ON SENSITIVITIES, RESIDUES AND PARTICIPATIONS: APPLICATIONS TO OSCILLATORY STABILITY ANALYSIS AND CONTROL.

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Abstract - This paper presents techniques for the evaluation and interpretation of eigenvalue sensitivities, in the context of the analysis and control of oscillatory stability in multimachine power systems. These techniques combine the numeric power of modal analysis of state-space models with the insight that can be obtained from transfer function descriptions. Relationships with tools from Selective Modal Analysis (namely, participations) are stressed. Examples of applications to a detailed multimachine power system model are given.

1. INTRODUCTION

Analysis and control of oscillatory behavior and dynamic stability in power systems have received a great deal of attention. The pioneering work [1] considers the local oscillation of a single machine by means of a transfer function model. The usually complex pattern of oscillations in a large power system can be studied through linear, time-invariant, state-space models based on the perturbations of the system state variables from their nominal values at a specific operating point; see [2] for confirmation of the usefulness of these models in accounting for observed oscillations in actual systems.

Eigenvalue sensitivities are one important outcome of the modal analysis of the above mentioned models. They have been put to use in [3], [4], [5], [6], [7] for the problems of Power System Stabilizers (PSS) siting and tuning. A general method to obtain sensitivities was put forward in [8]. [9], [6] present different specific procedures in the power system model context; those do not lend themselves to much flexibility nor insight. An insightful approach was used in [4], based on the relationships between sensitivities, open-loop residues and eigenvectors; our approach will extend this in several significant ways.

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Selective Modal Analysis (SMA), [10] is a framework for studying selected modes of the large models that arise in the oscillatory stability problem. It contains iterative procedures to obtain reduced order models that exactly preserve selected eigenvalues and eigenvectors of the full order model. In this context, the concept of *participation factor* was developed as a means to determine which state variables are more significant for a given dynamic mode, and the converse. We will give a summary of properties, emphasizing the relationships with sensitivities; introduce their use to relate system eigenvalues with whole subsystems, and discuss some important aspects of applications.

In order to address the problem of PSS siting, [4] and [11] have both used sensitivities of the swing eigenvalues with respect to gain changes in *ideal stabilizers* [4], i.e., static feedbacks that sense machine velocity and act upon the back e.m.f. of the same machine. Another method, discussed in [11], uses right eigenvectors ('mode shapes'), possibly weighted by machine inertias. We will show that the participation factors in the velocities are related to both approaches.

Parts **2** and **3** of this paper contain mainly mathematical results on eigenvalue sensitivities, and participations; some of the material is not novel, [10], [7], but it may be convenient at the present time to review it. Since insight is so important, some small examples from PSS application will be provided. Part **4** contains numerical results and pertinent discussions of a detailed multimachine model, used also in [12]. Further information can be found in [13].

2. EIGENVALUE SENSITIVITIES AND RESIDUES

Consider a linear, time-invariant system described by the state space model

$$\dot{x}(t) = Ax(t) \quad (1)$$

Eigenanalysis of the matrix A will produce eigenvalues λ_h (which we assume to be distinct), and corresponding right and left eigenvectors v_h, w_h^T, such that they meet

$$Av_h = \lambda_h v_h \quad (2)$$

$$w_h^T A = \lambda_h w_h^T \quad (3)$$

$$w_h^T v_h = 1 \quad (4)$$

Throughout this paper we will use the convenient normalization (4). Whenever it be necessary, we also assume some normalization that uniquely defines v_h. When dealing with only one mode at a time, we will drop the subindex 'h'.

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Assume now that the system model depends on a parameter q, and we are interested in the effect of small variations of this parameter on a specific eigenvalue. The eigenvalue sensitivity is then defined as a differentiation with respect to q (this operation will be denoted by (.)'), and is known to be, see for instance [8],

$$\lambda' = w^T A' v \tag{5}$$

Complex models of a system often arise from the connection of much simpler partial models, initially described by transfer functions. It may be difficult and cumbersome to trace and interpret the influence of the parameter q on the matrix A, whereas this influence may be very clear in terms of block diagrams. For this reason primarily, we propose the following hybrid formulation.

