$_n =_n$
$(k)$
$k =$
$k =$
$k$
$(k) =$
$0$
$_n =_n$
$() \subseteq$
$M_n()$
$m =$
$n^2$
$^m \subseteq^m$
$m$
$_n()$
$_n() = M_n() \backslash V(\det = 0),$

$V(f =$
$0) =$
$\{x \in^m :$
$f(x) =$
$0\}$
$f :^m \to$
$f \in$
$k[x_1, \dots, x_m]$
**Beispiel:**
$^m =$
$M_n()$
$G :=_n$
$()$

$$\mu \colon G \times G \longrightarrow G \qquad i \colon G \longrightarrow G \qquad e \colon \{x\} \longrightarrow G$$
$$(g,h) \longmapsto gh \qquad g \longmapsto g^{-1} \qquad x \longmapsto e =_n$$

$e$
$\mu$
$i$

$$A^{-1} = red \frac{1}{\det(A)} \cdot adj(A)$$

$^n$
$\mu \circ$
$(\mu, ) =$
$\mu \circ$
$(, \mu)$

$G \times G \times G (\mu, ) (, \mu) G \times G \mu G \times G \mu G (g,h,l) [mapsto][mapsto](gh,l)[d, endanchor = [xshift = -1.5em, yshift = -0.5em]nor$

$e \circ$
$p =$
$\mu \circ$
$(, i) \circ$
$\Delta$
$e \circ$
$p =$
$\mu \circ$
$(i, ) \circ$
$\Delta$

$G \times G (, i) G p \Delta G \times G \mu \{x\} e G (g,g) [mapsto] g [mapsto][mapsto][mapsto](g, g^{-1})[d, endanchor = [xshift = 0.5em, yshift = -$

$\mu \circ$
$(e, ) ==$
$\mu \circ$
$(, e)$

$G (e, ) [bendright, swap, endanchor = [xshift = 0.5em]] rr G \times G \mu G g [mapsto][bendright, mapsto, endanchor = [xshift = 2$

$f :^n \to^m$
$[] f_1(x_1, \dots, x_n), \dots, f_m(x_1, \dots, x_n) = f(x_1, \dots, x_n)$

$f_j \in$
$k[x_1, \dots, x_n]$
$j \in$
$\{1, \dots, n\}$
$U \subseteq^n$
$f \colon U \to^m$
$f =$
$\frac{h}{g}$
$h, g \in$
$k[x_1, \dots, x_n]$
$g(x) \neq$
$0$
$x \in$
$U$
$[ f_j =$
$\frac{h_j}{g_j}$
$]$
$f$

$g_1, \dots, g_m$
$k[x_1, \dots, x_n]$
$M \subseteq^n$
$M = V(I) = \{x \in^n \colon g_i(x) = 0 fr 1 \leq i \leq m\}$

$V(0) =^n$
$V(1) =$
$\emptyset$
$GL_n()$
$M_n()$
$GL_n()$

$_n() \hookrightarrow M_n() \times A \mapsto *A, \dfrac{1}{\det(A)}$

(1)

**Ab jetzt immer, wenn nichts anderes gesagt, offen und abgeschlossen bezglich Zariski-Topologie!**

$V$
$k$
$[\cdot,\cdot] \colon V \times V \longrightarrow V (v, w) \longmapsto [v, w]$

(2)

$[v, w] =$
$-[w, v]$
$*[u, v], w+$
$*[v, w], u+$
$*[w, u], v =$
$0$
$(V, [\cdot,\cdot])$
$k$
$V$
$[v, w] :=$
$0$
$_n(k) :=$
$M_n(k)$
$[A, B] :=$
$AB-$
$BA$
$_n(k) :=_n$
$(k)$
$_n(k)$
$n =$
$2$
$k =$

$_2() := \{A \in M_2() : (A) = 0\} = *h = (\,1\,) 00 - 1, e = (\,0\,) 100, f = (\,0\,) 010$

$[h, e] =$
$2e, [h, f] =$
$-2f$
$[e, f] =$
$h$
$[h, \cdot] \colon V \to$
$V$
$2$
$-2$
$G \leq_n$
$()$
$G$

**Ab jetzt: "algebraische Gruppe" = "lineare algebraische Gruppe" = "affine algebraische Gruppe"**

$G$
$(G) :=:= T_e G,$

$T_e G$
$e \in$
$G$

$T_e V(f_1, \dots, f_n) = *x \in^n \colon \dfrac{d}{dt} f_i(e + tx)\Big|_{t=0} = 0 ber k =$

$f =$
$x_1^2 -$
$x_2$
$2$

$T_{(0,0)} V(f) = *x \in^2 \colon \dfrac{d}{dt} f((0,0) + t(x_1, x_2))\big|_{t=0} = 0 = *x \in^2 \colon \dfrac{d}{dt}(tx_1)^2 - tx_2\big|_{t=0} = 0 = *x \in^2 \colon 2tx_1 - x_2\big|_{t=0} = 0 = \{x \in^2 \colon$

(3)