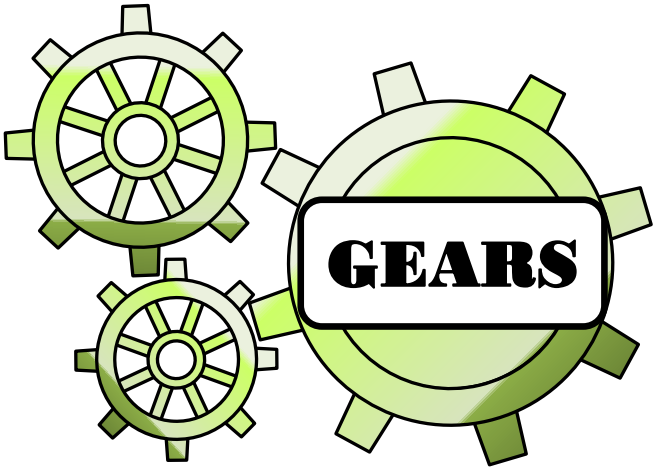

GEARS: The Fitzhugh-Nagumo example

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The Fitzhugh-Nagumo example in GEARS performs parameter estimation on the Fitzhugh-Nagumo model. The Fitzhugh-Nagumo model Fitzhugh (1961) is a simplified version of the Hodgkin–Huxley model, describing the activation and deactivation dynamics of a spiking neuron Hodgkin and Huxley (1952). We consider the model in the form as described by equations 1-5 and figure 1.

$$\frac{dV}{dt} = g\left(V - \frac{V^3}{3} + R\right) \tag{1}$$

$$\frac{dR}{dt} = -\frac{1}{g}\left(V - a + b \cdot R\right) \tag{2}$$

$$V(t_0, \boldsymbol{\theta}) = V_0,\ R(t_0, \boldsymbol{\theta}) = R_0 \tag{3}$$

$$y(t_i) = V(t_i) \tag{4}$$

$$\boldsymbol{\theta} = \{a, b, g\} \in [10^{-5}, 10^{5}] \tag{5}$$

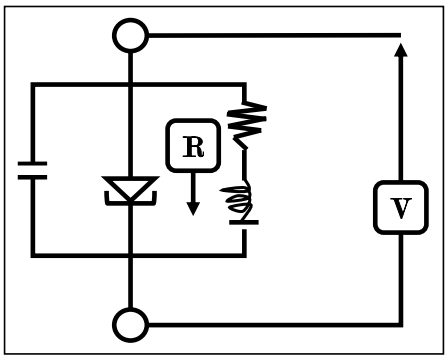


Figure 1: A visualisation of the structure of the Fitzhugh-Nagumo model.

Where y is the observation function considered in the example. Synthetic data was generated for the Fitzhugh-Nagumo model for parameter values $\{a, b, g\} = \{0.2, 0.2, 3\}$ for the initial conditions $V_0 = -1, R_0 = 1$. This data was generated with a standard deviation of 10% of the nominal signal level and a detection threshold of 0.1. This set-up for generating data was used to set up one fitting set of data and two data sets for cross-validation. Initial conditions for the cross-validation sets were varied randomly within a meaningful range.

A selection of the expected results achieved by running the FHN example in `GEARS` can be found below. For the full collection of the expected results of the example please consult the expected results folder in the FHN example folder.

Parameter	Value	Confidence	Coeff of variation	Bounds status
a	0.15482	± 0.0088454	2.915	Bounds not active
b	0.55983	± 0.028386	2.587	Bounds not active
g	2.7406	± 0.09037	1.6824	Bounds not active