2.1 Hybrid formulation

Consider the system (1) partitioned into two subsystems, as in Fig. 1. The input u(t) will be used later; for the moment, let it be set to 0. The subindices "1" and "2" will denote the (multidimensional) connecting outputs, and the partitioned state variables, eigenvectors and model matrices.

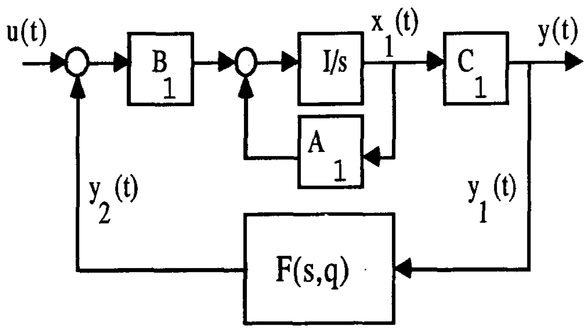

Figure 1. Hybrid representation of the system.

For subsystem 1, which does not depend on q, we have a state space description

$$\dot{x}_1(t) = A_1 x_1(t) + B_1 y_2(t) \quad ; \quad y_1(t) = C_1 x_1(t) \tag{6}$$

Note that (6) does not include a direct link between y_2 and y_1. Whereas this restriction appears to have no relevance in applications, its removal would not permit this simple theory to be developed.

For subsystem 2, which does depend on q, we have a transfer matrix description

$$y_2(s) = F(s,q)\, y_1(s) \tag{7}$$

The proposed hybrid formulation for the sensitivity is (see Appendix A for a proof)

$$\lambda' = w_1^T B_1 F'(\lambda) C_1 v_1 \tag{8}$$

2.2 Residues: the effect of the location of the feedback

We use now the input u(t) and the output y(t) in Fig.1, but merely to define the location of the specific feedback used. The closed-loop transfer matrix J(s) so defined can be expanded (in principle) in terms of all the (distinct) eigenvalues and residue matrices,

$$J(s) = \Sigma_h [R_h /(s-\lambda_h)] \tag{9}$$

$$R_h = \lim_{s\to\lambda_h} [J(s) /(s-\lambda_h)] \tag{10}$$

For each specific mode, the residue matrix can also be expressed as a function of the eigenvectors,

$$R = C_1 v_1 w_1^T B_1 \tag{11}$$

And then (8) becomes

$$\lambda' = \text{trace}\,[R\,F'(\lambda)] = \text{trace}\,[F'(\lambda)\,R] \tag{12}$$

where trace [.] stands for the sum of the terms in the principal diagonal of the square matrix [.] .

The important property (12) will be used extensively in the applications to be shown later on. It lays the foundation of a useful conceptual, as well as algorithmic, separation of the effect of the feedback location (which determines R) and the effect of the parameter q in the extended frequency response F(λ). See [14] for related material and root-locus interpretations.

2.3 Particular cases

The expression (12) can be seen as a manifold extension of some ideas which have been applied to PSS siting and tuning, [4]. At this point it will add insight to study some simple, but useful, cases.

Consider first a static, monovariable feedback. Then the sensitivity will coincide with the residue:

$$\text{For } f(s,q) = q\,, \qquad \lambda' = r \tag{13}$$

Secondly, consider a dynamic, monovariable feedback, where the parameter is a transfer function gain:

$$\text{For } f(s,q) = q\,d(s), \qquad \lambda' = r\,d(\lambda) \tag{14}$$

If we interpret the sensitivity as the shift introduced on the eigenvalue by a unitary increment of a real gain q, we can see from (14) that by shaping the phase of d(λ) it is possible to orient the shift in a desired direction; and then the amount of shift will be adjusted by the magnitude of q. Note also that, if λ is imaginary (which is almost the case in many PSS applications), d(λ) is a frequency response.

Last, note well that the residue matrix R can be evaluated from the forward loop (subsystem 1) if the feedback is a new addition to the system (i.e., F(s,q) = 0 for the nominal value of q). If this is not the case, (12), (13), (14) equally apply, but then the residues must be evaluated for the closed-loop system including F(s,q).

