December 9, 2019

Small Signal Stability: The enhancement of damping of electromechanical oscillations in multimachine power systems by the application of power system stabilisers (PSS)

abstract-Using a two-axis model and a dynamic model, I investigate low-frequency oscillations in this report. This report analyzes both the electromechanical oscillations and their damping. Two types of electromechanical oscillation exist. The initial mode type is Local mode. Local mode is typically between 1 and 3 Hz between a remote power plant and the remainder of the system. Inter-area oscillations comprise the second type of electromechanical oscillations. This oscillation has a frequency lower than 1 Hz. I analyze A linearized multimachine system that computes eigenvalues and identifies the machines that contribute to a specific eigenvalue. In this framework, both local and inter-area oscillations are studied. Due to the size of the power system, it is frequently necessary to construct reduced-order models for dynamic stability studies by retaining a small number of modes. Important are the correct definition and determination of which state variables significantly contribute to the selected modes. This necessitates a tool for identifying the state variables that play a significant role in a particular mode. Participation factor analysis helps determine how each dynamic variable influences a particular mode or eigenvalue. A participation factor is a measure of the sensitivity of an eigenvalue to an entry on the diagonal of the system A matrix. Through a participation factor analysis, the electromechanical oscillatory modes associated with the rotor angles of the machines can be identified. This subject is presented in this report in clear detail. In addition, we use the power system stabilizer (PSS) to dampen low-frequency oscillations. The application of stabilizers maximizes the enhancement of the overall damping characteristics of the system. **KEY WORDS**:Small signal stability, PSS, inter-area modes, electromechanical oscillations, damping torque

1 What does this report provide?

This report begins by demonstrating how to determine the eigenvalue of the multimachine system. Consequently, a simple two-area system is selected to demonstrate the calculation of eigenvalues using load flow data. Using the two-axis model to represent the generator and constant power load, the Eigenvalues of the linearized system are determined. We model four machines with the excitation system and transmission lines in mind. Next, a PSS is installed on bus 12 and the results of small signal stability are analyzed. Gain, Washout (High pass filter), and Lead-Lag blocks are included in the installed PSS (Phase compensator). In addition, I demonstrate how the compact form of the power system's matrix A is altered by installing the PSS in the two-area system. Using participation factors, I determine the frequencies and damping ratios of system modes and identify the associated dominant state variables. Additionally, I determine the mode shapes of rotor angle modes. In addition, I discuss the controllability and observability of inter-area modes. Using an Ostojic approach, I design a PSS to dampen the low-frequency oscillations. Included in the installed PSS are gain and Lead-Lag blocks (Phase compensator). We also provide the outcomes of designing PSSs with varying quantities of K_PSS.

2 Problems

- **Problem #1 (8.1 a of book)**

The single line diagram for the two-area system is given in Figure 1. The transmission line data, machine data, excitation system data, and load-ow results are given in Figures 2, 3, 4, and 5. Using the two-axis model for the generator and constant power load representation, obtain eigenvalues of the linearized system.

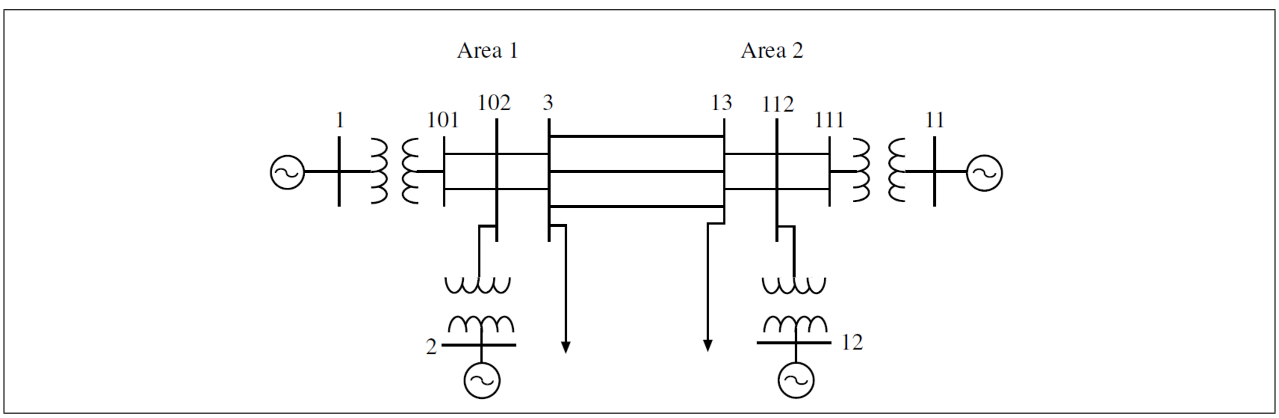

Figure 1: Two-area system

From Bus Number	To Bus Number	Series Resistance (R_s) pu	Series Reactance (X_s) pu	Shunt Susceptance (B) pu
1	101	0.001	0.012	0.00
2	102	0.001	0.012	0.00
3	13	0.022	0.22	0.33
3	13	0.022	0.22	0.33
3	13	0.022	0.22	0.33
3	102	0.002	0.02	0.03
3	102	0.002	0.02	0.03
11	111	0.001	0.012	0.00
12	112	0.001	0.012	0.00
13	112	0.002	0.02	0.03
13	112	0.002	0.02	0.03
101	102	0.005	0.05	0.075
101	102	0.005	0.05	0.075
111	112	0.005	0.05	0.075
111	112	0.005	0.05	0.075

Figure 2: Transmission Line Data on 100 MVA Base (Table 8.8)

Variable	Machine at Bus 1	Machine at Bus 2	Machine at Bus 11	Machine at Bus 12
X_1 (pu)	0.022	0.022	0.022	0.022
R_s (pu)	0.00028	0.00028	0.00028	0.00028
X_d (pu)	0.2	0.2	0.2	0.2
X'_d (pu)	0.033	0.033	0.033	0.033
T'_{do} (sec)	8.0	8.0	8.0	8.0
X_q (pu)	0.19	0.19	0.19	0.19
X'_q (pu)	0.061	0.061	0.061	0.061
T'_{qo} (sec)	0.4	0.4	0.4	0.4
H (sec)	54.0	54.0	63.0	63.0
D (pu)	0.0	0.0	0.0	0.0

Figure 3: Machine Data (Table 8.9)

Variable	Machine at Bus 1	Machine at Bus 2	Machine at Bus 11	Machine at Bus 12
K_A (pu)	200	200	200	200
T_A (sec)	.1	.1	.1	.1

Figure 4: Excitation System Data (Table 8.10)

Bus Number	Bus Type	Voltage Magnitude (pu)	Angle (degrees)	Real Power Gen. (pu)	Reactive Power Gen. (pu)	Real Power Load (pu)	Reactive Power Load (pu)
1	*PV*	1.03	8.2154	7.0	1.3386	0.0	0.0
2	*PV*	1.01	-1.5040	7.0	1.5920	0.0	0.0
11	Swing	1.03	0.0	7.2172	1.4466	0.0	0.0
12	*PV*	1.01	-10.2051	7.0	1.8083	0.0	0.0
101	*PQ*	1.0108	3.6615	0.0	0.0	0.0	0.0
102	*PQ*	0.9875	-6.2433	0.0	0.0	0.0	0.0
111	*PQ*	1.0095	-4.6977	0.0	0.0	0.0	0.0
112	*PQ*	0.9850	-14.9443	0.0	0.0	0.0	0.0
3	*PQ*	0.9761	-14.4194	0.0	0.0	11.59	-0.7350
13	*PQ*	0.9716	-23.2922	0.0	0.0	15.75	-0.8990

Figure 5: Load-Flow Results for the System (Table 8.11)

The admittance matrix of the two-area system is as follows:

Table 1: The admittance matrix of the two-area system

Bus (real number)	1 (11)	2(2)	3(1)	4(12)	5(101)	6(102)	7(111)	8(112)	9(3)	10(13)
1 (11)	6.896552	0	0	0	0	0	-6.89655	0	0	0
2 (2)	0	6.896552	0	0	0	-6.89655	0	0	0	0
3 (1)	0	0	6.896552	0	-6.89655	0	0	0	0	0
4 (12)	0	0	0	6.896552	0	0	0	-6.89655	0	0
5 (101)	0	0	-6.89655	0	10.85695	-3.9604	0	0	0	0
6 (102)	0	-6.89655	0	0	-3.9604	20.75794	0	0	-9.90099	0
7 (111)	-6.89655	0	0	0	0	0	10.85695	-3.9604	0	0
8 (112)	0	0	0	-6.89655	0	0	-3.9604	20.75794	0	-9.90099
9 (3)	0	0	0	0	0	-9.90099	0	0	11.25113	-1.35014
10 (13)	0	0	0	0	0	0	0	-9.90099	-1.35014	11.25113

Remark: To calculate the admittance matrix of two-area system, Shunt Susceptance of lines (B) are also considered.

In this problem, I consider simulation techniques for a multimachine power system using a two-axis machine model with no saturation and neglecting both the stator and the network transients. The resulting differential algebraic model is systematically derived. I must use the two-axis model in the form suitable for simulation after neglecting the subtransient reactances and saturation. I also neglect the turbine governor dynamics resulting in T_{Mi} being a constant. The limit constraints on V_{Ri} are also deleted, since I wish to concentrate on modeling and simulation. I assume a linear damping term $T_{FWi} = D_i(\omega_i - \omega s)$.

To calculate the eigenvalues of the linearized system, we need to calculate state space equations of this system. To form these equations, we need to calculate the initial condition of the aforementioned power system. To do so , first step is to calculate the rotor angles δ_i at all the machines by using the complex stator algebraic equation. The initial conditions of the state variables for the model are computed by systematically solving the load flow equations of the network first, and then computing the other algebraic and state variables. The load-flow equations are part of the network equations. Of note, load flow has been the traditional mechanism for computing a the steady-state operating point. In addition, In steady state, all the derivatives are zero in the differential equations. The initial conditions are obtained by the following equations:

$I_{G(i)}e^{j\gamma_i} = \frac{P_{G(i)} - jQ_{G(i)}}{\bar{V}_i^*}$ (1)
$\delta_i(0)$=Angle of $(V_i e^{j\theta_i} + (R_{s(i)} + jX_{q(i)})I_{G(i)}^{j\gamma_i})$ (2)
$I_{d(i)} + jI_{q(i)} = I_{G(i)}e^{j\gamma_i}e^{-j(\delta_i - \pi/2)}$ (3)
$V_{d(i)} + jV_{q(i)} = V_i e^{j\theta_i}e^{-j(\delta_i - \pi/2)}$ (4)
$E'_{d(i)} = (X_{q(i)} - X'_{q(i)})I_{q(i)}$ (5)
$E'_{q(i)} = (V_{q(i)} + R_{s(i)}I_{q(i)} + X'_{d(i)})I_{d(i)}$ (6)
$E_{fd(i)} = E'_{q(i)} + (X_{d(i)} - X'_{d(i)})I_{d(i)}$ (7)

According to these equation, the initial condition for 4 machine can be obtained as table 2 presents these values:

Table 2: The initial condition for 4 machines of Two-area system

	Machine 1 (bus 11)	Machine 2 (bus 2)	Machine 3 (bus 1)	Machine 4 (bus 12)
$\delta_i(0)$	0.797613340589030	0.761037656357729	0.933480561497469	0.593875770064631
$I_{d(i)}$	5.995744299078084	6.022450646560420	5.742735026470497	6.117447143701314
$I_{q(i)}$	3.888635755841996	3.774800614237417	3.859687086459257	3.717104362751911
$E'_{d(i)}$	0.501634012503618	0.486949279236627	0.497899634153244	0.479506462794997
$E'_{q(i)}$	0.918317692242255	0.912625133905325	0.915482291463785	0.926606875788978
$E_{fd(i)}$	1.919606990188295	1.918374391880915	1.874519040884358	1.948220548787098

One important point about load flow should be emphasized. Load flow is normally used to evaluate operation at a specific load level (specified by a given set of powers). For a specified load and generation schedule, the solution is independent of the actual load model. II t is certainly possible to evaluate the voltage at a constant impedance load for a specific case where that impedance load consumes a specific amount of power. Thus, the use of "constant power" in load- flow analysis does not require or even imply that the load is truly a constant power device. It merely gives the voltage at the buses when the loads (any type) consume a specific amount of power. The load characteristic is important when the analyst wants to study the system in response to a change, such as contingency analysis or dynamic analysis. For these purposes, standard load flow is computed on the basis of constant PQ loads and usually provides the "initial conditions" for the dynamic system. After calculation of initial condition, we write equations of wo-area system system. The equations of system include:

- Differential Equations:
  $\Delta\dot{X}_i = A_{1i}\Delta X_i + B_{1i}\Delta I_{gi} + B_{2i}\Delta V_{gi} + E_i\Delta U_i$ (8)

- Stator Algebraic Equations:
  $0 = C_1\Delta X_i + D_1\Delta I_{gi} + D_2\Delta V_{gi}$ (9)

- Network Equations:
  PV buses: $0 = C_2\Delta X_i + D_3\Delta I_g + D_4\Delta V_g + D_5\Delta V_l$ (10)
  PQ buses: $0 = D_6\Delta V_g + D_7\Delta V_l$ (11)

Where:

$$\Delta X_i = \begin{bmatrix} \Delta\delta_i \\ \Delta\omega_i \\ \Delta E'_{qi} \\ \Delta E'_{di} \\ \Delta E'_{fdi} \end{bmatrix}, \ \Delta I_{gi} = \begin{bmatrix} \Delta I_{di} \\ \Delta I_{qi} \end{bmatrix}, \ \Delta V_{gi} = \begin{bmatrix} \Delta\theta_i \\ \Delta V_i \end{bmatrix}, \ \Delta V_{li} = \begin{bmatrix} \Delta\theta_i \\ \Delta V_i \end{bmatrix} \ and \ \Delta U_i = \begin{bmatrix} \Delta T_{Mi} \\ \Delta V_{refi} \end{bmatrix} \quad (12)$$

Where for ΔV_{gi}, i=1,...,m (PV buses) and for ΔV_{li}, i=m+1,...n (PQ buses). Now, We must calculate the matrixes A_1, B_1, B_2, E_i, C_1, D_1, D_2, C_2, D_3, D_4, D_5, D_6, and D_7. Related equations are linearized analytically. The linearization of the differential equations yields the following matrixes.

Where:

$$A_1 = \begin{bmatrix} A_{11} & 0_{5\times5} & 0_{5\times5} & 0_{5\times5} \\ 0_{5\times5} & A_{12} & 0_{5\times5} & 0_{5\times5} \\ 0_{5\times5} & 0_{5\times5} & A_{13} & 0_{5\times5} \\ 0_{5\times5} & 0_{5\times5} & 0_{5\times5} & A_{14} \end{bmatrix}_{20\times20} \quad \text{and } A_{1i} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & -\frac{D_i}{M_i} & -\frac{I_{qio}}{M_i} & -\frac{I_{dio}}{M_i} & 0 \\ 0 & 0 & -\frac{1}{T'_{doi}} & 0 & \frac{1}{T'_{doi}} \\ 0 & 0 & 0 & \frac{1}{T'_{qoi}} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{T'_A} \end{bmatrix} \tag{13}$$

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. As a result, A_1 will be:

Table 3: Results for the Matrix A_1 of the two-area system to calculate eigenvalues.

0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-11.63477097	-17.93922499	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-0.125	0	0.125	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-2.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	-10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-13.17653986	-21.02231856	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-0.125	0	0.125	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	-2.5	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-10	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	-13.47284955	-20.04592686	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	-0.125	0	0.125	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	-2.5	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	-10	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-11.12155025	-18.30335905	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.125	0	0.125
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2.5	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-10

$$B_1 = \begin{bmatrix} B_{11} & 0_{5\times2} & 0_{5\times2} & 0_{5\times2} \\ 0_{5\times2} & B_{12} & 0_{5\times2} & 0_{5\times2} \\ 0_{5\times2} & 0_{5\times2} & B_{13} & 0_{5\times2} \\ 0_{5\times2} & 0_{5\times2} & 0_{5\times2} & B_{14} \end{bmatrix}_{20\times8} \quad \text{and } B_{1i} = \begin{bmatrix} 0 & 0 \\ \frac{(I_{qio}(X'_{di}-X'_{qi})-E'_{dio}}{M_i} & \frac{(I_{dio}(X'_{di}-X'_{qi})-E'_{qio}}{M_i} \\ -\frac{(X_{di}-X'_{di})}{T'_{doi}} & 0 \\ 0 & -\frac{(X_{qi}-X'_{qi})}{T'_{qoi}} \\ 0 & 0 \end{bmatrix} \tag{14}$$

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. As a result, B_1 will be:

Table 4: Results for the Matrix B1 of the two-area system to calculate eigenvalues.

0	0	0	0	0	0	0	0
-1.826659043	-3.24989841	0	0	0	0	0	0
-0.020875	0	0	0	0	0	0	0
0	0.3225	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	-2.068716759	-3.774287604	0	0	0	0
0	0	-0.020875	0	0	0	0	0
0	0	0	0.3225	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	-2.11523738	-3.756921998	0	0
0	0	0	0	-0.020875	0	0	0
0	0	0	0	0	0.3225	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	-1.746083389	-3.284895343
0	0	0	0	0	0	-0.020875	0
0	0	0	0	0	0	0	0.3225
0	0	0	0	0	0	0	0

$$B_2 = \begin{bmatrix} B_{21} & 0_{5\times2} & 0_{5\times2} & 0_{5\times2} \\ 0_{5\times2} & B_{22} & 0_{5\times2} & 0_{5\times2} \\ 0_{5\times2} & 0_{5\times2} & B_{23} & 0_{5\times2} \\ 0_{5\times2} & 0_{5\times2} & 0_{5\times2} & B_{24} \end{bmatrix}_{20\times8} \quad and\ B_{2i} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -\frac{K_{Ai}}{T_{Ai}} \end{bmatrix} \tag{15}$$

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. As a result, B_2 will be:

Table 5: Results for the Matrix B2 of the two-area system to calculate eigenvalues.

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	-2000	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	-2000	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	-2000	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-2000

$$E_1 = \begin{bmatrix} E_{11} & 0_{5\times2} & 0_{5\times2} & 0_{5\times2} \\ 0_{5\times2} & E_{12} & 0_{5\times2} & 0_{5\times2} \\ 0_{5\times2} & 0_{5\times2} & E_{13} & 0_{5\times2} \\ 0_{5\times2} & 0_{5\times2} & 0_{5\times2} & E_{14} \end{bmatrix}_{20\times8} \quad \text{and } E_{1i} = \begin{bmatrix} 0 & 0 \\ 1/M_i & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{K_{Ai}}{T_{Ai}} \end{bmatrix} \tag{16}$$

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. As a result, E_1 will be:

Table 6: Results for the Matrix E1 of the two-area system to calculate eigenvalues.

0	0	0	0	0	0	0	0
2.991993003	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	2000	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	3.490658504	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	2000	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	3.490658504	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	2000	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	2.991993003	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	2000

The algebraic equations consist of the stator algebraic equations and the network equations. There are several different ways of writing the stator algebraic equations as two real equations for computational purposes. The idea is to express I_{di}, I_{qi} in terms of the state and network variables. Of note, two popular method to present stator algebraic equations consist of the polar form and the rectangular form. we use Linearizied stator algebraic equations. For the stator algebraic equations, we have:

$$C_1 = \begin{bmatrix} C_{11} & 0_{2\times5} & 0_{2\times5} & 0_{2\times5} \\ 0_{2\times5} & C_{12} & 0_{2\times5} & 0_{2\times5} \\ 0_{2\times5} & 0_{2\times5} & C_{13} & 0_{2\times5} \\ 0_{2\times5} & 0_{2\times5} & 0_{2\times5} & C_{14} \end{bmatrix}_{8\times20} \quad \text{and } C_{1i} = \begin{bmatrix} -V_{i0}\cos(\delta_{j0} - \theta_{i0}) & 0 & 0 & 1 & 0 \\ V_{i0}\sin(\delta_{i0} - \theta_{i0}) & 0 & 1 & 0 & 0 \end{bmatrix} \tag{17}$$

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. As a result, C_1 will be:

Table 7: Results for the Matrix C1 of the two-area system to calculate eigenvalues.

-0.719369312	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.737161985	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-0.712827318	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0.715525831	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-0.724891323	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0.731732581	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.723690331	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.704536944	0	1	0	0

$$D_1 = \begin{bmatrix} D_{11} & 0_{2\times2} & 0_{2\times2} & 0_{2\times2} \\ 0_{2\times2} & D_{12} & 0_{2\times2} & 0_{2\times2} \\ 0_{2\times2} & 0_{2\times2} & D_{13} & 0_{2\times2} \\ 0_{2\times2} & 0_{2\times2} & 0_{2\times2} & D_{14} \end{bmatrix}_{8\times8} \quad \text{and } D_{1i} = \begin{bmatrix} -R_{si} & X'_{qi} \\ -X'_{di} & -R_{si} \end{bmatrix} \quad (18)$$

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. As a result, D_1 will be:

Table 8: Results for the Matrix D1 of the two-area system to calculate eigenvalues.

