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Warm Up

1. Consider two events A and B such that P (A) = 1/3 and P (B) = 1/2. Determine

the value of P (B \ A') for each of the following conditions:

(a) A and B are disjoint,

(b) A is a subset of B,

(c) P (A \ B) = 1/8.

1. In English the question asks the probability of something been both inside B (1/2) and outside A(1-1/3 =2/3). Since it uses conjunction the answer can never be bigger than the smaller of those two values.

Another way to look at it is to use De Morgan's Laws and it is equal to:

complement(B complement OR A)

Which in English is everything that is not (outside B or inside A). This means 1 – (outside B + A inside B) or:

1 - (1-P(B) + P(A & B))

1 – (1/2 + P(A & B))

Using the last formula we get :

a) 1-(1/2+0) = 1/2

b) 1-(1/2+1/3) = 1-5/6 = 1/6

c) 1-(1/2+1/8)= 1-5/8 = 3/8

2. Consider an experiment in which two balanced dice are rolled. Calculate the probability of each of the possible values of the sum of two numbers that may appear.

Starting in the middle with 7 we have 6 possible combinations as no matter which of the 6 numbers is rolled on the first dice you can make 7 with the second dice. If you move away from 7 their is one less combination on either side. If you roll a 1 you cannot make 8 and if you roll a 6 you cannot make 6. So the formula is 6 – (7-x)

so 2→1 3→2 4→3 …. 7→6 8→5 … 11→3 12→2

3. A committee composed of eight people is to be selected from a group of 20 people. What is the number of different groups of people that might be in the committee?

This is a combinations question so we end up with:

numerator = 20.19.18.17.16.15.14.13

denominator = 8 !

answer = 125, 970

4. Suppose that two dice were rolled and it was observed that the sum of two numbers is odd. What is the probability that the sum was less than 8?

There are 18 odd rolls. If an even number is rolled first there are three odd numbers that would make the total odd and vice vers). From question 2 we can get the odd rolls less than 8.

$3 \rightarrow 2$ ways, $5 \rightarrow 4$ ways, $7 \rightarrow$ 6 ways

So 10/18 = 5/9

5. Three different machines M1 , M2 and M3 were used for producing a large batch of similar manufactured items. Supposed that 20% of the items were produced by machine M1 , 30% by machine M2 and 50% by machine M3 . Suppose further that 1% of the items produced by machine M1 are defective, 2% by machine M2 and 4% by machine M3. Finally suppose that one item is selected at random from the entire batch and it is found to be defective. What is the probability that this item was produced by machine M2.

Note this question involves multiplying percentages which has an implied division by 100. Since the same division would be done on all machines I am going to ignore it and stick with natural numbers.

M1 defects = 20*1 =20

M2 defects = 30*2 = 60

M3 defects = 50*4 = 200

Total = 280

So if a defect is found the odds of it being M2 are 60/280 = 3/14

Probability

1. One ball is to be selected from a box containing red, white, blue, yellow, and green balls. If the probability that the selected ball will be red is 1/5 and the probability that it will be white is 2/5, what is the probability that it will be blue, yellow, or green?

Since the odds of selecting red and white balls are mutually exclusive we can sum their probabilities to 3/5. The answer is what is left over 1-3/5 = 2/5

2. Consider two events A and B with P (A) = 0.4 and P (B) = 0.7. Determine the maximum and minimum possible values of P (A n B) and the conditions under which each of these values is attained.

The mximum value of $P(A \cap B)$ is when one is a subset of the other. Here P(A) is smaller so that would mean $P(A \cap B) = 0.4$

The minimum value is when they are disjoint but that is subject to

$$P(A) + P(B) \leq 1$$

When $P(A) + P(B) > 1$ we must solve:

$$1 = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = 0.4 + 0.7 - 1$$

$$P(A \cap B) = 0.1$$

3. If 50% of the families in a certain city subscribe to the morning newspaper, 65% of the families subscribe to the afternoon newspaper, and 85% of the families subscribe to at least one of the two newspapers, what percentage of the families subscribe to both newspapers?

Similar to the above. P(A) = morning and P(B) = afternoon

$$0.85 = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = 0.5 + 0.65 – 0.85$$

$$P(A \cap B) = 0.3$$

4. If the probability that student A will fail a certain statistics examination is 0:5, the probability that student B will fail the examination is 0:2, and the probability that both students will fail is 0:1, what is the probability that at least one of these students will fail the examination?

Similar to 2 and 3.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.5 + 0.2 – 0.1$$

$$= 0.6$$

Finite Sample Space

1. **If two balanced dice are rolled, what is the probability that the sum of two numbers that appear will be odd?**

This was answered in warmup Q4. It is 18/36 although the product will be odd 3/4 of the time.

2. If two balanced dice are rolled, what is the probability that the di#erence between two numbers that appear will be less than 3?

So the number has to be within two. First of all x is rolled there are 6 equal combinations. Within one is 5 above and 5 below (1 has nothing below it and 6 has nothing above it) and within two is four above and four below (1 and 2 have no number two below it; 5 and 6 have no number two above it). 3 and 4 have 5 numbers with in 2. 5+5+4+4+3+3 = 24 and out of 36 rolls gives 24/36 which is 2/3.