2.4 Relationships between sensitivities for different parameters.

Sensitivities of the same eigenvalue for different parameters and feedbacks may be related; this is important for the evaluation of sensitivities, providing also useful insights. Two cases are of interest:

a) If we consider different parameters in the same feedback, they share a common residue, and the corresponding sensitivities are related in the same way as the different derivatives. For instance, a PSS parametrization used in [6] is:

$$f(s,q_k) = d(s)\, \Sigma_k\, (q_k\, s^k) \qquad (15)$$

where d(s) stands for a fixed transfer function. Then, from (12),

$$(\lambda')_k = r\, d(\lambda)\, \lambda^k \;\; ; \;\; (\lambda')_{k+1} / (\lambda')_k = \lambda \qquad (16)$$

b) The residue matrices corresponding to different feedback locations can be related by a generalized frequency response. If, as in Fig. 2, we consider different measuring (output) points related by a transfer matrix K(s), the corresponding residues are also related,

$$R_b = R_a K(\lambda) \qquad (17)$$

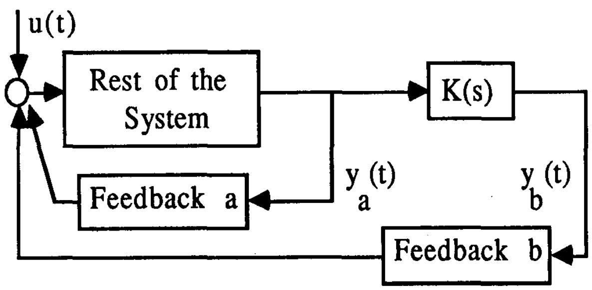

Figure 2. Feedbacks with a common entry point.

Similarly, for the different actuating (input) points in Fig. 3,

$$R_b = K(\lambda)\, R_a \qquad (18)$$

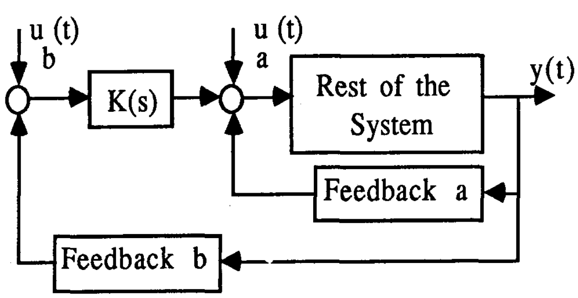

Figure 3. Feedbacks with a common measuring point

3. PARTICIPATIONS

The concept of *participation factor* was developed in [10], initially to measure the degree of participation of a state variable in a mode. Right eigenvectors account for the mode shape, but they have the same dimensions as the state variables, and are therefore unit dependent. By using the left eigenvectors, one can define a new state variable associated exclusively with the mode. The participation factors can be seen as right eigenvectors weighted by left eigenvectors (see (20) below).

Generalized participations can be defined by taking the i-th component of the h-th right eigenvector and the j-th component of the corresponding left eigenvector.

$$p_{ihj} = v_{ih} w_{hj} \qquad (19)$$

The *participation factor* is a very important particular case, when i=j:

$$p_{ih} = v_{ih} w_{hi} \qquad (20)$$

3.1 Properties of participations

a) p_{ih} is adimensional, and independent of the units used for the state variables. This is not, in general, a property of p_{ihj}; nevertheless, the dimension is independent of the considered mode.

b) From (4), the sum of p_{ih} of one mode in all the states is 1. From the orthogonality of right and left eigenvectors, the sum of p_{ih} of one state in all the modes is 1 , and the sum of p_{ihj}, for any pair i=j in all the modes is 0.

$$\Sigma_i\, p_{ih} = \Sigma_h\, p_{ih} = 1 \qquad (21)$$

$$\Sigma_h\, p_{ihj} = 0 \;\; , i \neq j \qquad (22)$$

c) From (5), the participations represent the sensitivity of the h-th eigenvalue to variations in an element of the matrix A.

$$p_{ihj} = \lambda_h' \;\; , \text{for } q = a_{ji} \qquad (23)$$

Properties d) and e) link the participations to residues and feedback sensitivities, as shown in Fig. 4, and follow directly from (11) and (13).

