Unit I Continued…
Circle Drawing Algorithm

A Simple Circle Drawing Algorithm

- The equation for a circle is:

$$x^2 + y^2 = r^2$$

- where r is the radius of the circle
- So, we can write a simple circle drawing algorithm by solving the equation for y at unit x intervals using:

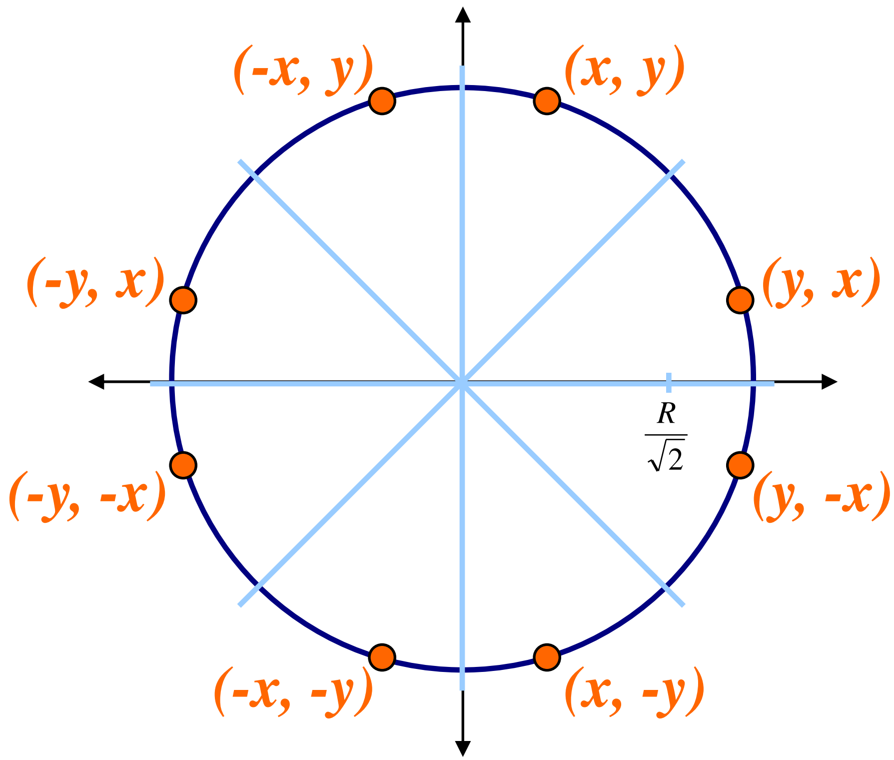
$$y = \pm\sqrt{r^2 - x^2}$$

A Simple Circle Drawing Algorithm (cont…)

- However, unsurprisingly this is not a brilliant solution!
- Firstly, the resulting circle has large gaps where the slope approaches the vertical
- Secondly, the calculations are not very efficient
  - The square (multiply) operations
  - The square root operation – try really hard to avoid these!
- We need a more efficient, more accurate solution

Eight-Way Symmetry

- The first thing we can notice to make our circle drawing algorithm more efficient is that circles centred at $(0, 0)$ have *eight-way symmetry*

