

Motivating clustering approaches

Goal: Structure documents by topic

Discover groups (*clusters*) of related articles



SPORTS

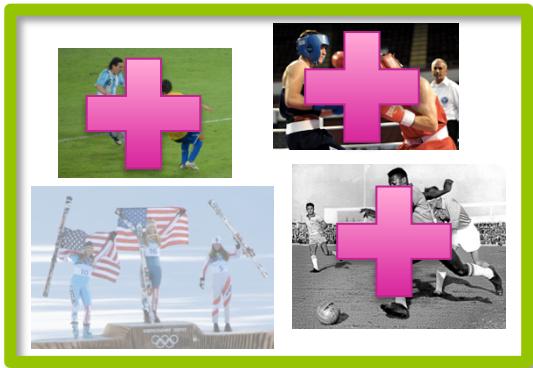
WORLD NEWS

Why might clustering be useful?



Learn user preferences

Set of clustered documents read by user



Cluster 1



Cluster 2



Cluster 3



Cluster 4



Use feedback
to learn user
preferences
over topics

Clustering: An unsupervised learning task

What if some of the labels are known?

Training set of labeled docs



SPORTS



WORLD NEWS

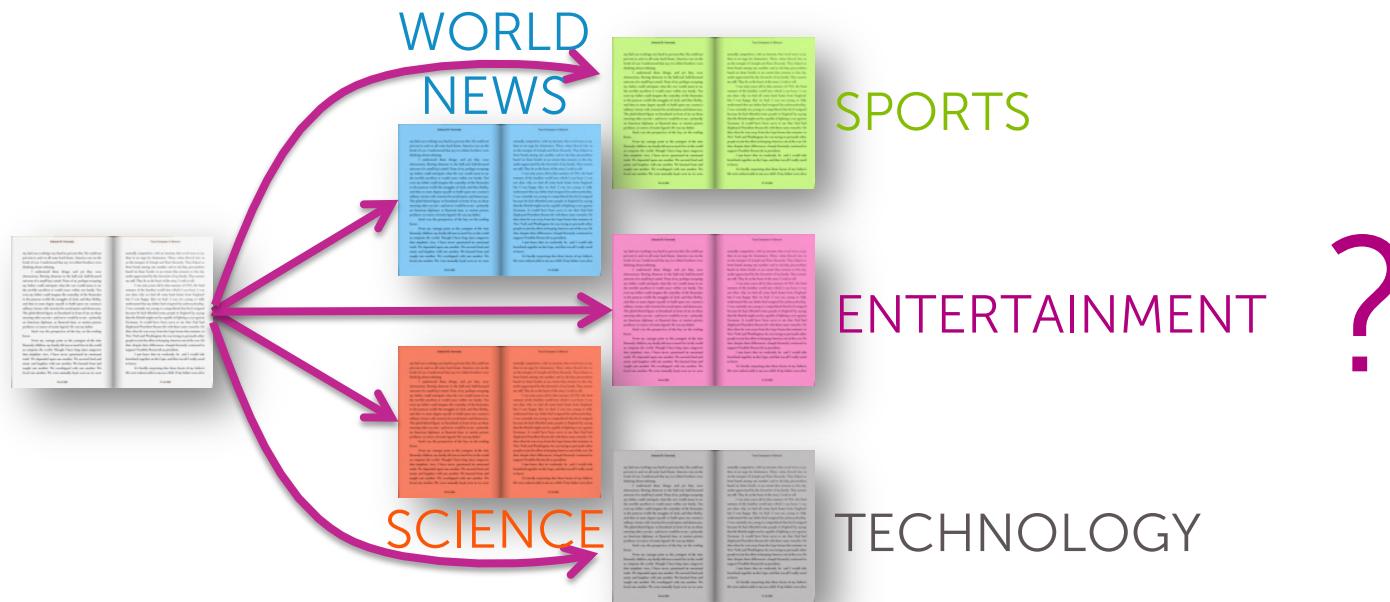


ENTERTAINMENT



SCIENCE

Multiclass classification problem



Example of
supervised learning

Clustering

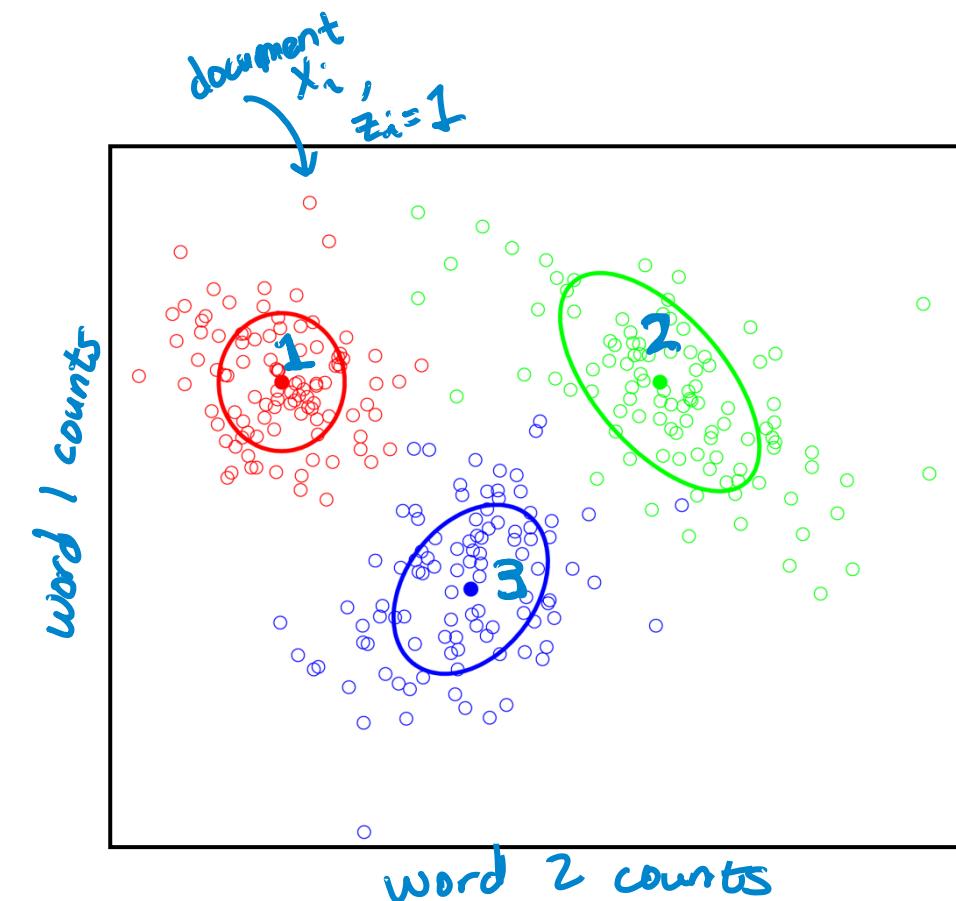
No labels provided

...uncover cluster structure
from input alone

Input: docs as vectors \mathbf{x}_i

Output: cluster labels z_i

An unsupervised
learning task

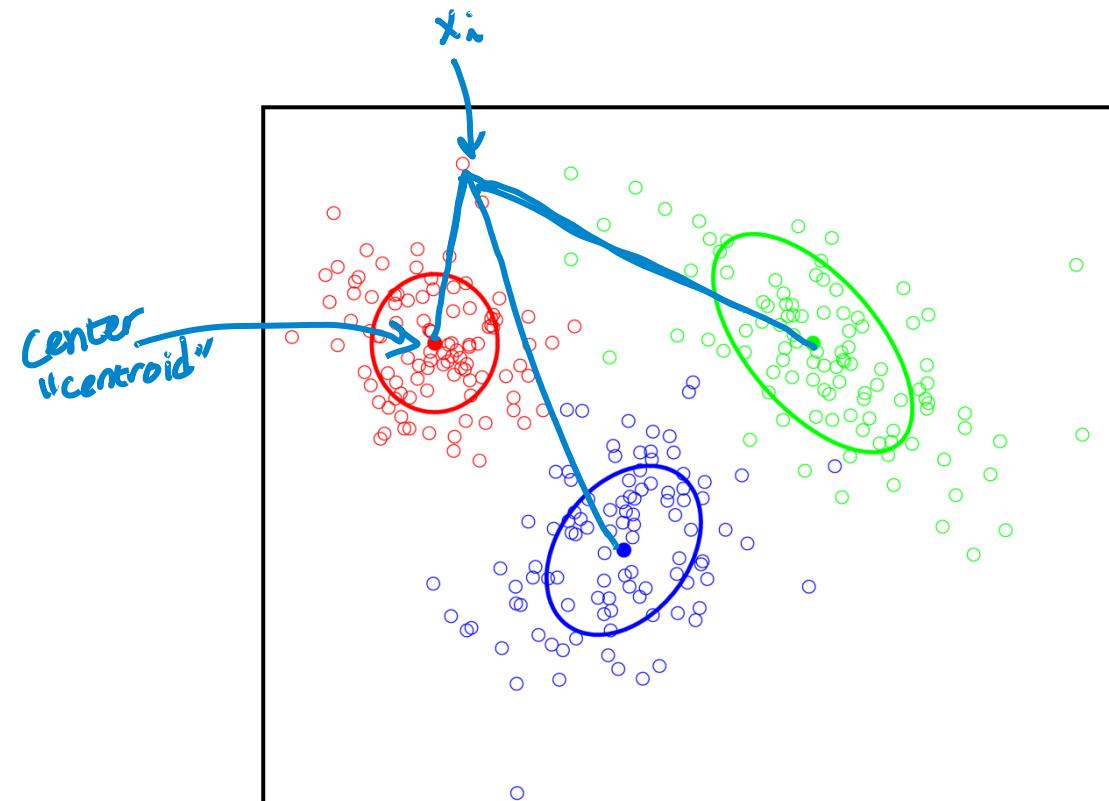


What defines a cluster?

Cluster defined by
center & shape/spread

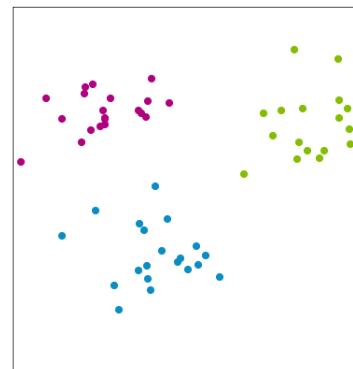
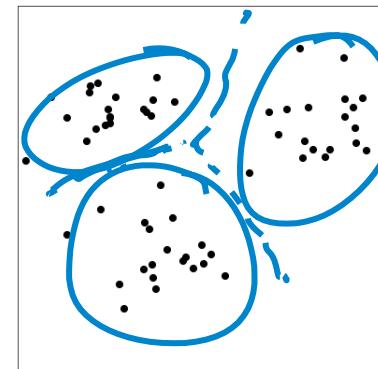
Assign observation x_i (doc)
to cluster k (topic label) if

- Score under cluster k is higher than under others
- For simplicity, often define score as **distance to cluster center** (ignoring shape)

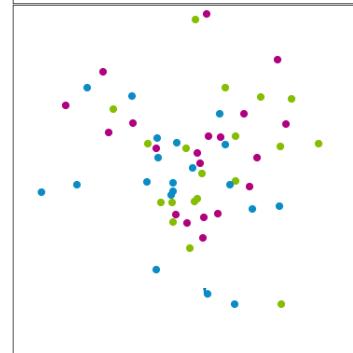
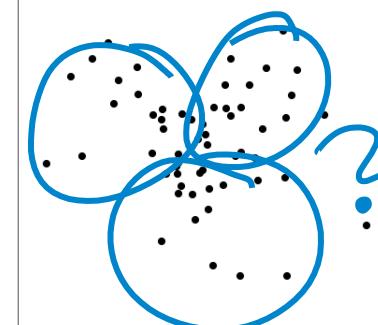


