirms the initial hypothesis positing that larger game boards, despite inherently leading to extended average game durations, may counter intuitively enhance the likelihood of a player securing a swift victory. For instance, examining Figure 3.2 for a balanced Ns/Nl ratio of 1.0, the probability of winning within N/4 turns increases from approximately 10% on an 8x8 board to over 25% on a 20x20 board in the fixed snake density configuration. This seemingly paradoxical effect can be intuitively explained by the proportionally greater number of pathways and tile options available on larger boards. The increased tile count provides players with more avenues to circumvent snake

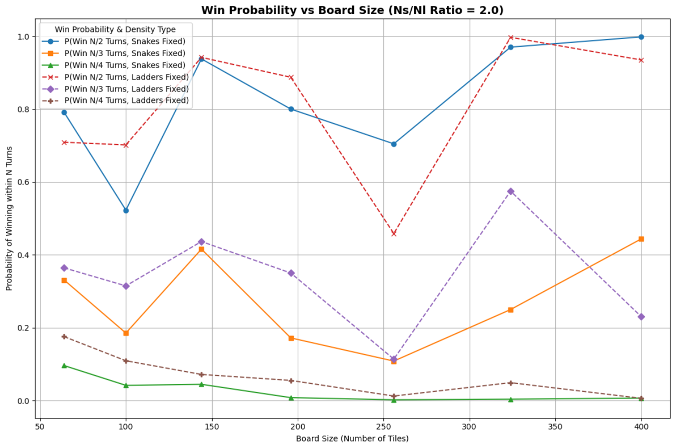


Figure 3.4: Win Probability vs Board Size for Ns/Nl Ratio = 2.0

encounters and capitalise on ladder climbs, thereby statistically improving the chances of a quicker, luck-favoured game resolution, even when the density of entities remains constant.

Impact of Ns/Nl Ratio on Win Probability: The N_S/N_L ratio emerges as a significant modulator, substantially influencing the baseline win probabilities and the scaling relationship between board size and win likelihood, as detailed in the preceding subsections.

- **Low Ns/Nl Ratio (0.5) - High Baseline Win Probability:** Figure 3.1, representing a low N_S/N_L ratio indicative of ladder abundance relative to snakes, demonstrates consistently elevated win probabilities across the spectrum of board sizes. Notably, the incremental increase in win probability associated with board size expansion is less pronounced in this configuration.
- **Balanced Ns/Nl Ratio (1.0) - Moderate and Scaling Probabilities:** In contrast, Figure 3.2, depicting a balanced configuration with an equal number of snakes and ladders, portrays a more graduated and discernible increase in win probability as board size scales.
- **Elevated Ns/Nl Ratios (1.5 and 2.0) - Reduced and Fluctuating Probabilities:** Figures 3.3 and 3.4, characterising higher N_S/N_L ratios where snakes outnumber ladders, illustrate a departure from strictly linear scaling patterns and reveal suppressed win probabilities, particularly for swift victories.
- **Density Configuration - Minor Influence:** Comparative analysis within each figure, contrasting fixed snake density and fixed ladder density lines, reveals that the specific choice between fixing snake or ladder density exerts a comparatively subordinate influ-

ence on win probability distributions, with the N_S/N_L ratio being the dominant factor.

