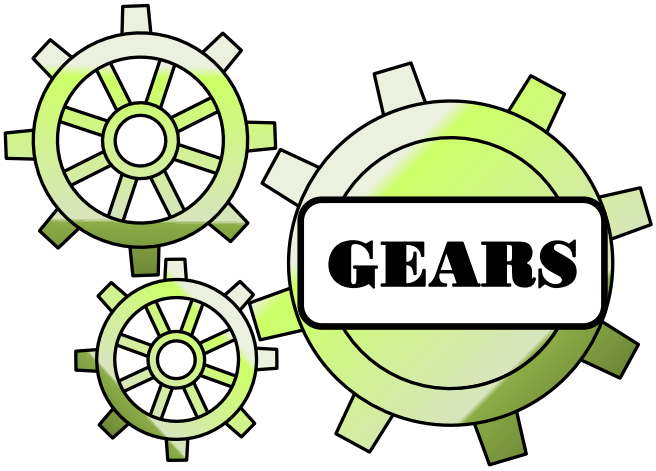


GEARS: The Fitzhugh-Nagumo example

Jake Alan Pitt and Julio R. Banga

September 17, 2018

The Fitzhugh-Nagumo example in GEARS performs parameter estimation on the Fitzhugh-Nagumo model. The Fitzhugh-Nagumo model Fitzhugh (1961) is a simplified version of the Hodgkin–Huxley model, describing the activation and deactivation dynamics of a spiking neuron Hodgkin and Huxley (1952). We consider the model in the form as described by equations 1-5 and figure 1.

$$\frac{dV}{dt} = g\left(V - \frac{V^3}{3} + R\right) \tag{1}$$

$$\frac{dR}{dt} = -\frac{1}{g}\left(V - a + b \cdot R\right) \tag{2}$$

$$V(t_0, \boldsymbol{\theta}) = V_0,\ R(t_0, \boldsymbol{\theta}) = R_0 \tag{3}$$

$$y(t_i) = V(t_i) \tag{4}$$

$$\boldsymbol{\theta} = \{a, b, g\} \in [10^{-5}, 10^{5}] \tag{5}$$

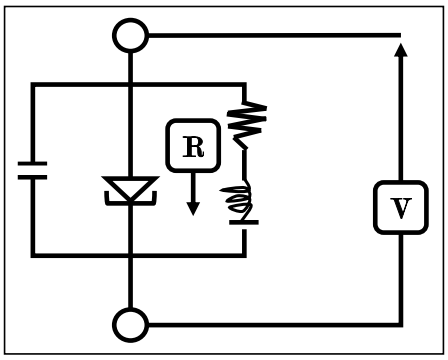


Figure 1: A visualisation of the structure of the Fitzhugh-Nagumo model.

Where y is the observation function considered in the example. Synthetic data was generated for the Fitzhugh-Nagumo model for parameter values $\{a, b, g\} = \{0.2, 0.2, 3\}$ for the initial conditions $V_0 = -1, R_0 = 1$. This data was generated with a standard deviation of 10% of the nominal signal level and a detection threshold of 0.1. This set-up for generating data was used to set up one fitting set of data and two data sets for cross-validation. Initial conditions for the cross-validation sets were varied randomly within a meaningful range.

A selection of the expected results achieved by running the FHN example in `GEARS` can be found below. For the full collection of the expected results of the example please consult the expected results folder in the FHN example folder.

Parameter	Value	Confidence (95%)	Coeff of variation (%)	Bounds status
a	0.15482	± 0.0088454	2.915	Bounds not active
b	0.55983	± 0.028386	2.587	Bounds not active
g	2.7406	± 0.09037	1.6824	Bounds not active