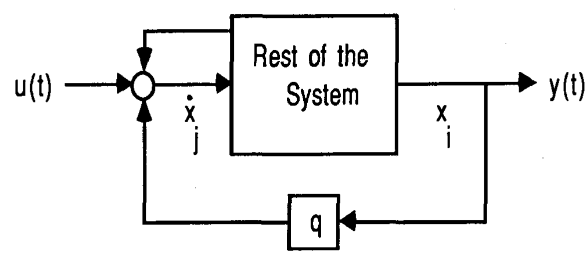

Figure 4. Interpretation of participacions.

d) p_{ihj} represents the residue in the h-th eigenvalue of the closed-loop transfer function defined with x_i as output and input added to $\dot{x}_j$.

e) p_{ihj} is the sensitivity of λ_h to small variations in q, under the conditions in d). When i=j, this property provides an interesting insight: a large p_{ih} indicates that the h-th eigenvalue is very sensitive to a local feedback around the i-th state variable.

3.2 A SMA application: reduced order model for one eigenvalue

Participations play an important role in the model order reduction by SMA techniques. We shall present now a sensitivity-based approach to one of these techniques, using a hybrid formulation consistent with the material of this paper. See [10] for a state-variable approach.

Refer to Fig. 5, where we have represented: a) a full order model, with eigenvalue λ ; b) a reduced order model (obtained by substituting the Laplace variable s by a parameter q), with eigenvalue λ_R. To the shown input and output, there correspond the residue matrices R and R_R. To subsystem 1 correspond, for each case, reduced right and left eigenvectors v_1, v_R and w_1^T, w_R^T

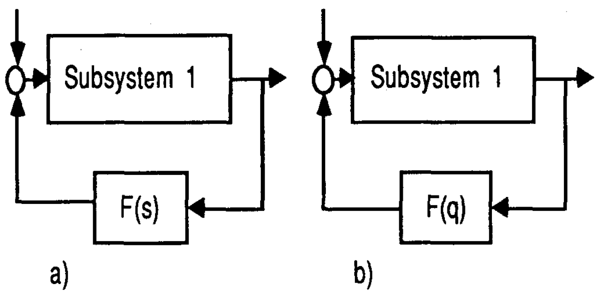

Figure 5. Full and reduced order models.

A main result form SMA, [10], is that, when q is set equal to λ , the reduced order model b) will conserve the eigenvalue λ , and the reduced components of the corresponding right and left eigenvectors. Nevertheless, it is important to notice that the normalization (4) implies a difference between both models: in general $w_1^T v_1 \neq 1$, whereas $w_R^T v_R$ will be set equal to 1. Assuming, without loss of generality, the same normalization in both models, in order to uniquely define right eigenvectors, we have:

$$\lambda_R = \lambda = q \quad (24)$$

$$v_R = v_1 \quad (25)$$

$$w_R^T = w_1^T / p_{S1} \quad (26)$$

$$p_{S1} = w_1^T v_1 \quad (27)$$

The scalar p_{S1} will be termed *participation of the subsistem 1* (in the considered mode), and can be seen to be equal to the sum of the participation factors of the state variables of subsystem 1.

The difference mentioned above is transmitted to the residue matrices,

$$R_R = R/p_{S1} \quad (28)$$

3.4 Algorithm to obtain λ. Local convergence.

An unknown λ may be iteratively approximated by using a first guess for q, which will result in a hopefully better approximation λ_R. The local convergence of this algorithm can be studied as the sensitivity of λ_R with respect to q, in the vicinity of $\lambda_R = \lambda = q$. Then, see Appendix B,

$$\lambda'_R = -p_{S2}/p_{S1} \quad (29)$$

$$p_{S2} = w_2^T v_2 = 1 - p_{S1} \quad (30)$$

The *participation ratio* of a subsystem is defined as $p_S/(1-p_S)$. We can thus state that the participation of a subsystem measures its influence in the considered eigenvalue, in the sense of the approximation to the true eigenvalue obtained when "freezing" the dynamic subsystem by substituting s by a close enough approximation. That is to say that if Mod $[p_{S2}] \ll 1$ (and consequently $p_{S1} \cong 1$), λ_R will be a much better approximation to λ than q. It should be well noted that "freezing" a subsystem is not the same thing as suppressing it; for an extreme example, consider a static subsystem which has no state variables and consequently zero participation, but can influence very much the eigenvalue.