3.3.2 Average Game Time vs. Board Size for Varying Ns/Nl Ratios

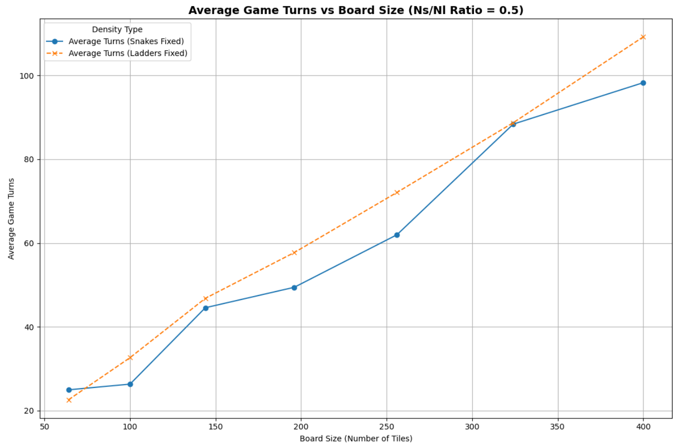


Figure 3.5: Average Game Turns vs Board Size for Ns/Nl Ratio = 0.5

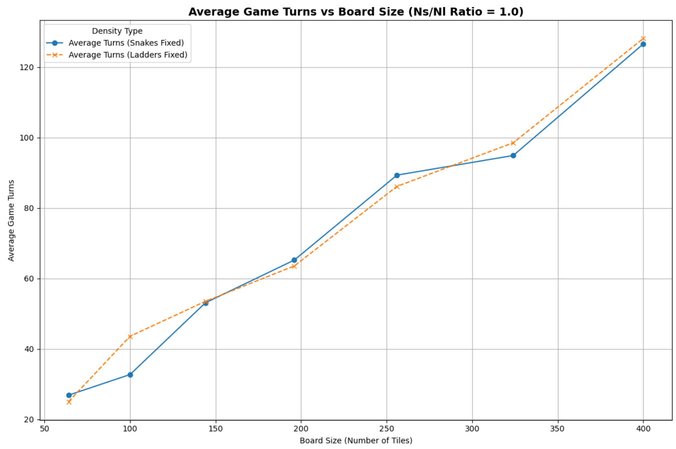


Figure 3.6: Average Game Turns vs Board Size for Ns/Nl Ratio = 1.0

Complementing the win probability analysis, Figures 3.5, 3.6, 3.7, and 3.8 depict the relationship between board size and average game time for the same varying N_S/N_L ratios (0.5, 1.0, 1.5, and 2.0). These figures provide a direct measure of how board dimensions and entity balance influence the typical duration of a Snakes and Ladders game.

Consistent Increase in Average Game Time with Board Size: A highly consistent and pronounced trend across all N_S/N_L ratios and density configurations is the *unambiguous increase in average game time as the board size scales upwards.* This observation directly validates the initial hypothesis that larger board dimensions, even with constant entity density and lengths, inherently lead to longer gameplay durations.

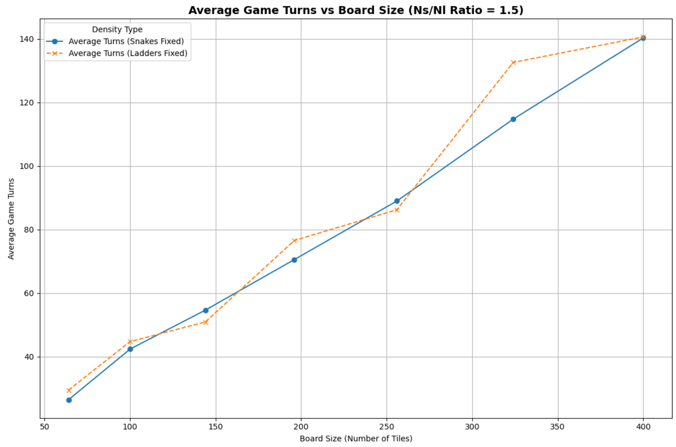


Figure 3.7: Average Game Turns vs Board Size for Ns/Nl Ratio = 1.5

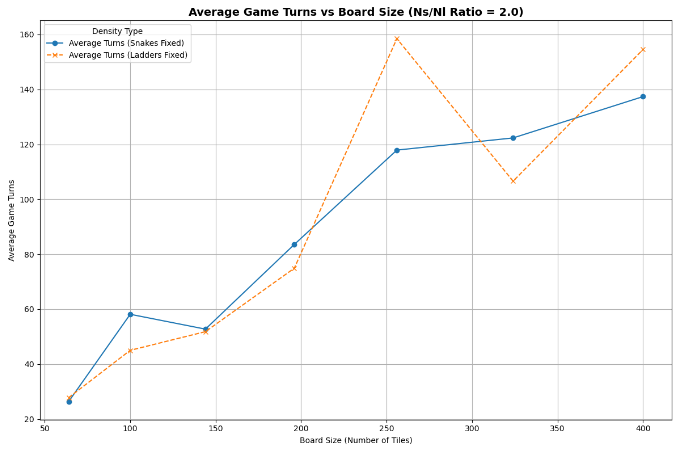


Figure 3.8: Average Game Turns vs Board Size for Ns/Nl Ratio = 2.0

Influence of Ns/Nl Ratio on Average Game Time Magnitude: While board size dictates the overall scaling of game time, the N_S/N_L ratio exerts a substantial influence on the *absolute magnitude* of average game times across different board dimensions, as detailed below:

- **Low Ns/Nl Ratio (0.5) - Shorter Game Times:** Figure 3.5 illustrates that at a low N_S/N_L ratio, average game times are consistently lower across all board sizes compared to higher ratios, indicating quicker game completion due to ladder abundance.
- **Balanced Ns/Nl Ratio (1.0) - Moderate Game Times:** Figure 3.6 demonstrates moderately increased average game times compared to the low Ns/Nl ratio scenario, with game times scaling more visibly with board size, offering a conventionally paced gameplay experience.
- **Elevated Ns/Nl Ratios (1.5 and 2.0) - Prolonged Game Times:** Figures 3.7 and 3.8 reveal a marked elongation of aver-

age game times, particularly at higher N_S/N_L ratios, reflecting the impeding effect of a higher density of snakes and leading to protracted gameplay.

- **Density Configuration - Minimal Impact on Average Game Time Scaling:** Similar to the win probability analysis, the density configuration (fixed snake vs. fixed ladder density) exhibits a comparatively negligible influence on the scaling of average game times, with the overall game duration primarily governed by board size and the N_S/N_L ratio.

Game Duration and Player Engagement: The observed scaling of average game time with board size has direct implications for player engagement and game experience. The simulations quantitatively demonstrate that board size is a primary lever for controlling the typical time investment required to play Snakes and Ladders. Designers can leverage this predictable scaling to tailor game sessions to different player preferences and contexts. Furthermore, the modulatory effect of the N_S/N_L ratio on average game time offers an additional layer of control over game pacing.

Finally, this chapter lays the groundwork for a more advanced analytical approaches. The observed complexities and probabilistic nature of \textit{Snakes and Ladders} dynamics suggest the utility of Markov models for deeper investigation. As the research moves forward, employing Markov models will allow to gain more nuanced insights into game mechanics, predict expected game durations for specific board configurations, and further quantify the elements that contribute to the overall player experience.

Chapter 4

Markov Model: Analytical Validation of Game Dynamics

Building upon the empirical insights into Snakes and Ladders game dynamics from agent-based simulations in Chapter 3, this chapter introduces a complementary analytical approach: a Markov Chain model. This chapter details the development, validation, and application of this model to rigorously predict expected game length, win probabilities, and steady-state distributions. Crucially, this Markovian framework serves to analytically verify the empirical findings from our simulations, thereby establishing a more robust and multifaceted understanding of the game's underlying mechanics.

4.1 What are Markov Chains

At its core, a Markov chain is a mathematical construct describing a system transitioning between discrete states in a sequential, chain-like manner. The defining characteristic, and analytical power, of a Markov chain lies in its 'memoryless' property: future state transitions depend solely on the current state, irrespective of the sequence of states that preceded it. In the context of Snakes and Ladders, we conceptualise each tile on the board as a distinct state. The probabilistic transition from one tile to the next is governed by the outcome of a dice roll, rendering the game dynamics amenable to Markovian analysis.

By adopting this framework, we can mathematically represent the inherent randomness of dice rolls and the deterministic rules of snake and ladder placements. This allows us to move beyond empirical observation and derive analytical expressions for key performance indicators, offering a powerful means of validating and extending our simulation-based research.

4.2 Fundamental Concepts

To ensure clarity and precision in our model description, it is essential to define the core concepts underpinning our Markovian approach:

1. **States:** Each tile on the Snakes and Ladders board, indexed from 0 to N (where N is the board size), is defined as a unique state within our Markov model. State 0 represents the starting position, and state N corresponds to the final tile, the absorbing goal state.
2. **Transitions:** Game progression is modelled as probabilistic transitions between these states. Transitions occur in discrete steps, driven by dice rolls. Each roll of a fair six-sided die produces an outcome $k \in \{1, 2, 3, 4, 5, 6\}$, each with an equal probability of $\frac{1}{6}$. Adding this outcome to the current tile number determines a tentative next position, subject to board boundaries and entity adjustments.
3. **Absorbing State:** The final tile, state N, is designated as an absorbing state. Once a player reaches or surpasses tile N, the game concludes, and the player remains in this state indefinitely. This is mathematically represented by a self-loop in the transition matrix, with a probability of 1.
4. **Memorylessness (Markov Property):** The defining 'memoryless' property dictates that the probability of transitioning to any future state depends exclusively on the current state. The history of previous moves or states is irrelevant. This assumption is valid for Snakes and Ladders, as each dice roll and subsequent move are probabilistically independent of prior game events. This Markovian property significantly simplifies the mathematical analysis, allowing us to construct a tractable and predictive model.