Hope for unsupervised learning

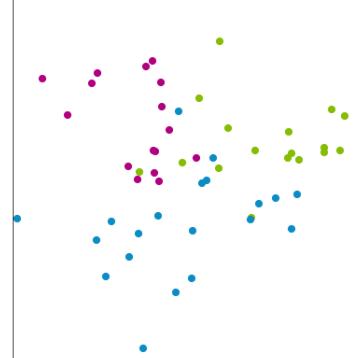
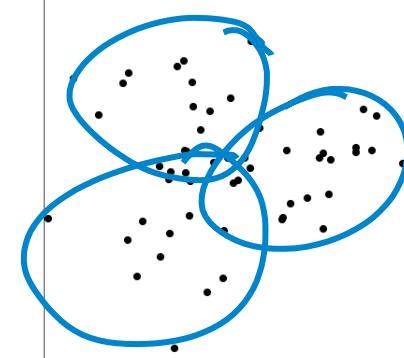
Easy



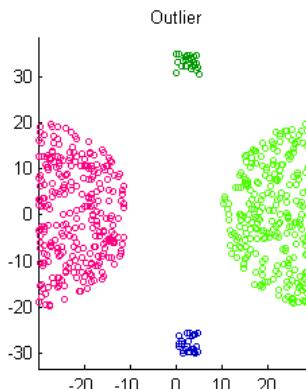
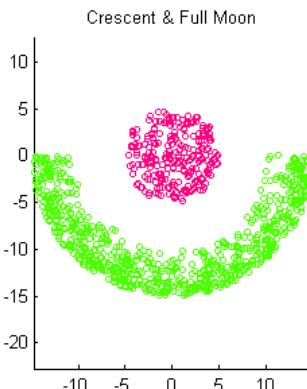
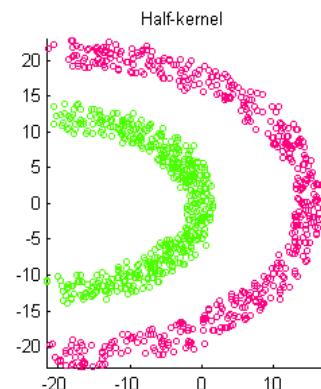
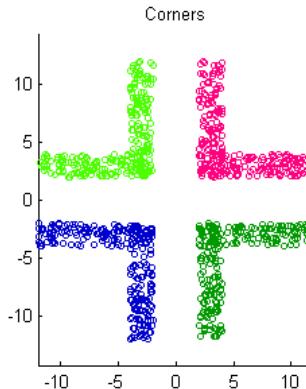
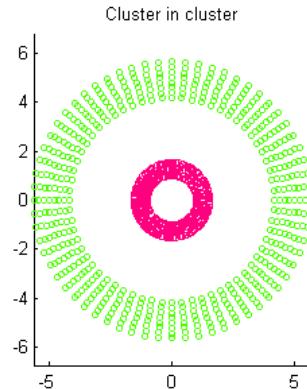
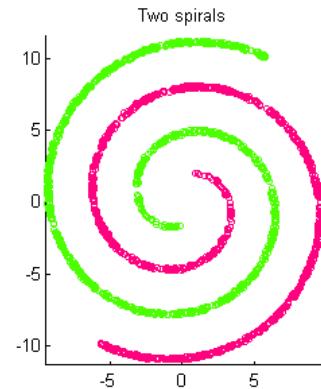
Impossible



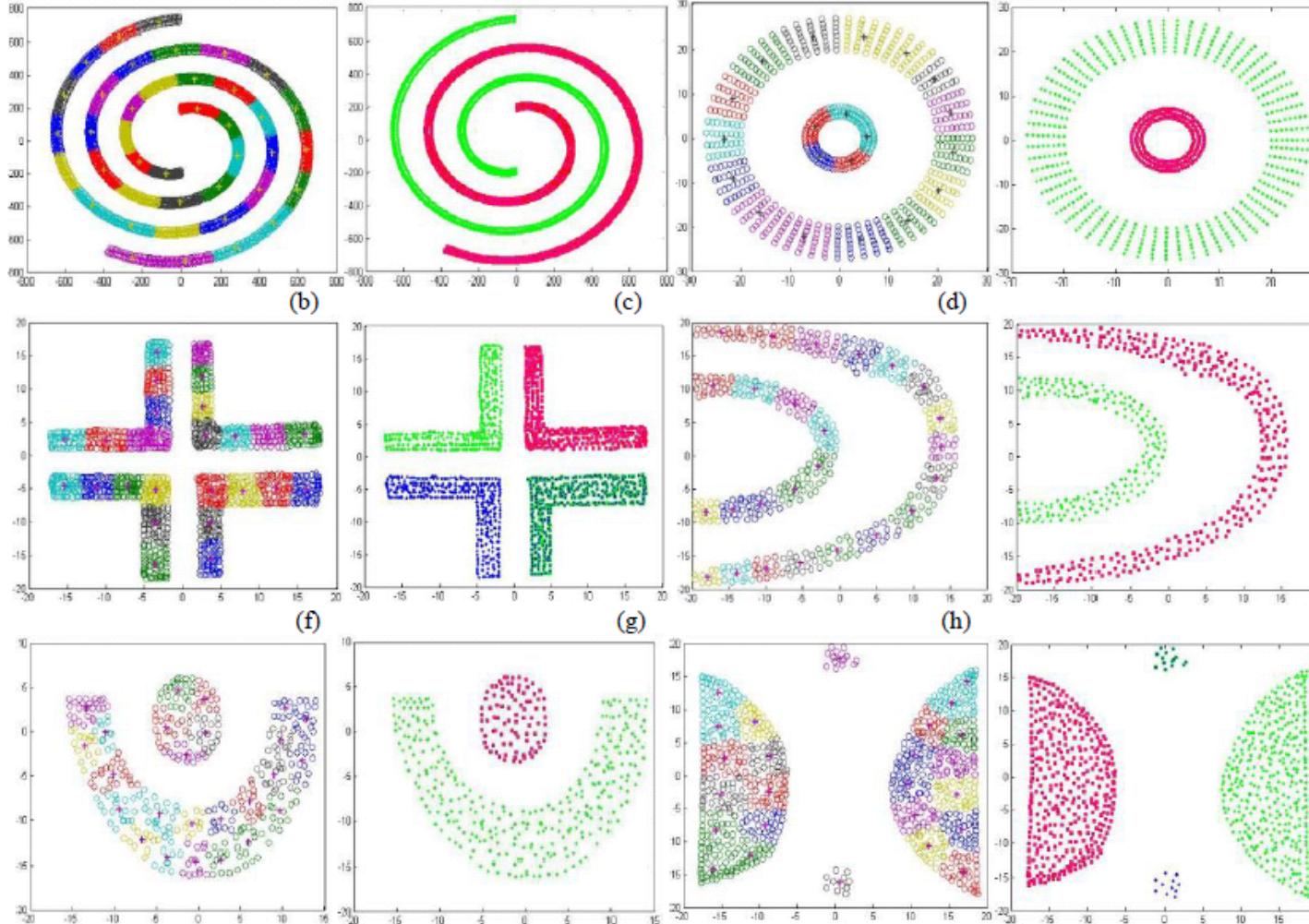
In between



Other (challenging!) clusters to discover...



Other (challenging!) clusters to discover...

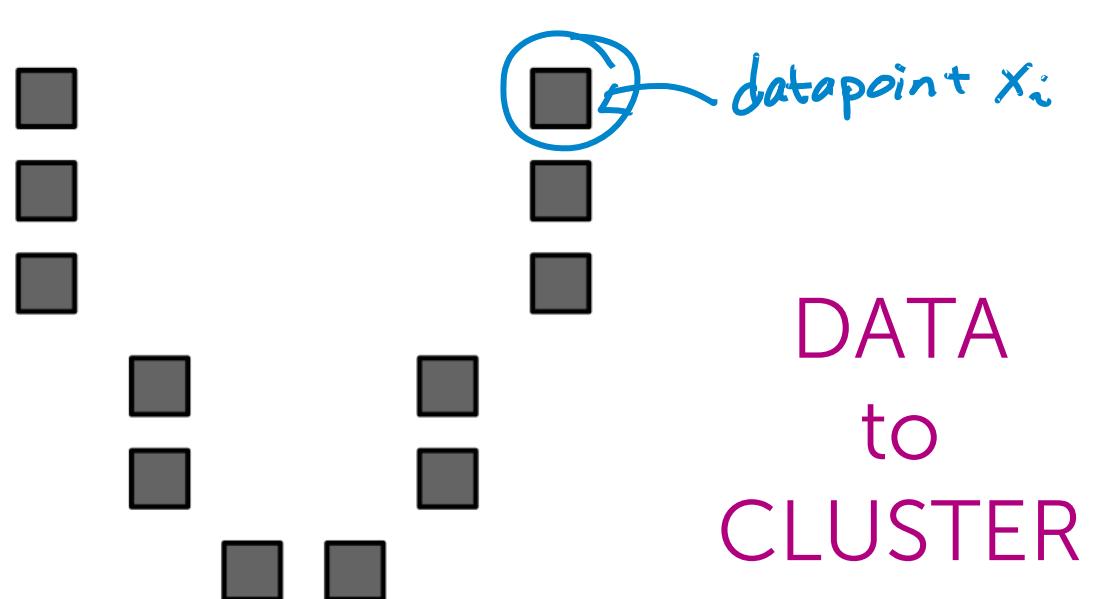


k-means: A clustering algorithm

k-means

Assume

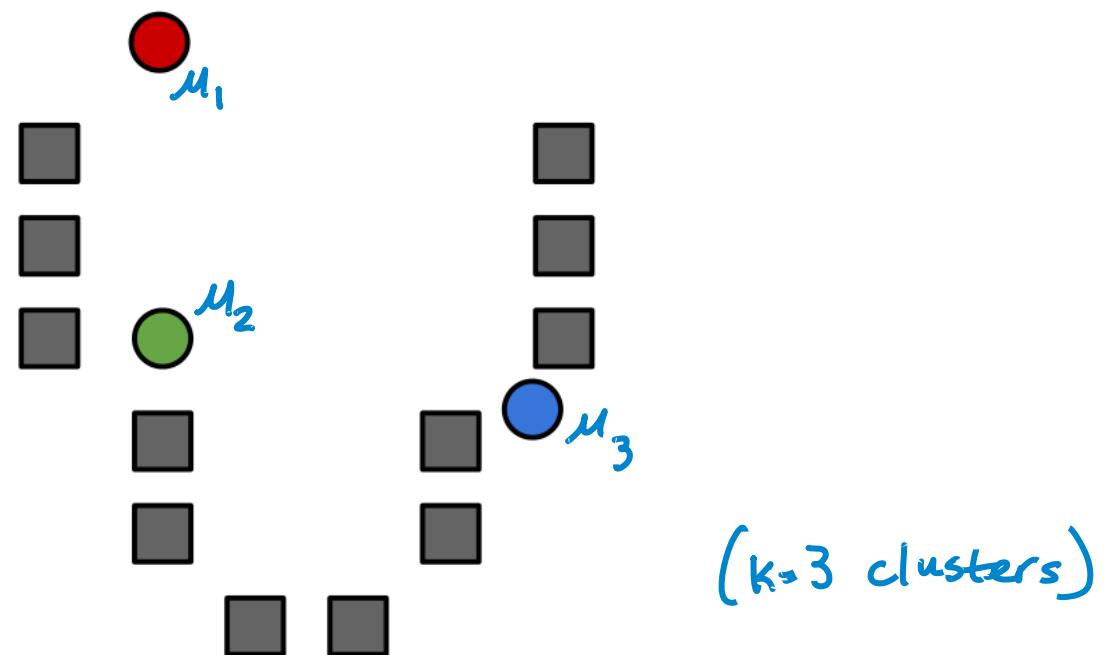
- Score = distance to cluster center
(smaller better)



k-means algorithm

0. Initialize cluster centers

$$\mu_1, \mu_2, \dots, \mu_k$$



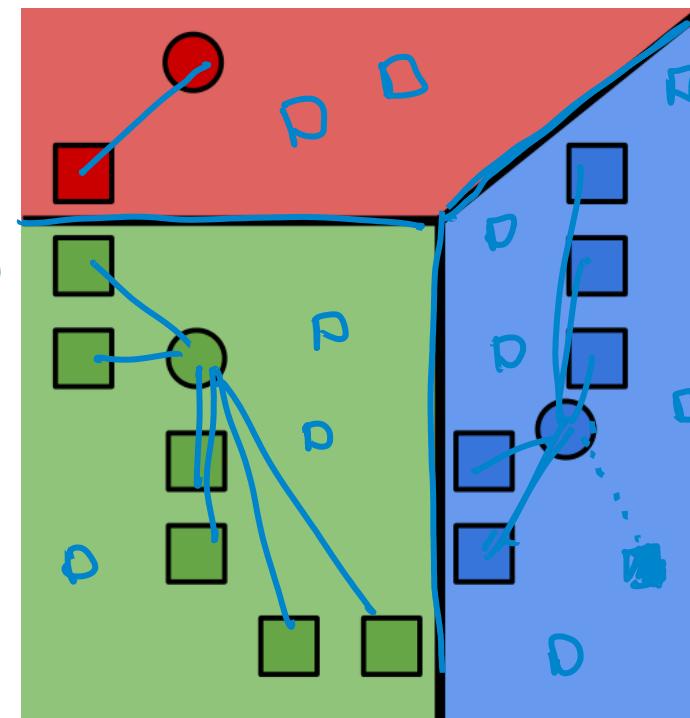
k-means algorithm

0. Initialize cluster centers
1. Assign observations to closest cluster center

$$z_i \leftarrow \arg \min_j \|\mu_j - \mathbf{x}_i\|_2^2$$

Inferred label for obs i, whereas supervised learning has given label y_i

return index j of the cluster whose center is closest to obs x_i (whereas min returning minimum value of $\|\cdot\|_2^2$)