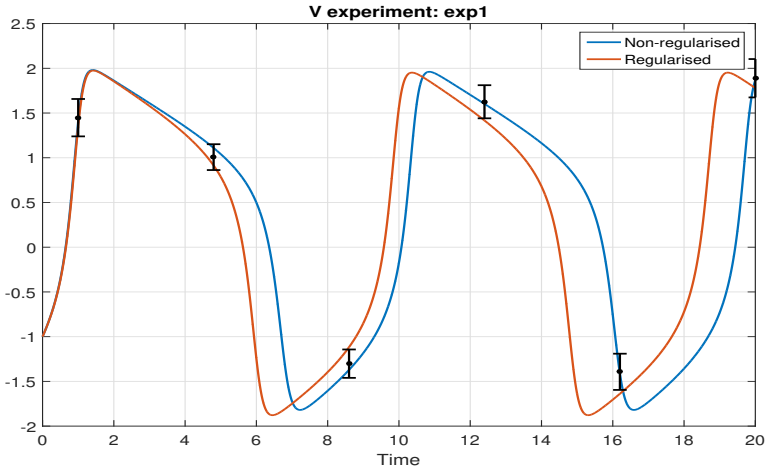
Table 1: A summary of the regularised results from the `GEARS` analysis of the FHN model.

Experiment	Regularised estimation	Non-regularised estimation
Experiment 1	0.27798	0.10946

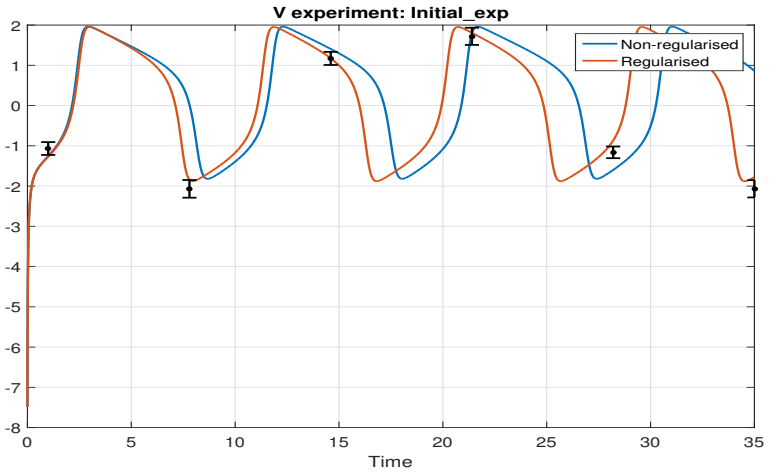
Table 2: The NRMSEs calculated for the fitting of the FHN model.

Experiment	Regularised estimation	Non-regularised estimation
All experiments	0.58761	1.5691
Initial experiment	0.31972	1.8314
Follow up experiment	0.76703	1.2531

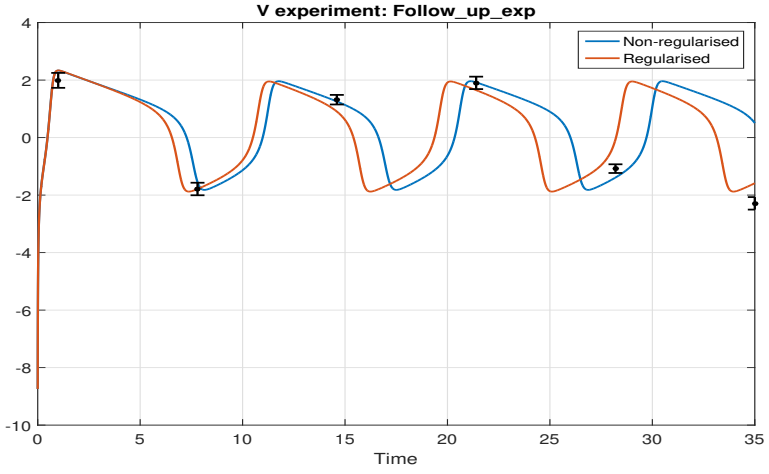
Table 3: The NRMSEs calculated for the cross-validation of the FHN model.



(a) A comparison of the FHN model fits with and without regularisation.



(b) A comparison of the FHN model predictions for the first cross-validation data set with and without regularisation.



(c) A comparison of the FHN model predictions for the second cross-validation data set with and without regularisation.

Figure 2: Figures showing the comparison between the regularised and non-regularised fits for both fitting and cross-validation.

References

Fitzhugh, R. (1961). Impulses and physiological states in the theoretical models of nerve membrane. *Biophysical journal*, 1:445–466.

Hodgkin, A. L. and Huxley, A. F. (1952). A quantitative description of membrane current and its application to conduction and excitation in nerve. *The Journal of Physiology*, 117(4):500–544.