4.3 Constructing the Transition Matrix

The core of our Markov model is the transition matrix, P, a square matrix of size $(N-(N_s+N_l)) \times (N-(N_s+N_l))$, where N is the board size. Each entry $P(i, j)$ quantifies the probability of transitioning from state i to state j in a single turn. To maintain the state space and focus on board position transitions, we do not treat snake heads or ladder bases as distinct states. For baseline analysis within this chapter, we configure the board with 10 snakes (N_S) and 10 ladders (N_L), with their lengths assigned randomly using the fixed start and end points methodology detailed in Chapter 2.

4.3.1 Methodology

For each non-absorbing state $i < N$:

1. **Dice Roll Simulation:** We simulate the roll of a fair six-sided die, generating an outcome $k \in \{1, 2, 3, 4, 5, 6\}$, each with a probability of $\frac{1}{6}$.
2. **Tentative Movement:** A tentative next state is calculated by adding the dice roll outcome k to the current state i. Boundary conditions are applied: if the tentative state exceeds the board size N, the excess is reflected back onto the board, ensuring the player remains within valid tile positions.
3. **Entity-Based Adjustment:** We then check if the tentative next state coincides with the start of a snake or the base of a ladder, based on the pre-defined board layout. If so, the state is immediately updated to the corresponding snake tail or ladder top, respectively.
4. **Probability Assignment:** For each dice outcome k, the transition probability $(\frac{1}{6})$ is added to the matrix entry $P(i, j)$, where j represents the final state reached after all movement and entity-based adjustments.

For the absorbing state N, the transition matrix row is configured to represent absorption: $P(N, N) = 1$, and $P(N, j) = 0$ for all $j \neq N$. This ensures that once the final state is reached, the probability of transitioning to any other state is zero.

Mathematically, for non-absorbing states $i < N$, the transition probability is expressed as:

$$P(i, j) = \sum_{k=1}^{6} \frac{1}{6} \cdot \mathbf{1}\{f(i, k) = j\}$$

where $f(i, k)$ is a function encapsulating the game's movement rules: it computes the next state j reached from state i after rolling k, incorporating boundary reflections and snake/ladder adjustments. The indicator function $\mathbf{1}\{\cdot\}$ evaluates to 1 if the condition is true, and 0 otherwise.

For the absorbing state N, the transition probabilities are deterministic:

$$P(N, N) = 1, \quad P(N, j) = 0 \quad \forall j \neq N.$$

This formulation guarantees that each row of the transition matrix sums to unity, a fundamental property of stochastic matrices and Markov chains.

4.4 Analytical Derivation of Game Metrics

With the transition matrix constructed, we can analytically derive key game metrics using matrix-based methods. Specifically, we leverage the concept of the fundamental matrix to calculate expected game turns and estimate win probabilities.

4.4.1 Fundamental Matrix and Expected Turns

To calculate the expected number of turns to reach the absorbing state, we first partition the transition matrix P into submatrices corresponding to transient (non-absorbing) and absorbing states. Let Q be the submatrix representing transitions between transient states, and R be the submatrix for transitions from transient to absorbing states. The fundamental matrix N, central to our analysis, is then computed as:

$$N = (I - Q)^{-1}$$

where I is the identity matrix of the same dimension as Q. The entry $N(i, j)$ of the fundamental matrix provides the expected number of times the system visits transient state j before absorption, starting from transient state i.

The expected number of turns, t, to reach the absorbing state from a starting state i is derived by summing the entries in the i^{th} row of the fundamental matrix N and multiplying by a column vector of ones, $\mathbf{1}$:

$$t_i = (N \cdot \mathbf{1})_i = \sum_j N(i, j)$$

For a standard Snakes and Ladders game starting at state 0, the expected number of turns to reach the final tile is:

$$t = t_0 = \sum_j N(0, j)$$

This sum represents the total expected number of steps spent in transient states before absorption, effectively quantifying the average game duration in turns.

4.4.2 Win Probabilities within Turn Limits

Beyond expected game duration, we extend the Markov model to estimate win probabilities within specified turn limits, providing a more nuanced measure of game hardness. Agent-based simulations empirically estimate win probabilities for turn limits of $N/2$, $N/3$, and $N/4$ (where N is board size) by tracking game completion rates within these thresholds over numerous trials. The Markov model, through iterative state transitions, provides a computationally derived estimate of these win probabilities. While a direct analytical derivation of win probability distributions from the fundamental matrix is mathematically complex, our iterative approach offers robust estimates, allowing for a direct comparison with simulation results and further validation of the model's predictive power.