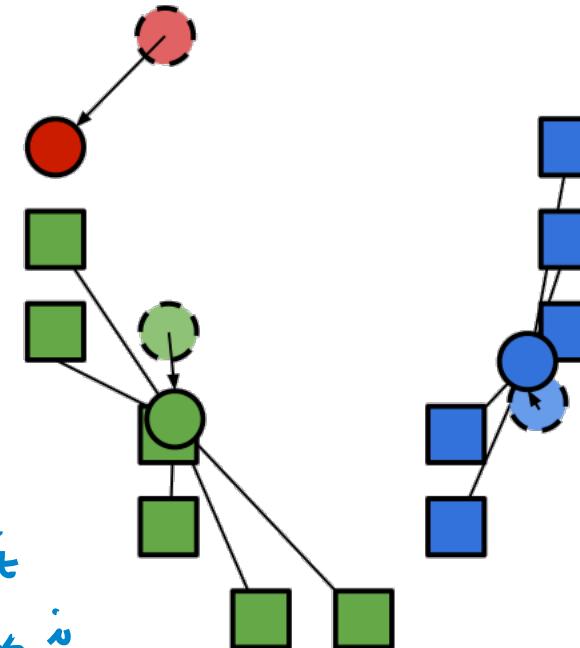
Voronoi tessellation
(for visualization only...
you don't
need to
compute this)

k-means algorithm

0. Initialize cluster centers
1. Assign observations to closest cluster center
2. Revise cluster centers as mean of assigned observations

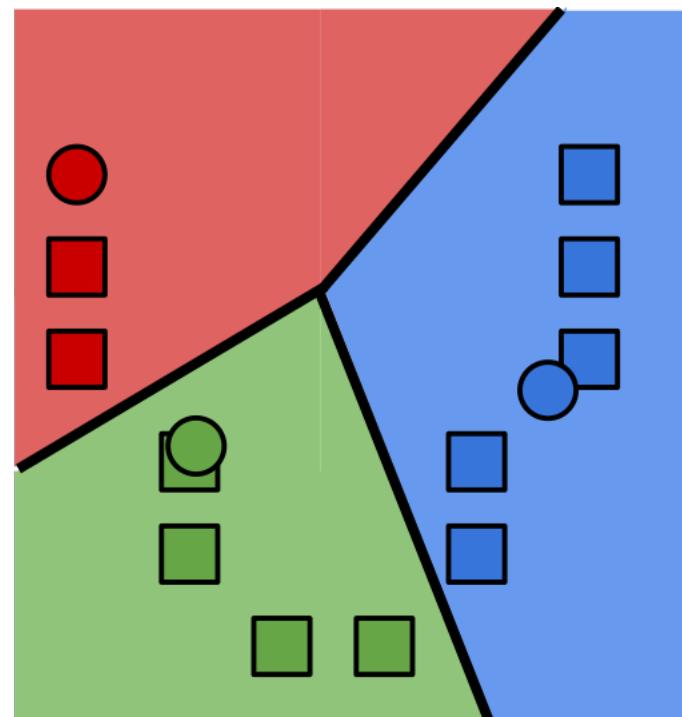
$$\underline{\underline{\mu_j}} = \frac{1}{n_j} \sum_{\substack{i: z_i=j \\ \text{\# of obs. in cluster } j}} \mathbf{x}_i$$

all obs. i such that $z_i=j$
 $(\text{obs. } i \text{ is in cluster } j)$



k-means algorithm

0. Initialize cluster centers
1. Assign observations to closest cluster center
2. Revise cluster centers as mean of assigned observations
3. Repeat 1.+2. until convergence



k-means as coordinate descent

A coordinate descent algorithm

1. Assign observations to closest cluster center

$$z_i \leftarrow \arg \min_j \|\mu_j - \mathbf{x}_i\|_2^2$$

2. Revise cluster centers as mean of assigned observations

$$\mu_j = \frac{1}{n_j} \sum_{i:z_i=j} \mathbf{x}_i$$

equivalent to

$$\mu_j \leftarrow \arg \min_{\mu} \sum_{i:z_i=j} \|\mu - \mathbf{x}_i\|_2^2$$

A coordinate descent algorithm

1. Assign observations to closest cluster center

$$z_i \leftarrow \arg \min_j \|\mu_j - \mathbf{x}_i\|_2^2$$

2. Revise cluster centers as mean of assigned observations

$$\mu_j \leftarrow \arg \min_{\mu} \sum_{i:z_i=j} \|\mu - \mathbf{x}_i\|_2^2$$

A coordinate descent algorithm

1. Assign observations to closest cluster center

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2. Revise cluster centers as mean of assigned observations

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Alternating minimization

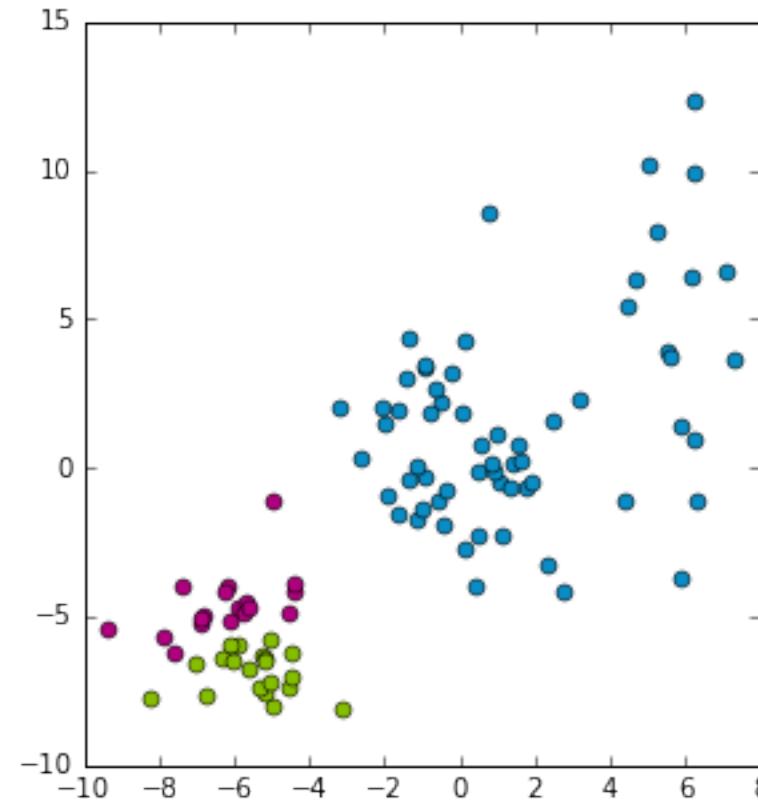
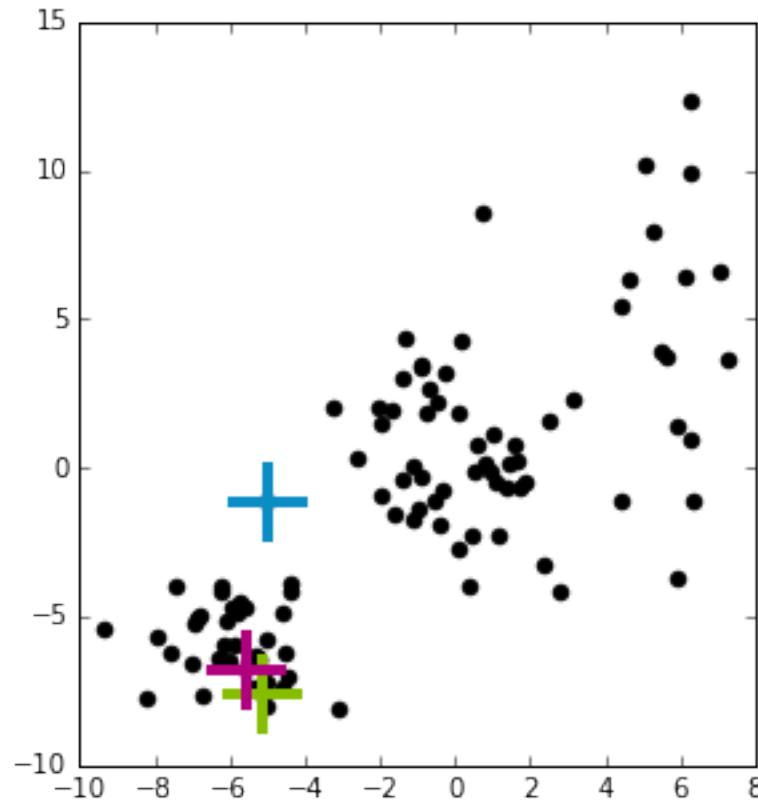
1. (\mathbf{z} given $\boldsymbol{\mu}$) and 2. ($\boldsymbol{\mu}$ given \mathbf{z})
= **coordinate descent**