-0.00028	0.061	0	0	0	0	0	0
-0.033	-0.00028	0	0	0	0	0	0
0	0	-0.00028	0.061	0	0	0	0
0	0	-0.033	-0.00028	0	0	0	0
0	0	0	0	-0.00028	0.061	0	0
0	0	0	0	-0.033	-0.00028	0	0
0	0	0	0	0	0	-0.00028	0.061
0	0	0	0	0	0	-0.033	-0.00028

$$D_2 = \begin{bmatrix} D_{21} & 0_{2\times2} & 0_{2\times2} & 0_{2\times2} \\ 0_{2\times2} & D_{22} & 0_{2\times2} & 0_{2\times2} \\ 0_{2\times2} & 0_{2\times2} & D_{23} & 0_{2\times2} \\ 0_{2\times2} & 0_{2\times2} & 0_{2\times2} & D_{24} \end{bmatrix}_{8\times8} \quad \text{and } D_{2i} = \begin{bmatrix} V_{i0}\cos(\delta_{i0}-\theta_{i0}) & -\sin(\delta_{i0}-\theta_{i0}) \\ -V_{i0}\sin(\delta_{i0}-\theta_{i0}) & -\cos(\delta_{i0}-\theta_{i0}) \end{bmatrix} \quad (19)$$

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. As a result, D_2 will be:

Table 9: Results for the Matrix D2 of the two-area system to calculate eigenvalues.

0.719369312	-0.715691248	0	0	0	0	0	0
-0.737161985	-0.698416808	0	0	0	0	0	0
0	0	0.712827318	-0.708441416	0	0	0	0
0	0	-0.715525831	-0.705769622	0	0	0	0
0	0	0	0	0.724891323	-0.710419981	0	0
0	0	0	0	-0.731732581	-0.703777984	0	0
0	0	0	0	0	0	0.723690331	-0.69756133
0	0	0	0	0	0	-0.704536944	-0.71652508

The dynamic circuit, together with the static network and the loads make the algebraic equations of the the network equations. The network equations can be expressed either in power-balance or current balance form. The latter form is more popular with the industry software packages. We use Linearizied network algebraic equations. For the network algebraic equations, we have:

$$C_2 = \begin{bmatrix} C_{21} & 0_{2\times5} & 0_{2\times5} & 0_{2\times5} \\ 0_{2\times5} & C_{22} & 0_{2\times5} & 0_{2\times5} \\ 0_{2\times5} & 0_{2\times5} & C_{23} & 0_{2\times5} \\ 0_{2\times5} & 0_{2\times5} & 0_{2\times5} & C_{24} \end{bmatrix}_{8\times20} \quad and$$

$$C_{1i} = \begin{bmatrix} I_{di0}V_{i0}cos(\delta_{i0}-\theta_{i0}) - I_{qi0}V_{i0}sin(\delta_{i0}-\theta_{i0}) & 0 & 0 & 0 & 0 \\ -I_{di0}V_{i0}sin(\delta_{i0}-\theta_{i0}) - I_{qi0}V_{i0}cos(\delta_{i0}-\theta_{i0}) & 0 & 0 & 0 & 0 \end{bmatrix} \quad (20)$$

It is worth to mention that i here is related to the PV buses. I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. As a result, C_2 will be:

Table 10: Results for the Matrix C2 of the two-area system to calculate eigenvalues.

1.4466	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-7.2172	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1.592	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-7	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1.3386	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-7	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.8083	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-7	0	0	0	0

$$D_3 = \begin{bmatrix} D_{31} & 0_{2\times2} & 0_{2\times2} & 0_{2\times2} \\ 0_{2\times2} & D_{32} & 0_{2\times2} & 0_{2\times2} \\ 0_{2\times2} & 0_{2\times2} & D_{33} & 0_{2\times2} \\ 0_{2\times2} & 0_{2\times2} & 0_{2\times2} & D_{34} \end{bmatrix}_{8\times8} \quad and\ D_{3i} = \begin{bmatrix} V_{i0}sin(\delta_{i0}-\theta_{i0}) & V_{i0}cos(\delta_{i0}-\theta_{i0}) \\ V_{i0}cos(\delta_{i0}-\theta_{i0}) & -V_{i0}sin(\delta_{i0}-\theta_{i0}) \end{bmatrix} \quad (21)$$

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. As a result, D_3 will be:

Table 11: Results for the Matrix D3 of the two-area system to calculate eigenvalues.

0.737161985	0.719369312	0	0	0	0	0	0
0.719369312	-0.737161985	0	0	0	0	0	0
0	0	0.715525831	0.712827318	0	0	0	0
0	0	0.712827318	-0.715525831	0	0	0	0
0	0	0	0	0.731732581	0.724891323	0	0
0	0	0	0	0.724891323	-0.731732581	0	0
0	0	0	0	0	0	0.704536944	0.723690331
0	0	0	0	0	0	0.723690331	-0.704536944

$$D_4 = \begin{bmatrix} [D_{41}]_{2\times2} & [D_{412}]_{2\times2} & [D_{413}]_{2\times2} & [D_{414}]_{2\times2} \\ [D_{421}]_{2\times2} & [D_{42}]_{2\times2} & [D_{423}]_{2\times2} & [D_{424}]_{2\times2} \\ [D_{431}]_{2\times2} & [D_{432}]_{2\times2} & [D_{43}]_{2\times2} & [D_{434}]_{2\times2} \\ [D_{441}]_{2\times2} & [D_{442}]_{2\times2} & [D_{443}]_{2\times2} & [D_{44}]_{2\times2} \end{bmatrix}_{8\times8} \quad and\ D_{4i} = \begin{bmatrix} AD_{4i} & BD_{4i} \\ CD_{4i} & DD_{4i} \end{bmatrix} \quad (22)$$

$$D_4 = \begin{bmatrix} AD_{41} & BD_{41} & AD_{412} & BD_{412} & AD_{413} & BD_{413} & AD_{414} & BD_{414} \\ CD_{41} & DD_{41} & CD_{412} & DD_{412} & CD_{413} & DD_{413} & CD_{414} & DD_{414} \\ AD_{421} & BD_{421} & AD_{42} & BD_{42} & AD_{423} & BD_{423} & AD_{424} & BD_{424} \\ CD_{421} & DD_{421} & CD_{42} & DD_{42} & CD_{423} & DD_{423} & CD_{424} & DD_{424} \\ AD_{431} & BD_{431} & AD_{432} & BD_{432} & AD_{43} & BD_{43} & AD_{434} & BD_{434} \\ CD_{431} & DD_{431} & CD_{432} & DD_{432} & CD_{43} & DD_{43} & CD_{434} & DD_{434} \\ AD_{441} & BD_{441} & AD_{442} & BD_{442} & AD_{443} & BD_{443} & AD_{44} & BD_{44} \\ CD_{421} & DD_{441} & CD_{442} & DD_{442} & CD_{443} & DD_{443} & CD_{44} & DD_{44} \end{bmatrix} \quad (22)$$

We have:
$AD_{4i} = -I_{di0}V_{i0}cos(\delta_{i0} - \theta_{i0}) + I_{qi0}V_{i0}sin(\delta_{i0} - \theta_{i0})$
$+ \sum_{k=1,k\neq i}^{n} V_{i0}V_{k0}Y_{ik}sin(\theta_{i0} - \theta_{k0} - \alpha_{ik})$ (23)

$BD_{4i} = I_{di0}sin(\delta_{i0} - \theta_{i0}) + I_{qi0}cos(\delta_{i0} - \theta_{i0})$
$+ \sum_{k=1}^{n} -V_{k0}Y_{ik}cos(\theta_{i0} - \theta_{k0} - \alpha_{ik}) - V_{i0}Y_{ii}cos(\theta_{i0} - \theta_{i0}) - \alpha_{ii})$ (24)

$CD_{4i} = I_{di0}V_{i0}sin(\delta_{i0} - \theta_{i0}) + I_{qi0}V_{i0}cos(\delta_{i0} - \theta_{i0})$
$+ \sum_{k=1,k\neq i}^{n} -V_{i0}V_{k0}Y_{ik}cos(\theta_{i0} - \theta_{k0} - \alpha_{ik})$ (25)

$DD_{4i} = I_{di0}cos(\delta_{i0} - \theta_{i0}) - I_{qi0}sin(\delta_{i0} - \theta_{i0})$
$+ \sum_{k=1}^{n} -V_{k0}Y_{ik}sin(A\theta_{i0} - \theta_{k0} - \alpha_{ik}) - V_{i0}Y_{ii}sin(\theta_{i0} - \theta_{i0} - \alpha_{ii})$ (26)
And for i, k while both of them are PV buses and there are synchronise machine in these buses, we have:
$AD_{4ik} = -V_{i0}V_{k0}Y_{ik}sin(\theta_{i0} - \theta_{k0} - \alpha_{ik})$ (26')
$BD_{4ik} = -V_{i0}Y_{ik}cos(\theta_{i0} - \theta_{k0} - \alpha_{ik})$ (27')
$CD_{4ik} = V_{i0}V_{k0}Y_{ik}cos(\theta_{i0} - \theta_{k0} - \alpha_{ik})$ (28')
$DD_{4ik} = -V_{i0}Y_{ik}sin(A\theta_{i0} - \theta_{k0} - \alpha_{ik})$ (29')
Because there is no line between the synchronise machines, all of $AD_{4ik}, BD_{4ik}, CD_{4ik}$, and DD_{4ik} are equal to 0. I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. As a result, D_4 will be:

Table 12: Results for the Matrix D4 of the two-area system to calculate eigenvalues.

-87.79598708	-7.103421289	0	0	0	0	0	0
7.31657952	-85.24393621	0	0	0	0	0	0
0	0	-84.41947799	-6.965501169	0	0	0	0
0	0	7.035188647	-83.58877221	0	0	0	0
0	0	0	0	-87.79866286	-7.103494316	0	0
0	0	0	0	7.316504303	-85.24133837	0	0
0	0	0	0	0	0	-84.42608812	-6.965446604
0	0	0	0	0	0	7.035243757	-83.58222754

$$D_5 = \begin{bmatrix} AD_{515} & BD_{515} & AD_{516} & BD_{516} & ... & AD_{51(10)} & BD_{51(10)} \\ CD_{515} & DD_{515} & CD_{516} & DD_{516} & ... & CD_{51(10)} & DD_{51(10)} \\ AD_{525} & BD_{525} & AD_{526} & BD_{526} & ... & AD_{52(10)} & BD_{52(10)} \\ CD_{525} & DD_{525} & CD_{526} & DD_{526} & ... & CD_{52(10)} & DD_{52(10)} \\ AD_{535} & BD_{535} & AD_{536} & BD_{536} & ... & AD_{53(10)} & BD_{53(10)} \\ CD_{535} & DD_{535} & CD_{536} & DD_{536} & ... & CD_{53(10)} & DD_{53(10)} \\ AD_{545} & BD_{545} & AD_{546} & BD_{546} & ... & AD_{54(10)} & BD_{54(10)} \\ CD_{545} & DD_{545} & CD_{546} & DD_{546} & ... & CD_{54(10)} & DD_{54(10)} \end{bmatrix}_{8\times 12} \quad (27)$$

It is worth to mention that i here is related to the PV buses. Where:
$AD_{5ik} = -V_{i0}V_{k0}Y_{ik}sin(\theta_{i0} - \theta_{k0} - \alpha_{ik})$ (28)
$BD_{5ik} = -V_{i0}Y_{ik}cos(\theta_{i0} - \theta_{k0} - \alpha_{ik})$ (29)
$CD_{5ik} = V_{i0}V_{k0}Y_{ik}cos(\theta_{i0} - \theta_{k0} - \alpha_{ik})$ (30)
$DD_{5ik} = -V_{i0}Y_{ik}sin(A\theta_{i0} - \theta_{k0} - \alpha_{ik})$ (31)

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. As a result, D_5 will be:

Table 13: Results for the Matrix D5 of the two-area system to calculate eigenvalues.

0	0	0	0	86.34938708	0.0984443	0	0	0	0	0	0
0	0	0	0	-0.09937952	85.5367876	0	0	0	0	0	0
0	0	82.82747799	0.035634073	0	0	0	0	0	0	0	0
0	0	-0.035188647	83.87592708	0	0	0	0	0	0	0	0
86.46006286	0.313122579	0	0	0	0	0	0	0	0	0	0
-0.316504303	85.53627113	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	82.61778812	0.035780464	0	0	0	0
0	0	0	0	0	0	-0.035243757	83.87592702	0	0	0	0

$$D_6 = \begin{bmatrix} AD_{651} & BD_{651} & AD_{652} & BD_{652} & ... & AD_{654} & BD_{654} \\ CD_{651} & DD_{651} & CD_{652} & DD_{652} & ... & CD_{654} & DD_{654} \\ AD_{661} & BD_{661} & AD_{662} & BD_{662} & ... & AD_{654} & BD_{664} \\ CD_{661} & DD_{661} & CD_{662} & DD_{662} & ... & CD_{654} & DD_{664} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ AD_{6(10)1} & BD_{6(10)1} & AD_{6(10)2} & BD_{6(10)2} & ... & AD_{6(10)4} & BD_{6(10)4} \\ CD_{6(10)1} & DD_{6(10)1} & CD_{6(10)2} & DD_{6(10)2} & ... & CD_{6(10)4} & DD_{6(10)4} \end{bmatrix}_{12\times 8} \quad (32)$$

It is worth to mention that i here is related to the PQ buses and k here is related to the PV buses. Where:
$AD_{5ik} = -V_{i0}V_{k0}Y_{ik}sin(\theta_{i0} - \theta_{k0} - \alpha_{ik})$ (33)
$BD_{5ik} = -V_{i0}Y_{ik}cos(\theta_{i0} - \theta_{k0} - \alpha_{ik})$ (34)
$CD_{5ik} = V_{i0}V_{k0}Y_{ik}cos(\theta_{i0} - \theta_{k0} - \alpha_{ik})$ (35)
$DD_{5ik} = -V_{i0}Y_{ik}sin(A\theta_{i0} - \theta_{k0} - \alpha_{ik})$ (36)

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. As a result, D_6 will be:

Table 14: Results for the Matrix D6 of the two-area system to calculate eigenvalues.

0	0	0	0	85.31989718	13.59076929	0	0
0	0	0	0	-13.99849237	82.83485163	0	0
0	0	81.69085435	13.53927959	0	0	0	0
0	0	-13.67467238	80.88203401	0	0	0	0
85.17481008	13.78087726	0	0	0	0	0	0
-14.19430358	82.69399037	0	0	0	0	0	0
0	0	0	0	0	0	81.48406587	13.50486212
0	0	0	0	0	0	-13.63991074	80.67729294
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

$$D_7 = \begin{bmatrix} AD_{755} & BD_{755} & AD_{755} & BD_{755} & ... & AD_{75(10)} & BD_{75(10)} \\ CD_{755} & DD_{755} & CD_{755} & DD_{755} & ... & CD_{75(10)} & DD_{75(10)} \\ AD_{766} & BD_{766} & AD_{766} & BD_{766} & ... & AD_{75(10)} & BD_{76(10)} \\ CD_{766} & DD_{766} & CD_{666} & DD_{766} & ... & CD_{75(10)} & DD_{76(10)} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ AD_{7(10)1} & BD_{7(10)1} & AD_{7(10)2} & BD_{7(10)2} & ... & AD_{7(10)(10)} & BD_{7(10)(10)} \\ CD_{7(10)1} & DD_{7(10)1} & CD_{7(10)2} & DD_{7(10)2} & ... & CD_{7(10)(10)} & DD_{7(10)(10)} \end{bmatrix}_{12\times12} \quad (37)$$

if i=k:
$AD_{7ii} = \sum_{k=1,k\neq i}^{n} V_{i0}V_{k0}Y_{ik}sin(\theta_{i0}-\theta_{k0}-\alpha_{ik})$ (38)
$BD_{7ii} = \sum_{k=1}^{n} -V_{k0}Y_{ik}cos(\theta_{i0}-\theta_{k0}-\alpha_{ik}) - V_{i0}Y_{ii}cos(\theta_{i0}-\theta_{i0})-\alpha_{ii})$ (39)
$CD_{7ii} = \sum_{k=1,k\neq i}^{n} -V_{i0}V_{k0}Y_{ik}cos(\theta_{i0}-\theta_{k0}-\alpha_{ik})$ (40)
$DD_{7ii} = \sum_{k=1}^{n} -V_{k0}Y_{ik}sin(A\theta_{i0}-\theta_{k0}-\alpha_{ik}) - V_{i0}Y_{ii}sin(\theta_{i0}-\theta_{i0}-\alpha_{ii})$ (41)
It is worth to mention that i is related to the PQ buses.

if $i \neq k$:
$AD_{7ik} = -V_{i0}V_{k0}Y_{ik}sin(\theta_{i0}-\theta_{k0}-\alpha_{ik})$ (42)
$BD_{7ik} = -V_{i0}Y_{ik}cos(\theta_{i0}-\theta_{k0}-\alpha_{ik})$ (43)
$CD_{7ik} = V_{i0}V_{k0}Y_{ik}cos(\theta_{i0}-\theta_{k0}-\alpha_{ik})$ (44)
$DD_{7ik} = -V_{i0}Y_{ik}sin(A\theta_{i0}-\theta_{k0}-\alpha_{ik})$ (45)

It is worth to mention that both i and k are related to the PQ buses. I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. As a result, D_7 will be:

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. As a result, A' will be:

Table 16: Results for the Matrix $A^{'} of the two - area system to calculate eigenvalues$.

0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-79.2775739	0	-67.43794942	35.08165238	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-0.464203942	0	-0.757551122	-0.002903513	0.125	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3.83614152	0	0.044856673	-7.786679346	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	-10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-89.12738405	0	-76.38745247	40.56110027	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-0.450536963	0	-0.757551122	-0.002903513	0.125	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	3.80058557	0	0.044856673	-7.786679346	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-10	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-91.71345155	0	-78.09100765	41.24635042	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-0.460753533	0	-0.757551122	-0.002903513	0.125	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	3.865091076	0	0.044856673	-7.786679346	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	-10	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-76.39262546	0	-64.48800503	35.3024235	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.44355439	0	-0.757551122	-0.002903513	0.125
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3.857521909	0	0.044856673	-7.786679346	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-10

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. As a result, $B^{'}$ will be:

Table 17: Results for the Matrix $B^{'}$ of the two-area system to calculate eigenvalues.

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
79.2775739	1.027299884	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.464203942	0.443862354	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-3.83614152	3.752301483	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-2000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	89.12738405	0.984097446	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0.450536963	0.448492335	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-3.80058557	3.713644127	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-2000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	91.71345155	1.933578548	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0.460753533	0.447238267	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	-3.865091076	3.724193503	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-2000	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	76.39262546	0.845082291	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0.44355439	0.455264122	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	-3.857521909	3.655642147	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-2000	0	0	0	0	0	0	0	0	0	0	0	0

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. As a result, K_1 will be:

Table 18: Results for the Matrix K_1 of the two-area system to calculate eigenvalues.

-112.745397	-14.4077056	0	0	0	0	0	0
0.088288813	-109.1167021	0	0	0	0	0	0
0	0	-108.2628921	-14.1300006	0	0	0	0
0	0	0.082816572	-107.1430323	0	0	0	0
0	0	0	0	-112.637145	-14.40960015	0	0
0	0	0	0	0.08633743	-109.2218011	0	0
0	0	0	0	0	0	-108.0524529	-14.12747016
0	0	0	0	0	0	0.085372322	-107.3513881

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. As a result, K_2 will be:

Table 19: Results for the Matrix K_2 of the two-area system to calculate eigenvalues.

26.39600991	0	22.43742941	-11.68994807	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.011090707	0	21.6956889	12.18420948	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	25.43541414	0	21.78090397	-11.58571582	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-0.047627925	0	21.50046376	11.8286223	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	26.17708217	0	22.27367647	-11.78122449	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0.230166873	0	21.86377122	12.09597437	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25.43466475	0	21.44943131	-11.76531951	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.050128565	0	21.83116159	11.64999457	0

Now, We have already calculated the matrixes A_1, B_1, B_2, E_i, C_1, D_1, D_2, C_2, D_3, D_4, D_5, D_6, and D_7 , K_1, and K_2. If we ΔI_g from equations (8)-(10), we come up with the following set of equations:
$\Delta\dot{X} = (A_1 - B_1 D_1^{-1} C_1)\Delta X + (B_2 - B_1 D_1^{-1} D_2)\Delta v_g + E_1 \Delta U$
$0 = K_2 \Delta X + K_1 \Delta V_g + D_5 \Delta V_l$
$0 = \Delta D_6 \Delta V_g + \Delta D_7 \Delta V_l$ These equation can be written in more compact forms:

$$\begin{bmatrix}\Delta\dot{X}\\ 0\end{bmatrix} = \begin{bmatrix}A' & B'\\ C' & D'\end{bmatrix}\begin{bmatrix}\Delta X\\ \delta V_N\end{bmatrix} + \begin{bmatrix}E_1\\ 0\end{bmatrix}\Delta U \text{ and } \Delta V_N = \begin{bmatrix}\Delta V_g\\ \Delta V_l\end{bmatrix}$$

These differential-algebraic equations (DEA) can be rewritten by the reordering of the algebraic variables as follows:

$$\begin{bmatrix}\Delta\dot{X}\\ 0\\ 0\end{bmatrix} = \begin{bmatrix}A' & B_1' & B_2'\\ C_1' & D_{11}' & D_{12}'\\ C_2' & D_{21}' & D_{22}'\end{bmatrix}\begin{bmatrix}\Delta X\\ \delta y_c\\ \delta y_b\end{bmatrix} + \begin{bmatrix}E_1\\ 0\\ 0\end{bmatrix}\Delta U$$

and We also have:

$$J'_{AE} = \begin{bmatrix}D_{11}' & D_{12}'\\ D_{21}' & D_2'\end{bmatrix}$$

According to defined equation, we calculated the needed matrixes.I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. As a result, $C^{'}$ will be:

Table 20: Results for the Matrix $C^{'}$ of the two-area system to calculate eigenvalues.