4.5 Validation: Markov Model vs. Agent-Based Simulations

To rigorously validate our Markov Chain model, we compared its analytical predictions against empirical results from the agent-based simulations detailed in Chapter 3. This section presents a comparative analysis of expected game turns, win probabilities, and game turn distributions.

4.5.1 Comparative Analysis of Expected Turns

A key validation point is the comparison of analytically calculated expected game turns with the average game turns observed in agent-based simulations. For a representative board configuration (Figure 4.1), simulations across 10,000 games yielded an average game duration of **33.99 turns**. Strikingly, the expected number of turns derived from our Markov model's fundamental matrix is **33.72 turns**. This remarkable concordance, with a negligible percentage difference, provides strong empirical validation for the Markov chain model's accuracy in predicting average game length in Snakes and Ladders. This analytical verification enhances the credibility of the simulation-based findings presented in Chapter 3, demonstrating convergent validity across methodological approaches.

4.5.2 Comparative Analysis of Win Probabilities

Further validation is achieved by comparing win probabilities within specified turn limits, calculated using both agent-based simulations and the Markov model. Figure 4.2 presents a comparison of win probabilities for varying board sizes at a fixed Ns/Nl ratio of 0.5 (Ns fixed). The plot juxtaposes win probabilities derived from agent-based simulations against those predicted by the Markov model for turn limits of N/2, N/3, and N/4.

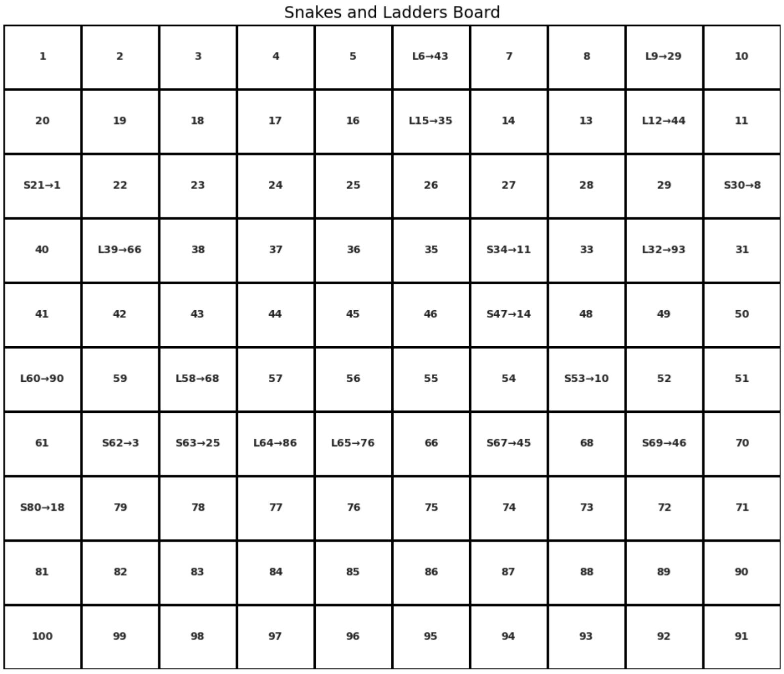


Figure 4.1: Representative Board Layout for Model Validation: Snakes and Ladders configuration used for comparing Markov model predictions with agent-based simulation results.

As illustrated in Figure 4.2, the Markov model's predictions for win probabilities closely align with the empirical results from agent-based simulations. Both methodologies demonstrate a similar trend: win probabilities within limited turns generally *increase* as board size expands, despite the longer average game times associated with larger boards, as discussed in Chapter 3. This counter-intuitive finding, validated by both simulation and analytical methods, underscores the nuanced impact of board size on game dynamics. The close quantitative agreement between the predicted and simulated win probabilities further bolsters confidence in the Markov model's ability to capture essential aspects of game hardness beyond just average game duration. Further comparisons across other Ns/Nl ratios and density configurations would similarly strengthen this validation.

