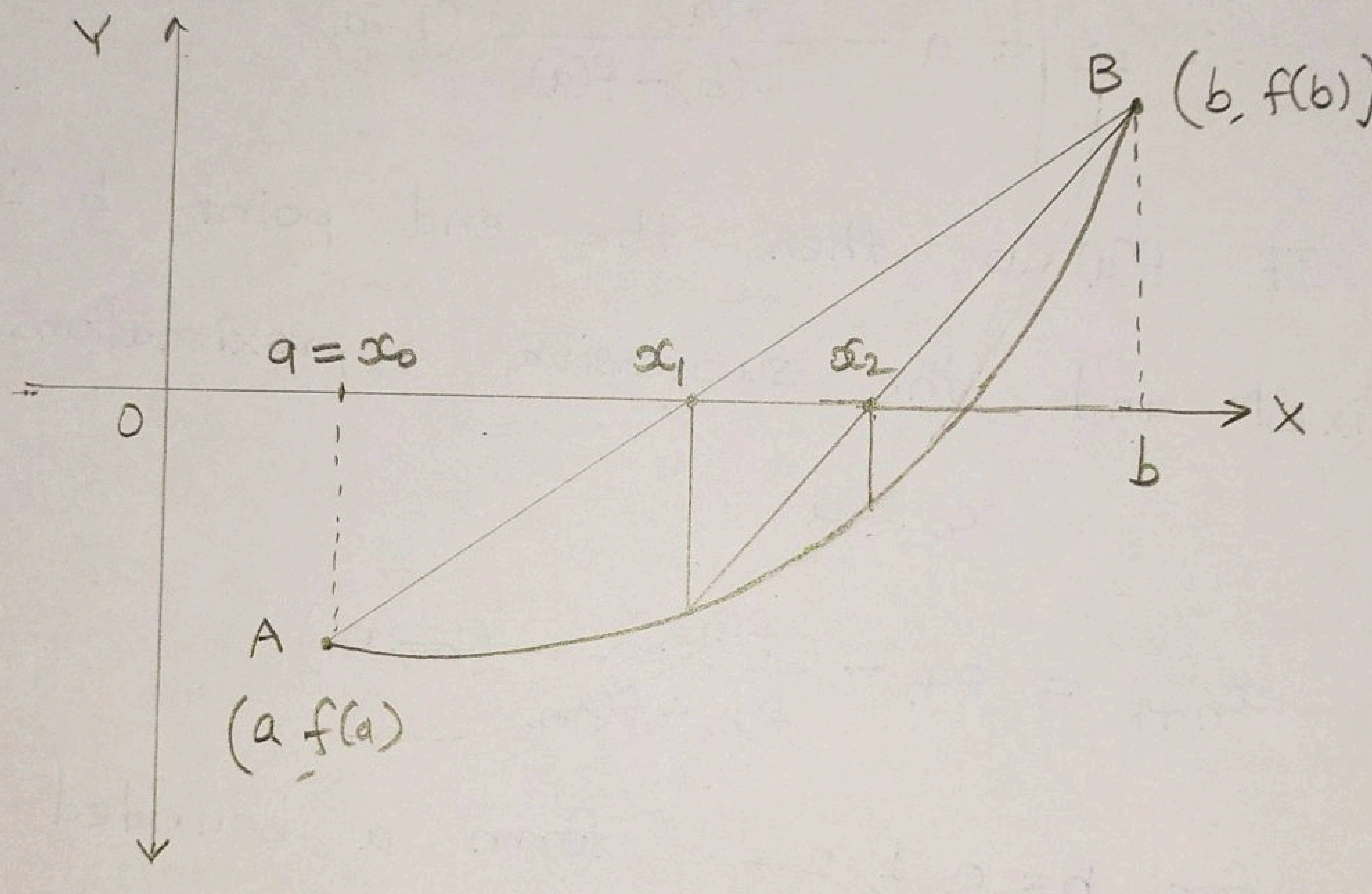
Regula-Falsi Method - (Method of false position)

To find root ξ of the equation

$f(x) = 0$ in the interval $[a, b]$

Assume that $f(a) < 0$ and $f(b) > 0$

so that $f(a) \cdot f(b) < 0$.



