Ex. Use Trapezoidal rule to numerically evaluate

$$\int_0^1 x e^{x^2} dx \quad \text{by taking} \quad h = 0.1$$

Soln: Let us say

$$I = \int_0^1 x e^{x^2} dx \qquad h = 0.1$$

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
y	0	0.1010	0.2081	0.3282	0.4694	0.6420	0.8599	1.1426

x	0.8	0.9	1
y	1.5171	2.0231	2.7182

By Trapezoidal rule

$$I = \int_0^1 x e^{x^2} dx = \frac{h}{2}\left[(y_0 + y_{10}) + 2(y_1 + y_2 + \cdots + y_9)\right]$$

$$= 0.05\,[17.301]$$

$$= 0.8650$$

Actual value:

$$\int_0^1 x e^{x^2} dx = \frac{1}{2} \int_0^1 2x e^{x^2} dx$$

$$= \frac{1}{2} \int_0^1 e^{x^2} d(x^2)$$

$$= \frac{1}{2} \left[e^{x^2} \right]_0^1$$

$$= \frac{1}{2} \left[e - 1 \right]$$

$$= 0.8591$$

Error = 0.0059

Percentage error $= \frac{0.0059}{0.8591}$

≈ 0.0068 OR

0.68 %

Ex. 2) Evaluate

$$\int_0^{\pi} \frac{\sin^2\theta}{5+4\cos\theta}\, d\theta$$

By Simpson's $\frac{3}{8}$th rule, taking $h = \frac{\pi}{6}$

Soln: Let us say

$$I = \int_0^{\pi} \frac{\sin^2\theta}{5+4\cos\theta}\, d\theta$$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$y=f(\theta)$	0	0.0295	0.1071	0.2	0.25	0.1627	0

By Simpson's $\frac{3}{8}$th rule,

$$I = \frac{3h}{8}\left[(y_0+y_6) + 3(y_1+y_2+y_4+y_5) + 2y_3\right]$$

$$= \frac{3}{8}\times\frac{\pi}{6}\left[(0+0) + 1.6479 + 0.4\right]$$

$$I = 0.4021$$

$$\therefore \int_0^{\pi} \frac{\sin^2\theta}{5+4\cos\theta}\, d\theta = 0.4021$$

Practice Examples -

Ex. 1) Evaluate numerically $\int_0^1 \frac{1}{1+x^2}\,dx$

$[h = 0.25]$ to calculate π.

Ans: 3.1416

Ex. 2) A river is 80 feet wide. The depth d in feet at a distance x feet from one bank is given by

x	0	10	20	30	40	50	60	70	80
d	0	4	7	9	12	15	14	8	3

Find approximately the area of cross section of the river.

Ans: 710 sq. feet

Ex. 3) A solid of revolution is formed by rotating about x axis, the area between x axis, the lines $x=0$ and $x=1$ and a curve through the points.

x	0.00	0.25	0.50	0.75	1.00
y	1.000	0.9886	0.9589	0.8489	0.9415

Estimate the volume of the solid formed.

Ans :.

Ex. 4) Use Simpson's $\frac{1}{3}$rd rule to evaluate

$$\int_0^1 \frac{\sin x}{2+3\sin x}\,dx$$ by dividing the interval into six parts.

Ans :

Ex. 5) Use Simpson's $\frac{3}{8}$th rule to evaluate

$$\int_0^{\pi/2} \sqrt{\sin x + \cos x}\,dx$$ taking $h = \frac{\pi}{6}$

Ex. 6) The speeds of an electric train at various times after leaving one station until it stops at the next stations are given in the following table. Find the distance between two stations.

Speed (mph)	0	13	33	$39\frac{1}{2}$	40	40	36	15	0
Time (min.)	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{4}$	$3\frac{1}{2}$

Ans : 1.666 miles