(28) shows that, given the hypothesis of a good participation of subsystem 1, the residue matrix of the reduced order model is a good approximation to that of the full order model.

3.5 Properties of the participation of a subsystem

It is obvious from the definition (27) that the joint participation of two or more subsystems is the sum of the individual participations. Also, that the participation of a static susbsystem, as mentioned above, is 0.

An important property of subsystem participation is that it does not depend on the actual set of state variables used to model the subsystem (that is to say, the participation of a subsystem is invariant for internal state variable transformations). This can be proven from the properties b) and e) of participation factors: the individual participations of the external state variables cannot change, because they are sensitivities; and the sum of all participations is always 1. This property highlights and cautions against some situations that may arise, [15]. For instance, inside a subsystem with little participation there can exist variables with large participation factors (obviously, they must largely cancel). It will then be safe to say that the subsystem is not very active in the considered mode, but it will be dangerous to freeze only some parts of it. These phenomena can be made easily to appear with suitable state transformations.

These transformations can also be used to advantage to redistribute the participation factors in the full-order system in order to improve the convergence of SMA algorithms, see [16] and [17].

4. APPLICATIONS TO PSS ANALYSIS, SITING AND DESIGN

A resumed overview (a fuller account can be found in [13]) will be given here of the results obtained when applying the concepts presented in Sections 2, 3 to a detailed multimachine Power System Model. This is EPRI 39-bus New England Test System, [18], [12]. The system contains 10 generation plants (machine 10 is a fictitious one, acting virtually as an infinite bus). Each of these plants is modelled as described in Fig. 6, and the connection to the network is made by means of a Jacobian matrix. No PSSs are included in this first model; the number of state variables is 93.

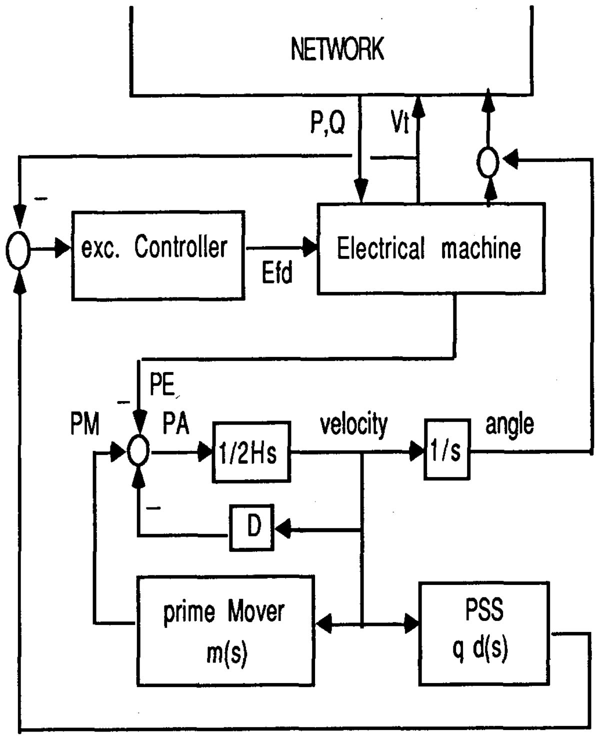

Figure 6. Model of one plant in a power system.

4.1 General analysis of participations

The question we try to answer with a participation analysis is this: Is it possible to assign each eigenvalue to a limited number of significative physical subsystems with which it seems to be mainly related? To answer this question, we have tried a number of different participation based tools:

a) Select the most significative participation factors (in the state variables); we have called significative to Mod[p_{ih}]> 0.2 . Then assign the eigenvalue to the subsystems associated to the selected state variables.

b) As the reader may have noticed, there is a logical flaw in assigning subsystem significance by means of state variable participation factors. So we compute now the subsystem participations, by adding up all corresponding participation factors, selecting also Mod[p_S]> 0.2. We find that, in our model, the results coincide almost entirely with those of procedure a). One must be cautioned, however, that this may not be the case for other modelling procedures, since they can change the sharing of subsystem participation between state variables.

c) Running along the same lines, in the modes where more than one subsystem is involved, there can exist cancellations between them. So, we compute the global participation ratio for the initially selected group of subsystems, and approve the selection if Mod[$p_S/(1-p_S)$]> 10. Again, some small corrections have been neccessary in a limited number of modes: in 3 cases, some subsystems have been disregarded since they do not contribute appreciably to the overall participation ratio; in 1 case, one subsystem was eliminated because it actually made the ratio worse; in 4 cases, subsystems with small p_S's have been added to improve the participation ratio.