26.39600991	0	22.43742941	-11.68994807	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.011090707	0	21.6956889	12.18420948	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	25.43541414	0	21.78090397	-11.58571582	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-0.047627925	0	21.50046376	11.8286223	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	26.17708217	0	22.27367647	-11.78122449	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0.230166873	0	21.86377122	12.09597437	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25.43466475	0	21.44943131	-11.76531951	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.050128565	0	21.83116159	11.64999457	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

For voltage-dependent loads, only the appropriate diagonal elements of $D'11$ and $D'22$ will be affected. Now, $D'22$ is the load-flow Jacobian JLF modified by the load representation. The stator algebraic variables are eliminated to obtain J'_{AE}. I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. As a result, J'_{AE} will be:

Table 21: The results for the matrix J'_{AE} of the two-area system to calculate eigenvalues.

-112.745397	-14.4077056	0	0	0	0	0	0	0	0	0	0	86.34938708	0.0984443	0	0	0	0	0	0
0.088288813	-109.1167021	0	0	0	0	0	0	0	0	0	0	-0.09937952	85.5367876	0	0	0	0	0	0
0	0	-108.2628921	-14.1300006	0	0	0	0	0	0	82.82747799	0.035634073	0	0	0	0	0	0	0	0
0	0	0.082816572	-107.1430323	0	0	0	0	0	0	-0.035188647	83.87592708	0	0	0	0	0	0	0	0
0	0	0	0	-112.637145	-14.40960015	0	0	86.46006286	0.313122579	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0.08633743	-109.2218011	0	0	-0.316504303	85.53627113	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	-108.0524529	-14.12747016	0	0	0	0	0	0	82.61778812	0.035780464	0	0	0	0
0	0	0	0	0	0	0.085372322	-107.3513881	0	0	0	0	0	0	-0.035243757	83.87592702	0	0	0	0
0	0	0	0	85.31989718	13.59076929	0	0	-124.9419514	-10.9740701	39.62205421	-2.942414224	0	0	0	0	0	0	0	0
0	0	0	0	-13.99849237	82.83485163	0	0	11.09285832	-123.7611981	2.905634046	40.1235992	0	0	0	0	0	0	0	0
0	0	81.69085435	13.53927959	0	0	0	0	38.26208599	10.57978637	-215.7756655	-20.49788378	0	0	0	0	95.82272512	-4.22693883	0	0
0	0	-13.67467238	80.88203401	0	0	0	0	-10.69404807	37.85327067	20.24280546	-218.7036491	0	0	0	0	4.125914992	98.16896335	0	0
85.17481008	13.78087726	0	0	0	0	0	0	0	0	0	0	-124.6277562	-10.95995879	39.45294612	-3.177627157	0	0	0	0
-14.19430358	82.69399037	0	0	0	0	0	0	0	0	0	0	11.06434083	-123.5951169	3.12996275	40.0537524	0	0	0	0
0	0	0	0	0	0	81.48406587	13.50486212	0	0	0	0	38.05190506	10.77805633	-214.662754	-20.44785086	0	0	95.12678307	-4.509830363
0	0	0	0	0	0	-13.63991074	80.67729294	0	0	0	0	-10.88044787	37.69381383	20.13860743	-218.1720597	0	0	4.381751181	97.90735186
0	0	0	0	0	0	0	0	0	0	93.10823255	23.31039064	0	0	0	0	-105.9568953	0.890958054	12.84866278	-0.730599452
0	0	0	0	0	0	0	0	0	0	-23.01901076	94.28681777	0	0	0	0	22.30916033	-111.0931937	0.709850428	13.22423094
0	0	0	0	0	0	0	0	0	0	0	0	0	0	92.37541089	23.48423409	12.45366938	3.319417728	-104.8290803	5.279726631
0	0	0	0	0	0	0	0	0	0	0	0	0	0	-23.13197058	93.78214304	-3.240083644	12.75859992	26.37205422	-110.7386147

At the end,in the more compact form, we have:
$A_{SYS} = A'_{20\times20} - B'_{20\times20} J'^{-1}_{20\times20} C'_{20\times20}$ (55)
I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. A_{SYS} will be:

Table 22: Results for the Matrix A_{SYS} of the two-area system to calculate eigenvalues.

0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-26.11170796	0	-28.68116519	7.918812997	0	4.353334748	0	5.417510598	-1.056811354	0	2.507649153	0	3.634387676	-0.283677484	0	19.25072406	0	13.23169624	-10.50090888	0
-0.187334502	0	-0.267296837	0.017754361	0.125	0.036738681	0	0.096416034	0.018868534	0	0.010486621	0	0.057081841	0.022395044	0	0.1401092	0	0.24921846	0.0048557	0
0.964461206	0	0.455956112	-4.911824586	0	-0.112980761	0	0.299620628	0.26871494	0	-0.157783975	0	0.135011397	0.222616048	0	-0.693696471	0	0.851036781	1.084209938	0
157.3121671	0	-1202.779827	-820.8704444	-10	-51.37891254	0	-295.510497	-114.4537261	0	19.16985618	0	-163.5317734	-109.883554	0	-125.1031107	0	-784.4804957	-303.0418404	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
4.752034099	0	7.112207002	-0.377388886	0	-34.87302612	0	-31.11860447	15.19604562	0	21.71121108	0	21.569976	-8.028062352	0	8.409780948	0	10.5743756	-2.039110376	0
0.027494323	0	0.095333401	0.028271283	0	-0.172802764	0	-0.241860662	0.022887381	0.125	0.071110656	0	0.2127492	0.053712066	0	0.074197784	0	0.15802537	0.01641642	0
-0.173229377	0	0.199547906	0.271641436	0	1.516526302	0	0.543538493	-5.400930487	0	-1.252999926	0	-0.041568641	1.140792198	0	-0.090296999	0	0.434578851	0.313245566	0
-15.66068362	0	-267.774412	-136.0891448	0	-15.69098455	0	-1293.555596	-694.4961661	-10	174.238969	0	-467.6856702	-425.2084422	0	-142.8873008	0	-471.5578358	-120.5096231	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
3.104690728	0	3.057068774	-1.140032748	0	22.37267173	0	14.0869373	-12.97025524	0	-30.22250739	0	-34.15178892	8.852276399	0	4.745144929	0	4.067482054	-2.159970666	0
0.019205032	0	0.058377695	0.015131188	0	0.123279693	0	0.215522804	0.004114241	0	-0.190440934	0	-0.287284602	0.010713045	0.125	0.047956209	0	0.095701801	0.007190087	0
-0.099461952	0	0.245358259	0.229476511	0	-0.84819142	0	0.665419318	1.149173339	0	0.937850514	0	0.363525706	-4.887482127	0	0.009802858	0	0.483275943	0.247956153	0
-16.49082416	0	-196.6495474	-95.34797847	0	-49.74815331	0	-661.6881304	-316.666782	0	176.483688	0	-1140.613258	-806.1684772	-10	-110.2447106	0	-344.0605824	-82.47053154	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
18.49584354	0	20.9075541	-5.276614695	0	6.96904585	0	11.57215131	-0.102550691	0	3.403780535	0	7.328628714	0.963648946	0	-28.86866992	0	-22.13476435	14.52883855	0
0.076514276	0	0.245648694	0.067628646	0	0.053493619	0	0.155779366	0.035910202	0	0.012027768	0	0.091132983	0.040134733	0	-0.142035664	0	-0.184018097	0.027324107	0.125
-1.187527879	0	-0.03537693	1.073403866	0	-0.24491022	0	0.143138512	0.304961122	0	-0.235400232	0	0.028879983	0.234976889	0	1.667838331	0	0.596586312	-5.498970571	0
137.1228575	0	-551.8030652	-436.3910837	0	-57.86348492	0	-393.4147121	-162.1185258	0	34.35236201	0	-215.744934	-153.3872978	0	-113.6117345	0	-1454.932051	-669.8865388	-10

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. As a result, Therefore, the eigenvalues will be:

Table 23: The eigenvalues of the two-area system without PSS

1	-4.518876715648844 +16.868851117054234i
2	-4.518876715648844 -16.868851117054234i
3	-5.115947359758870 +11.579302530150612i
4	-5.115947359758870 -11.579302530150612i
5	-5.285653549382072 + 7.276982115462068i
6	-5.285653549382072 - 7.276982115462068i
7	-5.350854444968339 + 6.963912337514744i
8	-5.350854444968339 - 6.963912337514744i
9	-0.761250340488480 + 7.513532561316059i
10	-0.761250340488480 - 7.513532561316059i
11	-0.774912052131946 + 6.838598425066129i
12	-0.774912052131946 - 6.838598425066129i
13	-0.017995854646413 + 4.617574423493768i
14	-0.017995854646413 - 4.617574423493768i
15	-4.674243210509141 + 0.000000000000000i
16	-4.525540498420993 + 0.084406218322709i
17	-4.525540498420993 - 0.084406218322709i
18	-4.303363127764723 + 0.000000000000000i
19	-0.000000000000001 + 0.000000237331902i
20	-0.000000000000001 - 0.000000237331902i

A real eigenvalue corresponds to a non-oscillatory mode. A negative real eigenvalue represents a decaying mode. The larger its magnitude, the faster decay. A positive real eigenvalue represents a aperiodic instability. In addition, complex eigenvalue occur in conjugate pairs, and each pair corresponds to an oscillatory mode. According to the table of eigenvalues, the two-area system is stable.

- **Problem #2 (8.2 of book)**

With three tie lines in service, add a PSS at bus 12 with the following parameters. KPSS = 25, TW = 10 sec, T1 = 0:047 sec, T2 = 0:021 sec, T3 = 3:0 sec, and T4 = 5:4 sec. What are the new eigenvalues?

The Power System Stabilizer (PSS) is a supplementary excitation controller used to damp generator electro-mechanical oscillations in order to protect the shaft line and stabilise the grid. It also damps generator rotor angle swings, which are of greater range in frequencies in power system. The figure 6 show a power system stablizer connected to the exciter of the machine at bus 12. This PSS consist of gain, washout (high pass filter), and lead-lag blocks (phase compensator)

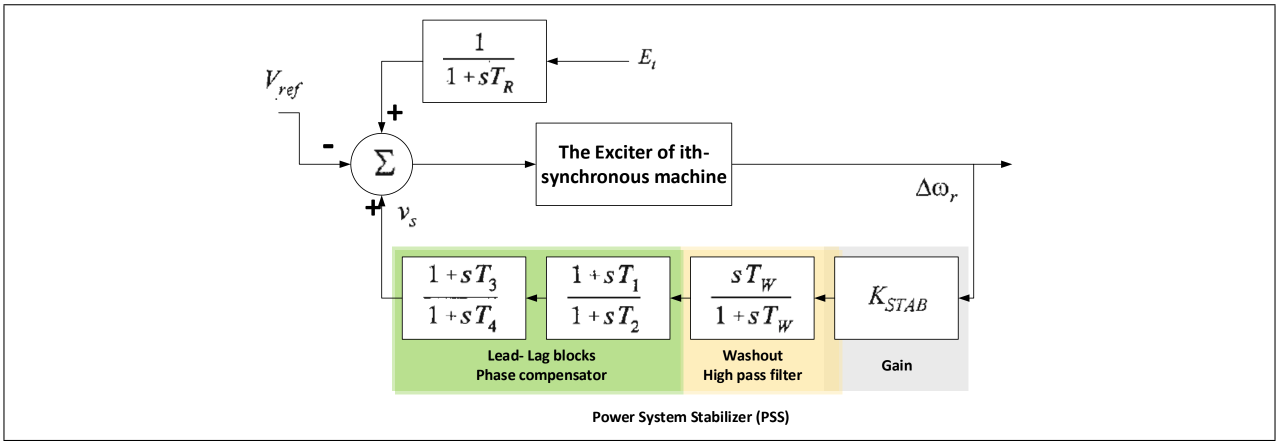

Figure 6: Power System Stabilizer connected to the exciter of the machine at bus 12 of the two-area system.

According to this figure, We have:

$\frac{\Delta V_{si}}{\Delta w_{ri}} = K_{PSS} \frac{sT_W}{1+sT_w} \frac{1+sT_1}{1+sT_2} \frac{1+sT_3}{1+sT_4}$ (56)

$\frac{\Delta V_{si}}{\Delta w_{ri}} = K_{PSS} \frac{T_W T_1 T_3 s^3 + T_W(T_1+T_3)s^2 + T_W s}{T_W T_2 T_4 s^3 + [T_W(T_2+T_4)+T_2T_4]s^2 + (T_W+T_2+T4)s+1}$ (57)

We assume that:

$X_1 = \Delta w_{ri} \frac{1}{T_W T_2 T_4 s^3 + [T_W(T_2+T_4)+T_2T_4]s^2 + (T_W+T_2+T4)s+1}$ (58)

According to this assumption, we have:

$T_W T_2 T_4 s^3 X_1 + [T_W(T_2+T_4)+T_2T_4]s^2 X_1 + (T_W+T_2+T4)sX_1 + v = \Delta w_{ri}$ (59)

In continue, we assume that:

$\frac{dx_1}{dt} = X_2 \Longrightarrow X_2(s) = sX_1(s)$ (60)

$\frac{dx_2}{dt} = X_3 \Longrightarrow X_3(s) = s^2 X_1(s)$ (61)

by using equation (59) and definitions declare in equations (60) and (61) , we obtain will be:

$T_W T_2 T_4 \dot{X}_3 + [T_W(T_2+T_4)+T_2T_4]X_3 + (T_W+T_2+T4)X_2 + X_1 = \Delta w_{ri} \Longrightarrow$

$\dot{X}_3 = \frac{-[T_W(T_2+T_4)+T_2T_4]X_3}{T_W T_2 T_4} - \frac{(T_W+T_2+T4)X_2}{T_W T_2 T_4} - \frac{X_1}{T_W T_2 T_4} + \frac{\Delta w_{ri}}{T_W T_2 T_4}$

(62)

Therefore, we have the following space-state for PSS:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{-1}{T_W T_2 T_4} & \frac{-(T_W+T_2+T4)}{T_W T_2 T_4} & \frac{-[T_W(T_2+T_4)+T_2T_4]}{T_W T_2 T_4} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_W T_2 T_4} \end{bmatrix} \Delta w_{ri} (63)$$

From equations (57) and (58), we have:

$\Delta V_{si} = K_{PSS}(T_W T_1 T_3 s^3 + T_W(T_1+T_3)s^2 + T_W s)\frac{\Delta w_{ri}}{T_W T_2 T_4 s^3 + [T_W(T_2+T_4)+T_2T_4]s^2 + (T_W+T_2+T4)s+1}$
$= K_{PSS}(T_W T_1 T_3 s^3 + T_W(T_1+T_3)s^2 + T_W s)X_1$
$= K_{PSS}(T_W T_1 T_3 \dot{X_3} + T_W(T_1+T_3)X_3 + T_W X_2)$
$= K_{PSS}(T_W T_1 T_3 \dot{X_3}) + T_W(T_1+T_3)X_3 + T_W X_2)$
(64)

By using Equation (62), we finally obtain:

$\Delta V_{si} = K_{PSS}[-\frac{T_1T_3}{T_2T_4}X_1 + (T_W - (T_W + T_2 + T4\frac{T_1T_3}{T_2T_4})X_2 + ((T_1+T_3)T_w - [T_W(T_2+T_4)+T_2T_4]\frac{T_2T_4}{T_2T_4})X_3] + K_{PSS}\frac{T_W T_2 T_4}{T_2T_4}\Delta w_{ri}$ (65)

$\Delta V_{si} = \begin{bmatrix} -\frac{T_1T_3}{T_2T_4} & (T_W - (T_W + T_2 + T4\frac{T_1T_3}{T_2T_4}) & ((T_1+T_3)T_w - [T_W(T_2+T_4)+T_2T_4]\frac{T_2T_4}{T_2T_4}) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} +$

$K_{PSS}\frac{T_W T_2 T_4}{T_2T_4}\Delta w_{ri}$ (66)

Accrediting to the PSS and Figure 2 and Equation (66) , we have: $\frac{d\Delta E_{fd}}{dt} = -\frac{d\Delta E_{fd}}{T_A} + \frac{K_A}{T_A}(\Delta V_{ref} - \Delta V + \Delta V_{si}) =$
$-\frac{d\Delta E_{fd}}{T_A} + \frac{K_A}{T_A}(\Delta V_{ref} - \Delta V + K_{PSS}[-\frac{T_1T_3}{T_2T_4}X_1 + (T_W - (T_W + T_2 + T4\frac{T_1T_3}{T_2T_4})X_2 + ((T_1+T_3)T_w - [T_W(T_2+T_4)+T_2T_4]\frac{T_2T_4}{T_2T_4})X_3] + K_{PSS}\frac{T_W T_2 T_4}{T_2T_4}\Delta w_{ri})$ (67)

As a result, according to the equations (63) and (67) the matrix A_1 for the Machine with PSS (Bus 12) will be:

$$\begin{bmatrix} \Delta\dot{\delta_i} \\ \Delta\dot{\omega_i} \\ \Delta\dot{E}'_{qi} \\ \Delta\dot{E}'_{di} \\ \Delta\dot{E}'_{fdi} \\ \Delta\dot{X_1} \\ \Delta\dot{X_2} \\ \Delta\dot{X_3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{D_i}{M_i} & -\frac{I_{qio}}{M_i} & -\frac{I_{dio}}{M_i} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{T'_{doi}} & 0 & \frac{1}{T'_{doi}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{T'_{qoi}} & 0 & 0 & 0 & 0 \\ 0 & \frac{K_A K_{PSS} T_1 T_3}{T_A T_2 T_4 W} & 0 & 0 & -\frac{1}{T'_A} & -\frac{K_A K_{PSS} T_1 T_3}{T_A T_2 T_4} & A & B \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & \frac{-1}{T_W T_2 T_4} & 0 & 0 & 0 & \frac{-1}{T_W T_2 T_4} & \frac{-(T_W+T_2+T4)}{T_W T_2 T_4} & C \end{bmatrix} \begin{bmatrix} \Delta\delta_i \\ \Delta\omega_i \\ \Delta E'_{qi} \\ \Delta E'_{di} \\ \Delta E'_{fdi} \\ \Delta X_1 \\ \Delta X_2 \\ \Delta X_3 \end{bmatrix}$$

Where:
$A = \frac{K_A K_{PSS}}{T_A}[T_W - (T_W + T_2 + T4)\frac{T_1T_3}{T_2T_4}]$
$B = \frac{K_A K_{PSS}}{T_A}[((T_1+T_3)T_w - [T_W(T_2+T_4)+T_2T_4]\frac{T_2T_4}{T_2T_4})]$
$C = \frac{-[T_W(T_2+T_4)+T_2T_4]}{T_W T_2 T_4}$

It is noted that A_1 for the Machines without PSS are similar to Equation (13). I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. As a result, A_1 will be:

Table 24: Results for the matrix A_1 with Power System Stabilizer connected to the exciter of the machine at bus 12 of the two-area system

0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-11.63477097	-17.93922499	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-0.125	0	0.125	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-2.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	-10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-13.17653986	-21.02231856	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-0.125	0	0.125	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	-2.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-10	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	-13.47284955	-20.04592686	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	-0.125	0	0.125	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	-2.5	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	-10	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-11.12155025	-18.30335905	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.125	0	0.125	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2.5	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	158.3128534	0	0	-10	-59682.53968	-440364.4444	-1779598.476
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.002339138	0	0	0	-0.881834215	-13.59876543	-47.9042328

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. $A^{'}$ will be:

Table 25: Results for the matrix $A^{'}$ with Power System Stabilizer connected to the exciter of the machine at bus 12 of the two-area system

0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-79.2775739	0	-67.43794942	35.08165238	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-0.464203942	0	-0.757551122	-0.002903513	0.125	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3.83614152	0	0.044856673	-7.786679346	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	-10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-89.12738405	0	-76.38745247	40.56110027	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-0.450536963	0	-0.757551122	-0.002903513	0.125	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	3.80058557	0	0.044856673	-7.786679346	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-10	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-91.71345155	0	-78.09100765	41.24635042	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-0.460753533	0	-0.757551122	-0.002903513	0.125	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	3.865091076	0	0.044856673	-7.786679346	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	-10	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-76.39262546	0	-64.48800503	35.3024235	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.44355439	0	-0.757551122	-0.002903513	0.125	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3.857521909	0	0.044856673	-7.786679346	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	158.3128534	0	0	-10	-59682.53968	-440364.4444	-1779598.476
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.002339138	0	0	0	-0.881834215	-13.59876543	-47.9042328

I run the MATLAB code for introduced equations of the two-area system. Matlab code is provided in the appendix. $B^{'}$ will be:

Table 26: Results for the matrix B' with Power System Stabilizer connected to the exciter of the machine at bus 12 of the two-area system