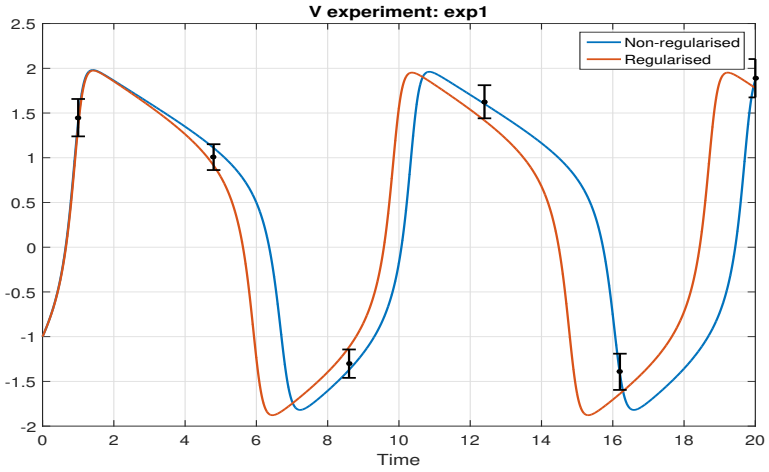
Table 1: A summary of the regularised results from the `GEARS` analysis of the FHN model.

Experiment	Regularised estimation	Non-regularised estimation
Experiment 1	0.27798	0.10946

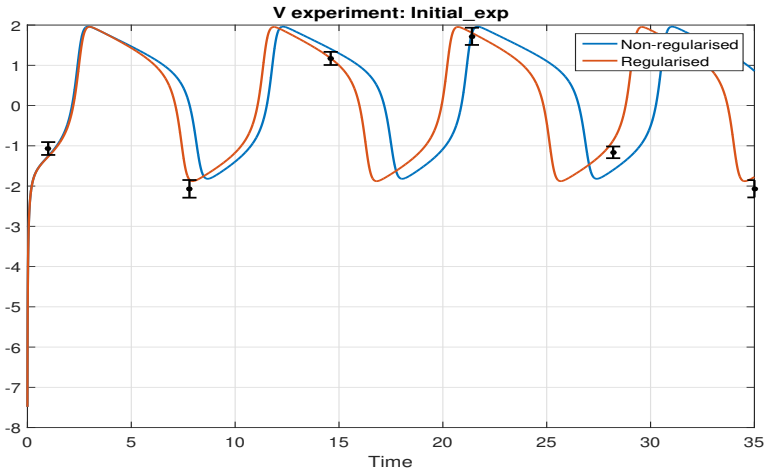
Table 2: The NRMSEs calculated for the fitting of the FHN model.

Experiment	Regularised estimation	Non-regularised estimation
All experiments	0.58761	1.5691
Initial experiment	0.31972	1.8314
Follow up experiment	0.76703	1.2531

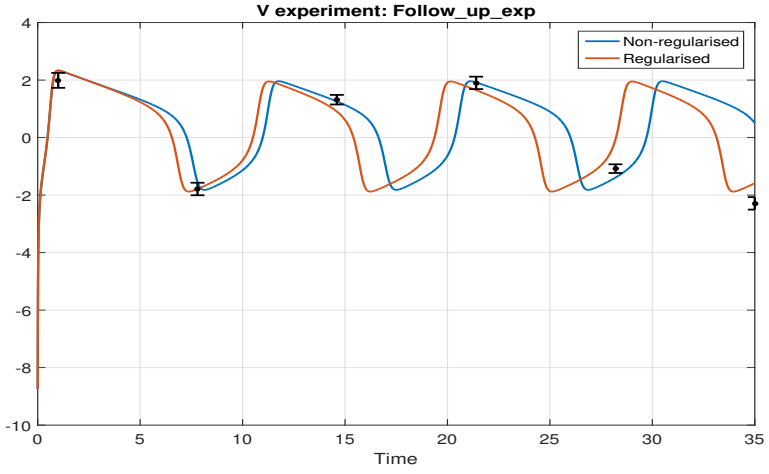
Table 3: The NRMSEs calculated for the cross-validation of the FHN model.



(a) A comparison of the FHN model fits with and without regularisation.



(b) A comparison of the FHN model predictions for the first cross-validation data set with and without regularisation.



(c) A comparison of the FHN model predictions for the second cross-validation data set with and without regularisation.

Figure 2: Figures showing the comparison between the regularised and non-regularised fits for both fitting and cross-validation.

References

Fitzhugh, R. (1961). Impulses and physiological states in the theoretical models of nerve membrane. *Biophysical journal*, 1:445–466.

Hodgkin, A. L. and Huxley, A. F. (1952). A quantitative description of membrane current and its application to conduction and excitation in nerve. *The Journal of Physiology*, 117(4):500–544.