# S.E. (Computer/Information Technology)
# ENGINEERING MATHEMATICS - III
## (2019 Pattern) (Semester - IV)

ne : 2 ½Hours] [Max. Marks : 70

*structions to the candidates:*

1) *Q.1 is compulsory.*
2) *Attempt Q2 or Q.3, Q4 or Q5, Q6 or Q7, Q8 or Q9.*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Figures to the right indicate full marks.*
5) *Use of electronic pocket calculator is allowed.*
6) *Assume suitable data, if necessary.*

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1) Write the correct option for the following multiple choice questions.

a) For a given set of bivariate data, $\bar{x} = 2, \bar{y} = -3$. The regression coefficient of x on y is –0.11. By using the regression equation of x on y, the most probable value of x when y=0 is _____. [2]

i) 0.57 ii) 0.87

iii) 0.77 iv) 1.77

b) If Probability density function $f(x)$ of a continuous random variable $x$ is defined by

$$f(x) = \begin{cases} \frac{1}{4}, -2 \le x \le 2 \\ 0, \text{otherwise} \end{cases}$$

then $P(x \le 1)$ is _____. [2]

i) $\frac{1}{4}$ ii) $\frac{1}{2}$

iii) $\frac{1}{3}$ iv) $\frac{3}{4}$

P.T.O.

c) Lagrange's polynomial through the points

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $y$ | 4 | 0 | 6 |

is given by _____. [2]

i) $y = 5x^2-3x+4$

ii) $y = 5x^3+3x+4$

iii) $y = 5x^2-9x+4$

iv) $y = x^2-9x+4$

d) Using Gauss elimination method, the solution of system of equations

$x+\frac{1}{4}y+\frac{1}{4}z=1, \frac{15}{4}y-\frac{9}{4}z=3, \frac{5}{4}y-\frac{19}{4}z=3$ is _____ [2]

i) $x=1, y=2, z=3$

ii) $x=\frac{1}{2}, y=1, z=\frac{1}{2}$

iii) $x=2, y=\frac{1}{2}, z=2$

iv) $x=1, y=\frac{1}{2}, z=-\frac{1}{2}$

e) The first four central moments of a distribution are 0,16,–64 and 162. The coefficient of Kurtosis $\beta_2$ is _____. [1]

i) 1.20

ii) 0.6328

iii) 1

iv) 0.3286

f) If $f(x)$ is continuous on $[a,b]$ and $f(a)f(b)<0$. then to find a root of $f(x)=0$, initial approximation $x_0$ by bisection method is _____ [1]

i) $x_0 = \frac{a-b}{2}$

ii) $x_0 = \frac{f(a)+f(b)}{2}$

iii) $x_0 = \frac{a+b}{2}$

iv) $x_0 = \frac{a-b}{a+b}$

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Q2) a) If marks scored by five students in statistics test of 100 marks, are given in following table. [5]

| Student | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Marks(/100)$x$ | 46 | 34 | 52 | 78 | 65 |

Find standard deviation and arithmetic mean $\bar{x}$.

b) Fit a law of the form $y=ap+b$ by least square method for the data, [5]

| $p$ | 100 | 120 | 140 | 160 | 180 | 200 |
|---|---|---|---|---|---|---|
| $y$ | 0.9 | 1.1 | 1.2 | 1.4 | 1.6 | 1.7 |

c) If the two lines of regression are $9x+y-\lambda=0$ and $4x+y=\mu$ and the means of $x$ & $y$ are 2 & $-3$ respectively. Find values of $\lambda,\mu$ and correlation coefficient between $x$ & $y$. [5]

OR

Q3) a) The first four moments of a distribution about 5 are 2,20,40 and 50. Find first four moments about mean, and $\beta_1, \beta_2$ [5]

b) Fit a parabola $y=ax^2 + bx + c$, by using least square method to the following data, [5]

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | 2 | 2 | 4 | 8 |

c) Calculate the coefficient of correlation from the following information.

$n=10$, $\Sigma x=40$, $\Sigma x^2=190$, $\Sigma y^2=200$, $\Sigma xy=150$, $\Sigma y=40$. [5]

Q4) a) Bag 1 contains 2 white and 3 red balls. Bag 2 contains 4 white and 5 red balls. One ball is drawn randomly from bag 1 and is placed in bag2. Later, one ball is drawn randomly from bag2. Find the probability that it is red. [5]

b) The expected number of matches those will be won by India in a series of five one day matches between India and England is three. If the probability of India's win in each match remains the same and the results of all the five matches are independent of each other, find the probability that India wins the series, using Binomial distribution. Assume that each match ends with a result.

c) The lifetime of an article has a normal distribution with mean 400 hours and standard deviation 50 hours. Find the expected number of articles out of 2,000 whose lifetime lies between 335 hours to 465 hours. (Given : Z=1.3,A=0.4032) [5]

OR

25) a) Find the expected value of the number of heads obtained when three fair coins are tossed simultaneously. [5]

b) On an average, 180 cars per hour pass a specified point on a particular road. Using Poisson distribution, find the probability that at least two cars pass the point in any one minute. [5]

c) The proportions of blood types O,A,B and AB in the general population of a country are known to be in the ratio 49:38:9:4 respectively. A research team observed the frequencies of the blood types as 88,80,22 and 10 respectively in a community of that country. Test the hypothesis at 5% level of significance that the proportions for this community are in accordance with the general population of that country. (Given : $\chi^2_{tab}$=7.815) [5]

26) a) Find the root of the equation $x^4+2x^3-x-1=0$, lying in the interval [0,1] using the bisection method at the end of fifth iteration. [5]

b) Find a real root of the equation $x^3+2x-5=0$ by applying Newton-Raphson method at the end of fifth iteration. [5]

c) Solve by Gauss-Seidel method, the system of equations:

$$20x_1 + x_2 - 2x_3 = 17$$

$$3x_1 + 20x_2 - x_3 = -18$$

$$2x_1 - 3x_2 + 20x_3 = 25$$

[5]

OR

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Q7) a) Solve by Gauss elimination method, the system of equations:

$$2x_1 + x_2 + x_3 = 10$$

$$3x_1 + 2x_2 + 3x_3 = 18$$ [5]

$$x_1 + 4x_2 + 9x_3 = 16$$

b) Solve by Jacobi's iteration method, the system of equations:

$$4x_1 + 2x_2 + x_3 = 14$$

$$x_1 + 5x_2 - x_3 = 10$$ [5]

$$x_1 + x_2 + 8x_3 = 20$$

c) Use Regula-Falsi method to find a real root of the equation $e^x - 4x = 0$ correct to three decimal places. [5]

Q8) a) Using Newton's forward interpolation formula, find $y$ at $x=8$ from the following data.

| $x$ | 0 | 5 | 10 | 15 | 20 | 25 |
|---|---|---|---|---|---|---|
| $y$ | 7 | 11 | 14 | 18 | 24 | 32 |

[5]

b) Evaluate $\int_0^1 \frac{dx}{x^2+1}$

using Simpson's $\frac{1}{3}^{rd}$ rule. (Take $h=0.2$) [5]

c) Use Euler's method, to solve $\frac{dy}{dx} = x + y$, $y(0) = 1$

Tabulate values of $y$ for $x=0$ to $x=0.3$ (Take $h=0.1$) [5]

OR

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Q9) a) Use Runge-Kutta method of 4th order, to solve

$\frac{dy}{dx} = xy,\ y(1) = 2$ at $x$=1.2 with h=0.2. [5]

b) Using Modified Euler's method, find y(0.2),

given $\frac{dy}{dx} + xy^2 = 0,\ y(0) = 2$ Take h=0.2 (Two iterations only) [5]

c) Using Newton's backward difference formula, find the value of $\sqrt{155}$ from the following data

| $x$ | 150 | 152 | 154 | 156 |
|---|---|---|---|---|
| $y = \sqrt{x}$ | 12.247 | 12.329 | 12.410 | 12.490 |

[5]

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Jayawant Shikshan Prasarak Mandal's

# JSPM's Group of Institute

Name of Institute

Varified & all the entries found correct

Jr.Superviso's Name, Signature & Date

A-

Roll No. (In Figures)

Centre

Roll No. (In Words)

Day & Date :

Examination :

Subject :

Section :

Course / Paper No. :

Medium of Answer :

Main Ans. Book + No. Of Suppliments :

Total

| Question No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total | Sign.of Examiner |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Marks Obtained | | | | | | | | | | | | |

Use of coloured pencil or ink is strictly prohibited except in case of diagrams and sketches
( Write on both sides and start writing on this page. )

SE (Comp / IT)

M III

(2019 Pat) (sem II)

June - 2022

SOLUTION

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**Q.1)** Write correct option for the following multiple choice questions.

