Functional Programming

Monad Transformers

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Reminder: Monad

Definition of a Monad – Previous lecture

- abstract datatype for instructions that produce values
- built-in combination >>=
- abstracts over different interpretations (computations)

Monad definition

The type class Monad

```haskell
class Monad m where
  (>>=) :: m a -> (a -> m b) -> m b
  return :: a -> m a
  fail :: String -> m a
```

with the following laws:

- `return x >>= f == f x`
- `m >>= return == m`
- `(m >>= f) >>= g == m >>= (\x -> f x >>= g)`

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  - If `f` and `g` are applicatives, what about `Comp f g`?
  - If `f` and `g` are monads, what about `Comp f g`?
- We sometimes want to use multiple functors, applicatives, monads at once!

Why combine functors, applicatives, monads

Lecture 12: Monadic interpreters.

Interpreters can have many features:

- Failure (**`Maybe`**).
- Keeping some state (`State`).
- Reading from the environment (`Reader`).
- ...

To implement an interpreter, we need to combine all these monads!

Let's start by combining functors!

To show that `Comp f g` is a functor ...

- Implement `fmap` (i.e., give an instance of the **`Functor`** class)
- Show that the functor laws hold
  - The identity function gets mapped to the identity function.
  - Functor composition commutes with function composition.

Let's combine applicatives!

To show that `Comp f g` is an applicative ...

- Implement `pure` and `(<*>)` (i.e., give an instance of the `Applicative` class)
- Show that the applicative laws hold ...

Let's combine Monads! – State alone

The State monad

```haskell
data ST s a = ST (s -> (a, s))
runST (ST sas) = sas

instance Monad (ST s) where
  return a = ST (\s -> (a, s))
  m >>= f = ST (\s ->
                let (a, s') = runST m s in
                runST (f a) s')
```

Let's combine Monads! – Maybe+State

The MaybeState monad

- Purpose: propagate state and signaling of errors
- Attention: the state is lost

```haskell
data MaybeState s a = MS { runMS :: s -> Maybe (a, s) }

....

instance Monad (MST s) where
  return a = MST (\s -> Just (a, s))
  ms >>= f = MST (\s -> case runMST ms s of
                      Nothing -> Nothing
                      Just (a,s') -> runMST (f a) s')
```

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```

We would have to write this again for each combination!

Alternative solution: Monad transformers

Monad transformers offer a better solution:

```haskell
class MonadTrans t where
  lift :: Monad m => m a -> t m a
```

A monad transformer `t` takes a monad `m` and yield a new monad `(t m)`.
Function `lift` lifts a computation from the underlying monad to the new monad.

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- What's the kind of `t` in `MonadTrans`?
- Answer: `t :: (* -> *) -> (* -> *)`

The MaybeT monad transformer

Definition

```
newtype MaybeT m a = MaybeT { runMaybeT :: m (Maybe a) }

instance (Monad m) => Monad (MaybeT m) where
  return = MaybeT . return . Just
  (MaybeT mmx) >>= f = MaybeT $ do
    mx <- mmx
    case mx of
      Nothing -> return Nothing
      Just x -> runMaybeT (f x)

instance MonadTrans MaybeT where
  lift mx = MaybeT $ mx >>= (return . Just)
```

A simple use of MaybeT

We can recover the “normal” monad by applying to `Identity`.

```
type MaybeLike = MaybeT Identity
```

The `StateT` monad transformer

Definition

```
newtype StateT s m a = StateT { runStateT :: s -> m (a,s) }

instance (Monad m) => Monad (StateT m) where
  return a = StateT $ \s -> return (a, s)
  m >>= f = StateT $ \s -> do
      (a, s') <- runStateT m s
      runStateT (f a) s'

instance MonadTrans StateT where
  lift ma = StateT $ \s -> do { a <- ma ; return (a, s) }
```

Let's combine Monads with transformers!

Demo!

The ReaderT monad transformer

Definition

```haskell
newtype ReaderT r m a = ReaderT { runReaderT :: r -> m a }

ask :: (Monad m) => ReaderT r m r
ask = ReaderT return

instance Monad m => Monad (ReaderT r m) where
   return = lift . return
   m >>= k = ReaderT $ \r -> do
               a <- runReaderT m r
               runReaderT (k a) r

instance MonadTrans (ReaderT r) where
   lift m = ReaderT (const m)
```

Back to interpreters

During lecture 12, a monadic interpreter for:

```haskell
data Term = Con Integer
          | Bin Term Op Term
            deriving (Eq, Show)

data Op = Add | Sub | Mul | Div
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Different interpreters with various features:

- Failure (⇒ exception/Maybe monad)
- Counting instructions (⇒ state monad)
- Traces (⇒ writer monad)

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- Monads do not always compose:
  if `m1` and `m2` are monads, there is no general definition that makes `Comp m1 m2` a monad

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- Order is important:
  `StateT s` **`Maybe`** $\neq$ `MaybeT (ST s)`
- You should not overdo it.
- It's all in the `mtl` library.