Ex. 1) Use Lagrange's interpolation formula to fit a polynomial to the following data.

x	-1	0	2	3
y	-8	3	1	2

Hence find $y(-2)$, $y(1)$ and $y(4)$.

Ans: $y = \frac{2}{3} x(x-2)(x-3)$

$+ \frac{1}{2}(x+1)(x-2)(x-3) - \frac{1}{6} x(x+1)(x-3)$

$+ \frac{1}{6}(x+1)(x)(x-2)$

$= \frac{1}{6}\left[7x^3 - 31x^2 + 28x + 18\right]$ is the required 3rd degree polynomial.

$y(-2) = -36.33$

$y(1) = 3.666$

$y(4) = 13.666$

Ex. 2) Express the function

$$\frac{x^2+6x-1}{(x^2-1)(x-4)(x-6)}$$ as a sum of

partial fractions using Lagrange's formula..

Ans : $$\frac{x^2+6x-1}{(x^2-1)(x-4)(x-6)} = \frac{3}{35}\cdot\frac{1}{(x+1)} + \frac{1}{5}\cdot\frac{1}{(x-1)}$$

$$-\frac{13}{10}\cdot\frac{1}{(x-4)} + \frac{71}{70}\,\frac{1}{(x-6)}$$

Ex. 3) Fit a polynomial of 3rd degree

x	0	1	3	4
y	-12	0	6	12

Ans : $y = x^3 - 7x^2 + 18x - 12$

Ex. 4) Determine $x(0)$ by inverse interpolation.

x	1	2	2.5	3
y	-6	-1	5.625	16

Ans : 2.122

Finite differences and difference operators.

Consider set of points (x_i, y_i) $i=0,1,\cdots n$

which satisfies the function $y = f(x)$.

Let us assume that the set of values

of x are evenly spaced and

$$x_i - x_{i-1} = h \quad \text{for} \quad i = 1, \cdots n$$

The forward difference operator Δ is

defined such that

$$\Delta f(x) = f(x+h) - f(x)$$

This difference $\Delta f(x)$ being the difference

between forward value of the function

(i.e., the value of the function at next

point $x+h$) and the present value (i.e., the

value of the function at current point x)

is called the first forward difference or

forward difference of the first order.

Now, $\Delta^2 f(x) = \Delta[\Delta f(x)]$

$$= \Delta[f(x+h) - f(x)]$$

$$= \Delta[f(x+h)] - \Delta f(x)$$

$$= f(x+2h) - f(x+h) - [f(x+h) - f(x)]$$

$$= f(x+2h) - 2f(x+h) + f(x)$$

Here $\Delta^2 f(x)$ gives forward difference of second order or second order forward difference of $f(x)$.

Now $y_0 = f(x_0)$, $y_1 = f(x_1) = f(x_0+h)$

$$y_2 = f(x_2) = f(x_1+h)$$

$$= f(x_0+2h)$$

$$= f(x_2)$$

$$\therefore \quad \Delta y_0 = \Delta f(x_0) = f(x_0+h) - f(x_0)$$

$$= y_1 - y_0$$

$\Delta y_1 = y_2 - y_1$ $\quad$ $\Delta y_2 = y_3 - y_2$

In general $\Delta y_r = y_{r+1} - y_r$

$$\Delta^2 y_0 = y_2 - 2y_1 + y_0.$$

Calculate $\Delta^3 y_0$ and $\Delta^4 y_0$?

Ex. 1) Prepare the forward difference table

for $y = x^2 + x + 1$ $\quad x = 0, 1, \cdots 5.$

Soln: Here x changes from $0, 1, \cdots 5$

$\therefore \quad x_{i+1} - x_i = 1 \qquad i = 0, 1, \cdots 4.$

i.e., $h = 1$

x	0	1	2	3	4	5
y	1	3	7	13	21	31

Forward difference table:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
0	1					
		$\Delta y_0 = 2$				
1	3		$\Delta^2 y_0 = 2$			
		$\Delta y_1 = 4$		$\Delta^3 y_0 = 0$		
2	7		$\Delta^2 y_1 = 2$		$\Delta^4 y_0 = 0$	
		$\Delta y_2 = 6$		$\Delta^3 y_1 = 0$		$\Delta^5 y_0 = 0$
3	13		$\Delta^2 y_2 = 2$		$\Delta^4 y_1 = 0$	
		$\Delta y_3 = 8$		$\Delta y_2 = 0$		
4	21		$\Delta^2 y_3 = 2$			
		$\Delta y_4 = 10$				
5	31					

Ex. 2) Determine $\Delta f(x)$ where

$f(x) = x^2 + x + 1$ with i) $h = 1$, ii) $h = 2$

Sol^n: We have

$$f(x) = x^2 + x + 1$$

i) $h = 1$

$$\Delta f(x) = f(x+h) - f(x)$$

$$= f(x+1) - f(x)$$

$$= (x+1)^2 + (x+1) + 1 - (x^2 + x + 1)$$

$$= x^2 + 3x + 3 - x^2 - x - 1$$

$$= 2x + 2$$

ii) Calculate $\Delta f(x)$ for $h = 2$.