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
79.2775739	1.027299884	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.464203942	0.443862354	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-3.83614152	3.752301483	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-2000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	89.12738405	0.984097446	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0.450536963	0.448492335	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-3.80058557	3.713644127	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-2000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	91.71345155	1.933578548	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0.460753533	0.447238267	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	-3.865091076	3.724193503	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-2000	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	76.39262546	0.845082291	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0.44355439	0.455264122	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	-3.857521909	3.655642147	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-2000	0	0	0	0	0	0	0	0	0	0	0	0

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. C' will be:

Table 27: Results for the matrix C' with Power System Stabilizer connected to the exciter of the machine at bus 12 of the two-area system

26.39600991	0	22.43742941	-11.68994807	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.011090707	0	21.6956889	12.18420948	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	25.43541414	0	21.78090397	-11.58571582	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-0.047627925	0	21.50046376	11.8286223	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	26.17708217	0	22.27367647	-11.78122449	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0.230166873	0	21.86377122	12.09597437	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25.43466475	0	21.44943131	-11.76531951	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.050128565	0	21.83116159	11.64999457	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. J'_{AE} will be:

Table 28: The results for the matrix J'_{AE} with Power System Stabilizer connected to the exciter of the machine at bus 12 of the two-area system

-112.745397	-14.4077056	0	0	0	0	0	0	0	0	0	0	86.34938708	0.0984443	0	0	0	0	0	0
0.088288813	-109.1167021	0	0	0	0	0	0	0	0	0	0	-0.09937952	85.5367876	0	0	0	0	0	0
0	0	-108.2628921	-14.1300006	0	0	0	0	0	0	82.82747799	0.035634073	0	0	0	0	0	0	0	0
0	0	0.082816572	-107.1430323	0	0	0	0	0	0	-0.035188647	83.87592708	0	0	0	0	0	0	0	0
0	0	0	0	-112.637145	-14.40960015	0	0	86.46006286	0.313122579	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0.08633743	-109.2218011	0	0	-0.316504303	85.53627113	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	-108.0524529	-14.12747016	0	0	0	0	0	0	82.61778812	0.035780464	0	0	0	0
0	0	0	0	0	0	0.085372322	-107.3513881	0	0	0	0	0	0	-0.035243757	83.87592702	0	0	0	0
0	0	0	0	85.31989718	13.59076929	0	0	-124.9419514	-10.9740701	39.62205421	-2.942414224	0	0	0	0	0	0	0	0
0	0	0	0	-13.99849237	82.83485163	0	0	11.09285832	-123.7611981	2.905634046	40.1235992	0	0	0	0	0	0	0	0
0	0	81.69085435	13.53927959	0	0	0	0	38.26208599	10.57978637	-215.7756655	-20.49788378	0	0	0	0	95.82272512	-4.22693883	0	0
0	0	-13.67467238	80.88203401	0	0	0	0	-10.69404807	37.85327067	20.24280546	-218.7036491	0	0	0	0	4.125914992	98.16896335	0	0
85.17481008	13.78087726	0	0	0	0	0	0	0	0	0	0	-124.6277562	-10.95995879	39.45294612	-3.177627157	0	0	0	0
-14.19430358	82.69399037	0	0	0	0	0	0	0	0	0	0	11.06434083	-123.5951169	3.12996275	40.0537524	0	0	0	0
0	0	0	0	0	0	81.48406587	13.50486212	0	0	0	0	38.05190506	10.77805633	-214.662754	-20.44785086	0	0	95.12678307	-4.509830363
0	0	0	0	0	0	-13.63991074	80.67729294	0	0	0	0	-10.88044787	37.69381383	20.13860743	-218.1720597	0	0	4.381751181	97.90735186
0	0	0	0	0	0	0	0	0	0	93.10823255	23.31039064	0	0	0	0	-105.9568953	0.890958054	12.84866278	-0.730599452
0	0	0	0	0	0	0	0	0	0	-23.01901076	94.28681777	0	0	0	0	22.30916033	-111.0931937	0.709850428	13.22423094
0	0	0	0	0	0	0	0	0	0	0	0	0	0	92.37541089	23.48423409	12.45366938	3.319417728	-104.8290803	5.279726631
0	0	0	0	0	0	0	0	0	0	0	0	0	0	-23.13197058	93.78214304	-3.240083644	12.75859992	26.37205422	-110.7386147

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. As a result, A_{SYS} will be:

Table 29: the results for the matrix A_{SYS} with Power System Stabilizer connected to the exciter of the machine at bus 12 of the two-area system

0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-26.11170796	0	-28.68116519	7.918812997	0	4.353334748	0	5.417510598	-1.056811354	0	2.507649153	0	3.634387676	-0.283677484	0	19.25072406	0	13.23169624	-10.50090888	0	0	0	0
-0.187334502	0	-0.267296837	0.017754361	0.125	0.036738681	0	0.096416034	0.018868534	0	0.010486621	0	0.057081841	0.022395044	0	0.1401092	0	0.24921846	0.0048557	0	0	0	0
0.964461206	0	0.455956112	-4.911824586	0	-0.112980761	0	0.299620628	0.26871494	0	-0.157783975	0	0.135011397	0.222616048	0	-0.693696471	0	0.851036781	1.084209938	0	0	0	0
157.3121671	0	-1202.779827	-820.8704444	-10	-51.37891254	0	-295.510497	-114.4537261	0	19.16985618	0	-163.5317734	-109.883554	0	-125.1031107	0	-784.4804957	-303.0418404	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4.752034099	0	7.112207002	-0.377388886	0	-34.87302612	0	-31.11860447	15.19604562	0	21.71121108	0	21.569976	-8.028062352	0	8.409780948	0	10.5743756	-2.039110376	0	0	0	0
0.027494323	0	0.095333401	0.028271283	0	-0.172802764	0	-0.241860662	0.022887381	0.125	0.071110656	0	0.2127492	0.053712066	0	0.074197784	0	0.15802537	0.01641642	0	0	0	0
-0.173229377	0	0.199547906	0.271641436	0	1.516526302	0	0.543538493	-5.400930487	0	-1.252999926	0	-0.041568641	1.140792198	0	-0.090296999	0	0.434578851	0.313245566	0	0	0	0
-15.66068362	0	-267.774412	-136.0891448	0	-15.69098455	0	-1293.555596	-694.4961661	-10	174.238969	0	-467.6856702	-425.2084422	0	-142.8873008	0	-471.5578358	-120.5096231	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
3.104690728	0	3.057068774	-1.140032748	0	22.37267173	0	14.0869373	-12.97025524	0	-30.22250739	0	-34.15178892	8.852276399	0	4.745144929	0	4.067482054	-2.159970666	0	0	0	0
0.019205032	0	0.058377695	0.015131188	0	0.123279693	0	0.215522804	0.004114241	0	-0.190440934	0	-0.287284602	0.010713045	0.125	0.047956209	0	0.095701801	0.007190087	0	0	0	0
-0.099461952	0	0.245358259	0.229476511	0	-0.84819142	0	0.665419318	1.149173339	0	0.937850514	0	0.363525706	-4.887482127	0	0.009802858	0	0.483275943	0.247956153	0	0	0	0
-16.49082416	0	-196.6495474	-95.34797847	0	-49.74815331	0	-661.6881304	-316.666782	0	176.483688	0	-1140.613258	-806.1684772	-10	-110.2447106	0	-344.0605824	-82.47053154	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
18.49584354	0	20.9075541	-5.276614695	0	6.96904585	0	11.57215131	-0.102550691	0	3.403780535	0	7.328628714	0.963648946	0	-28.86866992	0	-22.13476435	14.52883855	0	0	0	0
0.076514276	0	0.245648694	0.067628646	0	0.053493619	0	0.155779366	0.035910202	0	0.012027768	0	0.091132983	0.040134733	0	-0.142035664	0	-0.184018097	0.027324107	0.125	0	0	0
-1.187527879	0	-0.03537693	1.073403866	0	-0.24491022	0	0.143138512	0.304961122	0	-0.235400232	0	0.028879983	0.234976889	0	1.667838331	0	0.596586312	-5.498970571	0	0	0	0
137.1228575	0	-551.8030652	-436.3910837	0	-57.86348492	0	-393.4147121	-162.1185258	0	34.35236201	0	-215.744934	-153.3872978	0	-113.6117345	158.3128534	-1454.932051	-669.8865388	-10	-59682.53968	-440364.4444	-1779598.476
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.002339138	0	0	0	-0.881834215	-13.59876543	-47.9042328

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. At the end the eigenvalues of the system while PSS is installed at bus 12 will be:

Table 30: Eigenvalues of the system with Power System Stabilizer connected to the exciter of the machine at bus 12 of the two-area system

1	-47.749297748285748 + 0.000000000000000i
2	-4.619949152055119 +16.870741856393050i
3	-4.619949152055119 -16.870741856393050i
4	-4.999250014548639 +11.517418169409304i
5	-4.999250014548639 -11.517418169409304i
6	-0.961042059049763 + 8.630452102877301i
7	-0.961042059049763 - 8.630452102877301i
8	-0.756584960478967 + 7.480452655567410i
9	-0.756584960478967 - 7.480452655567410i
10	-5.352726306212475 + 7.090452296311369i
11	-5.352726306212475 - 7.090452296311369i
12	-5.257008478367309 + 5.347042377134976i
13	-5.257008478367309 - 5.347042377134976i
14	-0.086004758036597 + 4.662053248086060i
15	-0.086004758036597 - 4.662053248086060i
16	-4.513935762231415 + 0.000000000000000i
17	-4.436119483778652 + 0.000000000000000i
18	-4.401135373978134 + 0.000000000000000i
19	-4.155296459336419 + 0.000000000000000i
20	-0.205005581270590 + 0.000000000000000i
21	-0.057978907020322 + 0.000000000000000i
22	0.000000000000170 + 0.000000226902871i
23	0.000000000000170 - 0.000000226902871i

According to the results, the system become more stable when we install PSS at bus 12. As it is clear, the eigenvalues are moved to left side. Figure 7 show eigenvalues of the test power system in two different cases:1)Without Power System Stabilizer and 2)With Power System Stabilizer.

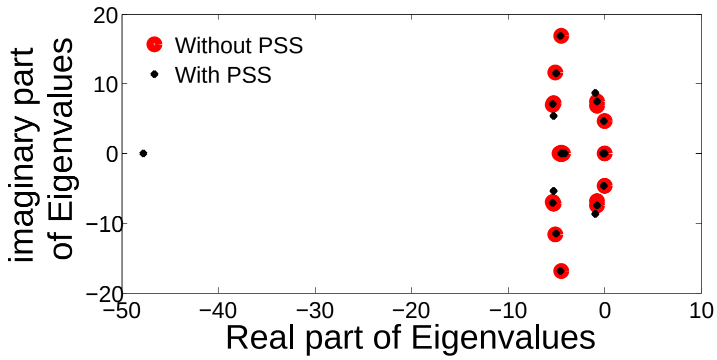

Figure 7: Eigenvalues of the test power system in two different cases:1)Without Power System Stabilizer and 2)With Power System Stabilizer

- **Problem #3**

In Problem 8.2, determine the frequencies and damping ratios of system modes and identify the associated dominant state variables using participation factors. Find the mode shapes of rotor angle modes, which are the magnitudes of the normalized right eigenvector components corresponding to the rotor speeds of the four machines. For each mode, indicate whether it is a local or an inter-area mode (see Kundur, p. 816) and whether it is identifiable and controllable (see Kundur, p. 716).

In many cases, instability and eventual loss of synchronism are initiated by some spurious disturbance in the system resulting in oscillatory behavior that, if not damped, may eventually build up. This is very much a function of the operating condition of the power system. Oscillations, even if undamped at low frequencies, are undesirable because they limit power transfers on transmission lines and, in some cases, induce stress in the mechanical shaft. The source of inter-area oscillations is dificult to diagnose. Electromechanical Oscillatory Modes are the modes associated with the rotor angles of the machines. These can be identified through a participation factor analysis.
The real component of the eigenvalue gives the damping, and the imaginary component give the frequency of oscillation. A negative real part presents a damped oscillation whereas a positive real part represents oscillation of increasing amplitude. Table 31 provide the mathematical notation and equation related to the Eigenvalue, frequency and damping ratios of system modes, mode shape, participation factor, mode Observability matrix, and mode Controllability matrix.

Table 31: The mathematical notation and equation related to the Eigenvalue, frequency and damping ratios of system modes, mode shape, participation factor, mode Observability matrix, and mode Controllability matrix.

Variable	Mathematical notation	Equation
Eigenvalue	λ	$\sigma \pm j\omega$
Frequency	f	$\frac{\omega}{2\pi}$
Damping ratio of the system mode	ζ	$\frac{-\sigma}{\sqrt{^2+\omega^2}}$
Mode shape	ψ	right eigenvector
Participation factor	P_{ki}	$\frac{w_{ki}v_{ki}}{w_i^t v_i}$
mode Observability matrix	$C^{'}$	$C\phi$
mode Controllability matrix	$B^{'}$	$\phi^{-1}B$

The damping ratio determines the rate of decay of the amplitude of the oscilation. Here, the electromechanical oscillations and their damping are calculated. According to the equations provided in the table, we calculate the wanted variables.

Table 32: The amount of eigenvalues, σ, ω, frequency and damping ratios of system modes, obsevability of the two-area system with Power System Stabilizer connected to the exciter of the machine at bus 12

Eigenvalue	σ	ω	Frequency	Damping ratio	Observability
1	-47.74929775	0	0	1	0.001285717
2	-4.619949152	16.87074186	2.68506196	0.264119648	-0.002495802
3	-4.619949152	16.87074186	2.68506196	0.264119648	-0.002495802
4	-4.999250015	11.51741817	1.833054033	0.398168528	-0.003059081
5	-4.999250015	11.51741817	1.833054033	0.398168528	-0.003059081
6	-0.961042059	8.630452103	1.373579113	0.110670738	-0.105555998
7	-0.961042059	8.630452103	1.373579113	0.110670738	-0.105555998
8	-0.75658496	7.480452656	1.190551017	0.100628217	-0.026526838
9	-0.75658496	7.480452656	1.190551017	0.100628217	-0.026526838
10	-5.352726306	7.090452296	1.128480532	0.60251028	0.003061915
11	-5.352726306	7.090452296	1.128480532	0.60251028	0.003061915
12	-5.257008478	5.347042377	0.851008225	0.701077808	0.045964226
13	-5.257008478	5.347042377	0.851008225	0.701077808	0.045964226
14	-0.086004758	4.662053248	0.741988819	0.01844469	-0.130683914
15	-0.086004758	4.662053248	0.741988819	0.01844469	-0.130683914
16	-4.513935762	0	0	1	0.021063676
17	-4.436119484	0	0	1	0.033010118
18	-4.401135374	0	0	1	-0.037508079
19	-4.155296459	0	0	1	0.072814515
20	-0.205005581	0	0	1	0.08104858
21	-0.057978907	0	0	1	-0.028524536
22	1.70023E-13	2.26903E-07	3.61127E-08	-7.49319E-07	-8.28649E-14
23	1.70023E-13	2.26903E-07	3.61127E-08	-7.49319E-07	-8.28649E-14

We have the following state equation in the normal form (decoulped) are: $\dot{Z} = \wedge Z + B^{'}\Delta U$
$\Delta y = C^{'}Z + D\Delta U$
According to mode contrlability matrix, if if the ith row of this matrix is zero, the inputs have no effect on the ith mode. In this case, the ith mode is uncontrollable. In addition, according to mode observability matrix, the ith columns of this matrix determines whether or not the variable Z_i contributes to the formation of the outputs. If the column is zero, then the corresponding mode is unobservable. This is why some poorly damped modes are sometimes not detected by observing the transient response of a few monitored quantities. By inspecting mode contrlability matrix and mode observability matrix, we can classify modes into controllable and observable; controllable and unobservable; uncontrollable and observable; uncontrollable and unobservable. I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix. Mode controlability matrix for this system will be:

Table 33: Mode contrlability matrix with Power System Stabilizer connected to the exciter of the machine at bus 12 of the two-area system

	ΔT_{M1}	ΔV_{ref1}	ΔT_{M2}	ΔV_{ref2}	ΔT_{M3}	ΔV_{ref3}	ΔT_{M4}	ΔV_{ref4}
$\Delta\delta_1$	0.051900324	5.746938678	0.022517424	3.047721148	0.011726064	1.916172741	6.243044244	-6.279956751
$\Delta\omega_1$	-0.759106246	404.3113041	0.611072759	504.6941868	-1.11128396	301.5998456	3.048447153	711.1266274
$\Delta E^{'}_{q1}$	-0.759106284	404.3113041	0.611072721	504.6941868	-1.111283997	301.5998455	3.048447114	711.1266273
$\Delta E^{'}_{d1}$	0.799104907	-487.6604269	-0.644650608	633.6118082	-1.869479089	505.578479	1.850349581	-544.0018482
$\Delta E^{'}_{fd1}$	0.799104867	-487.6604269	-0.644650648	633.6118082	-1.869479128	505.578479	1.850349539	-544.0018482
$\Delta\delta_2$	3.163440968	196.7546344	1.182025092	29.40426051	-0.26280682	-9.331412473	-8.184119763	-241.4075538
$\Delta\omega_2$	3.163440929	196.7546344	1.182025053	29.40426049	-0.262806858	-9.331412489	-8.184119804	-241.4075538
$\Delta E^{'}_{q2}$	0.49214777	2.821697908	-2.6885429	29.40319069	2.540267201	-44.75205106	-0.612959855	-7.855328881
$\Delta E^{'}_{d2}$	0.492147775	2.82169791	-2.688542895	29.40319069	2.540267206	-44.75205106	-0.61295985	-7.855328878
$\Delta E^{'}_{fd2}$	-0.696485929	265.4799755	2.426924559	-920.2112808	-2.089043519	839.5407321	1.106123688	-90.68807841
$\Delta\delta_3$	-0.696485978	265.4799755	2.42692451	-920.2112809	-2.089043567	839.5407321	1.106123638	-90.68807844
$\Delta\omega_3$	-4.009223237	720.5420826	-1.6121571	214.5086012	-0.701147806	-26.5778745	9.012721773	-808.725272
$\Delta E^{'}_{q3}$	-4.009223238	720.5420826	-1.612157101	214.5086012	-0.701147807	-26.5778745	9.012721773	-808.725272
$\Delta E^{'}_{d3}$	-1.558628311	20.64491753	1.279261274	-12.62484661	1.517687937	-17.10748301	-1.066725765	3.822875408
$\Delta E^{'}_{fd3}$	-1.55862831	20.64491753	1.279261275	-12.62484661	1.517687938	-17.10748301	-1.066725763	3.822875409
$\Delta\delta_4$	1.924417116	-205.6667944	-3.949087031	235.8368078	5.63955415	-265.7360707	-6.447384835	152.5429256
$\Delta\omega_4$	-7.046467346	834.6950567	-0.397150576	-12.80627453	-6.701013517	250.8316891	29.08176209	-775.9072784
$\Delta E^{'}_{q4}$	-6.239054494	697.8590436	-3.212646604	231.5793364	-1.087518687	7.430956466	23.97029903	-677.0393803
$\Delta E^{'}_{d4}$	1.613782198	-207.5043578	-0.396350286	-1.803601104	0.083332723	-35.02217839	-6.763604646	244.6422169
$\Delta E^{'}_{fd4}$	0.690037753	0.354211464	0.69220462	0.518674207	0.667182237	0.289951042	0.720873981	0.396944544
ΔX_1	17.60143834	7.549994479	17.67373625	12.51067553	17.03350176	7.079869195	18.30703547	12.06302371
ΔX_2	8.450681707	3.561986291	8.485967664	6.005166827	8.17912388	3.380453478	8.785489238	5.894711033
ΔX_3	8.44907134	3.561306756	8.484350533	6.004029743	8.177565388	3.379808265	8.783814897	5.893594531

The right eigenvalue provide the mode shape. The mode shape is the relative activity of the state variables when a particular mode is excited.
Magnitude: The magnitude of the elements of eigenvalue give the extents of the activities of the n state variables in the ith mode.
Angles: The angles of the elements give phase displacements of the state variables with regard to the mode.
I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix.