Convergence of k-means

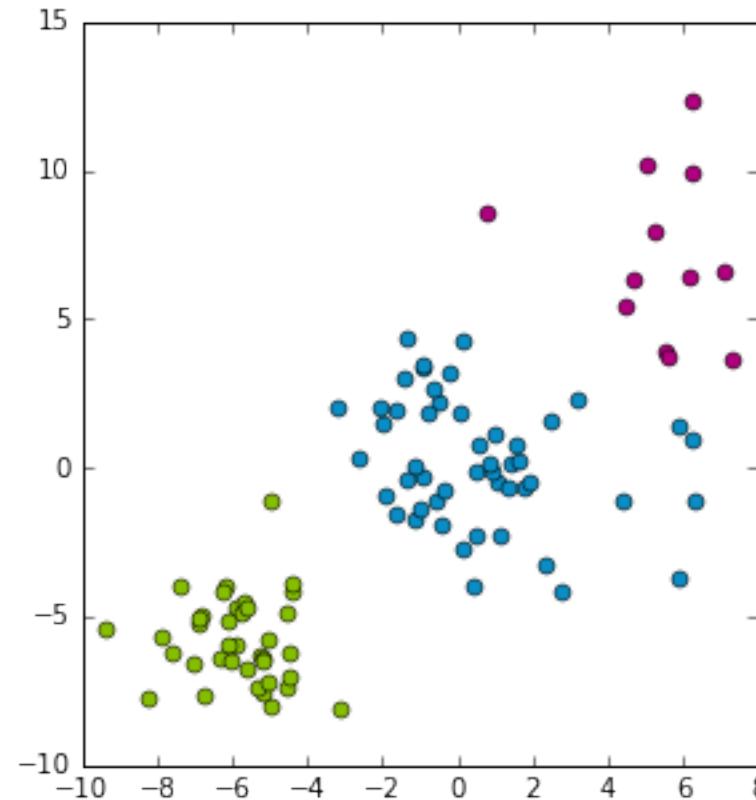
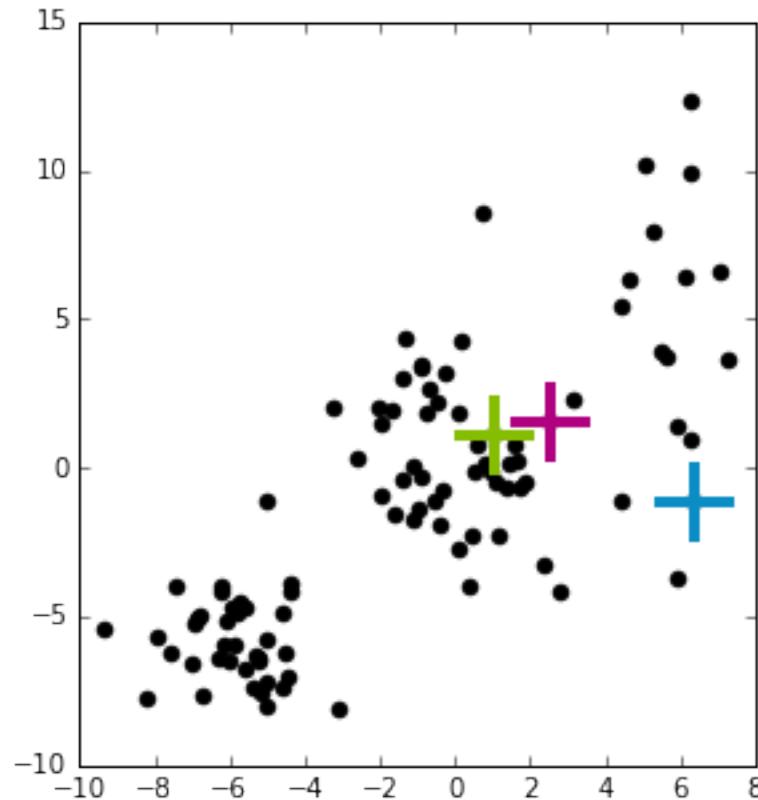
Converges to:

- Global optimum X
- Local optimum Local optimum
- neither X

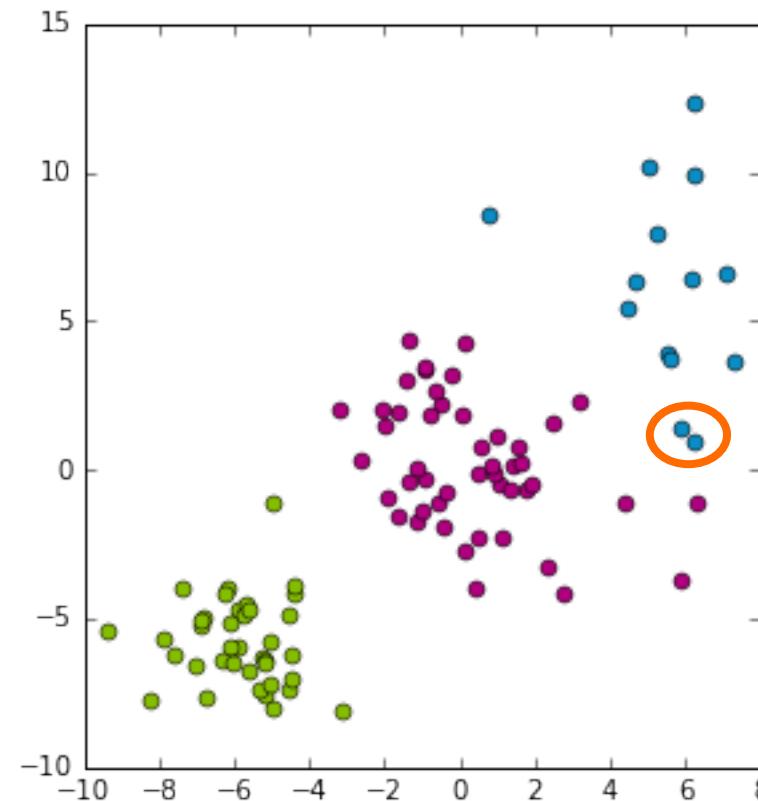
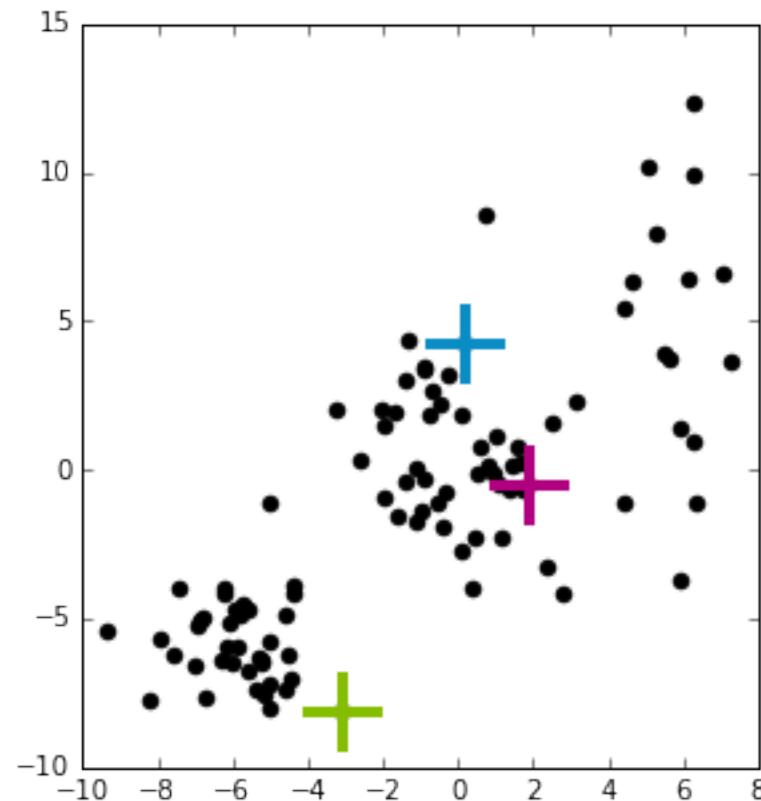
Convergence of k-means to local mode



Convergence of k-means to local mode



Convergence of k-means to local mode



Smart initialization with k-means++

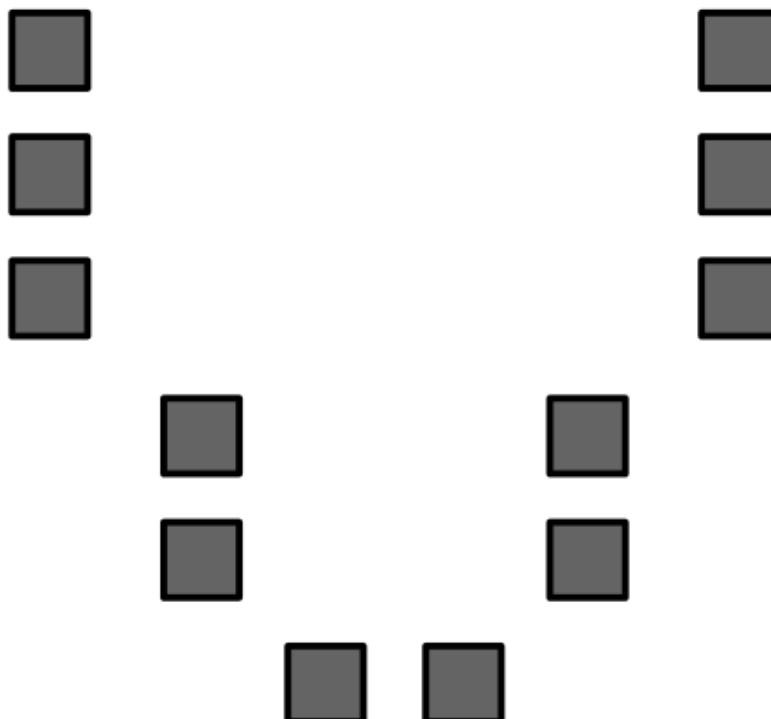
k-means++ overview

Initialization of k-means algorithm is critical to quality of local optima found

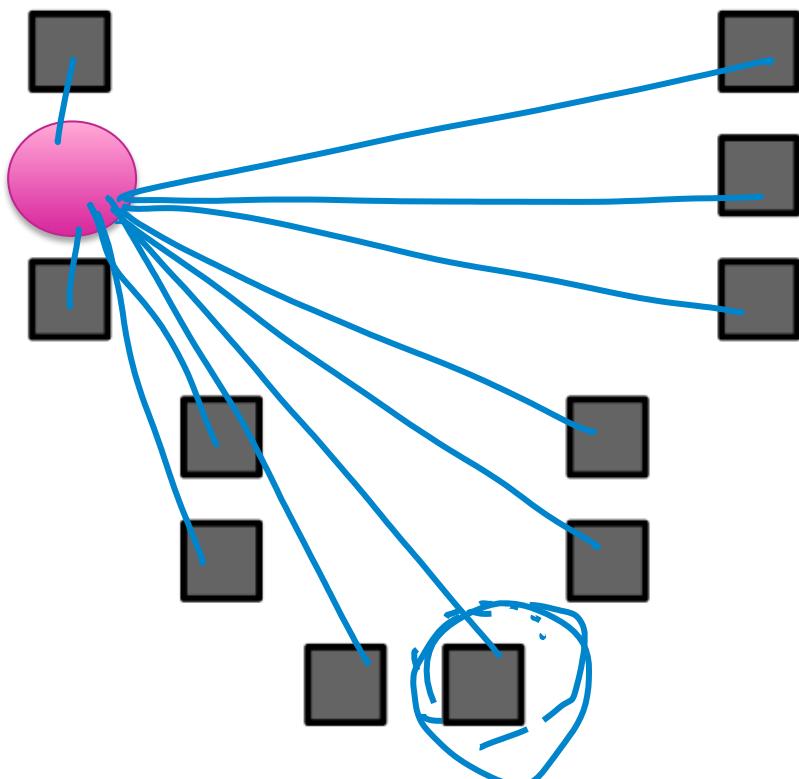
Smart initialization:

1. Choose first cluster center uniformly at random from data points
2. For each obs \mathbf{x} , compute distance $d(\mathbf{x})$ to nearest cluster center
3. Choose new cluster center from amongst data points, with probability of \mathbf{x} being chosen proportional to $d(\mathbf{x})^2$
4. Repeat Steps 2 and 3 until k centers have been chosen

k-means++ visualized

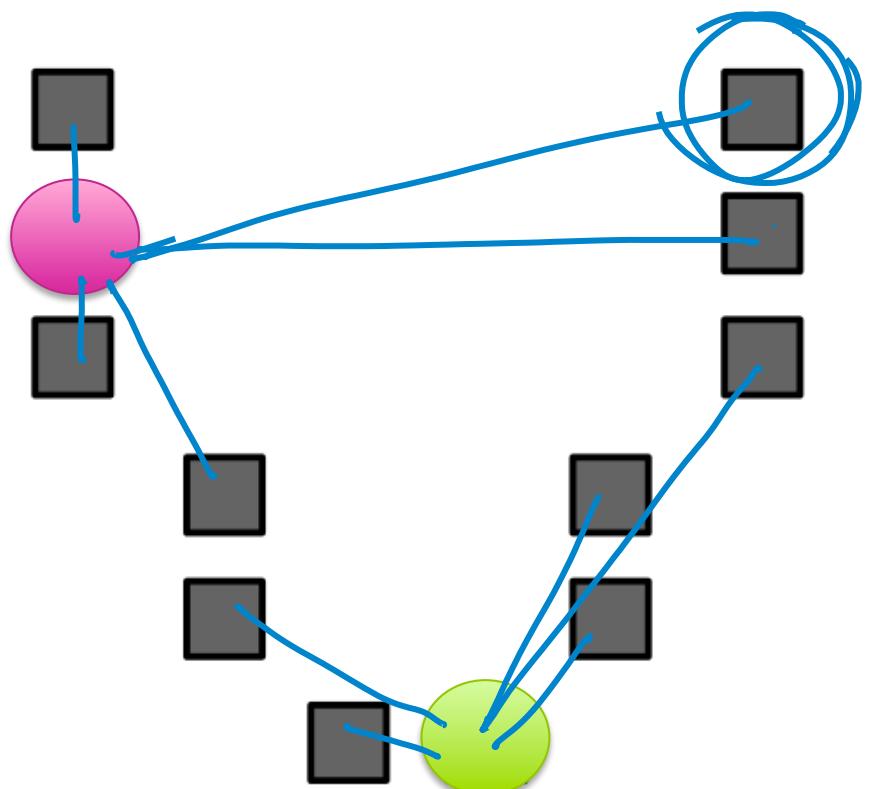


k-means++ visualized

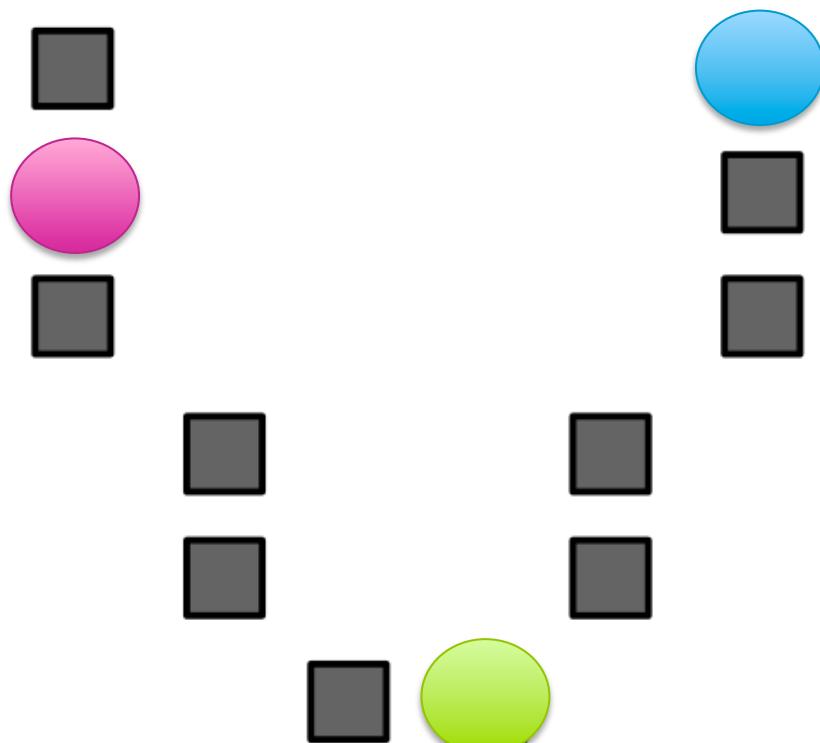


more likely to
select a datapoint
as a cluster center
if that datapoint is
far away
($dist^2$ increases
this effect)

k-means++ visualized



k-means++ visualized



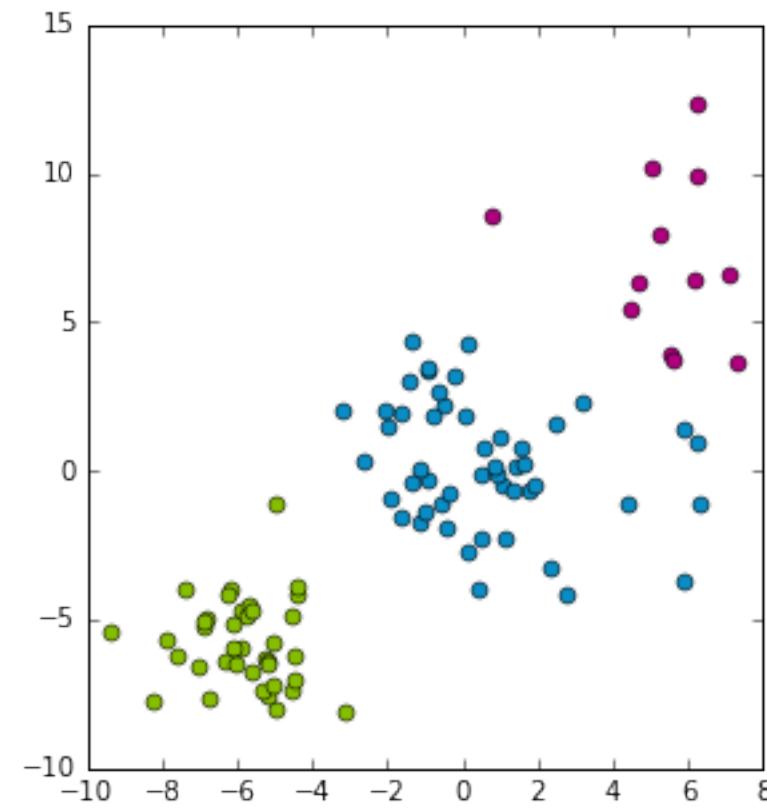
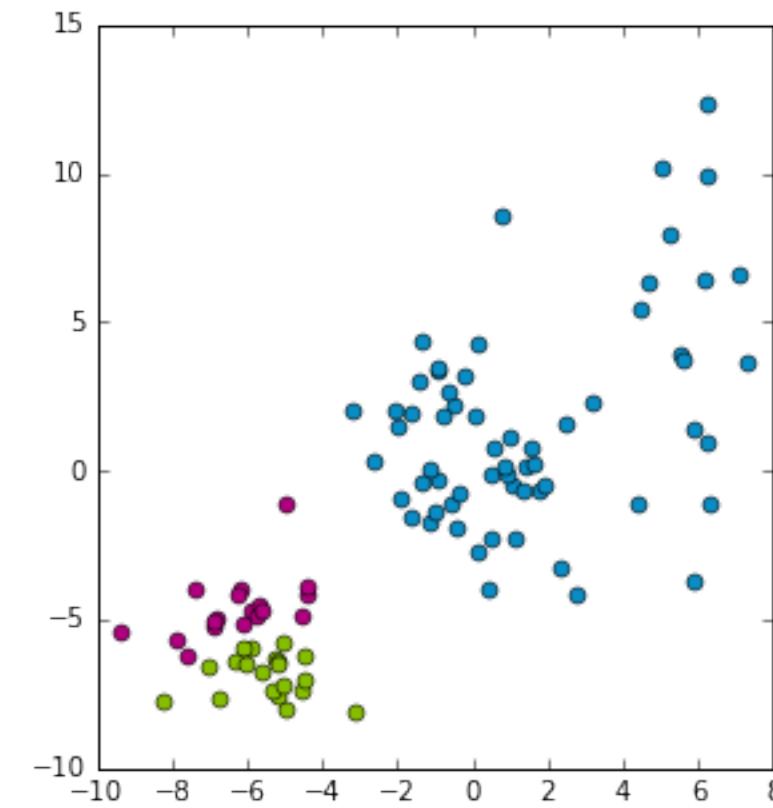
k-means++ pros/cons

Computationally costly relative to random initialization, but the subsequent k-means often converges more rapidly

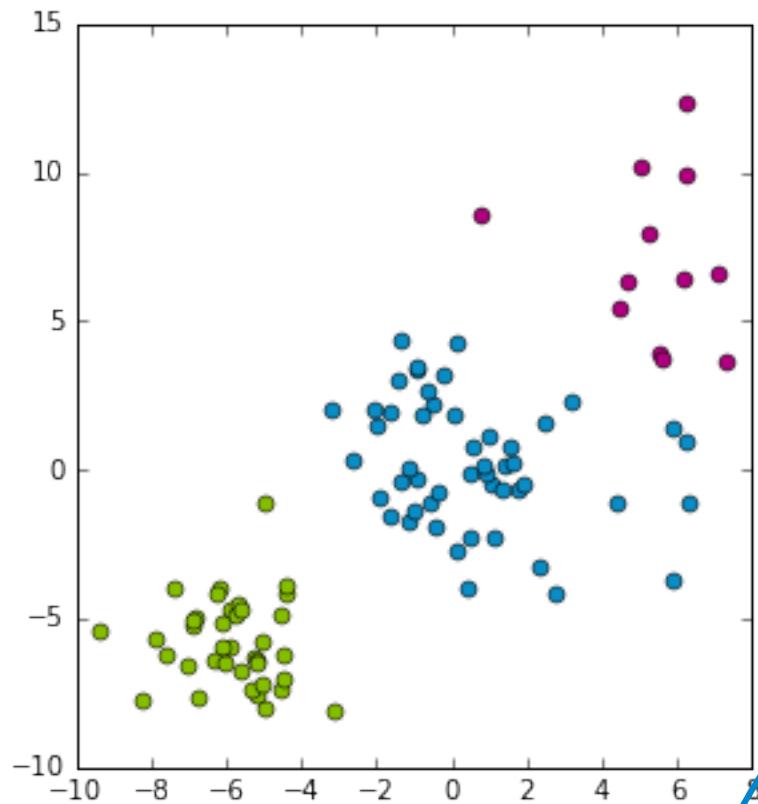
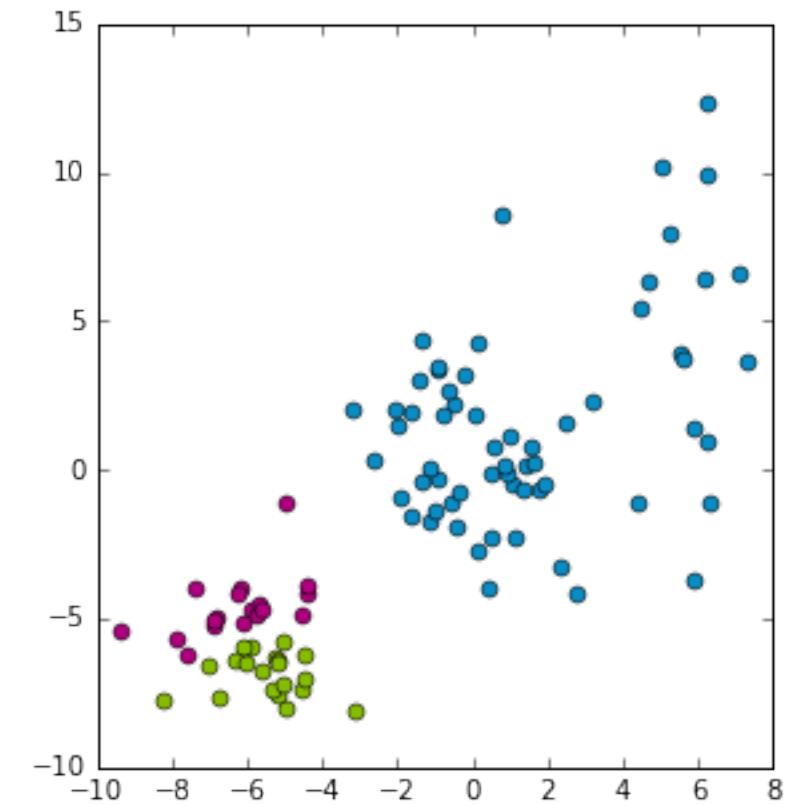
Tends to **improve** quality of local optimum and **lower** runtime

Assessing quality of the clustering and choosing the # of clusters

Which clustering do I prefer?