4.5.3 Comparative Distribution Analysis: Game Turns

A more granular validation of the Markov model is achieved by comparing the full distribution of game turns. Figure 4.3 presents a comparative view of game turn distributions, juxtaposing empirical distributions from 10,000 agent-based simulations against the distribution derived from the Markov model. The Markov-derived distribution is generated by probabilistically simulating game progression through the transition matrix for 10,000 iterations, mirroring the simulation approach.

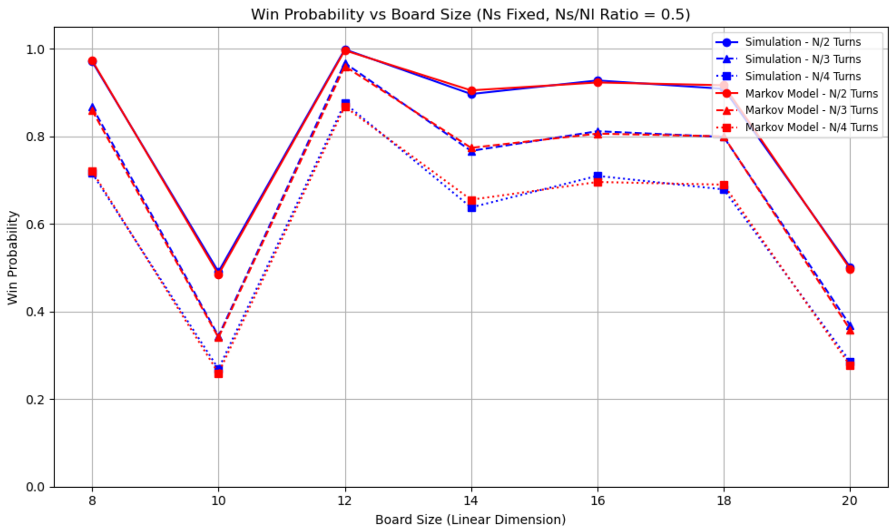


Figure 4.2: **Win Probability vs Board Size** Comparison of win probabilities within N/2, N/3, and N/4 turns, derived from agent-based simulations and Markov model predictions, demonstrating a high degree of agreement.

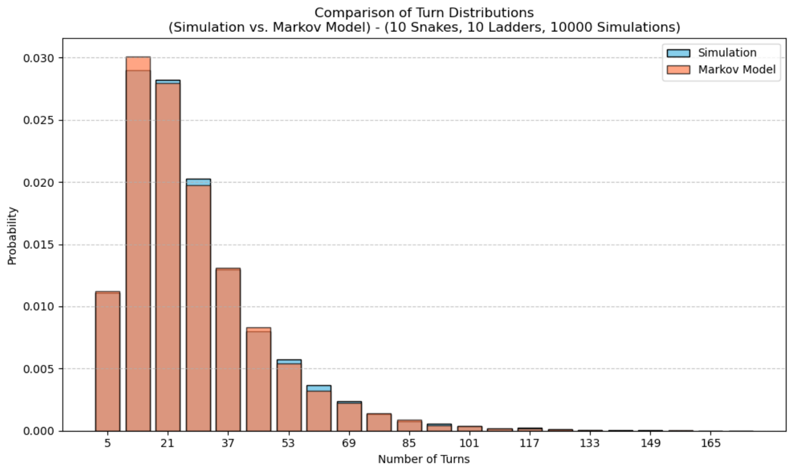
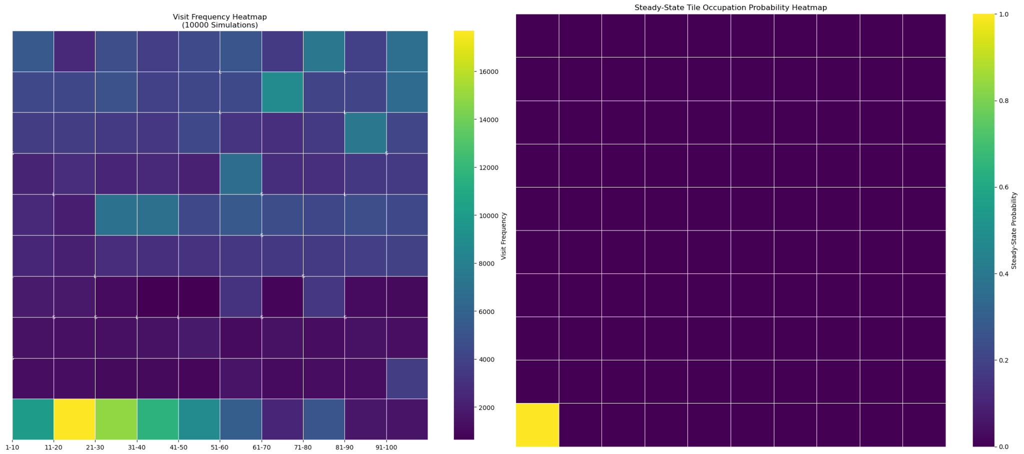