Table 34: Magnitude of mode shape with Power System Stabilizer connected to the exciter of the machine at bus 12 of the two-area system:Part A

SV/Mode	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}	λ_{11}	λ_{12}
$\Delta\delta_1$	1.63114E-05	8.26232E-05	8.26232E-05	0.001026307	0.001026307	0.010659778	0.010659778	0.00701294	0.00701294	0.000970815	0.000970815	0.007214519
$\Delta\omega_1$	0.000778857	0.001445235	0.001445235	0.01288592	0.01288592	0.092567336	0.092567336	0.052727603	0.052727603	0.008624764	0.008624764	0.054097828
$\Delta E'_{q1}$	0.000138733	0.003567166	0.003567166	0.005001771	0.005001771	0.0098727	0.0098727	0.004020634	0.004020634	0.002021001	0.002021001	0.010852169
$\Delta E'_{d1}$	4.95485E-05	0.000389117	0.000389117	0.000320654	0.000320654	0.001883359	0.001883359	0.001148848	0.001148848	0.000235308	0.000235308	0.00295999
$\Delta E'_{fd1}$	0.047619789	0.499923524	0.499923524	0.497068759	0.497068759	0.696233377	0.696233377	0.242735576	0.242735576	0.138574894	0.138574894	0.620546486
$\Delta\delta_2$	1.20719E-05	9.63972E-05	9.63972E-05	0.000520026	0.000520026	0.003853702	0.003853702	0.048719112	0.048719112	0.006976488	0.006976488	0.002432557
$\Delta\omega_2$	0.000576423	0.001686169	0.001686169	0.006529242	0.006529242	0.033464762	0.033464762	0.366300319	0.366300319	0.061979408	0.061979408	0.018240445
$\Delta E'_{q2}$	7.97535E-05	0.003550784	0.003550784	0.004682182	0.004682182	0.004075649	0.004075649	0.010128314	0.010128314	0.00961191	0.00961191	0.003743882
$\Delta E'_{d2}$	2.43251E-05	0.000232519	0.000232519	5.01626E-05	5.01626E-05	0.000895423	0.000895423	0.014282446	0.014282446	0.002470308	0.002470308	0.001089808
$\Delta E'_{fd2}$	0.027249968	0.498682801	0.498682801	0.468182125	0.468182125	0.28130385	0.28130385	0.50377999	0.50377999	0.652460693	0.652460693	0.215067256
$\Delta\delta_3$	4.94189E-06	0.000129849	0.000129849	0.001288115	0.001288115	0.002926325	0.002926325	0.047209796	0.047209796	0.006705103	0.006705103	0.001448983
$\Delta\omega_3$	0.000235972	0.002271298	0.002271298	0.016173076	0.016173076	0.025411606	0.025411606	0.354952346	0.354952346	0.059568412	0.059568412	0.010865147
$\Delta E'_{q3}$	5.50389E-05	0.003233064	0.003233064	0.006424909	0.006424909	0.00304325	0.00304325	0.011008873	0.011008873	0.010817361	0.010817361	0.001018528
$\Delta E'_{d3}$	2.58069E-05	0.000381368	0.000381368	0.000272824	0.000272824	0.000368738	0.000368738	0.009668764	0.009668764	0.001868549	0.001868549	0.000427881
$\Delta E'_{fd3}$	0.019105313	0.452842787	0.452842787	0.639422238	0.639422238	0.221052331	0.221052331	0.640007728	0.640007728	0.738608031	0.738608031	0.056663853
$\Delta\delta_4$	2.69264E-05	0.000195857	0.000195857	0.000494775	0.000494775	0.012477976	0.012477976	0.003576948	0.003576948	0.000539629	0.000539629	0.008307507
$\Delta\omega_4$	0.001285717	0.003425901	0.003425901	0.006212203	0.006212203	0.108356192	0.108356192	0.026893702	0.026893702	0.004794083	0.004794083	0.062293564
$\Delta E'_{q4}$	0.002624608	0.003864681	0.003864681	0.00354972	0.00354972	0.009273943	0.009273943	0.001638145	0.001638145	0.000621532	0.000621532	0.013005493
$\Delta E'_{d4}$	3.36717E-05	0.000144307	0.000144307	8.50395E-05	8.50395E-05	0.00380142	0.00380142	0.001419147	0.001419147	0.000237544	0.000237544	0.00355158
$\Delta E'_{fd4}$	0.998306171	0.544290688	0.544290688	0.352497198	0.352497198	0.603876353	0.603876353	0.093792221	0.093792221	0.041767968	0.041767968	0.746823754
ΔX_1	1.0188E-08	5.69442E-10	5.69442E-10	2.10666E-09	2.10666E-09	7.10743E-08	7.10743E-08	2.35305E-08	2.35305E-08	3.37975E-09	3.37975E-09	6.23333E-08
ΔX_2	4.86468E-07	9.96062E-09	9.96062E-09	2.64504E-08	2.64504E-08	6.17194E-07	6.17194E-07	1.76917E-07	1.76917E-07	3.00258E-08	3.00258E-08	4.67404E-07
ΔX_3	2.32285E-05	1.7423E-07	1.7423E-07	3.32101E-07	3.32101E-07	5.35959E-06	5.35959E-06	1.33017E-06	1.33017E-06	2.6675E-07	2.6675E-07	3.50481E-06

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix.

Table 35: Magnitude of mode shape with Power System Stabilizer connected to the exciter of the machine at bus 12 of the two-area system:Part B

SV/Mode	λ_{13}	λ_{14}	λ_{15}	λ_{16}	λ_{17}	λ_{18}	λ_{19}	λ_{20}	λ_{21}	λ_{22}	λ_{23}
$\Delta\delta_1$	0.007214519	0.074609201	0.074609201	0.017193151	0.020341985	0.019445075	0.018427955	0.536645396	0.503386144	0.5	0.5
$\Delta\omega_1$	0.054097828	0.347891252	0.347891252	0.077608779	0.090239477	0.085580406	0.076573616	0.110015301	0.029185778	1.13451E-07	1.13451E-07
$\Delta E'_{q1}$	0.010852169	0.001739379	0.001739379	0.015699443	0.020046247	0.016920242	0.027413096	0.050504246	0.004058821	2.07533E-14	2.07533E-14
$\Delta E'_{d1}$	0.00295999	0.010759245	0.010759245	0.017741637	0.022964162	0.017904818	0.03260356	0.027499311	0.002172578	1.09839E-14	1.09839E-14
$\Delta E'_{fd1}$	0.620546486	0.08894192	0.08894192	0.539723356	0.679398293	0.564384362	0.880181461	0.019260399	0.006464671	7.69035E-14	7.69035E-14
$\Delta\delta_2$	0.002432557	0.050543494	0.050543494	0.016757211	0.005742932	0.001652861	0.00144176	0.50631301	0.50098883	0.5	0.5
$\Delta\omega_2$	0.018240445	0.235676552	0.235676552	0.075640976	0.025476333	0.007274464	0.005990942	0.103796993	0.029046785	1.13451E-07	1.13451E-07
$\Delta E'_{q2}$	0.003743882	0.014381932	0.014381932	0.022079665	0.011398722	0.009109314	0.008535965	0.014858028	0.001199436	7.77888E-15	7.77888E-15
$\Delta E'_{d2}$	0.001089808	0.001426673	0.001426673	0.031338756	0.017049635	0.014814509	0.012241076	0.008024539	0.000636707	3.6068E-15	3.6068E-15
$\Delta E'_{fd2}$	0.215067256	0.574108953	0.574108953	0.77761687	0.398383827	0.320193787	0.280659292	0.007420585	0.002052705	8.52917E-15	8.52917E-15
$\Delta\delta_3$	0.001448983	0.074923074	0.074923074	0.002865022	0.008891191	0.010617413	0.001677619	0.494842404	0.500073875	0.5	0.5
$\Delta\omega_3$	0.010865147	0.349354792	0.349354792	0.012932524	0.039442384	0.046728671	0.006971002	0.101445455	0.028993737	1.13451E-07	1.13451E-07
$\Delta E'_{q3}$	0.001018528	0.009306864	0.009306864	0.003882436	0.014749587	0.017870576	0.00544498	0.005043414	0.000415917	1.74235E-15	1.74235E-15
$\Delta E'_{d3}$	0.000427881	0.006514264	0.006514264	0.008813358	0.021741671	0.02595462	0.008905634	0.002660909	0.00021589	7.97597E-16	7.97597E-16
$\Delta E'_{fd3}$	0.056663853	0.323166408	0.323166408	0.147433071	0.514878921	0.618413325	0.183101486	0.00287712	0.000739878	1.30514E-14	1.30514E-14
$\Delta\delta_4$	0.008307507	0.029424641	0.029424641	0.004666366	0.007441215	0.008522364	0.017523302	0.395348164	0.491981257	0.5	0.5
$\Delta\omega_4$	0.062293564	0.137202583	0.137202583	0.021063676	0.033010118	0.037508079	0.072814515	0.08104858	0.028524536	1.13451E-07	1.13451E-07
$\Delta E'_{q4}$	0.013005493	0.013369647	0.013369647	0.007355036	0.009267454	0.012524164	0.010321114	0.087295559	0.007113868	3.8119E-14	3.8119E-14
$\Delta E'_{d4}$	0.00355158	0.001753894	0.001753894	0.009277554	0.009813979	0.014490798	0.003347037	0.025351111	0.001998494	1.10791E-14	1.10791E-14
$\Delta E'_{fd4}$	0.746823754	0.474729022	0.474729022	0.25893707	0.316807397	0.42744412	0.314355583	0.049259834	0.004342004	1.86935E-13	1.86935E-13
ΔX_1	6.23333E-08	3.09093E-07	3.09093E-07	5.98236E-08	9.70075E-08	1.11954E-07	2.434E-07	0.001921187	0.00026245	3.00939E-10	3.00939E-10
ΔX_2	4.67404E-07	1.44125E-06	1.44125E-06	2.7004E-07	4.30337E-07	4.92723E-07	1.0114E-06	0.000393854	1.52166E-05	6.90713E-17	6.90713E-17
ΔX_3	3.50481E-06	6.72034E-06	6.72034E-06	1.21894E-06	1.90903E-06	2.16854E-06	4.20267E-06	8.07423E-05	8.8224E-07	3.61063E-19	3.61063E-19

Table 36: Angle of of mode shape: Part A

SV/Mode	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}	λ_{11}
$\Delta\delta_1$	0	-149.1055752	149.1055752	24.0015987	-24.0015987	-94.2734808	94.2734808	-80.53402748	80.53402748	134.0440977	-134.0440977
$\Delta\omega_1$	180	-43.79092645	43.79092645	137.4653329	-137.4653329	2.080501276	-2.080501276	15.24131964	-15.24131964	-98.90600673	98.90600673
$\Delta E'_{q1}$	180	-108.601552	108.601552	77.19943829	-77.19943829	-91.65952999	91.65952999	-69.01967432	69.01967432	-127.9664127	127.9664127
$\Delta E'_{d1}$	180	159.6317115	-159.6317115	-24.4274978	24.4274978	-165.6810144	165.6810144	-147.5651395	147.5651395	26.2289959	-26.2289959
$\Delta E'_{fd1}$	0	-2.834794421	2.834794421	-170.4487216	170.4487216	0	0	20.23636416	-20.23636416	-2.802919094	2.802919094
$\Delta\delta_2$	0	38.49693548	-38.49693548	-152.6759241	152.6759241	-84.54377071	84.54377071	78.63239507	-78.63239507	-51.48221725	51.48221725
$\Delta\omega_2$	180	143.8115842	-143.8115842	-39.21218999	39.21218999	11.81021137	-11.81021137	174.4077422	-174.4077422	75.56767829	-75.56767829
$\Delta E'_{q2}$	180	-109.3624112	109.3624112	-112.1635665	112.1635665	-54.43177115	54.43177115	-169.8350593	169.8350593	70.5001256	-70.5001256
$\Delta E'_{d2}$	180	158.9153563	-158.9153563	-40.9826378	40.9826378	179.1730677	-179.1730677	25.61895154	-25.61895154	-119.4991886	119.4991886
$\Delta E'_{fd2}$	0	-3.351321411	3.351321411	0.507162428	-0.507162428	38.30103891	-38.30103891	-70.5427508	70.5427508	-163.6680495	163.6680495
$\Delta\delta_3$	0	-149.3452489	149.3452489	-165.4476138	165.4476138	26.17123776	-26.17123776	-105.0522474	105.0522474	125.4101093	-125.4101093
$\Delta\omega_3$	180	-44.03060022	44.03060022	-51.98387961	51.98387961	122.5252198	-122.5252198	-9.276900314	9.276900314	-107.5399952	107.5399952
$\Delta E'_{q3}$	180	-109.8459234	109.8459234	-112.4566843	112.4566843	-12.67844026	12.67844026	-83.88155809	83.88155809	-125.4291963	125.4291963
$\Delta E'_{d3}$	180	157.9834079	-157.9834079	140.9018363	-140.9018363	21.96294713	-21.96294713	-159.0583209	159.0583209	38.33293286	-38.33293286
$\Delta E'_{fd3}$	0	-4.148737111	4.148737111	0	0	80.43482912	-80.43482912	0	0	0	0
$\Delta\delta_4$	180	31.4469478	-31.4469478	6.036746125	-6.036746125	96.70004621	-96.70004621	74.75007429	-74.75007429	-76.74382769	76.74382769
$\Delta\omega_4$	0	136.7615965	-136.7615965	119.5004803	-119.5004803	-166.9459717	166.9459717	170.5254214	-170.5254214	50.30606785	-50.30606785
$\Delta E'_{q4}$	0	-106.1133514	106.1133514	70.292105	-70.292105	150.7629626	-150.7629626	-153.7243956	153.7243956	53.76775793	-53.76775793
$\Delta E'_{d4}$	180	159.5374006	-159.5374006	-46.55203127	46.55203127	41.36893088	-41.36893088	36.12221025	-36.12221025	-107.091187	107.091187
$\Delta E'_{fd4}$	180	0	0	-177.0221609	177.0221609	-116.3529138	116.3529138	-53.51281783	53.51281783	-179.1748309	179.1748309
ΔX_1	180	-94.38724785	94.38724785	-121.3495357	121.3495357	-8.26004149	8.26004149	-27.92823129	27.92823129	148.16666	-148.16666
ΔX_2	0	10.92740086	-10.92740086	-7.885801553	7.885801553	88.09394059	-88.09394059	67.84711583	-67.84711583	-84.78344444	84.78344444
ΔX_3	180	116.2420496	-116.2420496	105.5779326	-105.5779326	-175.5520773	175.5520773	163.6224629	-163.6224629	42.2664511	-42.2664511

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix.

Table 37: Angle of mode shape with Power System Stabilizer connected to the exciter of the machine at bus 12 of the two-area system:Part B

SV/Mode	λ_{12}	λ_{13}	λ_{14}	λ_{15}	λ_{16}	λ_{17}	λ_{18}	λ_{19}	λ_{20}	λ_{21}	λ_{22}	λ_{23}
$\Delta\delta_1$	82.35383521	-82.35383521	117.6011492	-117.6011492	0	0	180	0	180	0	-180	180
$\Delta\omega_1$	-143.132624	143.132624	-151.341988	151.341988	180	180	0	180	0	180	-90.00003966	90.00003966
$\Delta E'_{q1}$	-151.2339555	151.2339555	-88.08479327	88.08479327	180	180	0	180	0	180	179.9997467	-179.9997467
$\Delta E'_{d1}$	-5.271523235	5.271523235	80.58634688	-80.58634688	0	0	180	0	180	0	-0.000255565	0.000255565
$\Delta E'_{fd1}$	-18.42843042	18.42843042	70.23926131	-70.23926131	0	0	180	0	0	180	179.999909	-179.999909
$\Delta\delta_2$	80.86551604	-80.86551604	-57.16563403	57.16563403	180	180	0	0	180	0	-180	180
$\Delta\omega_2$	-144.6209432	144.6209432	33.89122879	-33.89122879	0	0	180	180	0	180	-90.00004135	90.00004135
$\Delta E'_{q2}$	-150.7199942	150.7199942	-83.01010899	83.01010899	0	0	180	180	0	180	179.9998005	-179.9998005
$\Delta E'_{d2}$	-14.05961743	14.05961743	-25.03223864	25.03223864	180	180	0	0	180	0	-0.00022827	0.00022827
$\Delta E'_{fd2}$	-17.92971147	17.92971147	0	0	180	180	0	0	0	180	0.000287223	-0.000287223
$\Delta\delta_3$	-153.1624171	153.1624171	-70.79132614	70.79132614	0	180	0	180	180	0	-180	180
$\Delta\omega_3$	-18.64887631	18.64887631	20.26553669	-20.26553669	180	0	180	0	0	180	-90.00004299	90.00004299
$\Delta E'_{q3}$	-21.25645457	21.25645457	-42.47693636	42.47693636	0	0	180	180	0	180	179.9996909	-179.9996909
$\Delta E'_{d3}$	166.6684967	-166.6684967	-128.9916732	128.9916732	180	180	0	0	180	0	-0.000349347	0.000349347
$\Delta E'_{fd3}$	112.0707057	-112.0707057	33.37094987	-33.37094987	180	180	0	0	0	180	179.9998908	-179.9998908
$\Delta\delta_4$	-92.06338267	92.06338267	106.6756629	-106.6756629	180	180	0	180	180	0	180	-180
$\Delta\omega_4$	42.4501581	-42.4501581	-162.2674743	162.2674743	0	0	180	0	0	180	-90.00004185	90.00004185
$\Delta E'_{q4}$	46.39874217	-46.39874217	145.1589529	-145.1589529	0	0	180	0	180	0	-0.000242124	0.000242124
$\Delta E'_{d4}$	-155.1309715	155.1309715	-1.274510864	1.274510864	180	180	0	180	0	180	179.9997665	-179.9997665
$\Delta E'_{fd4}$	180	-180	-129.7724886	129.7724886	180	180	0	180	0	0	-3.58703E-05	3.58703E-05
ΔX_1	127.8057808	-127.8057808	13.52158091	-13.52158091	0	0	180	0	0	180	-90.00023527	90.00023527
ΔX_2	-97.6806784	97.6806784	104.5784437	-104.5784437	180	180	0	180	180	0	-0.000274785	0.000274785
ΔX_3	36.83286237	-36.83286237	-164.3646934	164.3646934	0	0	180	0	0	180	179.9974959	-179.9974959

The left eigenvalue identify which combination of the original state variables displays only the ith mode. As a result, the kth element of the right eigenvector measure the activity of state variable k in the ith mode and the kth element of the left eigenvector weighs the contribution of this activity to the ith mode.I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix.

Table 38: The activity of state variables in the ith mode with Power System Stabilizer connected to the exciter of the machine at bus 12 of the two-area system: part A

SV/Mode	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}	λ_{11}	λ_{12}
$\Delta\delta_1$	-0.828278889	-2.908170697	-2.908170697	3.9094336	3.9094336	6.738338557	6.738338557	0.882391249	0.882391249	-0.755989015	-0.755989015	2.330771469
$\Delta\omega_1$	0.017346405	-0.253712574	-0.253712587	0.267081141	0.267081128	1.057302261	1.057302248	0.164488276	0.164488277	-0.232783275	-0.232783291	-1.339984162
$\Delta E'_{q1}$	-0.867771598	-4.716898109	-4.71689811	4.718075137	4.718075137	-2.519802136	-2.519802136	-1.137848549	-1.137848549	4.790715021	4.79071502	8.961249088
$\Delta E'_{d1}$	0.301772081	-5.741337997	-5.741337997	6.848230897	6.848230897	-6.487562834	-6.487562834	-0.984034222	-0.984034222	-0.559903473	-0.559903473	-13.65737184
$\Delta E'_{fd1}$	0.002873469	0.202155652	0.202155652	-0.243830213	-0.243830213	0.098377317	0.098377317	0.001410849	0.001410849	0.132739988	0.132739988	0.360271041
$\Delta\delta_2$	-0.30801977	-0.263387321	-0.263387321	2.451396051	2.451396051	1.904646262	1.904646262	-0.007305676	-0.007305676	-2.624373709	-2.624373709	0.43956905
$\Delta\omega_2$	0.006450767	0.17505945	0.175059439	-0.184678795	-0.184678807	0.338625245	0.338625234	-0.770210806	-0.770210805	0.695262672	0.695262658	-0.461848989
$\Delta E'_{q2}$	-0.460197334	-2.243793773	-2.243793774	6.34116197	6.341161968	-2.388775223	-2.388775224	5.128255074	5.128255074	-9.597860474	-9.597860475	2.381761178
$\Delta E'_{d2}$	0.035815475	-5.605703394	-5.605703394	-5.785287416	-5.785287416	-2.387555772	-2.387555772	0.724428525	0.724428525	10.16905893	10.16905893	-4.760916193
$\Delta E'_{fd2}$	0.001523861	0.252347093	0.252347093	0.316805904	0.316805904	0.01470213	0.01470213	0.014701595	0.014701595	-0.46010564	-0.46010564	0.107254301
$\Delta\delta_3$	-0.160402958	-1.11597582	-1.11597582	-1.51266921	-1.51266921	1.357278291	1.357278291	-0.446584204	-0.446584204	4.055150072	4.055150072	-0.298316088
$\Delta\omega_3$	0.00335927	-0.318359404	-0.318359414	-0.535566309	-0.53556632	-0.075288608	-0.075288619	0.727732947	0.727732949	-0.598466884	-0.598466898	-0.200864051
$\Delta E'_{q3}$	-0.289336702	-3.272138079	-3.272138079	-0.440305354	-0.440305354	-0.638534712	-0.638534712	-5.442050699	-5.442050699	7.817455542	7.817455541	-1.067900797
$\Delta E'_{d3}$	-0.015243353	-5.532356441	-5.532356441	-7.257024372	-7.257024372	-1.558375278	-1.558375278	-0.598965424	-0.598965424	-10.71696413	-10.71696413	-2.043022285
$\Delta E'_{fd3}$	0.000958086	0.150799923	0.150799923	0.25278924	0.252789239	-0.004665706	-0.004665706	-0.022376026	-0.022376026	0.419770366	0.419770366	-0.013288937
$\Delta\delta_4$	1.296701617	4.287533838	4.287533838	-4.848160441	-4.848160441	-10.00026311	-10.00026311	-0.428501369	-0.428501369	-0.674787348	-0.674787348	-2.472024431
$\Delta\omega_4$	2.086583838	1.01886841	1.018868397	0.618433793	0.618433779	-2.735340542	-2.735340556	-0.204866741	-0.204866739	0.369694611	0.369694594	3.012280364
$\Delta E'_{q4}$	0.948255827	4.174633695	4.174633694	-12.02532468	-12.02532468	1.996061114	1.996061113	0.203267812	0.203267812	-2.593301742	-2.593301744	-10.1070127
$\Delta E'_{d4}$	-0.743893792	-6.740214655	-6.740214655	4.228826703	4.228826703	7.987903556	7.987903556	0.186163651	0.186163651	0.335115706	0.335115706	18.74821675
$\Delta E'_{fd4}$	-0.003139978	0.355563314	0.355563314	-0.272000924	-0.272000924	-0.120703777	-0.120703777	-0.003927664	-0.003927664	-0.045344039	-0.045344039	-0.404362636
ΔX_1	-796.8613163	-234.4341409	-234.4341345	-62.3052483	-62.30524163	533.3299656	533.3299721	31.23609804	31.23609724	-88.10549602	-88.10548792	-380.6038944
ΔX_2	-12240.1453	-3973.558873	-3973.558854	3574.42614	3574.42616	384.9225623	384.9225821	-19.06846609	-19.06846854	804.572508	804.5725327	6605.283645
ΔX_3	-42935.69679	-13924.12605	-13924.12605	10474.89006	10474.89007	5490.382126	5490.382127	192.7930074	192.7930073	1754.899326	1754.899327	17408.39627

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix.