k-means objective



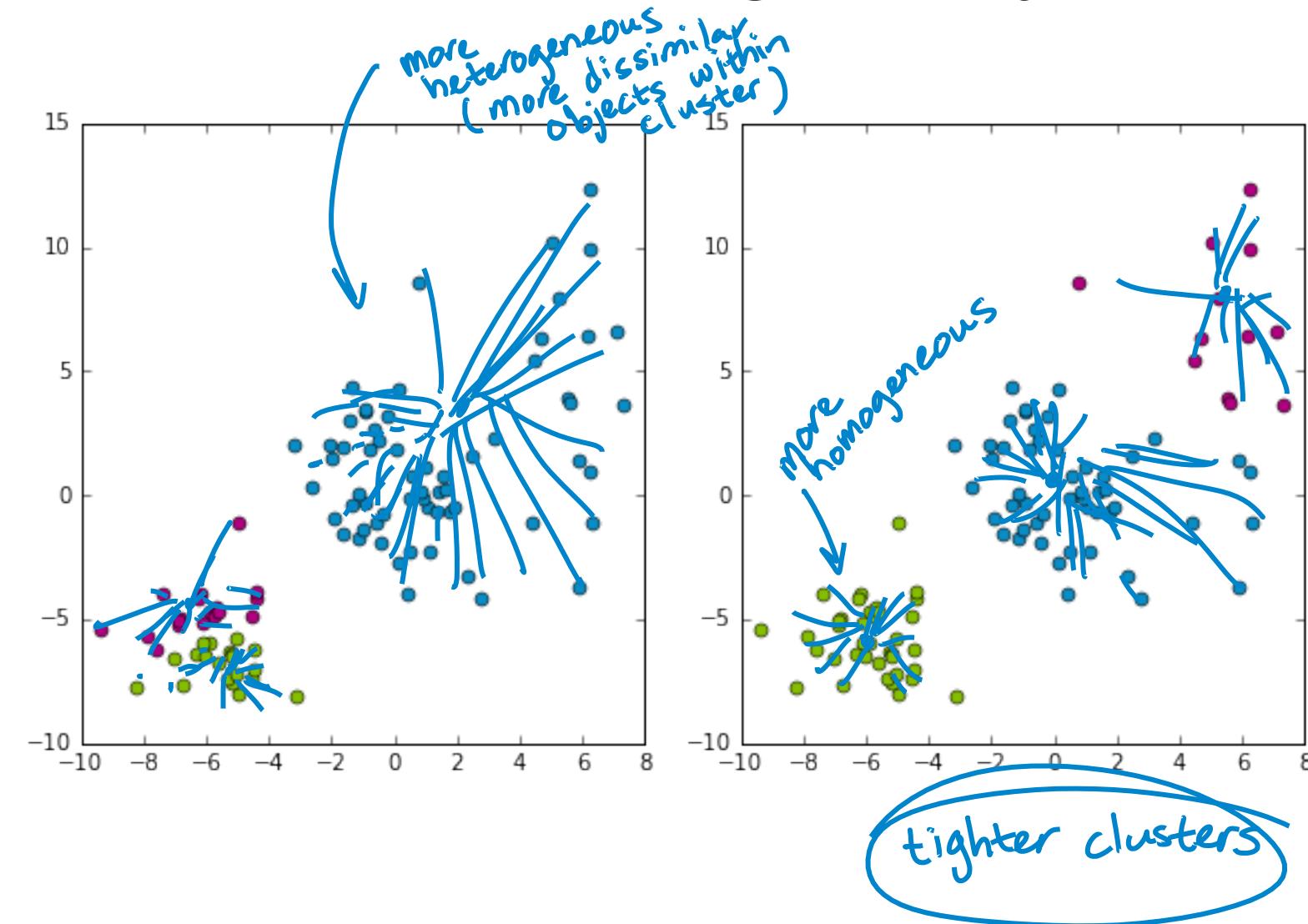
k-means is trying to minimize the **sum of squared distances**:

$$\min_{\{z_i\}, \{\mu_j\}} \sum_{j=1}^k \sum_{i:z_i=j} \|\mu_j - \mathbf{x}_i\|_2^2$$

Annotations explain the terms:

- k : sum over all clusters
- sum of squared distances in cluster j

Cluster heterogeneity



Measure of quality of
given clustering:

$$\sum_{j=1}^k \sum_{i:z_i=j} \|\mu_j - \mathbf{x}_i\|_2^2$$

Lower is better!

What happens as k increases?

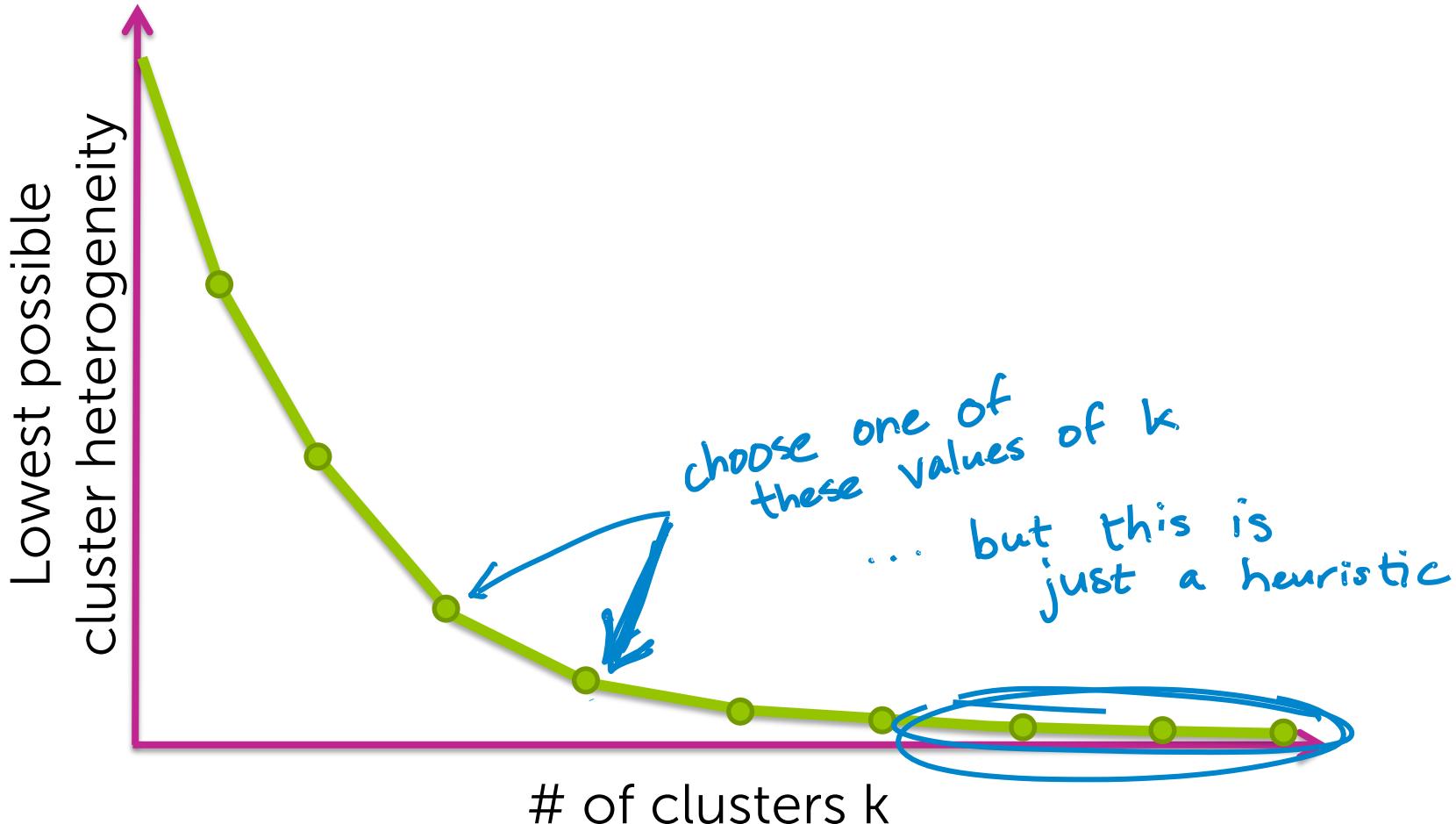
Can refine clusters more and more to the data
→ overfitting!

Extreme case of $k=N$:

- can set each cluster center equal to datapoint
- heterogeneity = 0 ! *(all distances to cluster centers are 0)*

Lowest possible cluster heterogeneity
decreases with increasing k

How to choose k?



MapReduce

Counting words on a single processor

(The “Hello World!” of MapReduce)

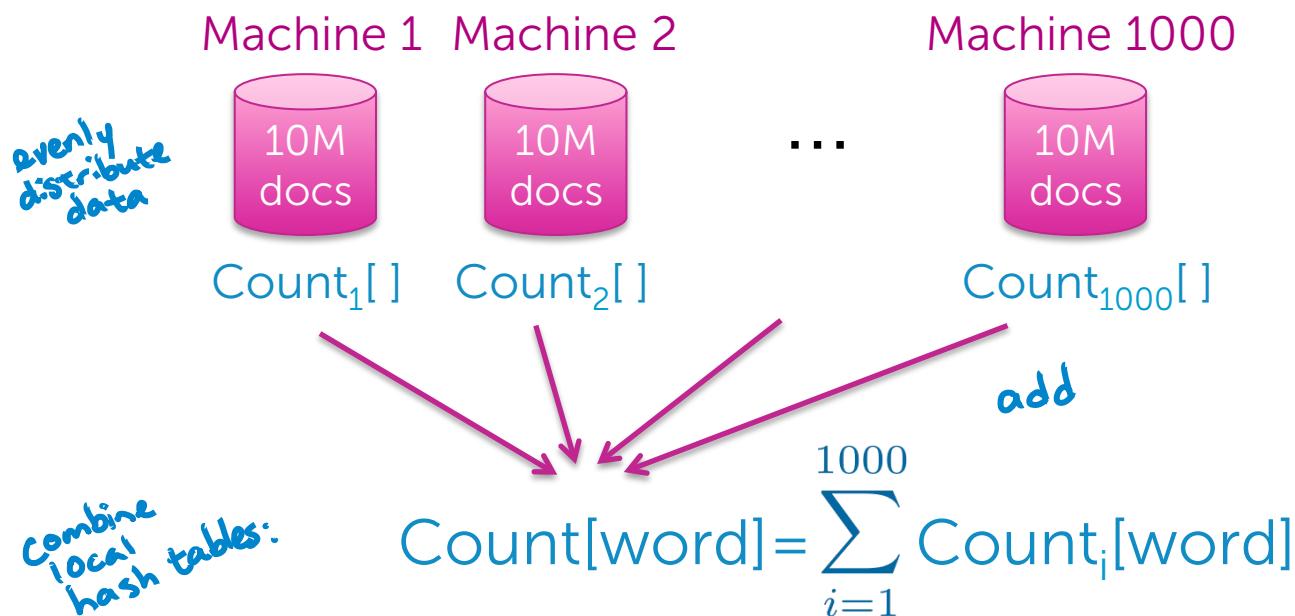
Suppose you have 10B documents and 1 machine and
want to count the # of occurrences of each word in the corpus

Code:

```
count[ ] ← init hash table
for d in documents
    for word in d
        count[word] += 1
```

Naïve parallel word counting

- Word counts are independent across documents (**data parallel**)
- Count occurrences in sets of documents separately, then merge