**a)** For a given set of bivariate data $\bar{x} = 2$, $\bar{y} = -3$ The regression coeff of $x$ on $y$ is $-0.11$. By using the regression eqn of $x$ on $y$, the most probable value of $x$ when $y = 0$ is

i) 0.57 ii) 0.87 iii) 0.77 iv) 1.67

Soln → Given $\bar{x} = 2$, $\bar{y} = -3$, $b_{xy} = -0.11$

The regression eqn of $x$ on $y$ is

$$x - \bar{x} = (y - \bar{y})\, b_{xy}$$

$$\Rightarrow x - 2 = (y + 3) \times (-0.11)$$

put $y = 0$, we get

$$x - 2 = 3 \times (0.11)$$

$$x = -0.33 + 2$$

$$\boxed{x = 1.67}$$

Correct option = 1.67 (iv)

**b)** If probability density function $f(x)$ of a continuous random variable $x$ is defined by

$$f(x) = \begin{cases} 1/4 & ; -2 \le x \le 2 \\ 0 & ; \text{otherwise} \end{cases}$$

then $P(x \le 1)$ is .....

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i) 1/4 ii) 1/2 iii) 1/3 iv) 3/4

Sol$^n$ → Given prob. density fun

$$f(x) = \begin{cases} 1/4 & ; -2 \le x \le 2 \\ 0 & ; \text{otherwise} \end{cases}$$

$$\therefore P(x \le 1) = \int_{-2}^{1} f(x)\,dx$$

$$= \int_{-2}^{1} \frac{1}{4}\,dx = \frac{1}{4}[x]_{-2}^{1}$$

$$= \frac{1}{4}[1+2]$$

$$\boxed{= 3/4}$$

correct option (iv)

c) Lagranges poly. through the points

| | | | |
|---|---|---|---|
| $x$ : | 0 | 1 | 2 |
| $y$ : | 4 | 0 | 6 |

given by

(i) $y = 5x^2 - 3x + 4$ ii) $y = 5x^3 + 3x + 4$
iii) $y = 5x^2 - 9x + 4$ iv) $y = x^2 - 9x + 4$

Sol$^n$ : →

| | | | |
|---|---|---|---|
| $x$ | $x_0 = 0$ | $x_1 = 1$ | $x_2 = 2$ |
| $y$ | $y_0 = 4$ | $y_1 = 0$ | $y_2 = 6$ |

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Lagranges interpolation formula is

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \times y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} \times y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} \times y_2$$

$$y = \frac{(x-1)(x-2)}{(0-1)(0-2)} \times 4 + 0 + \frac{(x-0)(x-1)}{(2-0)(2-1)} \times 6$$

$$= \frac{x^2-3x+2}{2} \times 4 + \frac{x^2-x}{2} \times 6$$

$$= 2[x^2-3x+2] + 3(x^2-x)$$

$$= 2x^2-6x+4+3x^2-3x$$

$$\boxed{y = 5x^2-9x+4}$$

correct option (iii)

d) Using Gauss Elimination method the solution of the system of eq$^n$ $x+\frac{1}{4}y+\frac{1}{4}z=1$

$$\frac{15}{4}y-\frac{9}{4}z=3$$

$$\frac{5}{4}y-\frac{19}{4}z=3 \text{ is}$$

i) $x=1, y=2, z=3$

ii) $x=\frac{1}{2}, y=1, z=\frac{1}{2}$

iii) $x=2, y=\frac{1}{2}, z=2$

iv) $x=1, y=\frac{1}{2}, z=-\frac{1}{2}$

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Sol$^n$ :- The matrix form of linear eq$^n$ is

$$\begin{bmatrix} 1 & 1/4 & 1/4 \\ 0 & 15/4 & -9/4 \\ 0 & 5/4 & -19/4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}$$

Apply $R_2 \to \frac{4}{15} R_2$ & $R_3 \to \frac{4}{5} R_3$

$$\begin{bmatrix} 1 & 1/4 & 1/4 \\ 0 & 1 & -9/15 \\ 0 & 1 & -19/5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 12/15 \\ 12/15 \end{bmatrix}$$

Apply $R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 1/4 & 1/4 \\ 0 & 1 & -9/15 \\ 0 & 0 & -48/15 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 12/15 \\ 24/15 \end{bmatrix}$$

Apply $R_3 \to \frac{-15}{48} R_3$

$$\begin{bmatrix} 1 & 1/4 & 1/4 \\ 0 & 1 & -9/15 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 12/15 \\ -24/48 \end{bmatrix}$$

$\Rightarrow \quad z = \frac{-24}{48} = -1/2 \quad \Rightarrow \quad z = -1/2$

& $\quad y - 9/15\, z = 12/15$

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$\Rightarrow \quad y = \frac{12}{15} - \frac{9}{15} \times \frac{1}{2}$

$= \frac{4}{5} - \frac{3}{10}$

$= \frac{8-3}{10} = \frac{5}{10} = \frac{1}{2}$

$\boxed{y = 1/2}$

& $x \Rightarrow \quad x + 1/4\, y + 1/4\, z = 1$

$\Rightarrow \quad x = 1 - \frac{1}{4} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{2}$

$\boxed{x = 1}$

$\therefore \quad \boxed{x = 1,\ y = 1/2 \quad z = -1/2}$

correct option (iv)

e) The first four central moments of distribution are 0, 16, −64, & 162. The coefficient of kurtosis $\beta_2$ is

(1) 1.20 (2) 0.6328 (3) 1 (4) 0.3286

$\Rightarrow$ Given $\mu_1 = 0$, $\mu_2 = 16$, $\mu_3 = -64$, $\mu_4 = 162$

$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{162}{(16)^2} = 0.6328$

correct option (ii)

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(f) If $f(x)$ is continuous on $[a,b]$ & $f(a)\cdot f(b)<0$ then to find root of $f(x)=0$, initial approximation $x_0$ by

i) $x_0 = \frac{a-b}{2}$  ii) $x_0 = \frac{f(a)+f(b)}{2}$

iii) $x_0 = \frac{a+b}{2}$  iv) $\frac{a-b}{a+b}$

→ By theory of bisection method

$$\boxed{x_0 = \frac{a+b}{2}}$$

correct option (iii)

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Q-2) Solve the following

a) If marks scored by five students in statistics test of 100 marks are given in the following table,

| Student | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| marks | 46 | 34 | 52 | 78 | 65 |

