CS 70 Discrete Mathematics and Probability Theory
Spring 2016 Walrand and Rao

HW 13

Due Thursday April 28 at 10PM

Before you start your homework, write down your team. Who else did you work with on this homework? List names and student ID's. (In case of hw party, you can also just describe the group.) How did you work on this homework? Working in groups of 3-5 will earn credit for your "Sundry" grade.

1. **Limiting Distribution**

   This problem invites you to test your understanding of the limiting distribution of a Markov chain.

   (a) (5 points) Construct a Markov chain that is not irreducible but that has a unique distribution and is such that its distribution converges to that unique invariant distribution, for any initial distribution.

   (b) (5 points) Show a Markov chain whose distribution converges to a limit that depends on the initial distribution.

2. **Aperiodicity**

   In this problem, we explore the concept of aperiodicity.

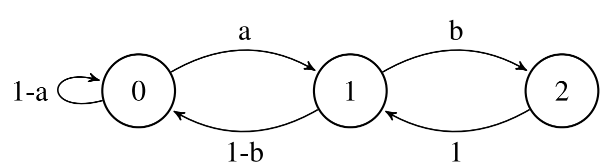
   (a) (5 points) Can you find a finite irreducible aperiodic Markov chain whose distribution does not converge?

   (b) (5 points) Construct a finite Markov chain that is a sequence of i.i.d. random variables. Is it necessarily irreducible and aperiodic?

3. **Function of a Markov Chain** (5 points)

   Show that a function $Y(n) = g(X(n))$ of a Markov chain $X(n)$ may not be a Markov chain.

4. **Analyze a Markov Chain** (20 points)



   Consider the Markov chain $X(n)$ with the state diagram shown above.

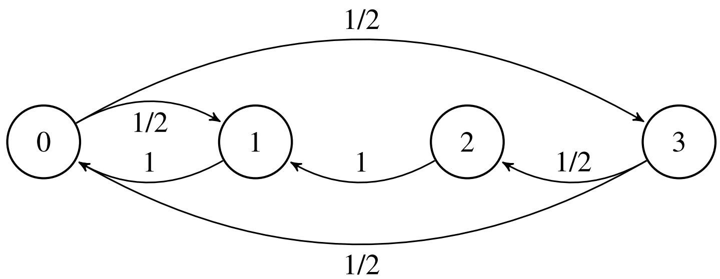
   a) Show that this Markov chain is aperiodic;

   b) Calculate $P[X(1) = 1, X(2) = 0, X(3) = 0, X(4) = 1 \mid X(0) = 0]$;

   c) Calculate the invariant distribution;

   d) Let $T_i = \min\{n \geq 0 \mid X(n) = i\}$. Calculate $E[T_2 \mid X(0) = 1]$.

5. **Period of States** (10 points)



Calculate explicitly $d(0), d(1), d(2)$ and $d(3)$, defined as

$$d(i) = \text{g.c.d}\{n > 0 \mid Pr[X_n = i | X_0 = i] > 0\}$$

for the Markov chain pictured above. That is, for each state i, identify the set $\{n > 0 \mid Pr[X_n = i | X_0 = i] > 0\}$ and finds its g.c.d.