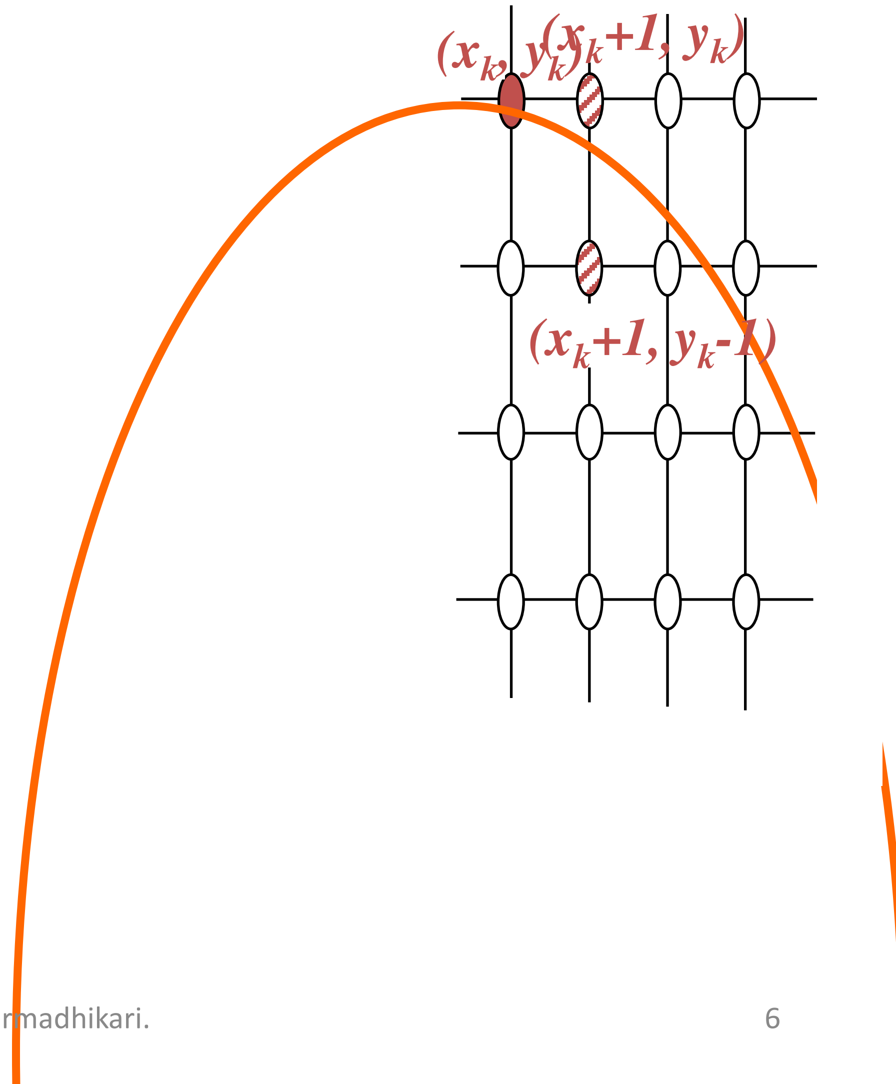


Mid-Point Circle Algorithm

- Similarly to the case with lines, there is an incremental algorithm for drawing circles – the *mid-point circle algorithm*
- In the mid-point circle algorithm we use eight-way symmetry so only ever calculate the points for the top right eighth of a circle, and then use symmetry to get the rest of the points

Mid-Point Circle Algorithm (cont…)

- Assume that we have just plotted point (x_k, y_k)
- The next point is a choice between (x_k+1, y_k) and (x_k+1, y_k-1)
- We would like to choose the point that is nearest to the actual circle
- So how do we make this choice?



Mid-Point Circle Algorithm (cont…)

- Let's re-jig the equation of the circle slightly to give us:

$$f_{circ}(x, y) = x^2 + y^2 - r^2$$

- The equation evaluates as follows:

$$f_{circ}(x, y) \begin{cases} < 0, \text{ if } (x, y) \text{ is inside the circle boundary} \\ = 0, \text{ if } (x, y) \text{ is on the circle boundary} \\ > 0, \text{ if } (x, y) \text{ is outside the circle boundary} \end{cases}$$

- By evaluating this function at the midpoint between the candidate pixels we can make our decision

Mid-Point Circle Algorithm (cont…)

- Assuming we have just plotted the pixel at (x_k, y_k) so we need to choose between (x_k+1, y_k) and (x_k+1, y_k-1)
- Our decision variable can be defined as:

$$\begin{aligned} p_k &= f_{circ}(x_k + 1, y_k - 1/2) \\ &= (x_k + 1)^2 + (y_k - 1/2)^2 - r^2 \end{aligned}$$