A cross-section of the results of this analysis is given in Table 1; some eigenvalues (identified by an arbitrary number, i.e. /13/) are shown together with the involved subsystems. These are identified by the plant number (1 to 10) and a capital letter implying the nature of the subsystem (Swing equation, prime Mover, Electrical machine, excitation Controller). As a general conclusion, it may be pointed out that a great majority of all eigenvalues have significant participations in a very small group of subsystems. On the other hand, many couplings do occur: /31/ is a global oscillation mode of the whole system against the rest of the network, modelled as plant 10; many eigenvalues involve subsystems of different nature and even different plants.

Table 1. Some eigenvalues and related subsystems.

Eigenvalue no.,	value	Subsystems with high participation
/13/	-0.042 + 8.9j	7S 6S
/23/	+0.00 + 7.0j	1S
/31/	-0.29 + 3.9j	10S -all others S
/11/	-10.2	9M
/39/	-4.02	7M 4M
/81/	-0.015	1M
/35/	-5.3 + 0.03j	1M 6E
/29/	-7.1	5E
/65/	-0.97	4E 7E
/38/	-4.2	5C 5E
/74/	-0.20 + 0.46j	6E 6C 4E
/5/	-49.4	8C
/56/	-1.29	8C 6C

4.2 PSS siting by means of participation factors in velocity

From property e), Fig.4, participation factors in velocity measure the eigenvalue sensitivity to the gain of a static feedback measuring velocity and acting upon acceleration. From Fig.6., this can be interpreted (correcting for the inertia 2H which relates the different entry points, (18)) as an extra damping D, or an extra torque P proportional to velocity. The correction term to convert participations to the ideal stabilizer sensitivities used in [4], [11] should also include the (static in their models) gains from torque to back e.m.f. In our detailed model, this last correction do not seem easy to perform. As additional properties participations are, as mentioned before, mode shapes weighted by left eigenvectors, and they are independent from the units used in the modeling; see [12] for a comparison between participations, mode shapes weighted by inertia (momenta) and mode shapes squared weighted by inertia (kinetic energies). In some situations, participation factors are actually proportional to kinetic energies, [19], [20].

Evaluating in our detailed model the participation factors of all the modes in the velocity variables (excepting the fictitious machine 10), we have found the following interesting properties:

a) The participations of non-swing modes are very small, i.e., these modes are not affected by our ideal stabilizers.

b) The most significant participations give a clear idea of the structure of oscillations, see Table 2.

c) The most significant participations are almost real and positive, showing that the damping coefficients essentially shift the eigenvalues to the left. Nevertheless, many less significant participations do not share this property, showing that small frequency shifts or even undampings can be produced.

Table 2. Participation factors (moduli) in velocity for the swing modes

Plant / Mode	8	1	4	9	5	2	3	6	7
/15/	0.45	0.04	0.04						
/23/	0.03	0.40		0.04	0.01	0.03	0.02		
/17/	0.03		0.40		0.04			0.02	0.04
/27/			0.02	0.32	0.13			0.02	0.01
/25/		0.01		0.04	0.15	0.17	0.13		
/19/						0.21	0.28		
/21/		0.02			0.13	0.05	0.02	0.19	0.14
/13/			0.01					0.24	0.28
/31/	0.01	0.02	0.03	0.09	0.07	0.02	0.03	0.05	0.03

We mention also that the participation factors in the angle variables are very similar to those for velocities, for the swing modes. It can be shown from (17), (18) that there exist a relationship between both:

$$p_{angle} = p_{velocity}(1 + (D+m(\lambda))/2H\lambda) \qquad (31)$$

For the swing modes, $\mathrm{Mod}(2H\lambda) \gg (\mathrm{Mod}(D+m(\lambda))$, leading to the above stated property.