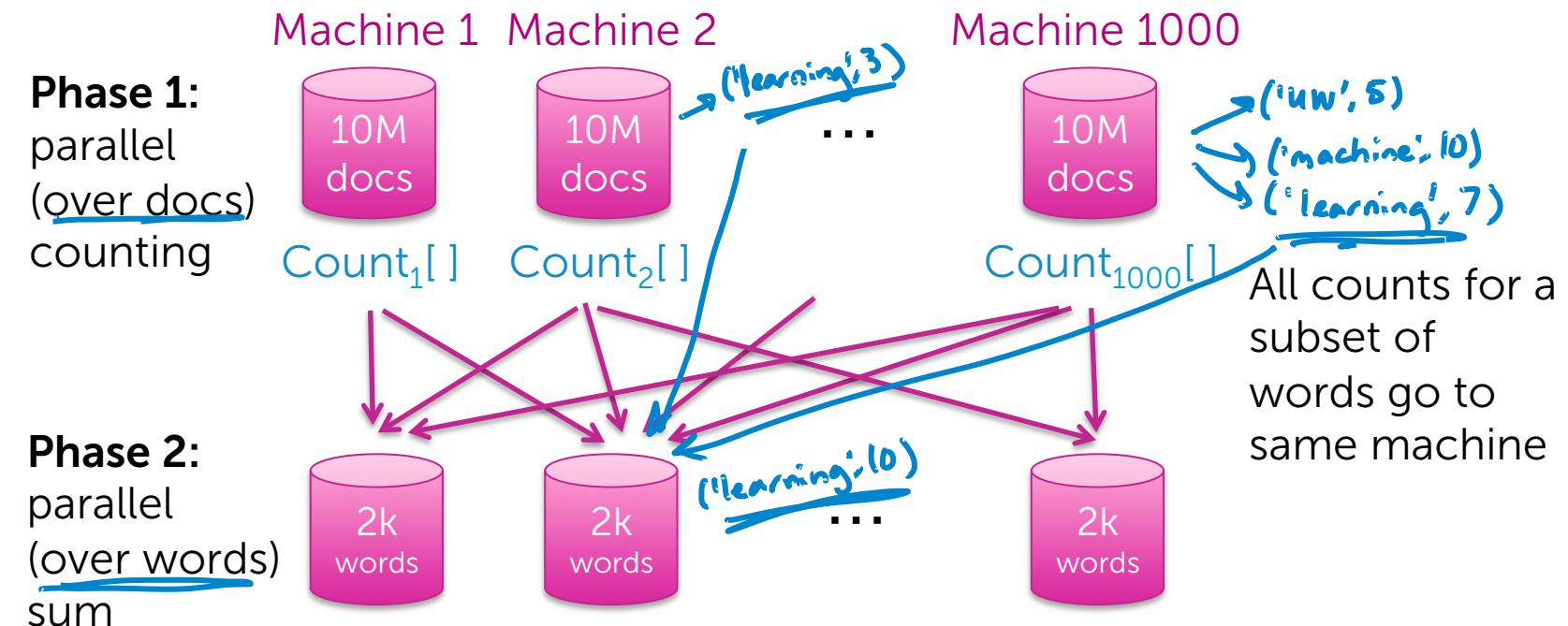
How do we do this for all words in vocab?

Back to sequential problem to merge counts...

have to cycle through
all words in vocab... ugh.

Counting words in parallel & merging tables in parallel

1. Generate pairs (word,count) in parallel
2. Merge counts for each word in parallel



Which words go to machine i ?
 $h: V \rightarrow [1, 2, \dots, \# \text{machines}]$

Send counts of 'learning' to machine
 $h["learning"]$

How to map words to machines? Use a hash function!

$h(\text{word index}) \rightarrow \text{machine index}$

MapReduce abstraction

Map:

- Data-parallel over elements,
e.g., documents
- Generate (key,value) pairs
 - “value” can be any data type

$('uw', 1)$
 $('machine', 1)$
 $('uw', 1)$
 $('learning', 1) \dots$

Reduce:

- Aggregate values for each key
- Must be commutative-associative operation
 $a+b = b+a$ $(a+b)+c = a+(b+c)$
- Data-parallel over keys
- Generate (key,value) pairs

$\text{reduce}('uw', [1, 17, 0, 0, 12, 2])$
 $\text{emit}('uw', 32)$

Word count example:

`map(doc)`

for `word` in `doc`
`emit(word, 1)`

\uparrow key
 \uparrow value

\downarrow key
 \downarrow list of values

`reduce(word, counts_list)`

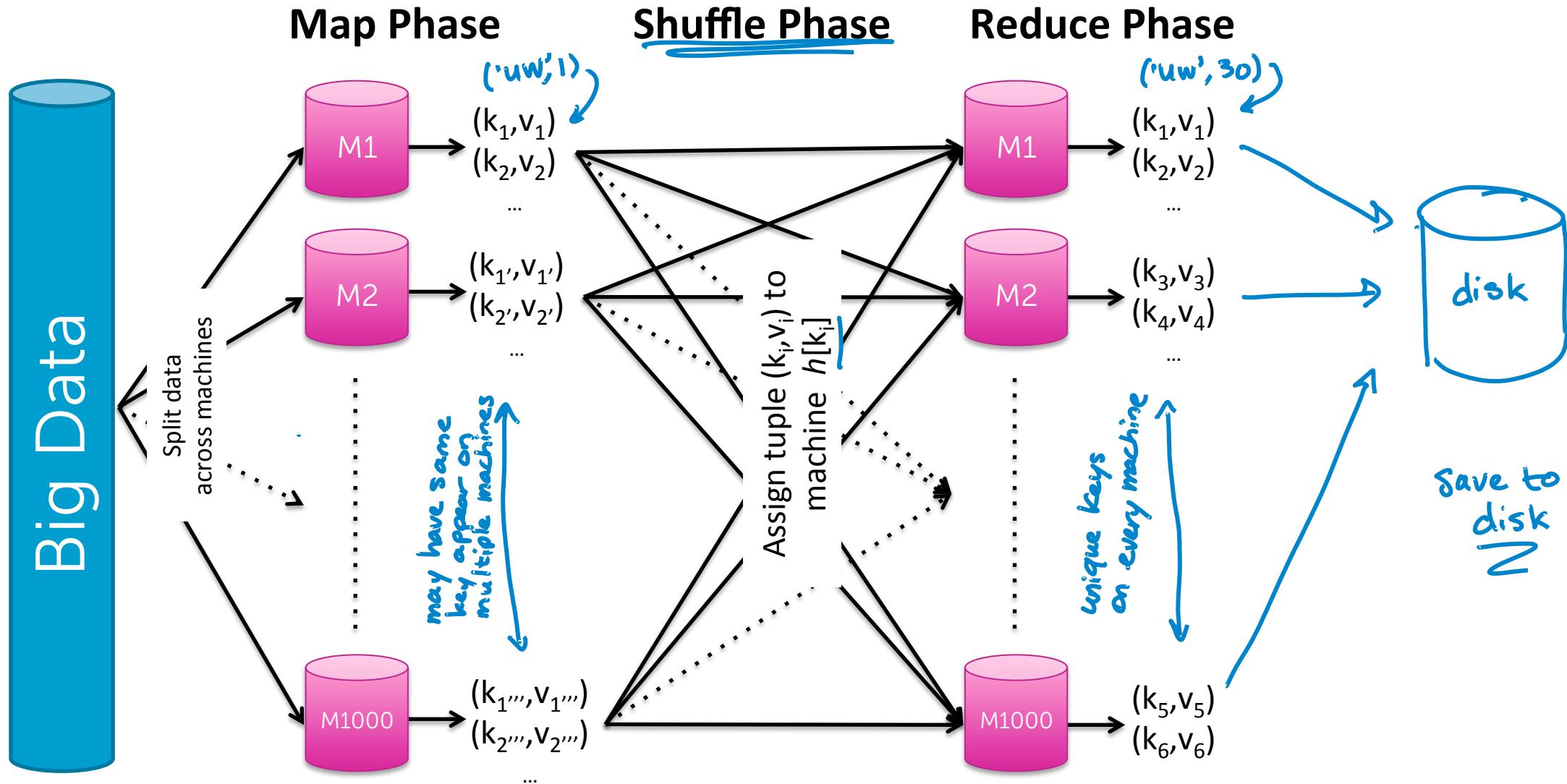
$c = 0$
for i in `counts_list`
 $c += \text{counts_list}[i]$
`emit(word, c)`

\uparrow key
 \uparrow value

MapReduce has long history in functional programming

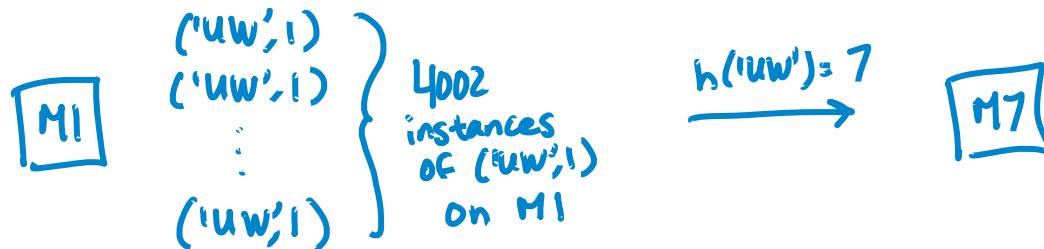
- Popularized by Google, and subsequently by open-source Hadoop implementation from Yahoo!

MapReduce – Execution overview

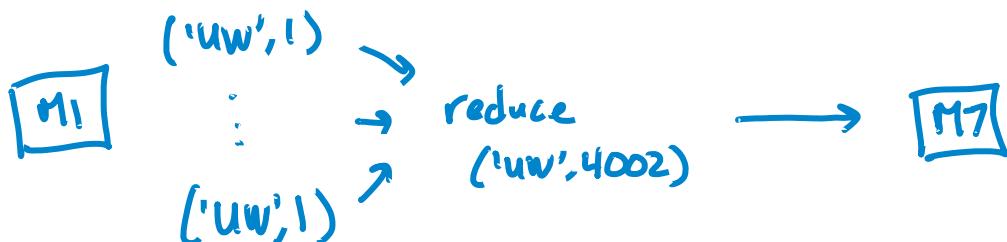


Improving performance: Combiners

- Naïve implementation of MapReduce is very wasteful in communication during shuffle:



- **Combiner:** Simple solution...Perform reduce locally before communicating for global reduce
 - Works because reduce is commutative-associative



Scaling up k-means via MapReduce

MapReducing 1 iteration of k-means

Classify: Assign observations to closest cluster center

$$z_i \leftarrow \arg \min_j \|\mu_j - \mathbf{x}_i\|_2^2$$

Map: For each data point, given $(\{\mu_j\}, \mathbf{x}_i)$, emit(z_i, \mathbf{x}_i)

Recenter: Revise cluster centers as mean of assigned observations

$$\mu_j = \frac{1}{n_j} \sum_{i:z_i=k} \mathbf{x}_i$$

Reduce: Average over all points in cluster j ($z_i=k$)

Classification step as Map

Classify: Assign observations to closest cluster center

$$z_i \leftarrow \arg \min_j \|\mu_j - \mathbf{x}_i\|_2^2$$

map([\mathbf{\mu}_1, \mathbf{\mu}_2, \dots, \mathbf{\mu}_k], \mathbf{x}_i)

set of cluster centers
a datapoint

$$z_i \leftarrow \arg \min_j \|\mu_j - \mathbf{x}_i\|_2^2$$

emit(z_i, x_i)
cluster label
datapoint

e.g. emit (2, [17, 0, 1, 7, 0, 0, 5])
z_i=2 (assigned to cluster 2)
datapoint x_i

Recenter step as Reduce

Recenter: Revise cluster centers as mean of assigned observations

$$\mu_j = \frac{1}{n_j} \sum_{i:z_i=k} \mathbf{x}_i$$

reduce(j , $\mathbf{x}_{\text{in_cluster}j}$: $[\mathbf{x}_1, \mathbf{x}_3, \dots]$)
 $\text{sum} = 0$ ← total mass in cluster
 $\text{count} = 0$ ← total # of obs. in cluster

for \mathbf{x} in $\mathbf{x}_{\text{in_cluster}j}$

$\text{sum} += \mathbf{x}$

$\text{count} += 1$

emit(j , sum/count)