Table 39: The activity of state variables in the ith mode with Power System Stabilizer connected to the exciter of the machine at bus 12 of the two-area system: part B

SV/Mode	λ_{13}	λ_{14}	λ_{15}	λ_{16}	λ_{17}	λ_{18}	λ_{19}	λ_{20}	λ_{21}	λ_{22}	λ_{23}
$\Delta\delta_1$	2.330771469	-0.888838342	-0.888838342	-2.903314025	10.44754151	9.17746914	-2.24122966	-0.047280054	-0.341081064	-0.415027292	-0.41502729
$\Delta\omega_1$	-1.339984162	-0.52093314	-0.52093314	0.643189043	-2.35510823	-2.085250362	0.539366969	0.23062813	5.882847428	2.824432309	2.823894083
$\Delta E'_{q1}$	8.961249088	1.093815147	1.093815147	-4.513204982	18.57657425	15.62887325	-4.851205819	0.013877997	0.300248817	0.140383036	0.140355855
$\Delta E'_{d1}$	-13.65737184	-0.307016996	-0.307016996	2.543508117	23.94037076	12.7785626	22.24529112	-0.002597363	-0.006947634	-0.002904875	-0.002904847
$\Delta E'_{fd1}$	0.360271041	0.010322459	0.010322459	-0.102833397	0.417347528	0.348929522	-0.103752179	0.000177106	0.003774997	0.001780993	0.001780653
$\Delta\delta_2$	0.43956905	1.157344007	1.157344007	5.10675139	0.504720652	4.050608967	0.47181726	-0.04065302	-0.293556047	-0.35723275	-0.357232751
$\Delta\omega_2$	-0.461848989	0.366481359	0.36648136	-1.131330099	-0.113775259	-0.920355457	-0.113545993	0.198302016	5.063152478	2.431050661	2.430587387
$\Delta E'_{q2}$	2.381761178	-0.830243726	-0.830243726	5.175263509	-0.285010325	5.186325418	-0.042166055	0.020321644	0.4975256	0.236700873	0.236655382
$\Delta E'_{d2}$	-4.760916193	-0.143194111	-0.143194111	-39.01877258	-19.77331781	-37.71111647	13.54311694	-0.002463761	-0.007956696	-0.004721327	-0.004721286
$\Delta E'_{fd2}$	0.107254301	-0.006312423	-0.006312423	0.117918404	-0.006403137	0.115789668	-0.000901801	0.000259337	0.006255338	0.003002583	0.003002015
$\Delta\delta_3$	-0.298316088	1.012383804	1.012383804	-7.292774455	8.516013989	1.371178814	-0.099199672	-0.039183461	-0.282921922	-0.344278408	-0.344278408
$\Delta\omega_3$	-0.200864051	0.434785567	0.434785567	1.615613256	-1.919698965	-0.311551155	0.023873067	0.191133632	4.879738805	2.343146392	2.342699917
$\Delta E'_{q3}$	-1.067900797	-0.90188994	-0.90188994	-5.831380618	5.582390191	0.166419677	-0.818777	0.011360275	0.281552835	0.133226505	0.133200709
$\Delta E'_{d3}$	-2.043022285	0.121816817	0.121816817	53.68942351	-98.13405074	-28.79719822	23.46223395	-0.002197757	-0.006172279	-0.002803204	-0.002803167
$\Delta E'_{fd3}$	-0.013288937	-0.008553742	-0.008553742	-0.132868035	0.125415845	0.003715478	-0.017511089	0.000144976	0.003539935	0.001690227	0.001689904
$\Delta\delta_4$	-2.472024431	-1.28088947	-1.28088947	5.08933709	-19.46827616	-14.59925692	1.868612073	0.127116535	0.917559033	0.11653845	0.116538448
$\Delta\omega_4$	3.012280364	-0.356526825	-0.356526824	-2.15487965	9.719862999	8.011482315	-2.260568336	0.24093438	6.118675897	2.936333483	2.935773876
$\Delta E'_{q4}$	-10.1070127	0.342779125	0.342779125	3.347441156	-17.26822155	-15.16260735	5.719444926	0.015552278	0.479723345	0.232344917	0.232300251
$\Delta E'_{d4}$	18.74821675	0.205575546	0.205575546	-59.35557056	272.145971	220.6732319	-39.42165748	0.005370389	0.007464254	-0.004496491	-0.004496449
$\Delta E'_{fd4}$	-0.404362636	0.001911438	0.001911438	0.076271463	-0.387953639	-0.33851969	0.122321108	0.000198472	0.006031512	0.002947356	0.002946797
ΔX_1	-380.6038943	28.72410012	28.72409991	387.3234832	-2009.845413	-1769.718959	682.6835245	-324.5886589	-8272.849406	-3971.342977	-3971.075034
ΔX_2	6605.283645	-85.41798393	-85.41798455	-2223.396522	11436.85975	10031.16299	-3766.579956	-3886.868416	-34822.50459	-15505.39836	-15504.58194
ΔX_3	17408.39627	-53.77324472	-53.77324475	-3179.419938	16146.05538	14078.54438	-5061.815365	-88.89178328	-952.1366917	-433.7746115	-433.7368308

Due to the large size of the power system, it is often necessary to construct reduced-order models for dynamic stability studies by retaining only a few modes. The appropriate definition and determination as to which state variables significantly participate in the selected modes become very important. This requires a tool for identifying the state variables that have significant participation in a selected mode. Participation factor analysis aids in the identification of how each dynamic variable affects a given mode or eigenvalue. a participation factor is a sensitivity measure of an eigenvalue to a diagonal entry of the system A matrix. The first columns and rows presents the state variables(SV). I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix.

Table 40: Participation factors with Power System Stabilizer connected to the exciter of the machine at bus 12 of the two-area system:part A

SV	$\Delta\delta_1$	$\Delta\omega_1$	$\Delta E'_{q1}$	$\Delta E'_{d1}$	$\Delta E'_{fd1}$	$\Delta\delta_2$	$\Delta\omega_2$	$\Delta E'_{q2}$	$\Delta E'_{d2}$	$\Delta E'_{fd2}$	$\Delta\delta_3$	$\Delta\omega_3$
$\Delta\delta_1$	1.35465E-05	1.35465E-05	0.000120711	1.49924E-05	0.0001372	3.72832E-06	3.72832E-06	3.68005E-05	8.73545E-07	4.16362E-05	7.94815E-07	7.94814E-07
$\Delta\omega_1$	0.00253508	0.00253508	0.569688543	0.045842317	0.563589141	0.001502128	0.001502129	0.680609897	0.021854011	0.674750566	0.004016332	0.004016331
$\Delta E'_{q1}$	0.00253508	0.00253508	0.569688543	0.045842317	0.563589141	0.001502128	0.001502128	0.680609897	0.021854011	0.674750566	0.004016332	0.004016332
$\Delta E'_{d1}$	0.037980068	0.037980069	0.813608983	0.041120388	0.804933217	0.008289557	0.008289557	0.851250183	0.005761116	0.847374039	0.058462202	0.0584622
$\Delta E'_{fd1}$	0.037980068	0.037980066	0.813608983	0.041120388	0.804933217	0.008289557	0.008289556	0.851250183	0.005761116	0.847374039	0.058462202	0.058462203
$\Delta\delta_2$	0.338315893	0.338315898	0.443885638	0.035567483	0.313095575	0.037547182	0.037547184	0.055935014	0.006859223	0.038614422	0.011167366	0.011167367
$\Delta\omega_2$	0.33831589	0.338315889	0.443885635	0.035567483	0.313095573	0.037547182	0.037547181	0.055935014	0.006859223	0.038614422	0.011167366	0.011167366
$\Delta E'_{q2}$	0.039515077	0.039515076	0.028058216	0.005477574	0.017806867	1	1	0.234787575	0.192826115	0.122762971	0.910980149	0.910980148
$\Delta E'_{d2}$	0.039515077	0.039515077	0.028058216	0.005477574	0.017806867	1	0.999999998	0.234787575	0.192826115	0.122762971	0.910980149	0.91098015
$\Delta E'_{fd2}$	0.009675115	0.009675115	0.055784916	0.00711094	0.05639829	0.135330142	0.135330144	0.956756954	0.211326813	0.957585241	0.111463484	0.111463482
$\Delta\delta_3$	0.009675115	0.009675115	0.055784916	0.00711094	0.05639829	0.135330142	0.135330142	0.956756954	0.211326813	0.957585241	0.111463484	0.111463484
$\Delta\omega_3$	0.275005377	0.275005378	0.740873453	0.28567631	0.740896155	0.034278687	0.034278687	0.077538557	0.035682648	0.077897837	0.011131459	0.011131459
$\Delta E'_{q3}$	0.275005377	0.275005378	0.740873453	0.28567631	0.740896155	0.034278687	0.034278687	0.077538557	0.035682648	0.077897837	0.011131459	0.011131459
$\Delta E'_{d3}$	1	0.999999997	0.009961553	0.106138383	0.00581191	0.541944543	0.541944545	0.070504104	0.011098118	0.032112268	0.881013354	0.881013356
$\Delta E'_{fd3}$	1	0.999999997	0.009961553	0.106138383	0.00581191	0.541944543	0.541944545	0.070504104	0.011098118	0.032112268	0.881013354	0.881013357
$\Delta\delta_4$	0.040821987	0.040821987	0.057944732	0.036903832	0.045388941	0.069982768	0.069982768	0.093447912	1	0.074988025	0.017086981	0.017086981
$\Delta\omega_4$	0.079572026	0.079572026	0.13942854	0.205842211	0.10616351	0.00108527	0.00108527	0.001216381	0.126225648	0.000955097	0.028349751	0.028349751
$\Delta E'_{q4}$	0.055807247	0.055807247	0.082697486	0.071550056	0.061584404	0.002093701	0.002093701	0.014774184	0.174708773	0.011594199	0.004552719	0.004552719
$\Delta E'_{d4}$	0.056945627	0.056945627	0.183360027	1	0.125911769	0.000937916	0.000937916	0.000496264	0.228578349	0.000348969	0.000229456	0.000229456
$\Delta E'_{fd4}$	0.016574113	0.016574113	0.000457846	4.66573E-05	2.22824E-06	0.013445496	0.013445496	0.000197235	1.29147E-05	1.25709E-06	0.012665859	0.012665859
ΔX_1	0.079078211	0.079078211	0.00056128	6.95201E-06	1.12399E-05	0.067735609	0.067735609	0.000274847	2.3333E-06	5.91392E-06	0.06516265	0.06516265
ΔX_2	0.757273931	0.757273929	6.99731E-09	3.872E-12	3.24115E-10	0.651819903	0.651819903	4.38604E-09	2.06567E-12	6.01135E-11	0.628182937	0.628182937
ΔX_3	0.757273927	0.757273929	6.99731E-09	3.872E-12	3.24115E-10	0.651819903	0.651819903	4.38604E-09	2.06567E-12	6.01135E-11	0.628182938	0.628182937

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix.

Table 41: Participation factors with Power System Stabilizer connected to the exciter of the machine at bus 12 of the two-area system:part B

State variables	$\Delta E'_{q3}$	$\Delta E'_{d3}$	$\Delta E'_{fd3}$	$\Delta\delta_4$	$\Delta\omega_4$	$\Delta E'_{q4}$	$\Delta E'_{d4}$	$\Delta E'_{fd4}$	ΔX_1	ΔX_2	ΔX_3
$\Delta\delta_1$	1.59674E-05	3.94436E-07	1.83535E-05	3.50089E-05	0.002689934	0.002495458	2.51152E-05	0.003143046	8.14011E-06	0.005970368	1
$\Delta\omega_1$	0.383333436	0.037345328	0.379013622	0.009220996	0.0375541	1	0.014855521	0.99417345	1.82037E-06	0.000202447	0.012267095
$\Delta E'_{q1}$	0.383333436	0.037345328	0.379013622	0.009220996	0.037554099	1	0.014855521	0.99417345	1.82037E-06	0.000202447	0.012267095
$\Delta E'_{d1}$	1	0.038348756	0.990768471	0.017536417	0.064829479	0.542055891	0.008737558	0.535867087	6.86617E-06	0.000531523	0.020372678
$\Delta E'_{fd1}$	1	0.038348756	0.990768471	0.017536417	0.064829481	0.542055891	0.008737558	0.535867087	6.86617E-06	0.000531523	0.020372678
$\Delta\delta_2$	0.005118874	0.00154186	0.003718936	0.530075056	1	0.480439021	0.096240541	0.312902328	0.000117722	0.003931623	0.104171667
$\Delta\omega_2$	0.005118874	0.00154186	0.003718936	0.530075052	1	0.480439017	0.09624054	0.312902325	0.000117722	0.003931623	0.104171667
$\Delta E'_{q2}$	0.247880481	0.128971553	0.151486259	0.013321247	0.021653101	0.005020706	0.002994351	0.003021815	2.76949E-06	7.39596E-05	0.001607205
$\Delta E'_{d2}$	0.247880481	0.128971553	0.151486259	0.013321247	0.021653101	0.005020706	0.002994351	0.003021815	2.76949E-06	7.39596E-05	0.001607205
$\Delta E'_{fd2}$	0.993290165	0.139245912	1	0.002001531	0.006555156	0.0062134	0.001904532	0.006156591	1.35694E-06	7.41738E-05	0.001636716
$\Delta\delta_3$	0.993290165	0.139245912	1	0.002001531	0.006555156	0.0062134	0.001904532	0.006156591	1.35694E-06	7.41738E-05	0.001636716
$\Delta\omega_3$	0.003454352	0.004144437	0.003360897	0.494792657	0.698687088	0.995756549	0.447659555	1	0.000261585	0.011880655	0.196283655
$\Delta E'_{q3}$	0.003454352	0.004144437	0.003360897	0.494792657	0.698687088	0.995756549	0.447659555	1	0.000261585	0.011880655	0.196283655
$\Delta E'_{d3}$	0.04387513	0.058285459	0.017382841	0.279946676	0.305052524	0.033016346	0.011398734	0.01337625	5.07096E-05	0.000649741	0.007048409
$\Delta E'_{fd3}$	0.04387513	0.058285459	0.017382841	0.279946676	0.305052523	0.033016346	0.011398734	0.01337625	5.07096E-05	0.000649741	0.007048409
$\Delta\delta_4$	0.018514858	0.386967798	0.01601991	0.019421585	0.037119477	0.020134572	0.450339063	0.016151057	1.89492E-05	0.000491009	0.003169392
$\Delta\omega_4$	0.030828544	0.798850692	0.02417745	0.054240579	0.120132411	0.059918515	1	0.04601804	7.29997E-05	0.001842757	0.011540671
$\Delta E'_{q4}$	0.000930039	0.233734579	0.000718541	0.038908895	0.093971412	0.059385536	1	0.045250287	6.19585E-05	0.001545654	0.009547365
$\Delta E'_{d4}$	0.006146937	0.288091938	0.004420811	0.04514732	0.226951198	0.081391179	0.181924948	0.053017527	0.000229107	0.005252516	0.02933111
$\Delta E'_{fd4}$	3.74264E-05	3.8201E-06	2.72469E-07	0.032828172	0.012755842	0.000886852	8.8934E-05	6.38642E-06	0.407350091	1	0.00468843
ΔX_1	5.39343E-05	6.13729E-07	1.20629E-06	0.207912471	0.080384812	0.00157179	6.87048E-06	1.20619E-05	1	0.244047807	0.000386887
ΔX_2	5.58094E-10	2.71333E-13	5.22565E-11	0.212640305	0.787359696	2.11111E-08	6.04322E-12	1.29411E-09	1	6.99345E-07	1.6917E-10
ΔX_3	5.58094E-10	2.71333E-13	5.22565E-11	0.212640302	0.787359696	2.11111E-08	6.04322E-12	1.29411E-09	1	6.99345E-07	1.6917E-10

The electromechanical oscillations is provided in this part. The electromechanical oscillation is of two type. Local mode have the frequency between 0.8 and 3 HZ while inter-are mode have the frequency between 0.1 and 0.8 HZ. I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix.

Table 42: Summary of results with Power System Stabilizer connected to the exciter of the machine at bus 12 of the two-area system

EV	Amount	Conrolability	Observability	mode	Dominant states
1	-47.7492977+ 0.000000000i				ΔX_3
2	-4.6199491+16.870741856i				$\Delta E'_{q4}, \Delta E'_{fd4}, \Delta E'_{q2}, \Delta E'_{fd2}$
3	-4.6199491-16.870741856i				$\Delta E'_{q4}, \Delta E'_{fd4}, \Delta E'_{q2}, \Delta E'_{fd2}$
4	-4.9992500+11.517418169i				$\Delta E'_{q3}, \Delta E'_{fd3}, \Delta E'_{q1}, \Delta E'_{fd1}$
5	-4.9992500-11.517418169i				$\Delta E'_{q3}, \Delta E'_{fd3}, \Delta E'_{q1}, \Delta E'_{fd1}$
6	-0.9610420+ 8.630452102i	yes	yes	Local	$\Delta\omega_4, \Delta\delta_4$
7	-0.9610420- 8.630452102i	yes	yes	Local	$\Delta\omega_4, \Delta\delta_4$
8	-0.7565849+ 7.480452655i	yes	yes	Local	$\Delta\omega_2, \Delta\delta_2, \Delta\omega_3, \Delta\delta_3$
9	-0.7565849- 7.480452655i	yes	yes	Local	$\Delta\omega_2, \Delta\delta_2, \Delta\omega_3, \Delta\delta_3$
10	-5.352726 + 7.09045229i				$\Delta E'_{fd3}, \Delta E'_{q3}, \Delta E'_{fd2}, \Delta E'_{q2}$
11	-5.352726 - 7.09045229i				$\Delta E'_{fd3}, \Delta E'_{q3}, \Delta E'_{fd2}, \Delta E'_{q2}$
12	-5.257008 + 5.34704237i				$\Delta E'_{fd4}, \Delta E'_{q4}, \Delta E'_{fd1}, \Delta E'_{q1}$
13	-5.257008 - 5.34704237i				$\Delta E'_{fd4}, \Delta E'_{q4}, \Delta E'_{fd1}, \Delta E'_{q1}$
14	-0.086004 + 4.66205324i	no	yes	Inter area	$\Delta\omega_1, \Delta\delta_1, \Delta\omega_3, \Delta\delta_3$
15	-0.086004 - 4.66205324i	no	yes	Inter area	$\Delta\omega_1, \Delta\delta_1, \Delta\omega_3, \Delta\delta_3$
16	-4.513935 + 0.00000000i				$\Delta E'_{fd1}$
17	-4.436119 + 0.00000000i				$\Delta E'_{fd2}$
18	-4.401135 + 0.00000000i				$\Delta E'_{fd3}$
19	-4.155296 + 0.00000000i				$\Delta E'_{fd4}$
20	-0.205005 + 0.00000000i				ΔX_2
21	-0.057978 + 0.00000000i				ΔX_1
22	0.000000 + 0.00000022i				ΔX_1
23	0.000000 - 0.00000022i				ΔX_1

The inter area mode has a small positive damping (0.0184446). The characteristics of the inter-area modes of oscillation are very complex and in some respects significantly differ from the characteristics of local plant modes. Load characteristics, in particular, have a major effect on the stability of inter-area modes. The manner in which excitation systems affect inter-area oscillations depend on the types and locations of the exciters and on the characteristics of loads.
Speed-governing systems normally do not have a very significant effect on the inter area oscillations. However, if they are not properly tuned , they may decrease damping of the oscillation slightly. A mode of oscillation in one part of the system may interact with a mode of oscillations in remote part due to mode coupling. This occurs when the frequencies of the two modes are nearly equal. The controllability of inter-area modes with PSS is a complex function of many factors including location of unit with PSS, chractristics and location of loads, types of exciters on other units. On some units, the PSS does not have the desired effect on the damping of inter-area oscilations. Here, the whole system is stable. In addition, because of PSS, 3 state variables are added to the state variables of the system without PSS. As a result, 3 eigen values are add so that the total number of eigenvlues are 23.

