# Predictor - Corrector Method -

## Milne's method -

In the methods so far explained to solve a differential equation $\frac{dy}{dx} = f(x, y)$ over an interval $(x_i, x_{i+1})$ only the value of $y$ at the beginning of the interval was required.

for example
$y(x_0) = y_0$.

In predictor - corrector method four prior values are required for finding the value at $x_{i+1}$. A predictor formula is used to predict the value of $y$ at $x_{i+1}$ and then a corrector formula is applied to improve this value.

$y(x_0) = y_0$, $y(x_1) = y_1$, $y(x_2) = y_2$

$y(x_3) = y_4$.

Milne's predictor formula is given by

$$\boxed{y_4^{(P)} = y_0 + \frac{4h}{3}\left[2f_1 - f_2 + 2f_3\right]}$$

Milne's corrector formula is given by

$$\boxed{y_4^{(c)} = y_2 + \frac{h}{3}\left[ f_2 + 4f_3 + f_4 \right]}$$

Using this improved value of $f_4$ is computed and again the corrector is applied to find a still better value of $y_4$. We repeat this step until $y_4$ remains unchanged.

$$y_5^{(P)} = y_1 + \frac{4h}{3}(2f_2 - f_3 + 2f_4)$$

$$y_5^{(c)} = y_3 + \frac{h}{3}\left[ f_3 + 4f_4 + f_5 \right]$$

Ex. 1) Using Milne's predictor corrector method to find $y$ at $x = 0.8$ for the differential equation

$$\frac{dy}{dx} = x - y^2 \quad \text{where}$$

$y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$

$y(0.6) = 0.1762$

Soln: We have

$$\frac{dy}{dx} = x - y^2 \quad \text{with}$$

$y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$

$y(0.6) = 0.1762$

| $x$ | $y$ | $y' = f(x, y)$ |
|---|---|---|
| $x_0 = 0$ | $y_0 = 0$ | $f_0 = f(x_0, y_0) = x_0 - y_0^2$ <br> $= 0$ |
| $x_1 = 0.2$ | $y_1 = 0.02$ | $f_1 = f(x_1, y_1) = x_1 - y_1^2$ <br> $= 0.1996$ |
| $x_2 = 0.4$ | $y_2 = 0.0795$ | $f_2 = f(x_2, y_2) = x_2 - y_2^2$ <br> $= 0.3937$ |
| $x_3 = 0.6$ | $y_3 = 0.1762$ | $f_3 = f(x_3, y_3) = x_3 - y_3^2$ <br> $= 0.5689$ |

Using Milne's predictor formula

$$y_4^{(P)} = y_0 + \frac{4h}{3}\left[2f_1 - f_2 + 2f_3\right]$$

$$= 0 + \frac{4}{3}(0.2)\left[2(0.1996) - 0.3937 + 2(0.5689)\right]$$

$$= 0.3049$$

Using corrector formula

$$y_4^{(C)} = y_2 + \frac{h}{3}\left(f_2 + 4f_3 + f_4^{(P)}\right)$$

$$= 0.3046$$

# Practice Session -

Ex. 1) Given $\frac{dy}{dx} = x^3 + y$, $y(0) = 2$

The value of $y(0.2) = 2.073$

$y(0.4) = 2.452$

$y(0.6) = 3.023$ are

got by RK method of 4th order.

Find $y(0.8)$ by Milnes predictor corrector method taking $h = 0.2$.

Ex. 2) From the data given below, find y at $x = 1.4$, using Milnes predictor-corrector formula.

$$\frac{dy}{dx} = x^2 + \frac{y}{2}$$

| $x$ | $y$ |
|---|---|
| 1 | 2 |
| 1.1 | 2.2156 |
| 1.2 | 2.4549 |
| 1.3 | 2.7514 |

3) If $\frac{dy}{dx} = 2e^x - y$, $y(0) = 2$, $y(0.1) = 2.010$

$y(0.2) = 2.04$, $y(0.3) = 2.09$, Find $y(0.4)$

using Milne's predictor- corrector method.