Counting Methods

1. If a man has six different sport shirts and four different pairs of slacks, how many di#erent combinations can he wear?

Their are six sports shirts. Each sports shirt can be paired with 4 different pairs of slacks. 6*4=24

2. In how many ways can the the letters a, b, c, d and e be arranged?

Permutation without replacement is n! → 5! = 120

3. Two pollsters will canvas a neighbourhood with 20 houses. Each pollster will visit 10 houses. How many different assignments of pollsters to houses are possible?

This is combinations so 20!/(10!*10!) or

numerator = 20.19.18.17.16.15.14.13.12.11

denominator = 10!

answer = 184,756

4. Suppose that a committee of 12 people is selected in a random manner from a group of 100 people. Determine the probability that two particular people A and B will both be selected.

Total possible committees is 100 choose 12. If we assign the two people to the committe we now have 98 choose 10 to get the number of committees they are on.

numerator → 98 choose 10 → 98!/(88!*10!)

denominator → 100 choose 12 → 100!/(88!*12!)

Because the denominator is a fraction we invert and multiply

numerator = 98! * 88! * 12!

denominator = 88! * 10! * 100!

After cancelling we have 12*11/100*99 = 1/75 (or 0.013333)

Conditional Probability:

1. **A box contains three coins with a head on each side, four coins with a tail on each side, and two fair coins. If one of these nine coins is selected at random and tossed once, what is the probability that a head will be obtained?**

We need to think of this in terms of 18 sides not 9 coins. So three coins have two heads which gives us 6 sides with heads. We also have two fair coins with one head which gives us another two heads. So 8 of the 18 sides are heads so our odds are 4/9.

2. 50% of the families in a certain city subscribe to the morning newspaper, 65% of the families subscribe to the afternoon newspaper, and 85% of the families subscribe to at least one of the two newspapers. If a family selected at random from the city subscribes to newspaper A, what is the probability that the family also subscribes to newspaper B?

P(A or B) = P(A) + P(B) - P(A ∩ B)

= P(A) + P(B) - P(A or B)

= 0.5 + 0.65 – 0.85

= 0.3

P (B | A) = P(A ∩ B) / P(A)

= 0.3/0.5

= 0.6

Independent Events

1. Suppose that the probability that the control system used in a spaceship will malfunction on a given flight is 0.001. Suppose further that a duplicate completely independent, control system is also installed in the spaceship to take control in case the first system malfunctions. Determine the probability that the spaceship will be under the control of either the original system or the duplicate system on a given flight.

It will not be under the control of one of the systems only if they both malfunction. The odds of both malfunctioning is 0.001*0.001 = 0.000001. Therefore it will be under control of one of the systems with 1- 0.000001 = 0.999999 probability.

2. Two students A and B are both registered for a certain course. Assume that student A attends class 80% of the time, student B attends class 50% of the time, and the absences of the two students are independent.

(a) What is the probability that at least one of the two students will be in class on a given day?

(b) If at least one of the two student is in class on a given day, what is the probability that A is in class that day?

a) One of them showing up → 1 – P(both miss)

1- (1-0.8)*(1-0.5)

1 -(0.2*0.5)

1-0.1 = 0.9

Another way to think of this would have been that A shows up 80% of the time. The other 20% that A misses then B shows up half the time so we can add 10% to A's 80%.

b) Since all of A's 80% is included in the 90% of the time where one of them shows up we have 80%/90% =8/9

Bayes' Theorem

1. In a certain city, 30% of the people are Conservatives, 50% are Liberals, and 20% are Independents. Records show that in a particular election, 65% of the Conservatives voted, 82% of the Liberals voted, and 50% of the Independents voted. If a person in the city is selected at random and it is learned that she did not vote in the last election, what is the probability that she is a Liberal?

Similar to warmup question 5 I am going to drop the percents here. Also P(did not vote) = 1- P(voted)

Conservatives who didn't vote = 30*35 = 1050

Liberals who didn't vote = 50*18 = 900

Independendents who didn't vote = 20*50 = 1000

If we have a person who didn't vote then the odds that they are Liberal is 900/(1050+900+1000) = 3/10

2. Suppose that a box contains five coins and that for each coin there is a different probability that a head will be obtained when the coin is tossed. Let pi denote the probability of a head when the ith coin is tossed, and suppose that p1 = 0, p2 = 1=4, p3 = 1=2, p4 = 3=4, p5 = 1. Suppose that one coin is selected at random from the box and when it is tossed once, a head is obtained. What is the posterior probability that the ith coin was selected?

I am going to convert the fractions to decimal so that p1=0, p2=0.25, p3=0.5, p4=0.75 and p5=1. Summing the probabilities give total = 2.5

Now $P(p_i|\text{head}) = p_i/\text{total}$

p1 = 0/2.5 = 0

p2 = 0.25/2.5 = 0.1

p3 = 0.5/2.5 = 0.2

p4 = 0.75/2.5 = 0.3

p5 = 1/2.5 = 0.4