- If $p_k < 0$ the midpoint is inside the circle and and the pixel at y_k is closer to the circle
- Otherwise the midpoint is outside and y_k-1 is closer

Mid-Point Circle Algorithm (cont…)

- To ensure things are as efficient as possible we can do all of our calculations incrementally
- First consider:

$$\begin{aligned} p_{k+1} &= f_{circ}\left(x_{k+1}+1,\, y_{k+1}-\tfrac{1}{2}\right) \\ &= [(x_k+1)+1]^2 + \left(y_{k+1}-\tfrac{1}{2}\right)^2 - r^2 \end{aligned}$$

- or:
- where y_{k+1} is either y_k or y_k-1 depending on the sign of p_k

$$p_{k+1} = p_k + 2(x_k+1) + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k) + 1$$

Mid-Point Circle Algorithm (cont…)

- The first decision variable is given as:

$$\begin{aligned} p_0 &= f_{circ}(1, r - \tfrac{1}{2}) \\ &= 1 + (r - \tfrac{1}{2})^2 - r^2 \\ &= \tfrac{5}{4} - r \end{aligned}$$

- Then if $p_k < 0$ then the next decision variable is given as:

$$p_{k+1} = p_k + 2x_{k+1} + 1$$

- If $p_k > 0$ then the decision variable is:

$$p_{k+1} = p_k + 2x_{k+1} + 1 - 2y_k + 1$$

The Mid-Point Circle Algorithm

- MID-POINT CIRCLE ALGORITHM
- Input radius r and circle centre (x_c, y_c), then set the coordinates for the first point on the circumference of a circle centred on the origin as:

$$(x_0, y_0) = (0, r)$$

- Calculate the initial value of the decision parameter as:

$$p_0 = 5/4 - r$$

- Starting with $k = 0$ at each position x_k, perform the following test. If $p_k < 0$, the next point along the circle centred on $(0, 0)$ is (x_k+1, y_k) and:

$$p_{k+1} = p_k + 2x_{k+1} + 1$$

The Mid-Point Circle Algorithm (cont…)

- Otherwise the next point along the circle is (x_k+1, y_k-1) and:

$$p_{k+1} = p_k + 2x_{k+1} + 1 - 2y_{k+1}$$

4. Determine symmetry points in the other seven octants
5. Move each calculated pixel position (x, y) onto the circular path centred at (x_c, y_c) to plot the coordinate values:

$$x = x + x_c \qquad y = y + y_c$$

6. Repeat steps 3 to 5 until $x >= y$

Pseudocode

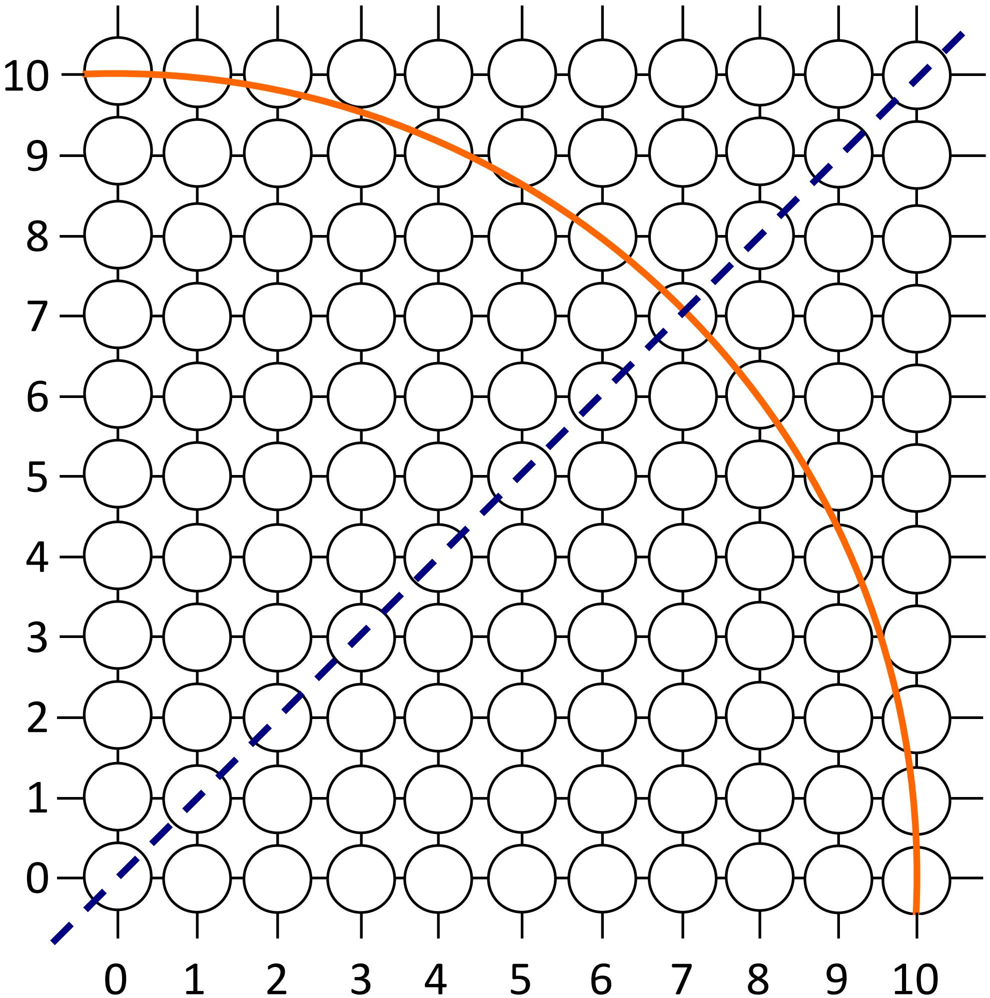
```
void circleMP(int xc,int yc,int r)
{
    int x = 0 , y = r , p = 1 – r;

    plotPoints(xc,yc,x,y);
    while (x < y){
            x = x +1;
             if (p < 0) then  p + = 2 * x + 1;
             else {
                         y - -;
                         p + = 2 *( x – y) + 1;
                   } // else complete
    plotPoints(xc,yc,x,y);
                      } // while complete

} // function def. complete
```

```
void plotPoints(int xc,int,yc,int x,int y)
{
  putpixel( xc + x, yc + y );
  putpixel( xc - x, yc + y );
  putpixel( xc + x, yc - y );
  putpixel( xc - x, yc - y );
  putpixel( xc + y, yc + x );
  putpixel( xc - y, yc + x );
  putpixel( xc + y, yc - x );
  putpixel( xc - y, yc - x );
}
```

Mid-Point Circle Algorithm Example

- To see the mid-point circle algorithm in action lets use it to draw a circle centred at (0,0) with radius 10

Mid-Point Circle Algorithm Example (cont…)