Geometrically, this method is equivalent to replacing the curve $y = f(x)$ by a chord that passes through the points $A(a, f(a))$ and $B(b, f(b))$.

The equation of the chord AB is

$$\frac{x-a}{b-a} = \frac{y-f(a)}{f(b)-f(a)}$$

If chord AB meets x axis at $x = x_1$

then $y = 0$

$$\frac{x_1-a}{b-a} = \frac{0-f(a)}{f(b)-f(a)}$$

$$\therefore x_1 = a - \frac{f(a)}{f(b)-f(a)}(b-a)$$

If $f(a) < 0$, then the end point b is fixed and the successive approximations

$$x_0 = a$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f(b)-f(x_n)}(b-x_n)$$

for $n = 0, 1, 2 \ldots$ form a bounded increasing monotonic sequence and

$$x_0 < x_1 < x_2 \cdots < x_n < x_{n+1} < \cdots \xi < b$$

If $f(a) > 0$ end point a is fixed. and successive approximations are

$$x_0 = b$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f(x_n) - f(a)} (x_n - a)$$

and $a < \xi < \cdots < x_{n+1} < x_n < \cdots < x_1 < x_0$

Note 1 - Fix the end point for which sign of f and f'' is same.

Note 2 - Successive approximations x_n lie on the side of root ξ where sign of f is opposite to the sign of f''.

Ex. 1. Using Regula-Falsi method, compute the real root of the equation

$$x^3 - 4x - 9 = 0$$

Soln: Here $f(x) = x^3 - 4x - 9$

$x = 2.5$ $\quad f(x) = -3.375$

$x = 3$ $\quad f(x) = 6$

$\therefore$ Root lies between 2.5 and 3.

$f'(x) = 3x^2 - 4$

$f''(x) = 6x$

$\therefore f''(2.5) = 15 > 0$ $\quad f''(3) = 18 > 0$

Since sign of f and f'' is same

at $x = 3$.

Fix the point $b = 3$.