 ↑ cluster label ↑ total mass
 ↓ total # obs

Some practical considerations

k-means needs an iterative version of MapReduce

- Not standard formulation

Mapper needs to get data point and all centers

- A lot of data!
- Better implementation:
mapper gets many data points

Summary of parallel k-means using MapReduce

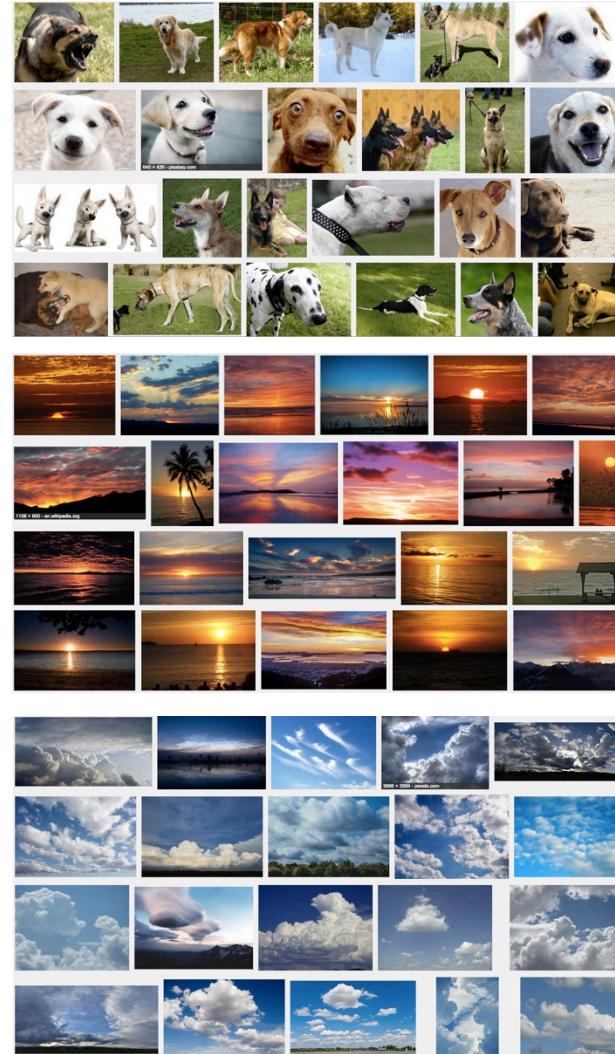
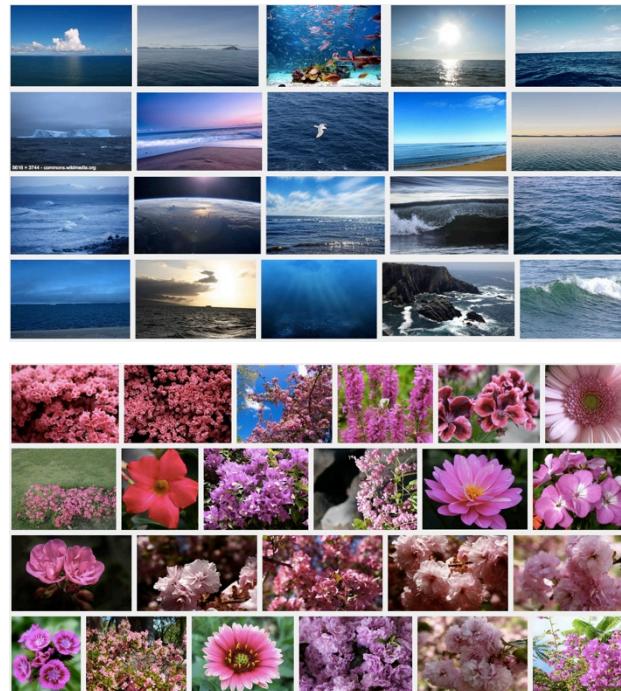
Map: classification step;
data parallel over data points

Reduce: recompute means;
data parallel over centers

Other examples

Clustering images

- For search, group as:
 - Ocean
 - Pink flower
 - Dog
 - Sunset
 - Clouds
 - ...



Structuring web search results

- Search terms can have multiple meanings
- Example: “**cardinal**”

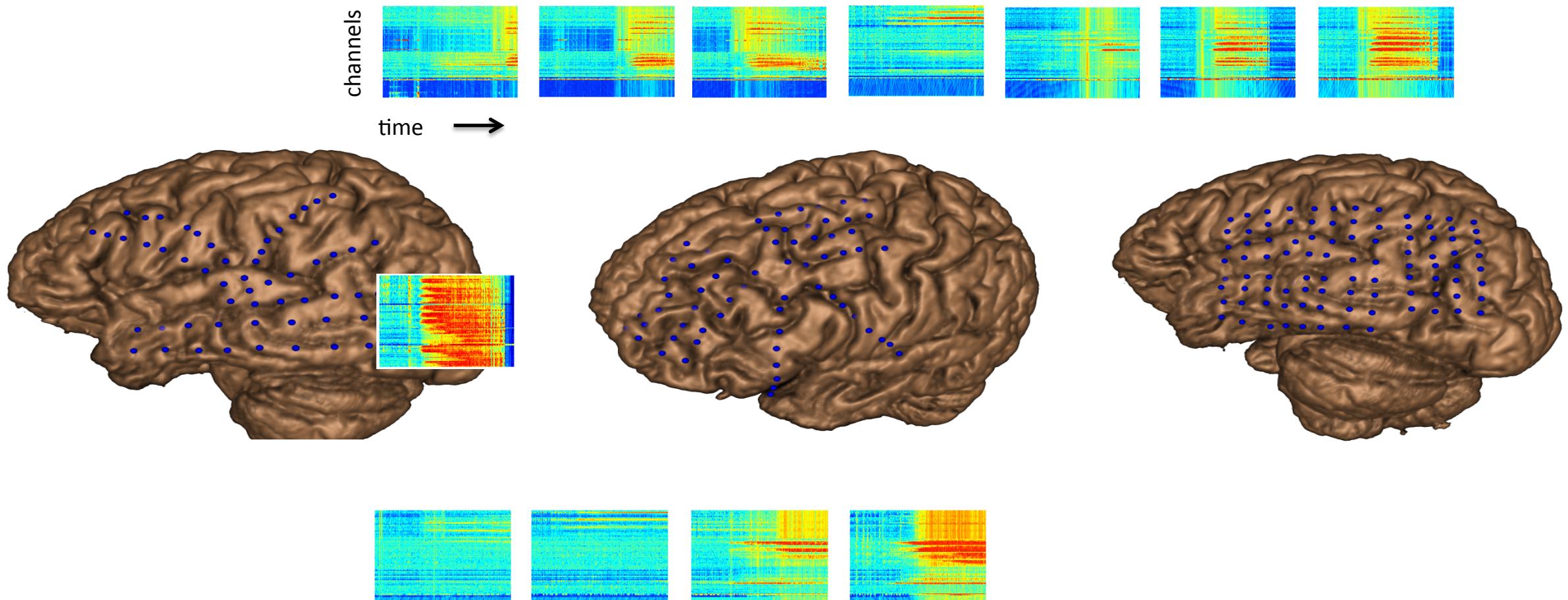


- Use clustering to **structure output**

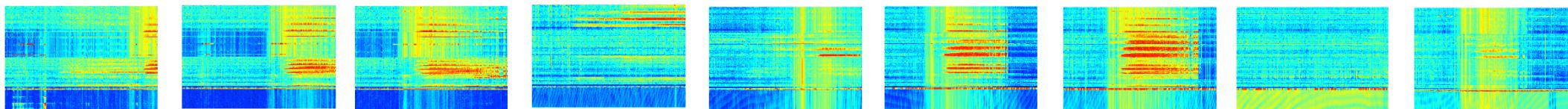
Grouping patients by medical condition

- Better characterize subpopulations and diseases

Example: Patients and seizures are diverse



Cluster seizures by observed time courses



Products on Amazon

- Discover product categories from purchase histories



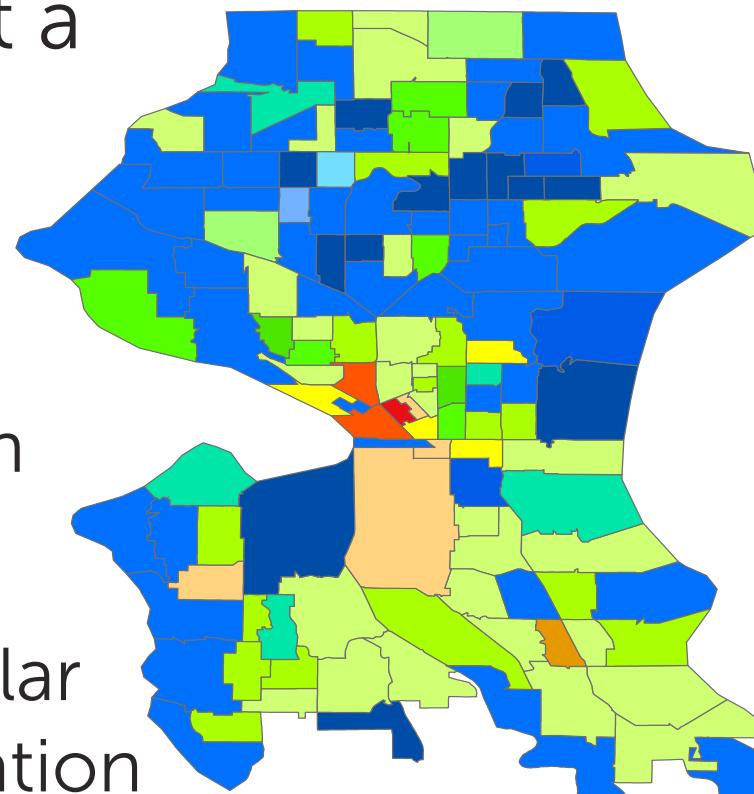
~~"furniture"~~
"baby"



- Or discovering groups of **users**

Discovering similar neighborhoods

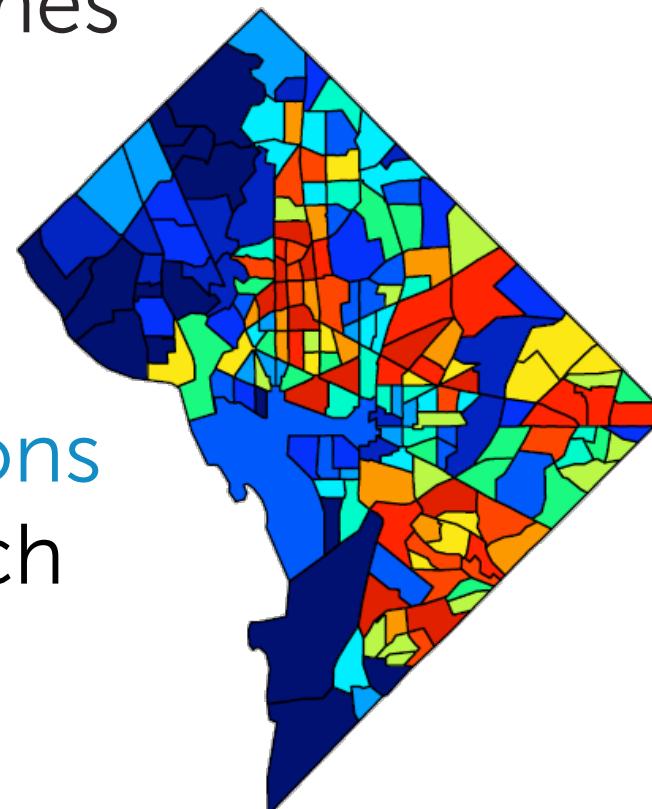
- **Task 1:** Estimate price at a small regional level
- **Challenge:**
 - Only a few (or no!) sales in each region per month
- **Solution:**
 - Cluster regions with similar trends and share information within a cluster



City of Seattle

Discovering similar neighborhoods

- **Task 2:** Forecast violent crimes to better task police
- Again, cluster regions and share information!
- Leads to improved predictions compared to examining each region independently



Washington, DC

Summary for k-means and MapReduce

What you can do now...

- Describe potential applications of clustering
- Describe the input (unlabeled observations) and output (labels) of a clustering algorithm
- Determine whether a task is supervised or unsupervised
- Cluster documents using k-means
- Interpret k-means as a coordinate descent algorithm
- Define data parallel problems
- Explain Map and Reduce steps of MapReduce framework
- Use existing MapReduce implementations to parallelize k-means, understanding what's being done under the hood