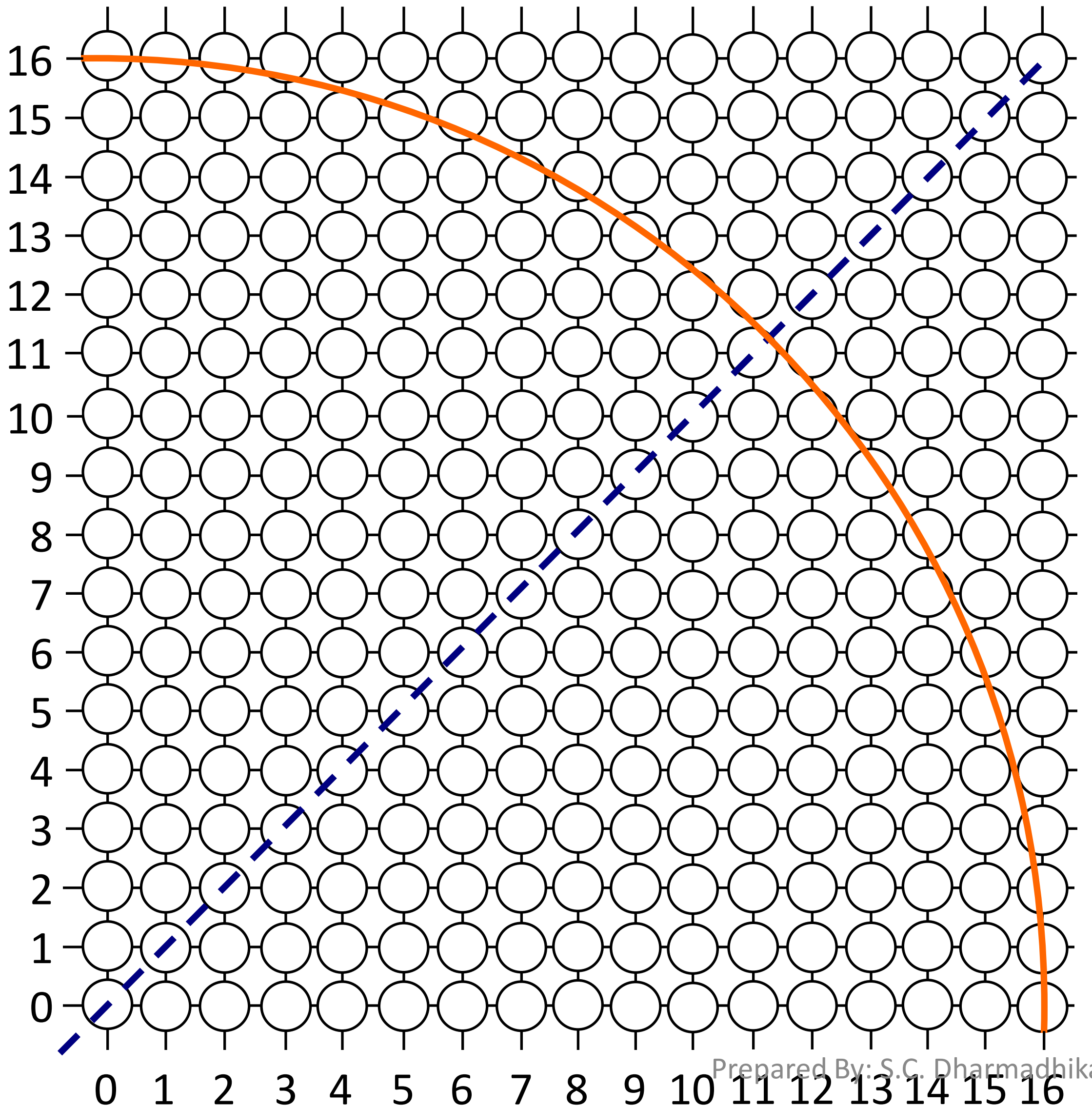


k	p_k	(x_{k+1},y_{k+1})	$2x_{k+1}$	$2y_{k+1}$
0				
1				
2				
3				
4				
5				
6				

Mid-Point Circle Algorithm Exercise

- Use the mid-point circle algorithm to draw the circle centred at (0,0) with radius 15

Mid-Point Circle Algorithm Example (cont…)



k	p_k	(x_{k+1},y_{k+1})	$2x_{k+1}$	$2y_{k+1}$
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				

Mid-Point Circle Algorithm Summary

- The key insights in the mid-point circle algorithm are:
  - Eight-way symmetry can hugely reduce the work in drawing a circle
  - Moving in unit steps along the x axis at each point along the circle's edge we need to choose between two possible y coordinates