Find standard deviation & arithmatic mean

=> Here $n$ = no. of student = 5

| Student | marks($x$) | $x_i - \bar{x}$ | $(x_i - \bar{x})^2$ |
|---|---|---|---|
| 1 | 46 | -9 | 81 |
| 2 | 34 | -21 | 441 |
| 3 | 52 | -3 | 9 |
| 4 | 78 | 23 | 529 |
| 5 | 65 | 10 | 100 |
| Total | 275 | — | 1160 |

$$\bar{\mu} = \frac{\Sigma x}{n} = \frac{275}{5} = 55$$

& $S.D = \sigma = \sqrt{\frac{1}{n}\Sigma(x_i - \bar{x})^2}$

$$= \sqrt{\frac{1}{5} \times 100}$$

$$\boxed{\sigma = 15.2315}$$

& $\boxed{\bar{x} = AM = 55}$

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(b) Fit a law of the form $y = ap + b$ by least square method for the following data

| | | | | | | |
|---|---|---|---|---|---|---|
| p : | 100 | 120 | 140 | 160 | 180 | 200 |
| y : | 0.9 | 1.1 | 1.2 | 1.4 | 1.6 | 1.7 |

⇒ Here $n$ = no. of points = 6

The eqn of lines are

$$\boxed{\begin{aligned} \Sigma y_i &= a \Sigma p_i + nb \quad -(1) \\ \Sigma p_i y_i &= a \Sigma p_i^2 + b \Sigma p_i \quad -(2) \end{aligned}}$$

| | $p_i$ | $y_i$ | $p_i^2$ | $p_i y_i$ |
|---|---|---|---|---|
| | 100 | 0.9 | 10000 | 90 |
| | 120 | 1.1 | 14400 | 132 |
| | 140 | 1.2 | 19600 | 168 |
| | 160 | 1.4 | 25600 | 224 |
| | 180 | 1.6 | 32400 | 288 |
| | 200 | 1.7 | 40000 | 340 |
| Total | 900 | 7.9 | 142000 | 1242 |

eqn (1) & (2) becomes.

$$7.9 = a \times 900 + 6b$$

$$900a + 6b = 7.9 \quad -(3)$$

& $1242 = a \times 142000 + b \times 900$

$$\Rightarrow 142000a + 900b = 1242 \quad -(4)$$

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on solving eqn (3) & (5) we get

$a = -0.019$
$b = 4.167$

$\therefore$ The required line is

$$\boxed{y = -0.019p + 4.167}$$

(c) If the two lines of regression are $ax + y - \lambda = 0$ & $4x + y = \mu$ and the means of $x$ & $y$ are $2$ & $-3$ resp. Find the values of $\lambda$, $\mu$ & corelation coeff bctn $x$ & $y$.

$\Rightarrow$ Given two lines of regression are

$$ax + y - \lambda = 0 \quad -(1)$$
$$4x + y = \mu \quad -(2)$$

given $\bar{x} = 2$ & $\bar{y} = -3$

The means $(\bar{x}, \bar{y}) = (2, -3)$ satisfies (1) & (2)

$\therefore$ we have

$\Rightarrow a\bar{x} + \bar{y} = \lambda$
& $4\bar{x} + \bar{y} = \mu$

$\Rightarrow a \times 2 - 3 = \lambda$
& $4 \times 2 - 3 = \mu$

$\Rightarrow \boxed{\lambda = 15}$ & $\boxed{\mu = 5}$

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(3)

(b) Fit the parabola $y = ax^2 + bx + c$ by using least square method to the following data

| | | | | |
|---|---|---|---|---|
| $x$ : | 0 | 1 | 2 | 3 |
| $y$ : | 2 | 2 | 4 | 8 |

Sol$^n$ ⇒ Here $n$ = no. of points $= 4$

The eqns. of parabola are

$$\Sigma y = a\Sigma x^2 + b\Sigma x + nc \quad -(1)$$

$$\Sigma xy = a\Sigma x^2 + b\Sigma x^2 + c\Sigma x \quad -(2)$$

$$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2 \quad -(3)$$

| | $x$ | $y$ | $x^2$ | $x^3$ | $x^4$ | $xy$ | $x_i^2 y_i$ |
|---|---|---|---|---|---|---|---|
| | 0 | 2 | 0 | 0 | 0 | 0 | 0 |
| | 1 | 2 | 1 | 1 | 1 | 2 | 2 |
| | 2 | 4 | 4 | 8 | 16 | 8 | 16 |
| | 3 | 8 | 9 | 27 | 81 | 24 | 72 |
| Total | 6 | 16 | 14 | 36 | 98 | 34 | 90 |

∴ so eqn (1), (2), (3) becomes.

$\Rightarrow 16 = a(14) + b(6) + 4c$

$\Rightarrow 14a + 6b + 4c = 16$

$\Rightarrow 7a + 3b + 2c = 8 \quad -(4)$

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Also $34 = a(36) + b(14) + c(6)$

$\Rightarrow 36a + 14b + 6c = 34$

$\Rightarrow 18a + 7b + 3c = 17$ — (5)

& $90 = a(198) + b(36) + c(14)$

$\Rightarrow 49a + 18b + 7c = 45$ — (6)

The matrix form of (4), (5), (6) is

$$\begin{bmatrix} 7 & 3 & 2 \\ 18 & 7 & 3 \\ 49 & 18 & 7 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 8 \\ 17 \\ 45 \end{bmatrix}$$

Apply $R_2 \to R_2 - \frac{18}{7}R_1$

$R_3 \to R_3 - 7R_1$

$$\begin{bmatrix} 7 & 3 & 2 \\ 0 & -5/7 & -\frac{15}{7} \\ 0 & -3 & -7 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 8 \\ -25/7 \\ -11 \end{bmatrix}$$

Apply $R_2 \to -\frac{1}{7 \times 5} R_2$

$$\begin{bmatrix} 7 & 3 & 2 \\ 0 & 1 & +3 \\ 0 & -3 & -7 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ -11 \end{bmatrix}$$

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Apply $R_1 \to R_1 - 3R_2$
$R_3 \to R_3 + 3R_2$

$$\begin{bmatrix} 7 & 0 & -7 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -7 \\ 5 \\ 4 \end{bmatrix}$$

Apply $R_1 \to \frac{1}{7} R_1$, $R_3 = \frac{1}{2} R_3$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ 2 \end{bmatrix}$$

Apply $R_1 \to R_1 + R_3$
$R_2 \to R_2 - 3R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$\Rightarrow$ $a = 1$
$b = -1$
$c = 2$

$\therefore$ The required parabola is

$$\boxed{y = x^2 - x + 2}$$

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(c) Calculate the correlation coefficient from the following data

$n=10$, $\sum_{i=1}^{10} x = 40$, $\sum_{i=1}^{10} x^2 = 190$, $\sum_{i=1}^{10} y^2 = 200$

$\sum_{i=1}^{10} xy = 150$ $\sum_{i=1}^{10} xy = 150$, $\sum_{i=1}^{10} y = 40$

$\Rightarrow$ The corelation coeff betn $x$ & $y$ is

$$r(x,y) = \frac{cov(x,y)}{\sigma_x \sigma_y}$$

$$= \frac{\Sigma xy - n\bar{x}\bar{y}}{\sqrt{(\Sigma x^2 - n(\bar{x})^2)(\Sigma y^2 - n(\bar{y})^2)}}$$

$$= \frac{150 - 10(4)(4)}{\sqrt{190-10(4)^2}\sqrt{200-10(4)^2}}$$

$$= \frac{150-160}{\sqrt{30}\sqrt{40}}$$

$\because \bar{x} = \frac{\Sigma x}{n} = \frac{40}{10} = 4$

& $\bar{y} = \frac{\Sigma y}{n} = \frac{40}{10} = 4$

$r(x,y) = -0.2887$

$\therefore$ The correlation coeff betn $x$ & $y$ is

$\boxed{= -0.2887}$

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Q-4 Solve the following.

a) Bag 1 contains 2 white & 3 red balls. Bag 2 contains 4 white & 5 red balls. One ball is drawn randomly from bag 1 & is placed in bag 2. Later one ball is drawn randomly from bag 2. Find the probability that it is red.

