Poisson Distribution -

Poisson distribution is the discrete probability distribution of a discrete random variable x.

When p be the probability of success is very small and n the no. of trials is very large and np is finite then we get another distribution called Poisson distribution. It is considered as limiting case of Binomial distribution with $n \to \infty$, $p \to 0$ and np remaining finite.

$$\therefore \quad p(r) = \frac{e^{-z} \cdot z^r}{r!} \qquad r = 0, 1, 2 \ldots$$

Now
$$p(r \geq 0) = \sum_{r=0}^{\infty} \frac{e^{-z} \cdot z^r}{r!}$$
$$= e^{-z} \left\{ 1 + z + \frac{z^2}{2!} + \cdots \right\}$$
$$= e^{-z} \cdot e^{z}$$
$$= e^0$$
$$= 1$$

This probability distribution is called Poisson probability distribution.

Mean & Variance of Poisson Distribution:

$$\text{Mean} = E(X) = \mu_1 = z$$

$$\text{Variance} = \sigma^2 = \mu_2 = E(X^2) - [E(X)]^2$$

$$= z$$

$\therefore$ Mean $= z$

$SD = \sigma = \sqrt{z}$

$\therefore$ variance of Poisson distribution = mean of Poisson distribution.

Ex. 1) A manufacture of cotter pins knows that 2 % of his product is defective. If he sells cotter pins in boxes of 100 pins and guarntees that not more than 5 pins will be defective in a box. Find the approximate probability that a box will fail to meet the guarnteed quality.

Soln.: Here $n = 100$

p is probability of defective pins $= \frac{2}{100} = 0.02$

$\therefore$ z = average no. of defective pins in a box

$\therefore$ $z = np = 100 \times 0.02$

$= 2$

As p is small, we can use Poisson distribution

$$p(r) = \frac{e^{-z} \cdot z^r}{r!}$$

$$= \frac{e^{-2} \cdot 2^r}{r!}$$

Probability that a box will fail to meet the guarnteed quality is

$$p(r > 5) = 1 - p(r \leq 5)$$

$$= 1 - \sum_{r=0}^{5} \frac{e^{-2} \cdot 2^r}{r!}$$

$$= 1 - e^{-2} \sum_{r=0}^{5} \frac{2^r}{r!}$$

$$= 0.0165$$

Ex. 2) In a Poisson distribution if

$p(r=1) = 2\,p(r=2)$, find $p(r=3)$

Soln: We have

$$p(r) = \frac{e^{-z} \cdot z^r}{r!}$$

Now $$p(r=1) = \frac{e^{-z} \cdot z}{1} = 2 \cdot \frac{e^{-z} \cdot z^2}{2}$$

$$\Rightarrow z = 1$$

$$\therefore p(r=3) = \frac{e^{-1} \cdot (1)}{3!} = \frac{1}{6e} = 0.0613$$

Note: We experience number of situations where change of occurrence of an event in a short time interval is very small. However there are infinitely many opportunities to ocaur. The no. of occurrences of such event follows Poisson distribution.

i) Nember of defectives in a production centre

ii) Number of accidents on a highway.

iii) No. of printing mistakes per page.

iv) Number of telephone calls during a particular time.

v) No. of dishonoured checks at a bank.

Ex. 3) Number of road accidents on a highway during a month follows a Poisson distribution with mean 5.

Find the probabability that in a certain month number of accidents on the highway will be

i) less than 3

ii) Between 3 & 5

iii) More than 3.

Soln: Let X = number of road accidents on a highway during a month

$$X \rightarrow P(z=5)$$

$$p(r) = \frac{e^{-5} \cdot 5^r}{r!}$$

i) $p(r<3) = p(r \leq 2) = 0.1246$

ii) $P(3 \leq r \leq 5) = p(r=3) + p(r=4) + p(r=5)$

$= 0.4913$

iii) $p(r>3) = 1 - p(r \leq 3)$

$= 0.7349$

Ex. 4) The average no of misprints per page of a book is 1.5. Assuming the distribution of number of misprints to be Poisson. Find

a) The probability that a particular book is free from misprints

b) Number of pages containing more than 1 misprint. if the book contains 900 pages.

Soln. Here $z = 1.5$

$$p(r) = \frac{e^{-1.5} \cdot (1.5)^r}{r!}$$

a) $$p(r=0) = \frac{e^{-1.5}(1.5)^0}{0!} = e^{-1.5} = 0.2231$$

b) $$p(r>1) = 1 - p(r \le 1)$$

$$= 1 - \{ p(r=0) + p(r=1) \}$$

$$= 1 - (0.2231 + 0.3346)$$

$$= 0.4421$$

$\therefore$ Number of pages in the book containing more than one misprint $= 900 \times 0.4421$

$$= 397.95 \approx \underline{398}$$

Exercise.

1) Determine the probability that 2 of 100 books bound will be defective if it is known that 5 % of book bound at this bindery are defective.

a) Use BD

b) Use PD

Soln: a) $p = 5\% = \frac{5}{100} = 0.05$

$q = 0.95$

$n = 100 \qquad r = 2$

$p(r=2) = {}^{100}C_2 (0.05)^2 (0.95)^{98}$

$= 0.081$

b) $\lambda = np$

$= 100 \times 0.05 = 5$

$\therefore p(r=2) = \frac{5^2 . e^{-5}}{2!}$

$= 0.084.$

2) Between 2 pm and 3 pm the average number of phone calls per minute coming into the company are 2. Find the probability that during one particular minute there will be

a) No phone calls at all

b) 2 or less calls.

Ans : a) 0.1353

b) $\frac{6}{65}$