- **Problem #4**

Design a PSS whose inputs are the speed variations of two machines that belong to different areas while its output acts on the AVR of one machine in one area. Justify your choice. Compare the performance of this PSS with that of the local PSS described in Problem 2.

The basic function of power system stabilizer (PSS) is to add damping to the generator rotor oscillation by controlling its excitation using auxiliary stabilizing signal. To provide damping, the stabilizer must producer a component of electrical torque in phase with the rotor speed deviation. Since the purpose of the PSS is to introduce a damping torque component, a logical signal to use for for controlling generator excitation is the speed variation. Hence, We use a stabilizing device, called the power system stabilizer (PSS) to damp out the low-frequency oscillations.

To design, We first analyse the system without PSS. So, we study Eigenvalue, Frequency,Damp Ratio, Dominant states, and Modes of two-area system without PSS I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix.

Table 43: Eigenvalue, Frequency,Damp Ratio, Dominant states, and Modes of two-area system without PSS

SV	Eigenvalue	Frequency	Damp Ratio	Dominant states	Mode
$\Delta\delta_1$	-4.51887 +16.8688511i	2.68476104	0.258759297	$\Delta E'_{q4}, \Delta E'_{fd4}$	
$\Delta\omega_1$	-4.51887 -16.8688511i	2.68476104	0.258759297	$\Delta E'_{q4}, \Delta E'_{fd4}$	
$\Delta E'_{q1}$	-5.11594 +11.5793025i	1.842903235	0.404131553	$\Delta E'_{q3}, \Delta E'_{fd3}$	
$\Delta E'_{d1}$	-5.11594 -11.5793025i	1.842903235	0.404131553	$\Delta E'_{q3}, \Delta E'_{fd3}$	
$\Delta E'_{fd1}$	-5.28565 + 7.2769821i	1.158167674	0.587684573	$\Delta E'_{q1}, \Delta E'_{fd1}, \Delta E'_{q4}, \Delta E'_{fd4}$	
$\Delta\delta_2$	-5.28565 - 7.2769821i	1.158167674	0.587684573	$\Delta E'_{q1}, \Delta E'_{fd1}, \Delta E'_{q4}, \Delta E'_{fd4}$	
$\Delta\omega_2$	-5.35085 + 6.9639123i	1.108341072	0.609281365	$\Delta E'_{q2}, \Delta E'_{fd2}, \Delta E'_{q3}, \Delta E'_{fd3}$	
$\Delta E'_{q2}$	-5.35085 - 6.9639123i	1.108341072	0.609281365	$\Delta E'_{q2}, \Delta E'_{fd2}, \Delta E'_{q3}, \Delta E'_{fd3}$	
$\Delta E'_{d2}$	-0.76125 + 7.5135325i	1.195815847	0.100801184	$\Delta\omega_2, \Delta\delta_2, \Delta\omega_3, \Delta\delta_3$	Local
$\Delta E'_{fd2}$	-0.76125 - 7.5135325i	1.195815847	0.100801184	$\Delta\omega_2, \Delta\delta_2, \Delta\omega_3, \Delta\delta_3$	Local
$\Delta\delta_3$	-0.77491 + 6.8385984i	1.088396743	0.112593898	$\Delta\omega_1, \Delta\delta_1, \Delta\omega_4, \Delta\delta_4$	Local
$\Delta\omega_3$	-0.77491 - 6.8385984i	1.088396743	0.112593898	$\Delta\omega_1, \Delta\delta_1, \Delta\omega_4, \Delta\delta_4$	Local
$\Delta E'_{q3}$	-0.01799 + 4.6175744i	0.734909795	0.003897223	$\Delta\omega_3, \Delta\delta_3, \Delta\omega_1, \Delta\delta_1$	Inter-area
$\Delta E'_{d3}$	-0.01799 - 4.6175744i	0.734909795	0.003897223	$\Delta\omega_3, \Delta\delta_3, \Delta\omega_1, \Delta\delta_1$	Inter-area
$\Delta E'_{fd3}$	-4.67424 + 0.0000000i	0	1	$\Delta E'_{d4}$	
$\Delta\delta_4$	-4.52554 + 0.0844062i	0.013433667	0.999826114	$\Delta E'_{d1}, \Delta E'_{d2}$	
$\Delta\omega_4$	-4.52554 - 0.0844062i	0.013433667	0.999826114	$\Delta E'_{d1}, \Delta E'_{d2}$	
$\Delta E'_{q4}$	-4.30336 + 0.0000000i	0	1	$\Delta E'_{d3}$	
$\Delta E'_{d4}$	-0.00000 + 0.0000002i	3.77725E-08	2.85386E-09	$\Delta\omega_1, \Delta\delta_1, \Delta\omega_3, \Delta\delta_3, , \Delta\omega_4, \Delta\delta_4$	
$\Delta E'_{fd4}$	-0.00000 - 0.0000002i	3.77725E-08	2.85386E-09	$\Delta\omega_1, \Delta\delta_1, \Delta\omega_3, \Delta\delta_3, , \Delta\omega_4, \Delta\delta_4$	

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix.

Table 44: Participation factor matrix of two-area system without PSS : Part A

SV	$\Delta\delta_1$	$\Delta\omega_1$	$\Delta E'_{q1}$	$\Delta E'_{d1}$	$\Delta E'_{fd1}$	$\Delta\delta_2$	$\Delta\omega_2$	$\Delta E'_{q2}$	$\Delta E'_{d2}$	$\Delta E'_{fd2}$
$\Delta\delta_1$	0.002194452	0.002194452	0.571971928	0.046147921	0.564268826	0.001568208	0.001568208	0.687791651	0.022070002	0.67996042
$\Delta\omega_1$	0.002194452	0.002194452	0.571971928	0.046147921	0.564268826	0.001568208	0.001568208	0.687791651	0.022070002	0.67996042
$\Delta E'_{q1}$	0.040014765	0.040014764	0.649258427	0.036897228	0.646887594	0.005517237	0.005517237	0.85676069	0.005239197	0.858664122
$\Delta E'_{d1}$	0.040014765	0.040014765	0.649258427	0.036897228	0.646887594	0.005517237	0.005517237	0.85676069	0.005239197	0.858664122
$\Delta E'_{fd1}$	0.092770033	0.092770034	1	0.109659621	0.998769799	0.039498107	0.039498107	0.254678457	0.061470971	0.251860184
$\Delta\delta_2$	0.092770033	0.092770034	1	0.109659621	0.998769799	0.039498107	0.039498107	0.254678457	0.061470971	0.251860184
$\Delta\omega_2$	0.037719531	0.037719531	0.28599608	0.036739886	0.287867039	0.143931846	0.143931846	0.997179679	0.220135249	1
$\Delta E'_{q2}$	0.037719531	0.037719531	0.28599608	0.036739886	0.287867039	0.143931846	0.143931846	0.997179679	0.220135249	1
$\Delta E'_{d2}$	0.004556018	0.004556018	0.000789502	0.000683755	0.000565573	1	0.999999998	0.245773787	0.191636474	0.128021193
$\Delta E'_{fd2}$	0.004556018	0.004556018	0.000789502	0.000683755	0.000565573	0.999999998	1	0.245773786	0.191636473	0.128021192
$\Delta\delta_3$	0.880620272	0.880620271	0.236128475	0.134415406	0.133386217	0.006163974	0.006163974	0.003393764	0.00253763	0.002104043
$\Delta\omega_3$	0.88062027	0.880620266	0.236128475	0.134415405	0.133386216	0.006163974	0.006163974	0.003393764	0.00253763	0.002104043
$\Delta E'_{q3}$	0.811714438	0.811714438	0.018589947	0.069093341	0.006367611	0.645576408	0.645576408	0.07489063	0.017764557	0.034167312
$\Delta E'_{d3}$	0.811714437	0.811714435	0.018589947	0.069093341	0.006367611	0.645576407	0.645576408	0.07489063	0.017764557	0.034167311
$\Delta E'_{fd3}$	0.050747962	0.050747962	0.020408408	0.116089907	0.016332354	0.000819537	0.000819537	0.000138096	0.036247777	0.00012073
$\Delta\delta_4$	0.027182983	0.027182983	0.028481153	0.687208594	0.023113514	0.073699689	0.073699689	0.082002094	1	0.065764055
$\Delta\omega_4$	0.027182983	0.027182983	0.028481153	0.687208594	0.023113514	0.073699689	0.073699689	0.082002094	1	0.065764055
$\Delta E'_{q4}$	9.5541E-05	9.5541E-05	0.008618302	0.36063025	0.00652427	0.005448431	0.005448431	0.028750568	0.037221274	0.022068106
$\Delta E'_{d4}$	0.961789047	0.961789043	2.96973E-10	1.51707E-13	8.1785E-11	0.827855312	0.827855307	2.22036E-11	5.59841E-14	1.1302E-10
$\Delta E'_{fd4}$	0.961789034	0.961789038	2.96973E-10	1.51707E-13	8.1785E-11	0.827855297	0.827855302	2.22036E-11	5.59841E-14	1.1302E-10

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix.

Table 45: Participation factor matrix of two-area system without PSS: Part B

SV	$\Delta\delta_3$	$\Delta\omega_3$	$\Delta E'_{q3}$	$\Delta E'_{d3}$	$\Delta E'_{fd3}$	$\Delta\delta_4$	$\Delta\omega_4$	$\Delta E'_{q4}$	$\Delta E'_{d4}$	$\Delta E'_{fd4}$
$\Delta\delta_1$	0.004000761	0.004000761	0.38644742	0.037734293	0.381001404	0.008072912	0.008072912	1	0.014886336	0.9897651
$\Delta\omega_1$	0.004000761	0.004000761	0.38644742	0.037734293	0.381001404	0.008072912	0.008072912	1	0.014886336	0.9897651
$\Delta E'_{q1}$	0.054620979	0.054620979	1	0.037361716	0.997641799	0.005601644	0.005601643	0.60462363	0.011217236	0.603681204
$\Delta E'_{d1}$	0.054620979	0.054620979	1	0.037361716	0.9976418	0.005601644	0.005601644	0.60462363	0.011217236	0.603681204
$\Delta E'_{fd1}$	0.038226004	0.038226004	0.403885183	0.049776054	0.403402882	0.083935254	0.083935253	0.695063006	0.135809747	0.686555236
$\Delta\delta_2$	0.038226004	0.038226004	0.403885183	0.049776054	0.403402882	0.083935254	0.083935253	0.695063006	0.135809747	0.686555236
$\Delta\omega_2$	0.095064741	0.095064741	0.789046742	0.116687715	0.793772461	0.055272607	0.055272607	0.425361274	0.084410282	0.425345953
$\Delta E'_{q2}$	0.095064741	0.09506474	0.789046742	0.116687715	0.793772461	0.055272607	0.055272607	0.425361274	0.084410282	0.425345953
$\Delta E'_{d2}$	0.844286979	0.844286981	0.22183959	0.11649973	0.137422588	0.024333105	0.024333105	0.010747084	0.004460475	0.006479227
$\Delta E'_{fd2}$	0.844286977	0.844286975	0.221839589	0.116499729	0.137422587	0.024333105	0.024333105	0.010747084	0.004460475	0.006479227
$\Delta\delta_3$	0.025916064	0.025916064	0.011248455	0.004014768	0.00643366	0.999999999	1	0.232805594	0.217547177	0.118594596
$\Delta\omega_3$	0.025916064	0.025916063	0.011248455	0.004014768	0.00643366	0.999999997	1	0.232805593	0.217547176	0.118594595
$\Delta E'_{q3}$	1	1	0.045585853	0.06263874	0.018085803	0.543949303	0.543949303	0.011923503	0.042171547	0.005989836
$\Delta E'_{d3}$	0.999999998	1	0.045585853	0.06263874	0.018085803	0.543949302	0.543949301	0.011923503	0.042171546	0.005989836
$\Delta E'_{fd3}$	0.006547068	0.006547068	0.00266674	0.019339097	0.002181351	0.076637602	0.076637602	0.071173752	1	0.060832498
$\Delta\delta_4$	0.065226437	0.065226437	0.043055905	0.412046907	0.034005623	0.01504298	0.01504298	0.029947598	0.253189113	0.024314285
$\Delta\omega_4$	0.065226437	0.065226437	0.043055905	0.412046907	0.034005623	0.01504298	0.01504298	0.029947598	0.253189113	0.024314285
$\Delta E'_{q4}$	0.012181858	0.012181858	0.03782471	1	0.028318613	0.002260669	0.002260669	0.01241233	0.087441407	0.00954892
$\Delta E'_{d4}$	0.797834761	0.797834764	1.74293E-10	1.38867E-13	1.34322E-11	0.999999994	1	1.61205E-10	9.75616E-14	1.00823E-10
$\Delta E'_{fd4}$	0.797834762	0.797834759	1.74293E-10	1.38867E-13	1.34322E-11	1	0.999999994	1.61205E-10	9.75616E-14	1.00823E-10

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix.

Table 46: Magnitude of mode shape of two-area system without PSS:Part A

SV/Mode	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}
$\Delta\delta_1$	8.25831E-05	8.25831E-05	0.000936038	0.000936038	0.004998324	0.004998324	0.002867309	0.002867309	0.002245358	0.002245358
$\Delta\omega_1$	0.0014422	0.0014422	0.011849415	0.011849415	0.044955087	0.044955087	0.025181393	0.025181393	0.016956937	0.016956937
$\Delta E'_{q1}$	0.003572845	0.003572845	0.004748165	0.004748165	0.009652051	0.009652051	0.005162786	0.005162786	0.000870304	0.000870304
$\Delta E'_{d1}$	0.000390677	0.000390677	0.000293088	0.000293088	0.001277513	0.001277513	0.00083522	0.00083522	0.000521542	0.000521542
$\Delta E'_{fd1}$	0.500144451	0.500144451	0.475625477	0.475625477	0.668689955	0.668689955	0.34809507	0.34809507	0.059394519	0.059394519
$\Delta\delta_2$	9.76094E-05	9.76094E-05	0.000519527	0.000519527	0.003629927	0.003629927	0.006374257	0.006374257	0.050466631	0.050466631
$\Delta\omega_2$	0.001704614	0.001704614	0.006576757	0.006576757	0.032647681	0.032647681	0.055980249	0.055980249	0.381123895	0.381123895
$\Delta E'_{q2}$	0.003553063	0.003553063	0.004733811	0.004733811	0.004697273	0.004697273	0.00905672	0.00905672	0.010794266	0.010794266
$\Delta E'_{d2}$	0.000233048	0.000233048	4.67515E-05	4.67515E-05	0.001330994	0.001330994	0.002270939	0.002270939	0.014625186	0.014625186
$\Delta E'_{fd2}$	0.49842475	0.49842475	0.476982922	0.476982922	0.322220291	0.322220291	0.60838629	0.60838629	0.535647709	0.535647709
$\Delta\delta_3$	0.000129502	0.000129502	0.001256049	0.001256049	0.003812945	0.003812945	0.005851539	0.005851539	0.047924737	0.047924737
$\Delta\omega_3$	0.002261576	0.002261576	0.015900472	0.015900472	0.034293747	0.034293747	0.051389613	0.051389613	0.36192752	0.36192752
$\Delta E'_{q3}$	0.003234638	0.003234638	0.006432988	0.006432988	0.006469954	0.006469954	0.008993197	0.008993197	0.010861536	0.010861536
$\Delta E'_{d3}$	0.000382092	0.000382092	0.000270068	0.000270068	0.000998418	0.000998418	0.001668094	0.001668094	0.009848181	0.009848181
$\Delta E'_{fd3}$	0.452513776	0.452513776	0.645231395	0.645231395	0.448251844	0.448251844	0.606023331	0.606023331	0.640988366	0.640988366
$\Delta\delta_4$	0.000195006	0.000195006	0.000541045	0.000541045	0.004849033	0.004849033	0.003534128	0.003534128	0.006520274	0.006520274
$\Delta\omega_4$	0.003405514	0.003405514	0.006849145	0.006849145	0.04361236	0.04361236	0.031037552	0.031037552	0.049241099	0.049241099
$\Delta E'_{q4}$	0.003877695	0.003877695	0.003582999	0.003582999	0.007174537	0.007174537	0.00545953	0.00545953	0.002080902	0.002080902
$\Delta E'_{d4}$	0.000144041	0.000144041	0.000109686	0.000109686	0.001832458	0.001832458	0.001265933	0.001265933	0.001632608	0.001632608
$\Delta E'_{fd4}$	0.544597601	0.544597601	0.359664262	0.359664262	0.49156958	0.49156958	0.3656971	0.3656971	0.119515471	0.119515471

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix.

Table 47: Magnitude of mode shape of two-area system without PSS:Part B

SV/Mode	λ_{11}	λ_{12}	λ_{13}	λ_{14}	λ_{15}	λ_{16}	λ_{17}	λ_{18}	λ_{19}	λ_{20}
$\Delta\delta_1$	0.049158835	0.049158835	0.071452073	0.071452073	0.013511978	0.008124203	0.008124203	0.00019775	0.5	0.5
$\Delta\omega_1$	0.338328932	0.338328932	0.329937771	0.329937771	0.063158271	0.036772806	0.036772806	0.00085099	1.18666E-07	1.18666E-07
$\Delta E'_{q1}$	0.011859536	0.011859536	0.004535006	0.004535006	0.007480037	0.013929996	0.013929996	0.013665101	7.25376E-16	7.25376E-16
$\Delta E'_{d1}$	0.010170754	0.010170754	0.008392053	0.008392053	0.005093214	0.0203997	0.0203997	0.021669685	3.54418E-16	3.54418E-16
$\Delta E'_{fd1}$	0.615447606	0.615447606	0.136675732	0.136675732	0.255043772	0.495156261	0.495156261	0.471445582	1.59812E-14	1.59812E-14
$\Delta\delta_2$	0.005352751	0.005352751	0.062537268	0.062537268	0.003657113	0.016671447	0.016671447	0.003513961	0.5	0.5
$\Delta\omega_2$	0.036839577	0.036839577	0.28877268	0.28877268	0.017094238	0.075460429	0.075460429	0.01512185	1.18666E-07	1.18666E-07
$\Delta E'_{q2}$	0.003518418	0.003518418	0.015689501	0.015689501	0.007776325	0.019151644	0.019151644	0.010985298	3.24308E-17	3.24308E-17
$\Delta E'_{d2}$	0.002227454	0.002227454	0.002408655	0.002408655	0.013506596	0.026117199	0.026117199	0.019361465	8.05036E-17	8.05036E-17
$\Delta E'_{fd2}$	0.200392444	0.200392444	0.629807219	0.629807219	0.289655434	0.672748194	0.672748194	0.384272226	1.32063E-14	1.32063E-14
$\Delta\delta_3$	0.010039557	0.010039557	0.088144319	0.088144319	0.011081168	0.01486711	0.01486711	0.006865435	0.5	0.5
$\Delta\omega_3$	0.069095871	0.069095871	0.407016046	0.407016046	0.051796072	0.067293412	0.067293412	0.02954446	1.18666E-07	1.18666E-07
$\Delta E'_{q3}$	0.003439201	0.003439201	0.010117839	0.010117839	0.006374164	0.01173889	0.01173889	0.02070265	4.48122E-16	4.48122E-16
$\Delta E'_{d3}$	0.001702787	0.001702787	0.007383119	0.007383119	0.005994116	0.014037702	0.014037702	0.031052556	3.36179E-16	3.36179E-16
$\Delta E'_{fd3}$	0.180710572	0.180710572	0.353191836	0.353191836	0.222146477	0.406095238	0.406095238	0.706367707	2.76282E-15	2.76282E-15
$\Delta\delta_4$	0.050446732	0.050446732	0.052664941	0.052664941	0.018590032	0.006548746	0.006548746	0.003197524	0.5	0.5
$\Delta\omega_4$	0.347192707	0.347192707	0.243186133	0.243186133	0.086894332	0.029641767	0.029641767	0.013760106	1.18666E-07	1.18666E-07
$\Delta E'_{q4}$	0.011733144	0.011733144	0.003413289	0.003413289	0.024317625	0.009819054	0.009819054	0.010154984	2.39717E-16	2.39717E-16
$\Delta E'_{d4}$	0.016162405	0.016162405	0.005982945	0.005982945	0.03674682	0.015390871	0.015390871	0.018729339	1.47301E-16	1.47301E-16
$\Delta E'_{fd4}$	0.549093987	0.549093987	0.15086875	0.15086875	0.885540147	0.349182121	0.349182121	0.35603174	1.19941E-14	1.19941E-14

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix.