=) let event of selecting bag 1 is A.
event of selecting bag 2 is B

$\therefore P(A) = P(B) = 1/2$

Let R be the event of selection red balls.
Given bag 1 contains 2 white & 3 red balls, while bag 2 contains 4 white & 5 red balls.
Suppose, one ball is drawn from bag 1 & it is placed in bag 2.

$\therefore$ There are two cases.
a) Ball is white
b) Ball is red.

$\therefore$ Bag 1 contains 1 white & 3 red balls
or 2 white & 2 red balls

& Bag 2 contains 5 white & 5 red balls
or 4 white & 6 red balls.

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$\therefore$ conditional probabilities for Bag 1 are

$P_1(R/A) = 3/4$ or $P_2(R/A) = 2/4$

and

condition probabilities for Bag 2 are

$P_1(R/B) = 5/10$ or $R_2(R/B) = 6/10$

$\therefore$ By bayes thm
Probability of red ball drawn randomly from bag 2 is.

$$P(B/R) = \sum_{i=1}^{2} \frac{P_i(R/B) \cdot P(B)}{P(R)}$$

$\therefore$ By Baye's thm.
Probability of red ball drawn randomly from bag 2 is

$$P(B/R) = \sum_{i=1}^{2} \frac{P_i(R/B)\, P(B)}{P(R)}$$

$$= \frac{P_1(R/B)\, P(B)}{P_1(R/B)P(A) + P_1(R/B) \cdot P(B)}$$

$$+ \frac{P_2(R/B)\, P(B)}{P_2(R/A)P(A) + P_2(R/B)\, P(B)}$$

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$$P(B/R) = \frac{\frac{5}{10}\,\frac{1}{2}}{\frac{3}{4}\,\frac{1}{2} + \frac{5}{10}\,\frac{1}{2}} + \frac{\frac{6}{10}\,\frac{1}{2}}{\frac{2}{4}\,\frac{1}{2} + \frac{6}{10}\,\frac{1}{2}}$$

$$= \frac{1/4}{5/8} + \frac{3/10}{11/20}$$

$$= 2/5 + 6/11$$

$$\boxed{P(B/R) = \frac{52}{55}}$$

$\therefore$ Prob. of a red ball drawn randomly from bag 2 is $\frac{52}{55}$ i.e. $\boxed{0.9455}$.

(b) The expected no. of matches those will be won by India in a series of five one day matches betn India & England is 3. If the probability of India's win in each match remain the same & the result of all the five matches are indpt of each other. find the probability that India wins the series, using Binomial distribution Assume that each match ends with a result.

$\Rightarrow$ Let $P$ = prob. of India win a match

$= \frac{3}{5} = 0.6$

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As. $p+q=1$

$\Rightarrow q = 1-p$

$q = 1-0.6 = 0.4$

Here $n=5$ = total no. of matches

$P(X=r) = {}^nC_r\, p^r q^{n-r}$

$P(X=r) = {}^5C_r\, p^r q^{5-r}$

probability that India wins a series is

$P(X \geq 3)$

$= P(X=3) + P(X=4) + P(X=5)$

$= {}^5C_3 (0.6)^3 (0.4)^2 + {}^5C_4 (0.6)^4 (0.4)^1 + {}^5C_5 (0.6)^5 (0.4)^0$

$= 10(0.6)^3 (0.4)^2 + 5(0.6)^4 (0.4)^1 + (0.6)^5 (0.4)^0$

$= 0.3456 + 0.2592 + 0.07776$

$= 0.6826$

$\boxed{\therefore P(X \geq 3) = 0.6826}$

The probability that, India wins the series is 0.6828.

(c) The lifetime of an article has a normal distribution with mean 400 hrs. & standard deviation 50 hrs. ~~of 2000 whose~~ Find the expected no. of articles out ~~of hrs.~~ 2000 whose lifetime lies 335 hours to 465 hours.

(Given $Z=1.3$, $A=0.4032$, $Z=0.7$, $A=0.2580$)

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Given $\mu$ = mean = 400

$\sigma$ = S.D = 50

n = total no. of articles = 2000

As we know that.

$$z = \frac{x-\mu}{\sigma} = \frac{x-400}{50}$$

probability of articles betn 335 & 465 hours.

$\Rightarrow P(335 \le x \le 465)$

$\Rightarrow P\left(\frac{335-\mu}{\sigma} \le z \le \frac{465-\mu}{\sigma}\right)$

$\Rightarrow P\left(\frac{335-400}{50} \le z \le \frac{465-400}{50}\right)$

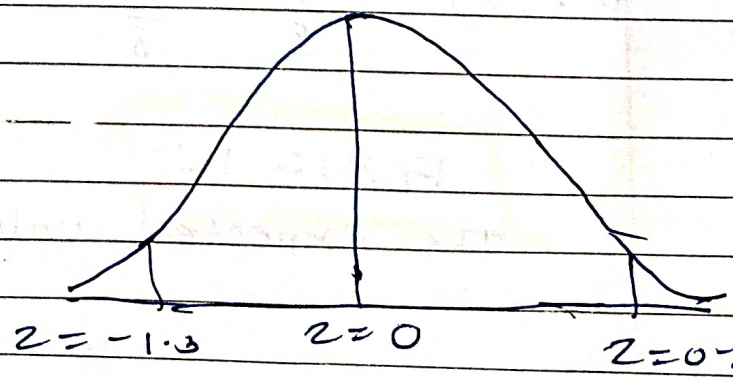
$\Rightarrow P\left(\frac{-65}{50} \le z \le \frac{35}{50}\right)$

$\Rightarrow P(-1.3 \le z \le 0.7)$

$\Rightarrow = P(-1.3 \le z \le 0) + P(0 \le z \le 0.7)$

$= 0.4032 + 0.2580$

$= 0.6612$



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No of articles betn 335 to 465 hours
$= 0.6612 * 2000$
$= 1322$

OR

Q.5] Solve the following

a) Find the expected values of the no. of heads obtained when three fair coins are tossed simultaneously.

⇒ If three coins are tossed then

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$\therefore \boxed{n(S) = 8}$

Let x be the random variable of getting heads.

| $X = x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(X=x)$ | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{3}{8}$ | $\frac{1}{8}$ |

$\therefore \boxed{E(X) = 1.5}$

$\therefore$ The expected value of no. of heads is 1.5

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(b) On an average 180 cars per hour pass a specified point on a particular road. Using Poission distribution find the probability that atleast two cars pass the point in any one minute

Soln : Given avarage $= \lambda = np = 180$ for 1 hr.
& $\lambda$ = avarage $= 3$ for 1 min.

By Poission distribution

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \quad ; \quad x = 0, 1, 2, 3 \ldots$$

$$\therefore P(X=x) = \frac{e^{-3}(3)^{x}}{x!}$$

The probability that atleast two cars pass the point in any one minute is

$$P(X \geq 2) = 1 - P(X < 2)$$
$$= 1 - [P(X=1) + P(X=0)]$$
$$= 1 - \left[\frac{e^{-3}(3)^{1}}{1!} + \frac{e^{-3}3^{0}}{0!}\right]$$
$$= 1 - e^{-3}[3+1]$$
$$= 1 - 4e^{-3}$$
$$= \boxed{0.8009}$$

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$\therefore$ The probability that atleast two cars pass the point in one minute is 0.8009.