Figure 4.3: **Distribution of Game Turns: Simulation vs. Markov Model (10 Snakes, 10 Ladders, 10,000 Games):** Probability distributions of game turns from agent-based simulations and Markov model-based predictions exhibit a high degree of congruence, demonstrating model validity.

Visual inspection of Figure 4.3 reveals a strong qualitative agreement between the two distributions. Both exhibit a characteristic right-skewed pattern, peaking in the 10-20 turn range and displaying a long tail extending towards higher turn counts. This close mirroring of distributional shapes and central tendencies provides compelling evidence that the Markov model accurately captures the stochasticity and overall dynamics of game progression in Snakes and Ladders. The histogram derived from agent-based simulations closely aligns with the distribution predicted by the Markov model, further solidifying the model's capacity to represent the game's probabilistic nature.

4.5.4 Steady-State Distribution and Gameplay Hotspots

The steady-state distribution, derived analytically from the Markov model, offers insights into the long-term probabilities of tile occupation, representing the equilibrium state of the game over infinite plays. Figure 4.4 (b) visualises this steady-state distribution as a heatmap, juxtaposed with the Tile Visit Frequency Heatmap (Figure 4.4 (a)) empirically derived from agent-based simulations.



(a) (b)

Figure 4.4: Heatmap Comparison: (a) Tile Visit Frequency Heatmap from Agent-Based Simulations, empirically highlighting gameplay hotspots; (b) Steady-State Tile Occupation Probability Heatmap from Markov Model, illustrating long-term equilibrium.

As theoretically expected for a well-formulated absorbing Markov chain, the Steady-State Heatmap (Figure 4.4 (b)) demonstrates a probability of 1.0 concentrated on tile 100 (bottom-right corner), with probabilities for all other tiles approaching zero. This analytically confirms that, in the long run, the game predictably terminates at the absorbing winning state.

In contrast, the Tile Visit Frequency Heatmap from simulations (Figure 4.4 (a)) empirically reveals tiles frequently visited during active gameplay. This heatmap highlights gameplay hotspots and tiles of relative importance during a typical game session. While the Steady-State Heatmap represents the game's theoretical long-term behaviour, the Tile Visit Frequency Heatmap offers more practical insights into tile occupation patterns and player experience during actual gameplay. Together, they provide complementary perspectives on game dynamics and tile importance.

4.5.5 Analytical Efficiency in Game Time Derivation

A significant advantage of the Markov model is its analytical efficiency in deriving expected game time. Unlike agent-based simulations, which require computationally intensive repeated trials to approximate average game durations, the Markov model offers a direct, deterministic calculation through fundamental matrix analysis. This analytical solution provides a rigorous and computationally efficient alternative for determining average game durations across diverse board configurations and parameter settings. The demonstrated close agreement between the analytical expected turns and simulation-based average turns validates the Markovian approach as not only accurate but also a computationally advantageous tool for analysing game time in Snakes and Ladders.

4.6 Conclusion: Validation and Utility of the Markov Model

This chapter has successfully developed and rigorously validated a Markov Chain model for Snakes and Ladders. Through comparative analysis, encompassing expected game turns, win probabilities, game turn distributions, and steady-state behaviour, we have demonstrated a strong convergence between the analytical predictions of the Markov model and the empirical findings from agent-based simulations. This robust validation reinforces the accuracy and reliability of both methodological approaches.

The Markov model provides a powerful analytical framework for understanding and quantifying Snakes and Ladders game dynamics. Its capacity to derive expected game time and steady-state distributions analytically, coupled with its validation against empirical simulations for win probabilities and turn distributions, establishes its utility as a valuable tool for game analysis and design. Moreover, the model's computational efficiency offers a significant advantage for rapid evaluation of diverse game configurations and parameter variations, paving the way for more normative investigations into optimal board design and game balancing, which we will address in subsequent chapters. The validated Markovian approach, therefore, not only enhances our understanding of Snakes and Ladders but also provides a robust analytical foundation for further prescriptive research in game design and mechanics.

References