We have also compared the results for the detailed model (order 93) with three reduced order models of equal size (order 20): a) Classic model as used in [4]; b) Truncated model as used in [11]; c) model obtained with a single iteration of a SMA algorithm. All of them show a very similar general picture; nevertheless some clear discrepancies, which would lead to wrong siting, can be detected in a) and b), but not in c).

4.3 Implementable stabilizers

At our level of modelling, the only difference between ideal and implementable stabilizers is the actuating point, which in the last case is the reference of the excitation controller, see Fig. 6. The residue corresponding to this feedback can be obtained via a generalized participation for velocity and one state variable of the said controller, by using again (18). Computing this residue for all the plants and modes, it was found that:

a) Many non-swing modes are more sensitive than the swing modes. This speaks of a strong atenuation of the swing mode high frequencies in the way through exciter and machine.

b) In order to assess the structure of oscillations, see Table 3, one must observe one important caution: generalized participations are dimensional, and therefore do not permit direct comparison of sensitivities for different plants. In our system, machines 5, 7 and 9 have excitation systems which differ from all the others; this is reflected in comparatively higher sensitivities. So, we can obtain the most sensitive eigenvalues for each PSS, but not in principle the most influential plant for each eigenvalue. One idea that suggests itself is to normalize by dividing each column in Table 3 by the total sum of the column. Doing this, or simply taking as significant only the higher sensitivities in each column, the structure of the oscillations is seen to coincide with that obtained from the participations in velocity, see Table 2, save for the global oscillation mode /31/, whose sensitivities are relatively enhanced; this comes from the same low-pass effect mentioned in a).

c) The most significant residues are about 180° out of phase with the corresponding participation factors. We have assessed that about 90° of this phase can be atributed to the exciters, which are rather slow in our model.

Table 3. Residues (moduli) for implementable PSS; swing modes

Plant / Mode	8	1	4	9	5	2	3	6	7
/15/	0.67	0.01	0.06	0.04	0.02				0.04
/23/	0.04	0.23		0.31	0.07	0.07	0.03	0.02	0.08
/17/	0.05		0.69		0.28			0.02	0.34
/27/	0.02	0.02	0.07	2.96	0.99	0.01	0.01	0.03	0.13
/25/			0.03	0.32	1.09	0.41	0.24		
/19/						0.42	0.43		0.02
/21/				0.07	0.86	0.11	0.02	0.29	1.36
/13/			0.02					0.28	2.48
/31/	0.09	0.23	0.35	1.97	1.13	0.15	0.18	0.31	1.15

Other types of PSS, such as those measuring accelerating or electrical power, can be equally studied. What is more interesting is that the corresponding residues can be related (in the same way as frequency responses are related in [21]). Using (17) we have:

$$r_{acc.\ power} = r_{velocity}\,(D + 2H\lambda) \qquad (32)$$

$$r_{elec.\ power} = r_{velocity}\,(D + 2H\lambda - m(\lambda)) \qquad (33)$$

The dominant term is again here $2H\lambda$ for the swing modes; since λ is almost imaginary, we find a phase lead of about 90°. Also, this term filters the low frequencies, so that the effect a) previously found is not so strong with these stabilizers.

4.4 Some ideas on stabilizer design

The sensitivity to the gain of the PSS is given by (14), where r is the residue discussed in the previous section. Since these have a phase around 180°, we must compensate it to obtain a damping effect. We could simply use a negative gain q; we have found that this adequately damps the swing eigenvalues, but has the adverse effect of creating unstable non-swing modes (some of the sensitive ones mentioned in 4.3 a)). An important conclusion is that the PSS should also atenuate the sensitivity of these eigenvalues, through a high-pass characteristic. Both requirements (180° phase lead and smaller gain at low frequencies) can be met by 3 lead-lag terms. It is important to notice that one cannot take for granted the usual number of 2 lead-lag terms, [6].

Another thing to notice is that the same ideas can be used to tune already existing PSSs; one must simply take into account the closed-loop residue obtained with the system model augmented by any PSSs which are present.