(c) The proportions of blood type O, A, B & AB in the general population of a country are known to be in the ratio 49 : 38 : 9 : 4 resp. A research team observed the frequencies of the blood types as 88, 80, 22, & 10 resp. in a community of that country. Test the hypo. at 5% level of significance that the proportions for this community are in accordance with the general population of that country.
(Given $\chi^2_{tab} = 7.815$)

$\Rightarrow$ let $H_0$ = proportions of blood types O, A, B, AB are in the ratio 49 : 38 : 9 : 4

| | | | | |
|---|---|---|---|---|
| Observed freq. ($O_i$) | 88 | 80 | 22 | 10 |
| Expected freq. ($e_i$) | $\frac{49}{100} \times 200 = 98$ | $\frac{38}{100} \times 200 = 76$ | $\frac{9}{100} \times 200 = 18$ | $\frac{4}{100} \times 2000 = 8$ |

we use the $\chi^2$ test

$$\chi^2_{k-p-1} = \sum_{i=1}^{4} \frac{(O_i - e_i)^2}{e_i}$$

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$$= \frac{(88-98)^2}{98} + \frac{(80-76)^2}{76} + \frac{(22-18)^2}{18} + \frac{(10-8)^2}{8}$$

$$= \frac{100}{98} + \frac{16}{76} + \frac{16}{18} + \frac{4}{8}$$

$$= 1.020 + 0.2105 + 0.8888 + 0.5$$

$$= 2.6193$$

Here $k=4$, $p=0$

$\therefore \boxed{\chi^2_3 = 2.6193}$

but given $\boxed{\chi^2 = 7.815 > \chi^2_3}$

$\therefore$ $\boxed{\text{we reject } H_0}$

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Q-6) Solve the following

a) Find the root of the eqn $x^4+2x^3-x-1=0$ lying in $[0,1]$. using the bisection method at the end of fifth iteration.

$\Rightarrow$ Here $f(x) = x^4+2x^3-x-1$ ; $[0,1]$

Here take $a=0$, $b=1$

$f(a) = f(0) = -1 < 0$

$f(b) = f(1) = 1+2-1-1 = 1 > 0$

$\therefore f(a) \cdot f(b) < 0$

$\therefore$ The root lies betn 0 & 1

$\therefore$ By bisection method
Initial approximation is.

$x_1 = \frac{a+b}{2} = \frac{0+1}{2} = \frac{1}{2} = 0.5$

$f(x_1) = f(0.5) = (0.5)^4 + 2(0.5)^3 - (0.5) - 1$

$= -1.8875 < 0$

Now $f(a) \cdot f(b) = -1.1875 < 0$

$\therefore$ The root lies betn $[0.5, 1]$

$\therefore$ Hence by bisection method

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$x_2 = \frac{a+x_1}{2} = \frac{0.5+1}{2} = 0.75$

$f(x_2) = f(0.75) = (0.75)^2 + 2(0.75)^3 - (0.75) - 1$

$= -0.5890 < 0$

As $f(x_2) \cdot f(b) < 0$

$\Rightarrow$ Root lies betn $[0.75, 1]$

Again $x_3 = \frac{0.75+1}{2} = \frac{1.75}{2} = 0.875$

$f(x_3) = f(0.875)$
$= (0.875)^4 + 2(0.875)^3 - 0.875 - 1$

$= 0.05150$

$f(x_2) \cdot f(x_3) = < 0$

$\Rightarrow$ Root lies betn $[0.75, 0.875]$

$\therefore x_4 = \frac{0.75 + 0.875}{2} = 0.8125$

$f(x_4) = f(0.8125) = (0.8125)^4 + 2(0.8125)^3$
$- 0.8125 - 1$

$= -0.304 < 0$

As $f(x_3) f(x_4) < 0$

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$\Rightarrow$ Root lies betn $[0.8125, 0.875]$

$\therefore x_5 = \frac{x_4 + x_5}{2} = \frac{0.8125 + 0.875}{2} = 0.8438$

$f(x_5) = (0.8438)^4 + 2(0.8438)^3 - 0.8438 - 1$

$= -0.14 < 0$

Now $f(x_3) \cdot f(x_5) < 0 \Rightarrow -0.007 < 0$

The root lies betn $[0.8438, 0.875]$

$\therefore x_6 = \frac{0.8438 + 0.875}{2} = 0.8594$

Soln of given eqn is $\boxed{0.8594}$

(b) Find the real root of the eqn $x^3 + 2x - 5 = 0$ by applying Newton-Raphson method at the end of fifth iteration.

Soln:- Given $f(x) = x^3 + 2x - 5$

$f(1) = 1 + 2 - 5 = -2 < 0$

$f(2) = 8 + 4 - 5 = 7 > 0$

Here $f(1) \cdot f(2) < 0$

$\Rightarrow$ Root lies betn 1 & 2

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(b) By Newton Raphson method.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

as. $f'(x) = 3x^2 + 2$

Take $x_0 = 1.5$ initial approximation

$n = 0$, $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$$= 1.5 - \frac{(1.5)^3 + 2 \times (1.5) - 5}{3 \times (1.5)^2 + 2}$$

$$\boxed{x_1 = 1.3429}$$

for $n = 1$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.3429 - \frac{(1.3429)^3 + 2(1.3429) - 5}{3 \times (1.3429)^2 + 2}$$

$$\boxed{x_2 = 1.3283}$$

for $n = 2$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.3283 - \frac{(1.3283)^3 + 2 \times 1.3283 - 5}{3(1.3283)^2 + 2}$$

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$$\boxed{x_3 = 1.3283}$$

for $n=3$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 1.3283 - \frac{(1.3283)^2 + 2(1.3283) - 5}{3(1.3283)^2 + 2}$$

$$\boxed{x_4 = 1.3283}$$

$\therefore$ Sol$^n$ of eq$^n$ $x^3 + 2x - 5 = 0$ is $\boxed{1.3283}$

(c) solve by Gauss-Seidal method, the system of eq$^n$

$$20x_1 + x_2 - 2x_3 = 17$$
$$3x_1 + 20x_2 - x_3 = -18$$
$$2x_1 - 3x_2 + 20x_3 = 25$$

$\Rightarrow$ Given eq$^n$ are as

$$20x_1 + x_2 - 2x_3 = 17 \quad -(1)$$
$$3x_1 + 20x_2 - x_3 = -18 \quad -(2)$$
$$2x_1 - 3x_2 + 20x_3 = 25 \quad -(3)$$

from eq$^n$ (1)

$$x_1 = \frac{1}{20}(17 - x_2 + 2x_3) \quad -(4)$$

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from (2) $x_2 = \frac{1}{20}\left[-18 - 3x_1 + x_3\right]$ — (5)

from (3)

$x_3 = \frac{1}{20}\left[25 - 2x_1 + 3x_2\right]$ — (6)