$x_0 = 2.5$

$$x_1 = x_0 - \frac{f(x_0)}{f(b) - f(x_0)} (b - x_0)$$

$x_1 = 2.68$

$x_2 = 2.7033$

$x_3 = 2.706$

$x_4 = \underline{2.70650}$

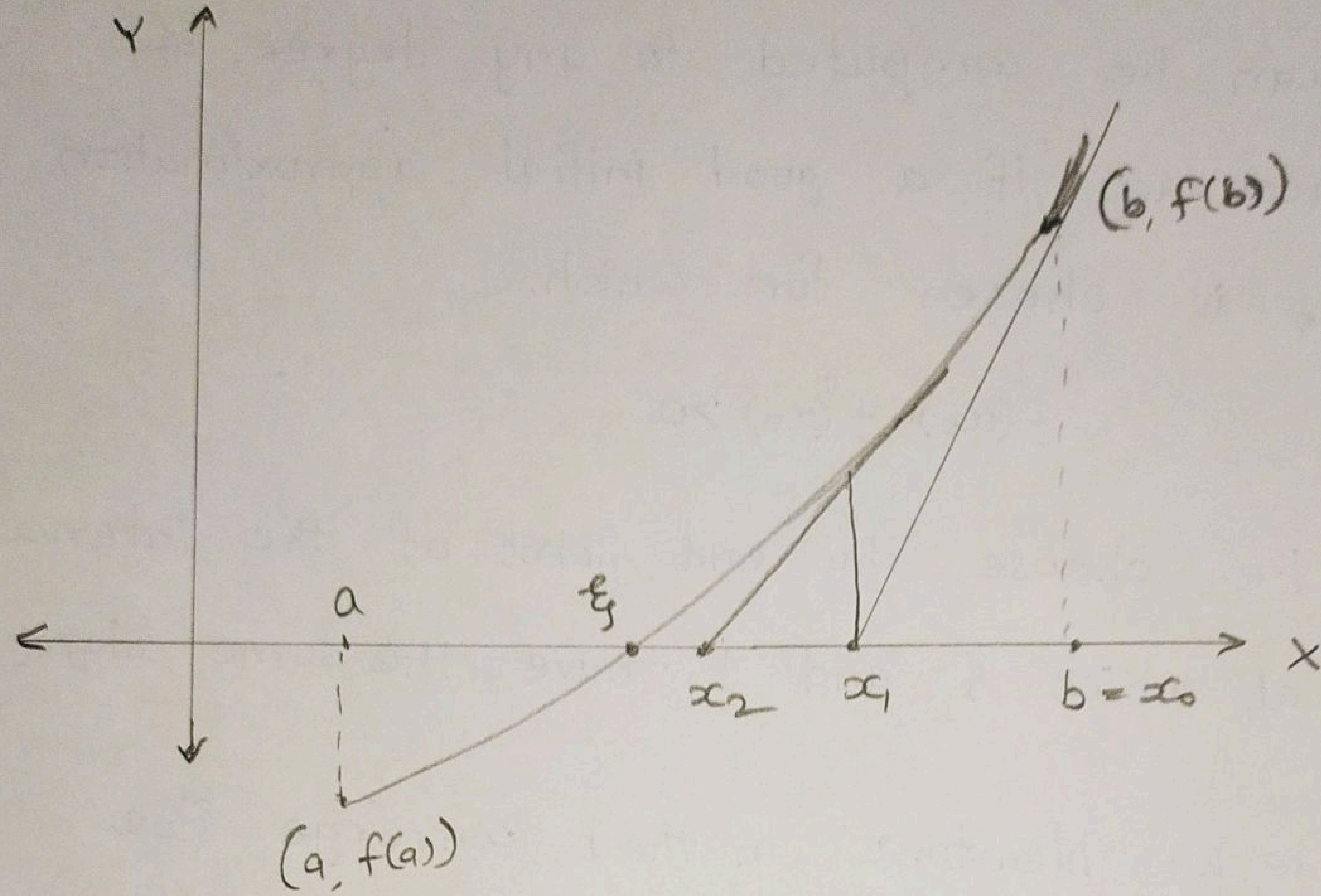
Ex. 2) Determine the root of

$xe^{x} - 2 = 0$ by the method of

false position.

Ans: $0.852\underline{6}$

Newton- Raphson method-



Suppose $\xi = x + h$ where h is a small quantity.

By Taylor's series

$$f(x+h) \approx f(x) + h f'(x)$$

$$\therefore \quad h = -\frac{f(x)}{f'(x)}$$

$$\therefore \quad \xi = x + h = x - \frac{f(x)}{f'(x)}$$

In general,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad n = 0, 1, 2, \ldots$$

Note 1. A root ξ of equation $f(x)=0$ can be computed to any degree of accuracy if a good initial approximation x_0 is chosen for which.

$$f(x_0) \cdot f''(x_0) > 0.$$

i.e. choose the end point of the interval at which f and f'' have the same sign.

Note 2. Newton's method converge slow if f' is small. (fails when $f'=0$ because tangent in this case is parallel to x axis and will never meet it.)

Ex. Find a positive root of $x^4 - x = 10$ using Newton's Raphson method.

Soln.

$$f(x) = x^4 - x - 10 = 0$$

Root lies between 1.8 and 1.9.

Since both f and f'' have same sign at $x = 1.9$.

$$\therefore x_0 = 1.9$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1.8572$$

$$\underline{x_2} = 1.8555$$

Ex. Apply Newton-Raphson method to evaluate approximately $\sqrt{12}$.