Sensitivities to other PSS parameters can be easily obtained and related between them, as introduced in **2.4** a). Similar ideas of design are extended in [3], [5], [6], [7], [12] to several PSSs; the insights provided by the hybrid formulation presented here can become very useful in this situation.

5. CONCLUSIONS

Techniques to obtain eigenvalue sensitivities to parameters in a feedback have been presented, that combine the numerical power of eigenanalysis with an intuitive vision of the effect of the parameters, useful in analysis as well as in design problems. The tools presented extend and clarify in several meaningful ways approaches previously used in PSS siting and design; they include a conceptual and algorithmic separation of the effects of the situation of the feedback and that of the parameters in a generalized frequency response, and also potentially useful relationships between sensitivities for different feedback placements and parameters.

A revision of participation concepts from Selective Modal Analysis has been presented, stressing the rapports with sensitivities. Some novel hints and cautions on the use of participations to assess the involvement of system modes with variables and subsystems have been given.

A summary of relevant results on the application of the above stated concepts to a detailed multimachine power system model, concerning overall system analysis, the use of participation factors in velocity to guide PSS placement, and the analysis and design of implementable PSSs show the potential of these techniques.

APPENDIX A

We proceed here to prove (8). Allow first a state description of subsystem 2, F(s),

$$\dot{x}_2(t) = A_2 x_2(t) + B_2 y_1(t) \quad ;$$

$$y_2(t) = C_2 x_2(t) + D_2 y_1(t) \tag{A1}$$

$$F(s) = C_2\, M_2(s)\, B_2 + D_2 \tag{A2}$$

$$M_2(s) = (sI - A_2)^{-1} \tag{A3}$$

From (6), (A1), the complete matrix A has the form

$$A = \begin{bmatrix} A_1 + B_1 D_2 C_1 & B_1 C_2 \\ B_2 C_1 & A_2 \end{bmatrix} \tag{A4}$$

Obtaining in (A2) and (A4) the derivatives with respect to q (only matrices with subindex "2" can depend on q),

$$F'(s) = C'_2 M_2(s) B_2 + C_2 M_2(s) A'_2 M_2(s) B_2 + C_2 M_2(s) B'_2 + D'_2 \tag{A5}$$

$$A' = \begin{bmatrix} B_1 D'_2 C_1 & B_1 C'_2 \\ B'_2 C_1 & A'_2 \end{bmatrix} \tag{A6}$$

It is interesting to note that the partitioned eigenvectors can be related by means of extended frequency responses, in the same way as the residues are in (17), (18). From (2), (3), (A4), it follows directly:

$$v_2 = M_2(\lambda) B_2 C_1 v_1 \tag{A7}$$

$$w_2^T = w_1^T B_1 C_2 M_2(\lambda) \tag{A8}$$

Substituting (A6), (A7), (A8) in (5),

$$\lambda' = w_1^T\,[\, I : B_1 C_2 M_2 \,] \begin{bmatrix} B_1 D'_2 C_1 & B_1 C'_2 \\ B'_2 C_1 & A'_2 \end{bmatrix} \begin{bmatrix} I \\ \text{-----} \\ M_2 B_2 C_1 \end{bmatrix} v_1 \tag{A9}$$

Straightforward multiplication of (A9) and substitution of (A5) with s=λ gives (8).

APPENDIX B

The local convergence ratio (29) of the algorithm outlined in **3.4** can be proven from sensitivity results. We assume the same model description as in Fig. 1, (6), (7), (A1), (A2), but now s is replaced by the parameter q. From (8) we have, for the reduced order model in Fig. 5 b),

$$\lambda'_R = w_R^T B_1 (\partial F(q)/\partial q) C_1 v_R \tag{B1}$$

From (A2), (A3),

$$\partial F(q)/\partial q = -C_2 M_2^2(q) B_2 \tag{B2}$$

Now (B1) becomes, taking into account (B2), (25), (26),

$$\lambda'_R = -w_1^T B_1 C_2 M_2^2(q) B_2 C_1 v_1 / p_{S1} \tag{B3}$$

And from (A7), (A8), for q=λ ,

$$\lambda'_R = -w_2^T v_2 / p_{S1} = -\,p_{S2}/p_{S1} \tag{B4}$$

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