Gauss seidal Iterations are:

$$x_1^{(n+1)} = \frac{1}{20}\left[17 - x_2^{(n)} + 2x_3^{(n)}\right]$$

$$x_2^{(n+1)} = \frac{1}{20}\left[-18 - 3x_1^{(n+1)} + x_3^{(n)}\right]$$

$$x_3^{(n+1)} = \frac{1}{20}\left[25 - 2x_1^{(n+1)} + 3x_2^{(n+1)}\right]$$

we take initial approximations as
$(x_1^{(0)}, x_2^{(0)}, x_3^{(0)}) = (0, 0, 0)$

first approximations is:

$$x_1^{(1)} = \frac{1}{20}\left(17 - x_2^{(0)} + 2x_3^{(0)}\right]$$

$$= \frac{1}{20}\left[17 - 0 + 2\right] = 0.85$$

$$x_2^{(1)} = \frac{1}{20}\left[-18 - 3x_1^{(1)} + x_3^{(0)}\right]$$

$$= \frac{1}{20}\left[-18 - 3(0.85) + 0\right] = -1.0275$$

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$$x_3^{(1)} = \frac{1}{20}\left[25 - 2x_1^{(1)} + 3x_2^{(1)}\right]$$

$$= \frac{1}{20}\left[25 - 2(0.85) + 3(-1.0275)\right]$$

$$\boxed{x_3^{(1)} = 1.0109}$$

second approximation is

$$x_1^{(2)} = \frac{1}{20}\left[17 - x_2^{(1)} + 2x_3^{(1)}\right]$$

$$= \frac{1}{20}\left[17 + 1.0275 + 2(1.0109)\right]$$

$$\boxed{= 1.0025}$$

$$x_2^{(2)} = \frac{1}{20}\left[-18 - 3x_1^{(2)} + x_3^{(1)}\right]$$

$$= \frac{1}{20}\left[-18 - 3(1.0025) + 1.0109\right]$$

$$= -0.9998$$

$$x_3^{(2)} = \frac{1}{20}\left[25 - 2x_1^{(2)} + 3x_2^{(2)}\right]$$

$$= \frac{1}{20}\left[25 - 2(1.0025) + 3(-0.9998)\right]$$

$$= 0.9998$$

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Third approximation

$$x_3^{(3)} = \frac{1}{20}\left[17 - x_2^{(2)} + 2x_3^{(2)}\right]$$

$$= \frac{1}{20}\left[17 + 0.9998 + 2(0.9998)\right]$$

$$= 0.9999$$

$$x_2^{(2)} = \frac{1}{20}\left[-18 - 3x_1^{(3)} + x_3^{(2)}\right]$$

$$= \frac{1}{20}\left[-18 - 3(0.9999) + 0.9998\right]$$

$$= -0.9999$$

$$x_3^{(3)} = \frac{1}{20}\left[25 - 2x_1^{(3)} + 3x_2^{(3)}\right]$$

$$= \frac{1}{20}\left[25 - 2(0.9999) + 3(-0.9999)\right]$$

$$= 1.0000$$

$\therefore$ Approximate sol$^n$ of given system of eq$^n$ is

$$\boxed{(0.9999, -0.9999, 1.0000) \approx (1, -1, 1)}$$

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Q-7) Solve the following

a) Solve by Gauss Elimination method, the system of eqn:

$$2x_1 + x_2 + x_3 = 10$$
$$3x_1 + 2x_2 + 3x_3 = 18$$
$$x_1 + 4x_2 + 9x_3 = 16$$

$\Rightarrow$ Given system of eqns are

$$\left.\begin{aligned} 2x_1 + x_2 + x_3 &= 10 \\ 3x_1 + 2x_2 + 3x_3 &= 18 \\ x_1 + 4x_2 + 9x_3 &= 16 \end{aligned}\right\} \quad -(1)$$

The matrix form of eqn (1) is

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 18 \\ 16 \end{bmatrix}$$

$\rightarrow R_{13}$

$$\begin{bmatrix} 1 & 4 & 9 \\ 3 & 2 & 3 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 16 \\ 18 \\ 10 \end{bmatrix}$$

$\rightarrow R_2 - 3R_1, \quad R_3 - 2R_1$

$$\begin{bmatrix} 1 & 4 & 9 \\ 0 & -10 & -24 \\ 0 & -7 & -17 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 16 \\ -30 \\ -22 \end{bmatrix}$$

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Apply $\rightarrow \frac{-1}{10} R_2$

$$\begin{bmatrix} 1 & 4 & 9 \\ 0 & 1 & 2.4 \\ 0 & -7 & -17 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 16 \\ 3 \\ -22 \end{bmatrix}$$

$\longrightarrow R_3 + 7R_2$

$$\begin{bmatrix} 1 & 4 & 9 \\ 0 & 1 & 2.4 \\ 0 & 0 & -0.2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 16 \\ 3 \\ -1 \end{bmatrix}$$

$\Rightarrow \quad -0.2x_3 = -1$

$\boxed{x_3 = 5}$

Also $\quad x_2 + 2.4x_3 = 3$

$\Rightarrow \quad x_2 = 3 - (2.4)(5)$

$\boxed{x_2 = -9}$

Again $\quad x_1 + 4x_2 + 9x_3 = 16$

$\Rightarrow x_1 + 4(-9) + 9(5) = 16$

$\Rightarrow \boxed{x_1 = 7}$

$\therefore$ The soln is

$\boxed{x_1 = 7, \quad x_2 = -9, \quad x_3 = 5}$

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(b) Solve by Jacobi Iterative method, the system of eqns.

$$4x_1 + 2x_2 + x_3 = 14$$
$$x_1 + 5x_2 - x_3 = 10$$
$$x_1 + x_2 + 8x_3 = 20$$

=) given linear eqn are

$$4x_1 + 2x_2 + x_3 = 10 \quad -(1)$$
$$x_1 + 5x_2 - x_3 = 10 \quad -(2)$$
$$x_1 + x_2 + 8x_3 = 20 \quad (3)$$

from eqn (1)

$$x_1 = \frac{1}{4}(10 - 2x_1 - x_3) - (4)$$

from eqn (2)

$$x_2 = \frac{1}{5}(10 - x_1 + x_3] \quad -(5)$$

from eqn (3)

$$\Rightarrow x_3 = \frac{1}{8}(20 - x_1 - x_2) \quad -(6)$$

from eqn (4), (5), (6)
Jacobi iterations are

$$x_1^{(n+1)} = \frac{1}{5}[14 - 2x_2^{(n)} - x_3^{(n)}]$$

$$x_2^{(n+1)} = \frac{1}{5}[10 - x_1^{(n)} + x_3^{(n)}]$$

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$$x_3^{(n+1)} = \frac{1}{8}\left(20 - x_1^{(n)} - x_2^{(n)}\right]$$

Let us take initial approximations as
$(x_1^{(0)}, x_2^{(0)}, x_3^{(0)}) = (0, 0, 0)$

First approximation is

$$x_1^{(1)} = \frac{1}{4}\left[14 - 2x_2^{(0)} - x_3^{(0)}\right]$$

$$= \frac{1}{4}\left[14 - 2x_0 - 0\right] = \frac{14}{4} = 3.5$$

$$x_2^{(1)} = \frac{1}{5}\left[10 - x_1^{(0)} + x_3^{(0)}\right]$$

$$= \frac{1}{5}\left[10 - x_1^{(0)} - x_3^{(0)}\right] =$$

$$= \frac{1}{5}\left(10 - 0 + 0\right]$$

$$= 2$$

$$x_3^{(1)} = \frac{1}{8}\left(20 - x_1^{(0)} - x_2^{(0)}\right]$$

$$= \frac{1}{8}\left[20 - 0 - 0\right] = \frac{20}{8} = 2.5$$

Second approximation is

$$x_2^{(1)} = \frac{1}{4}\left(14 - 2x_1^{(1)} - x_3^{(1)}\right]$$

$$= \frac{1}{4}\left[14 - 2(2) - 2.5\right] = 1.875$$

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$$x_2^{(2)} = \frac{1}{5}\left[10 - x_1^{(1)} + x_3^{(1)}\right]$$

$$= \frac{1}{5}\left[10 - 3.5 + 2.5\right] = 1.8$$

$$x_3^{(2)} = \frac{1}{8}\left[20 - x_1^{(1)} - x_2^{(1)}\right]$$

$$= \frac{1}{8}\left[20 - 3.5 - 2\right] = 1.8125$$

Third approximations is

$$x_3^{(1)} = \frac{1}{4}\left[14 - 2x_2^{(2)} - x_3^{(2)}\right]$$

$$= \frac{1}{4}\left[14 - 2(1.8) - 1.8125\right]$$

$$= 2.1469$$

$$x_2^{(1)} = \frac{1}{5}\left[10 - x_2^{(2)} + x_3^{(2)}\right]$$

$$= \frac{1}{5}\left[10 - 1.875 + 1.8125\right]$$

$$= 1.9875$$

$$x_3^{(3)} = \frac{1}{8}\left[20 - x_1^{(2)} - x_2^{(2)}\right]$$

$$= \frac{1}{8}\left[20 - 1.875 - 1.8\right]$$

$$= 2.0406$$

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Fourth approximation is

$$x_1^{(4)} = \frac{1}{4}\left[14 - 2x_2^{(3)} - x_3^{(3)}\right]$$

$$= \frac{1}{4}\left[14 - 2(1.9875) - 2.0406\right]$$

$$= 1.9961$$

$$x_2^{(4)} = \frac{1}{5}\left[10 - x_1^{(0)} + x_3^{(3)}\right]$$

$$= \frac{1}{5}\left[10 - 2.1469 + 2.0406\right]$$

$$= 1.9787$$

$$x_3^{(4)} = \frac{1}{8}\left[20 - x_1^{(3)} - x_2^{(3)}\right]$$

$$= \frac{1}{8}\left[20 - 2.1469 - 1.9875\right]$$

$$= 1.9832$$

Fifth approximation is

$$x_1^{(5)} = \frac{1}{4}\left[14 - 2x_2^{(4)} - x_3^{(4)}\right]$$

$$= \frac{1}{4}\left[14 - 2(1.9787) - 1.9832\right]$$

$$= 2.0149$$

$$x_2^{(5)} = \frac{1}{5}\left[10 - x_1^{(4)} + x_3^{(4)}\right]$$

$$= \frac{1}{5}\left[10 - 1.9961 + 1.9832\right] = 1.9974$$

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$$x_3^{(5)} = \frac{1}{8}\left(20 - x_1^{(4)} - x_2^{(4)}\right)$$

$$= \frac{1}{8}\left[20 - 1.9961 - 1.9787\right]$$

$$= 2.0032$$

Approximate sol^n is $\boxed{(2, 2, 3)}$

(c) Use Regula-Falsi method to find a real root of the eq^n. $e^x - 4x = 0$ correct to three decimal places.

$\Rightarrow$

$$f(x) = e^x - 4x$$
$$f(2) = e^2 - 4\times 2 = -0.6109 < 0$$
$$f(3) = e^3 - 12 = 8.0855 > 0$$

As $f(2)\cdot f(3) < 0$

$\Rightarrow$ Root lies bet^n 2 & 3 i.e. (2,3)

$\therefore$ By Regular falsi method

$$x_{n+1} = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$n = 0$

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = \frac{2(8.0855) - 3(-0.6109)}{8.0855 + 0.6109}$$

$$\boxed{x_1 = 2.0702}$$

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$f(x_1) = f(2.0702)$

$= e^{2.0702} - 4(2.0702)$

$= -0.3544 < 0$

& $f(b) = 8.0855 > 0$

$\therefore$ The root lies betn $(a, b)$

$= [2.0702, 3] = [a, b]$

$\therefore$ By Regula falsi method

$$x_2 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$= \frac{(2.0702)(+8.0855) - 3(-0.3544)}{8.0855 + 0.3544}$$

$$\boxed{x_2 = 2.1092}$$

$f(x_2) = f(2.1092)$

$= e^{2.1092} - 4(2.1092)$

$= -0.19525 < 0$

& $f(b) > 0$

$\therefore$ Root lies betn $\boxed{[2.1092, 3] = [a, b]}$

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$\therefore$ By Regula - falsi method

$$x_3 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$x_3 = \frac{2.1092(8.0855) - 3(-0.1952)}{8.0855 + 0.1952}$$

$$\boxed{x_3 = 2.1302}$$

$$f(x_3) = f(2.1302)$$

$$= e^{2.1302} - 4(2.1302)$$

$$= -0.10425 < 0$$

& $f(b) > 0$

$\therefore$ Root lies betn $[2.1302, 3] = [a, b]$

$\therefore$ By Regula falsi method

$$x_4 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$= \frac{2.1302(8.0855) - 3(-0.010425)}{8.0855 + 0.010425}$$

$$\boxed{x_4 = 2.1413}$$

$\therefore$ The root of the eqn $e^x - 4x = 0$ is

$$\boxed{2.1443}$$

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$f(x_4) = f(2.1413)$

$= e^{2.1413} - 4(2.1413)$

$= -0.0547 < 0$

& $f(b) > 0$

$\therefore$ Root lies betn $[2.1413, 3]$

$\therefore$ By Regula falsi method

$$x_5 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$= \frac{(2.1413)(8.0855) - 3(-0.0547)}{8.0855 + 0.0547}$$

$$\boxed{x_5 = 2.1471}$$

$f(x_5) = f(2.1471)$

$= e^{2.1471} - 4(2.1471)$

$= -0.0284 < 0$

& $f(b) > 0$

$\therefore$ Root lies betn $[2.1471, 3]$

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$\therefore$ by Regula falsi method

$$x_6 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$= \frac{(2.1443)(8.0855) - 3(-0.0284)}{8.0855 + 0.0284}$$

$$\boxed{x_6 = 2.1443}$$

$\therefore$ The root of the eqn

$$\boxed{e^x - 4x = 0 \text{ is } 2.1443.}$$

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Q-8) Solve the following.

a) Using Newton's forward interpolation formula, find y at $x=8$ from the following data

| $x$ | 0 | 5 | 10 | 15 | 20 | 25 |
|---|---|---|---|---|---|---|
| $y$ | 7 | 11 | 14 | 18 | 24 | 32 |

⇒ The forward diff table is

| $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ | $\Delta^4 y$ | $\Delta^5 y$ |
|---|---|---|---|---|---|---|
| 0 | 7 | | | | | |
| | | 4 | | | | |
| 5 | 11 | | -1 | | | |
| | | 3 | | 2 | | |
| 10 | 14 | | 1 | | -1 | |
| | | 4 | | 1 | | 0 |
| 15 | 18 | | 2 | | -1 | |
| | | 6 | 1 | 0 | | |
| 20 | 24 | | 2 | | | |
| | | 8 | | | | |
| 25 | 32 | | | | | |

∴ Newton's forward interpolation formula is

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \cdots$$

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where $u = \frac{x - x_0}{h} = \frac{x - 0}{5} = \frac{x}{5}$

$$y = 7 + \frac{x}{5}(4) + \frac{1}{2!}\frac{x}{5}\left(\frac{x}{5} - 1\right)(-1)$$
$$+ \frac{1}{3!}\frac{x}{5}\left(\frac{x}{5} - 1\right)\left(\frac{x}{5} - 2\right)$$
$$+ \frac{1}{4!}\frac{x}{5}\left(\frac{x}{5} - 1\right)\left(\frac{x}{5} - 2\right)\left(\frac{x}{5} - 3\right)(-1) + \cdots$$

At $x = 8$

$$y = 7 + \frac{8}{5}(4) + \frac{1}{2}\frac{8}{5}\left(\frac{8}{5} - 1\right)(-1)$$
$$+ \frac{1}{6}\frac{8}{5}\left(\frac{8}{5} - 1\right)\left(\frac{8}{5} - 2\right)$$
$$+ \frac{1}{24}\frac{8}{5}\left(\frac{8}{5} - 1\right)\left(\frac{8}{5} - 2\right)\left(\frac{8}{5} - 3\right)(-1) + \cdots$$

$$= 7 + 6.4 - 0.48 - 0.128 - 0.0224$$

$$\boxed{y = 12.7696}$$

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(D) Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ using simpson's $\frac{1}{3}^{rd}$ rule (take $h=0.2$)

$\Rightarrow$ Given $I = \int_0^1 \frac{dx}{1+x^2}$

$\therefore f(x) = y = \frac{1}{1+x^2}$

$a = 0 \quad b = 1, \quad h = 0.2$

| | $x_0$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ |
|---|---|---|---|---|---|---|
| $x$ | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
| $y$ | 1.0 | 0.9615 | 0.8621 | 0.7353 | 0.698 | 0.50 |
| | $y_0$ | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $y_5$ |

$\therefore$ By Simpson's $1/3^{rd}$ rule

$$I = \frac{h}{3}\left[(y_0+y_5) + 4(y_1+y_3) + 2(y_2+y_4)\right]$$

$$= \frac{0.2}{3}\left[(1+0.50) + 4(0.9615+0.7353) + 2(0.8621+0.6098)\right]$$

$$= 0.7491$$

$$\therefore \boxed{\int_0^1 \frac{dx}{1+x^2} = 0.7419}$$

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(c) Use Euler's method to solve $\frac{dy}{dx} = x+y$ ; $y(0)=1$
Tabulate values of $y$ for $x=0$ to $x=0.3$ (take $h=0.1$)

$\Rightarrow$ Given D.E is

$$\frac{dy}{dx} = x+y \quad ; \quad y(0)=1 \quad , \quad h=0.1$$

$\therefore f(x,y) = x+y$

By Euler's method.

$$y_{n+1} = y_n + f(x_n, y_n) \quad \text{—(1)}$$

At $x_1 = x_0 + h = 0 + 0.1 = 0.1$

from eqn (1)

$$y_1 = y_0 + h f(x_0, y_0)$$
$$= 1 + (0.1)(0+1)$$
$$= 1 + 0.1$$
$$\boxed{y_1 = 1.1}$$

At $x_2 = x_1 + h = 0.1 + 0.1 = 0.2$

$\therefore$ from (1)

$$y_2 = y_1 + h f(x_1, y_1)$$
$$= 1.1 + (0.1)(0.1 + 1.1)$$
$$\boxed{y_2 = 1.22}$$

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At $x_3 = x_2 + h$
$= 0.2 + 0.1$
$= 0.3$

$\therefore$ from ①

$y_3 = y_2 + h f(x_2, y_2)$

$= 1.22 + (0.1)(0.2 + 1.22)$

$\boxed{y_3 = 1.362}$

$\therefore$ The value of $y$ at $x = 0.3$ is

$\boxed{1.362}$

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Q-9 Solve the following

(a) Use Runge-Kutta method of 4th order to solve $\frac{dy}{dx} = xy$, $y(1) = 2$, at $x = 1.2$ with $h = 0.2$.

=) Given D.E is

$$\frac{dy}{dx} = xy \quad ; \quad y(1) = 2$$

$\therefore f(x,y) = xy$, $x_0 = 1$, $y_0 = 2$, $h = 0.2$

$\therefore$ By RK fourth order method

$$y_{n+1} = y_n + k$$

where $k_1 = h f(x_0, y_0)$

$$k_2 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$k_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$k_4 = h f(x_0 + h, y_0 + k_3)$$

$$k = \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$$

for $n = 0$

$x_1 = x_0 + h = 0 + 0.2 = 0.2$

$k_1 = h f(x_0, y_0)$

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$k_1 = hf(x_0, y_0)$

$= (0.2)(x_0\ y_0)$

$= (0.2)((1)(2))$

$\boxed{k_1 = 0.4}$

$k_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$

$= 0.2\ f(1 + \frac{0.2}{2}, 2 + \frac{0.4}{2})$

$= 0.2\ f(1.1, 2.2)$

$= 0.2\ [(1.1)(2.2)]$

$\boxed{k_2 = 0.484}$

$k_3 = hf(x_0 + h/2, y_0 + \frac{k_2}{2})$

$= (0.2)\ f(1 + \frac{0.2}{2}, 2 + \frac{0.484}{2})$

$= (0.2)\ f(1.1, 2.242)$

$= (0.2)\ [(1.1)(2.242)]$

$\boxed{k_3 = 0.49324}$

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$$k_4 = h f(x_0 + h, y_0 + k_3)$$

$$= (0.2) f(1 + 0.2, 2 + 0.49324)$$

$$= (0.2)\left[(1.2)(2.49324)\right]$$

$$\boxed{k_4 = 0.5984}$$

$$k = \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$$

$$= \frac{1}{6}\left[0.4 + 2(0.484) + 2(0.49324) + 0.5984\right]$$

$$\boxed{k = 0.4921}$$

$$y_1 = y_0 + k$$

$$= 2 + 0.4921$$

$$\boxed{y = 2.4921 \quad \text{at} \quad x = 0.2}$$

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(b) Using modified Euler's method find $y(0.2)$ given $\frac{dy}{dx} + xy^2 = 0$

$y(0) = 2$ Take $h = 0.2$
(Two iterations only)

⇒ Give D.E is

$$\frac{dy}{dx} + xy^2 = 0$$

$$y(0) = 2$$

Here $f(x,y) = -xy^2$

$x_0 = 0$, $y_0 = 2$, $h = 0.2$

By modified Euler's method.

$$y_1^{(k+1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(k)})\right] \quad \text{--- (1)}$$

At $x_1 = x_0 + h = 0 + 0.2 = 0.2$

$$y_1^{(0)} = y_1 = y_0 + hf(x_0, y_0)$$

$$= 2 + (0.2)f(0, 2)$$

$$y_1 = 2$$

$$y_1^{(0)} = y_1 = 2$$

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$\therefore$ The first iterations.

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$

$$= 2 + \frac{0.2}{2}\left[f(0, 2) + f(0.2, 2)\right]$$

$$= 2 + 0.1\left[-(0)(2)^2 + [-(0.2)(2)]\right]$$

$$\boxed{y_1^{(1)} = 1.92}$$

The second iteration is.

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$= 2 + \frac{0.2}{2}\left[f(0, 2) + f(0.2, 1.92)\right]$$

$$= 2 + (0.1)\left[-(0)(2)^2 + [-(0.2)(1.92)^2]\right]$$

$$\boxed{y_1^{(2)} = 1.9263.}$$

(c) Using Newton's backward interpolation formula find the value of $\sqrt{155}$ from the foll. data

| $x$ | 150 | 152 | 154 | 156 |
|---|---|---|---|---|
| $y = \sqrt{x}$ | 12.247 | 12.329 | 12.410 | 12.490 |

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⇒ The backward difference table is

| $x$ | $y$ | $\nabla y$ | $\nabla^2 y$ | $\nabla^3 y$ |
|---|---|---|---|---|
| 150 | 12.247 | | | |
| | | 0.082 | | |
| 152 | 12.329 | | −0.001 | |
| | | 0.081 | | 0 |
| 154 | 12.410 | | −0.001 | |
| | | 0.080 | | |
| 156 | 12.490 | | | |

Newton's backward interpolation poly is.

$$y = y_n + \frac{u}{1!}\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \cdots$$

where $u = \frac{x - x_n}{h} = \frac{x-156}{2}$

$$\therefore\ y = 12.490 + \frac{x-156}{2}(0.080) + \frac{1}{2!}\frac{(x-156)}{2}\left[\frac{x-156}{2}+1\right][-0.001]$$

$$y = 12.490 + 0.04(x-156) - \frac{0.01}{8}(x-156)(x-156)$$

At $x = 155$

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$$y = 12.490 + 0.04(155 - 156) - 0.000125(155 - 156)(155 - 154)$$

$$y = 12.490 - 0.04 + 0.000125$$

$$\boxed{y = 12.450125}$$