Table 48: Angle of mode shape in degree of two-area system without PSS:Part A

SV/Mode	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}
$\Delta\delta_1$	-141.5338022	141.5338022	15.1984871	-15.1984871	135.8731494	-135.8731494	128.0884789	-128.0884789	62.7689828	-62.7689828
$\Delta\omega_1$	-36.53734628	36.53734628	129.0352043	-129.0352043	-98.13398054	98.13398054	-104.373962	104.373962	158.5542909	-158.5542909
$\Delta E'_{q1}$	-106.1334312	106.1334312	69.76959022	-69.76959022	-124.1781778	124.1781778	-124.7008439	124.7008439	4.072842744	-4.072842744
$\Delta E'_{d1}$	161.6519215	-161.6519215	-30.57697177	30.57697177	44.93423118	-44.93423118	41.17768198	-41.17768198	37.57727534	-37.57727534
$\Delta E'_{fd1}$	-0.691703079	0.691703079	-177.4535649	177.4535649	0	0	1.115674675	-1.115674675	94.86356021	-94.86356021
$\Delta\delta_2$	37.90353244	-37.90353244	-153.9960793	153.9960793	-36.65129366	36.65129366	111.3528801	-111.3528801	84.48034864	-84.48034864
$\Delta\omega_2$	142.8999884	-142.8999884	-40.15936207	40.15936207	89.34157643	-89.34157643	-121.1095608	121.1095608	-179.7343433	179.7343433
$\Delta E'_{q2}$	-106.2539366	106.2539366	-111.3545985	111.3545985	86.26400544	-86.26400544	-126.2789206	126.2789206	-171.7485369	171.7485369
$\Delta E'_{d2}$	161.0891917	-161.0891917	-56.95078868	56.95078868	-99.26283822	99.26283822	39.96177149	-39.96177149	30.22929859	-30.22929859
$\Delta E'_{fd2}$	-0.565619027	0.565619027	1.716002402	-1.716002402	-148.8393826	148.8393826	0	0	-74.35768474	74.35768474
$\Delta\delta_3$	-143.7594888	143.7594888	-167.3992627	167.3992627	143.0450022	-143.0450022	-74.74541896	74.74541896	-101.1385913	101.1385913
$\Delta\omega_3$	-38.76303278	38.76303278	-53.56254544	53.56254544	-90.96212772	90.96212772	52.79214015	-52.79214015	-5.353283206	5.353283206
$\Delta E'_{q3}$	-106.5558584	106.5558584	-112.8697277	112.8697277	-112.4613608	112.4613608	36.62568652	-36.62568652	-83.44924001	83.44924001
$\Delta E'_{d3}$	160.5546468	-160.5546468	142.9041968	-142.9041968	52.27888031	-52.27888031	-158.8194023	158.8194023	-154.0384998	154.0384998
$\Delta E'_{fd3}$	-1.181340628	1.181340628	0	0	11.84522284	-11.84522284	162.5601821	-162.5601821	0	0
$\Delta\delta_4$	38.97574593	-38.97574593	18.83878746	-18.83878746	-44.59627722	44.59627722	-47.00965187	47.00965187	-67.19689545	67.19689545
$\Delta\omega_4$	143.9722019	-143.9722019	132.6755047	-132.6755047	81.39659287	-81.39659287	80.52790724	-80.52790724	28.58841263	-28.58841263
$\Delta E'_{q4}$	-105.8189217	105.8189217	71.68370927	-71.68370927	70.75763835	-70.75763835	68.69803121	-68.69803121	-37.81968045	37.81968045
$\Delta E'_{d4}$	159.6060505	-159.6060505	-31.90915367	31.90915367	-106.872944	106.872944	-115.237703	115.237703	-124.7994526	124.7994526
$\Delta E'_{fd4}$	0	0	-175.4366849	175.4366849	-164.6809651	164.6809651	-165.2973116	165.2973116	47.94446734	-47.94446734

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix.

Table 49: Angle of mode shape in degree of two-area system without PSS:Part B

SV/Mode	λ_{11}	λ_{12}	λ_{13}	λ_{14}	λ_{15}	λ_{16}	λ_{17}	λ_{18}	λ_{19}	λ_{20}
$\Delta\delta_1$	66.67364837	-66.67364837	119.7972803	-119.7972803	180	-105.5962581	105.5962581	180	0	0
$\Delta\omega_1$	163.1385125	-163.1385125	-149.9794247	149.9794247	0	73.33523766	-73.33523766	0	89.99999978	-89.99999978
$\Delta E'_{q1}$	95.77033287	-95.77033287	155.2174415	-155.2174415	0	86.74090231	-86.74090231	0	179.9999967	-179.9999967
$\Delta E'_{d1}$	14.46410266	-14.46410266	73.18059169	-73.18059169	180	-92.74495296	92.74495296	180	-4.46967E-06	4.46967E-06
$\Delta E'_{fd1}$	180	-180	-147.2000083	147.2000083	180	-94.0862063	94.0862063	180	-4.00535E-06	4.00535E-06
$\Delta\delta_2$	-120.1785533	120.1785533	-49.62428659	49.62428659	0	5.977686925	-5.977686925	0	7.22906E-21	-7.22906E-21
$\Delta\omega_2$	-23.71368913	23.71368913	40.59900841	-40.59900841	180	-175.0908174	175.0908174	180	89.99999977	-89.99999977
$\Delta E'_{q2}$	75.74401094	-75.74401094	-81.89184134	81.89184134	180	-178.5888138	178.5888138	0	-4.12178E-05	4.12178E-05
$\Delta E'_{d2}$	-149.0283222	149.0283222	-32.99090168	32.99090168	0	-2.535265245	2.535265245	180	179.9999882	-179.9999882
$\Delta E'_{fd2}$	173.5102046	-173.5102046	0	0	0	0	0	180	-2.31285E-06	2.31285E-06
$\Delta\delta_3$	89.58665455	-89.58665455	-63.46175759	63.46175759	180	122.7455196	-122.7455196	180	-2.32823E-21	2.32823E-21
$\Delta\omega_3$	-173.9484813	173.9484813	26.76153742	-26.76153742	0	-58.32298472	58.32298472	0	90.00000033	-90.00000033
$\Delta E'_{q3}$	126.0774196	-126.0774196	-40.2733334	40.2733334	0	-70.45159824	70.45159824	0	179.999998	-179.999998
$\Delta E'_{d3}$	23.93303422	-23.93303422	-120.2754057	120.2754057	180	99.99632187	-99.99632187	180	-3.25195E-06	3.25195E-06
$\Delta E'_{fd3}$	-145.0450428	145.0450428	33.46326558	-33.46326558	180	107.5349793	-107.5349793	180	-6.52263E-05	6.52263E-05
$\Delta\delta_4$	-109.4365017	109.4365017	125.4665578	-125.4665578	0	-144.6461604	144.6461604	0	3.54566E-21	-3.54566E-21
$\Delta\omega_4$	-12.97163756	12.97163756	-144.3101472	144.3101472	180	34.28533532	-34.28533532	180	89.99999952	-89.99999952
$\Delta E'_{q4}$	10.71337923	-10.71337923	115.6838111	-115.6838111	180	68.43484095	-68.43484095	0	-1.1669E-07	1.1669E-07
$\Delta E'_{d4}$	-159.6416676	159.6416676	86.94760658	-86.94760658	0	-105.0987144	105.0987144	180	-6.16514E-06	6.16514E-06
$\Delta E'_{fd4}$	111.6855855	-111.6855855	177.9637633	-177.9637633	0	-111.581837	111.581837	180	179.9999954	-179.9999954

According the participation factor and kind of mode provided, $-0.017995854646413+4.617574423493768i$ and $-0.017995854646413 - 4.617574423493768i$ are related to the Inter-area mode. In this modes, $\Delta\delta_1$, $\Delta\omega_1$, $\Delta\delta_3$, and $\Delta\omega_3$ have the highest participation factor among other sate variable. Their participation factor are equal to 0.81, 0.81, 1,and 1, respectively. These two machine are in two different area. Because the Machine 3 has the larger participation factor than Machine 1, the PSS is installed at machine 3. Therefore, I Design a PSS whose inputs are the speed variations of machines 1 and machine 3 that belong to different areas while its output acts on the AVR of machine 3 in one area. Reference [3] is used to design PSS.

Designing PSS base on the Ostojic: The K-th residue of the the transfer function G_{si} is equal to: the dynamic equation is as follows:
$\Delta\dot{x} = A\Delta x + b_i \Delta U_{si}$ (69)
$\Delta W_i = C_i^t \Delta x$ (70)
The transfer function between ΔW_i and ΔU_{si} in the laplas domain is as follows:
$\frac{\Delta U_{si}(s)}{\Delta W_i(s)} = G_{si}(s)$ (71)
$R_{ki} = C_i V_k w_k^t b_i$ (73)
Proof: We assume that:
$\Delta x = V\Delta z$ (73-a)
According to the equation (69), we have:
$V\ \Delta\dot{z} = AV\Delta\Delta z + b_i \Delta U_{si}$ (73-b)
$\Delta W_i = C_i^t V \Delta z$ (73-c)
Therefore:
$\Delta\dot{z} = V^{-1} A V \Delta z + V^{-1} b_i \Delta U_{si}$ (73-d)
$\Delta W_i = C_i^t V \Delta z$ (73-e)
In addition we have:
$A = \bar{V}\bar{\Lambda}\bar{V}^{-1} = \bar{W}^{-t}\bar{\Lambda}\bar{W}^t$ (73-f)
and
$\bar{V}A\bar{V}^{-1} = \bar{\Lambda}$(73-g)

In the equations (73-d),(73-e) and (73-f), V is right eigen ector while W is the left eigenvetores.Therefore:
$\Delta\dot{z} = \bar{\Lambda}\Delta z + V^{-1} b_i \Delta U_{si} \Longrightarrow (S - \bar{\Lambda})\Delta z = V^{-1} b_i \Delta U_{si} \Longrightarrow \Delta z = \frac{V^{-1} b_i}{(S-\bar{\Lambda})}\Delta U_{si}$ (73-h)

By sing equations (73-c) and (73-h), we obtain:
$\Delta W_i = C_i^t V \frac{V^{-1} b_i}{(S-\bar{\Lambda})}\Delta U_{si} \Longrightarrow = \frac{C_i^t V V^{-1} b_i}{(S-\bar{\Lambda})}\Delta U_{si}$ (73-i)

Therefore, we obtain:

$G_{si}(s) = \frac{C_i^t V V^{-1} b_i}{(S-\bar{\Lambda})}(73-j)$

Because we have $V^{-1} = W^t$, equation 73-j will be:

$G_{si}(s) = \frac{C_i^t V W^t b_i}{(S-\bar{\Lambda})}$ (73-k)

We can rewrite equation (73-K) as follows:
$G_{si}(s) = \sum_k \frac{R_{ki}}{(S-\Lambda_k)}$ (73-l)

It is noted that $\bar{\Lambda}$ is the diagonal matrix with eigenvalues as diagonal elements. As a result, we have:

$R_{ki} = C_i V_k w_k^t b_i$ (73-m)

According to the ostojic approach, we can rewrite the transfer function Without the washouts as follows:

$\frac{\Delta U_{si}(s)}{\Delta W_i(s)} = K_i(\frac{1+a_i\tau_i s}{1+\tau_i s})^{n_i}$ (74)

$\Phi_i = \frac{\pi - arg[R_{ki}]}{n_i}$ (75)

Typically, Φ_i of one transfer function of the phase compensation block is less than equal to 1. In othr words, the maximum phase lead of a feasible PSS transfer function, Φ_i, has to be smaller than 1 rad. Thus, the number of lead-lag pairs, n_i, for a typical PSS is between 1 and 4. In a multimachine power system, the phase lag of residue $arg[R_{ki}]$ may vary from few degrees to more than 180 degree. Therefore, a sequentially tuned stabilizer , while improving the damping of on eigenvalue , may introduce a negative damping on the other frequencies of electromechanical oscillations.

$n_i\Phi_i = \pi - arg[R_{ki}]$ (76)

$a_i = \frac{1+sin(\phi_i)}{1-sin(\phi_i)}$ (77)

$\tau_i = \frac{1}{\omega_i\sqrt{a_i}}$
(78)

It is worth to mention that: K_{PSS} is in the range of 0.1 t0 50
T_1 is the lead time constant, 0.2 to 1.5 sec
T_2 is the lag time constant, 0.02 to 0.15 sec
T_3 is the lead time constant, 0.2 to 1.5 sec
T_4 is the lag time constant, 0.02 to 0.15 sec
I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix.

Table 50: Matrix a_i to design a PSS whose inputs are the speed variations of machines 1 and machine 3 that belong to different areas while its output acts on the AVR ofmachine 3 in one area. :part A

SV	$\Delta\delta_1$	$\Delta\omega_1$	$\Delta E'_{q1}$	$\Delta E'_{d1}$	$\Delta E'_{fd1}$	$\Delta\delta_2$	$\Delta\omega_2$	$\Delta E'_{q2}$	$\Delta E'_{d2}$	$\Delta E'_{fd2}$
$\Delta\delta_1$	3.089358196	1.41634019	36.45089061	2936.365983	1.477310086	3.265044085	1.699411343	3.851057852	6.763180806	27.29013832
$\Delta\omega_1$	1.41634019	3.089358196	2936.365993	36.45089059	3.265044085	1.477310086	3.851057852	1.699411342	27.29013831	6.763180805
$\Delta E'_{q1}$	12.93187767	307.0463157	2.008254852	1.189679574	131.410626	10.3566308	67.73841424	8.242237435	5.404644717	1.960909923
$\Delta E'_{d1}$	307.0463173	12.93187769	1.189679575	2.008254851	10.35663079	131.4106257	8.242237433	67.73841415	1.960909923	5.404644715
$\Delta E'_{fd1}$	2.635638351	1.613767415	59.10838553	4826.087449	1.70295682	2.797657519	1.964225221	3.27648873	8.2736232	19.93694151
$\Delta\delta_2$	1.613767415	2.635638351	4826.087457	59.10838553	2.797657519	1.70295682	3.27648873	1.964225221	19.93694151	8.2736232
$\Delta\omega_2$	5.020709994	1.039953453	13.96432383	74.93608658	1.001004383	5.329750818	1.145163294	6.470564343	3.986648196	116.4441087
$\Delta E'_{q2}$	1.039953454	5.020709991	74.93608639	13.96432384	5.329750822	1.001004383	6.470564347	1.145163294	116.4441089	3.986648197
$\Delta E'_{d2}$	3.918165378	29.0469754	60.5753649	3.483546354	3.666891096	2.451243813	4.704767049	2.108725029	32.70448734	1.926965548
$\Delta E'_{fd2}$	29.04697488	3.918165202	3.483546391	60.57537742	2.451243783	3.666891249	2.108725003	4.704767238	1.92696556	32.70448942
$\Delta\delta_3$	1.10911913	8.332593624	41.71118333	7.75899106	7.191794773	1.373636253	9.007639091	1.196346598	271.1644938	2.946353175
$\Delta\omega_3$	8.332593625	1.10911913	7.758991058	41.7111833	1.373636253	7.191794776	1.196346599	9.007639095	2.946353175	271.1644937
$\Delta E'_{q3}$	2.224411878	1.917390377	15.4190424	83.80293378	8.994023248	1.511477162	11.09437343	1.710309273	1429.531325	8.099985096
$\Delta E'_{d3}$	1.917390378	2.224411874	83.80293363	15.41904243	1.511477162	8.994023235	1.710309272	11.09437341	8.099985086	1429.531356
$\Delta E'_{fd3}$	2.966096785	2.966096785	57.18884556	57.18884557	1.845654015	1.845654015	2.147013258	2.147013258	9.555078307	9.555078307
$\Delta\delta_4$	3.235205726	426.5162473	5.835616099	2.046123978	67.13230439	4.218402504	132.5137094	3.540195967	22.50213852	1.096915185
$\Delta\omega_4$	426.5162412	3.23520573	2.04612398	5.835616092	4.218402501	67.13230454	3.540195964	132.5137098	1.096915184	22.50213849
$\Delta E'_{q4}$	2.797836262	2.797836263	66.59715692	66.59715705	1.894759466	1.894759468	2.20334298	2.203342982	9.915805648	9.915805645
$\Delta E'_{d4}$	2.569473318	18.63633697	37.02092988	1.941434485	1.196859539	496.1382451	1.335861144	191.4740786	11.59050298	3.355416925
$\Delta E'_{fd4}$	18.63633697	2.569473318	1.941434485	37.02092988	496.138245	1.196859539	191.4740786	1.335861144	3.355416925	11.59050298

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix.

Table 51: Matrix α_i to design PSS whose inputs are the speed variations of machines 1 and machine 3 that belong to different areas while its output acts on the AVR ofmachine 3 in one area.:part B

SV	$\Delta\delta_3$	$\Delta\omega_3$	$\Delta E'_{q3}$	$\Delta E'_{d3}$	$\Delta E'_{fd3}$	$\Delta\delta_4$	$\Delta\omega_4$	$\Delta E'_{q4}$	$\Delta E'_{d4}$	$\Delta E'_{fd4}$
$\Delta\delta_1$	6.133152717	22.9529502	2.013190856	4.590070199	1.443280265	6.664374398	2.638446516	1.442063709	126.6322955	126.6322962
$\Delta\omega_1$	22.95295019	6.133152716	4.590070199	2.013190856	1.443280265	2.638446516	6.664374399	1.442063709	126.6322962	126.6322955
$\Delta E'_{q1}$	5.950959318	2.109678108	40.01963314	6.622230013	86.50443796	4.711215388	20.46989865	86.52741373	1.538810335	1.538810336
$\Delta E'_{d1}$	2.109678107	5.950959316	6.622230012	40.01963312	86.50443797	20.46989865	4.711215389	86.52741374	1.538810336	1.538810335
$\Delta E'_{fd1}$	7.444418721	17.1347857	2.309602622	3.917445551	1.259035666	5.493733646	3.076596733	1.258880995	306.2774309	306.2774333
$\Delta\delta_2$	17.1347857	7.444418721	3.917445551	2.309602622	1.259035666	3.076596733	5.493733646	1.258880995	306.2774333	306.2774309
$\Delta\omega_2$	3.672104383	83.46921711	1.347752557	7.99835761	2.162201806	12.79465156	1.739063403	2.160294096	28.36070381	28.3607039
$\Delta E'_{q2}$	83.46921721	3.672104384	7.998357612	1.347752557	2.162201807	1.739063403	12.79465156	2.160294097	28.3607039	28.36070381
$\Delta E'_{d2}$	32.82927656	1.832996064	7.30607358	1.23789921	4.153414054	1.000538377	55.67763915	3.998697033	14.53528735	14.53528646
$\Delta E'_{fd2}$	1.832996074	32.82927843	1.237899205	7.306073709	4.153413996	55.67763689	1.000538366	3.998696984	14.53528645	14.53528733
$\Delta\delta_3$	175.5849319	2.743148605	10.21300952	1.076082075	2.885486689	1.3362073	22.47346519	2.863126752	17.79007527	17.79007507
$\Delta\omega_3$	2.743148605	175.5849318	1.076082075	10.21300952	2.88548669	22.4734652	1.3362073	2.863126753	17.79007507	17.79007527
$\Delta E'_{q3}$	5579961.491	6.86163826	14.29363517	1.904043777	2.106792202	1.773317171	12.1226262	2.108737136	28.34031088	28.34031354
$\Delta E'_{d3}$	6.861638254	5579968.325	1.904043776	14.29363515	2.106792201	12.12262619	1.773317172	2.108737136	28.34031354	28.34031088
$\Delta E'_{fd3}$	8.722732824	8.722732824	2.703266213	2.703266213	1	4.068278165	4.068278165	1	65535	65535
$\Delta\delta_4$	26.2396343	1.16182977	1092.714363	3.004542295	11.42266218	2.027390211	705.4290838	11.4962153	3.225328974	3.225328949
$\Delta\omega_4$	1.161829769	26.23963427	3.004542294	1092.714368	11.42266218	705.4290836	2.027390211	11.4962153	3.225328949	3.225328974
$\Delta E'_{q4}$	9.011942815	9.011942813	2.743318492	2.743318491	1	4.066807258	4.066807253	1	65535	65535
$\Delta E'_{d4}$	10.13222715	3.671522552	3.322802542	11.76874266	5.828427124	186.0997472	1.341438456	5.828427124	5.828431961	5.828422323
$\Delta E'_{fd4}$	3.671522552	10.13222715	11.76874266	3.322802542	5.828427124	1.341438456	186.0997472	5.828427124	5.828422323	5.828431961

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix.

Table 52: Matrix τ_i to design a PSS whose inputs are the speed variations of machines 1 and machine 3 that belong to different areas while its output acts on the AVR ofmachine 3 in one area.:part A

SV	$\Delta\delta_1$	$\Delta\omega_1$	$\Delta E'_{q1}$	$\Delta E'_{d1}$	$\Delta E'_{fd1}$	$\Delta\delta_2$	$\Delta\omega_2$	$\Delta E'_{q2}$	$\Delta E'_{d2}$	$\Delta E'_{fd2}$
$\Delta\delta_1$	0.033727206	0.049811624	0.009818845	0.00109398	0.048772909	0.032807261	0.045474218	0.030208174	0.022794968	0.011347799
$\Delta\omega_1$	0.049811624	0.033727206	0.00109398	0.009818845	0.032807261	0.048772909	0.030208174	0.045474218	0.011347799	0.022794968
$\Delta E'_{q1}$	0.024015233	0.00492851	0.060940805	0.079177648	0.007533597	0.026835417	0.01049301	0.030081192	0.037147879	0.061672106
$\Delta E'_{d1}$	0.00492851	0.024015233	0.079177648	0.060940805	0.026835417	0.007533597	0.030081192	0.01049301	0.061672106	0.037147879
$\Delta E'_{fd1}$	0.084645872	0.108175328	0.017874098	0.001978113	0.105304486	0.082158295	0.098051231	0.075917929	0.047775014	0.030776514
$\Delta\delta_2$	0.108175328	0.084645872	0.001978113	0.017874098	0.082158295	0.105304486	0.075917929	0.098051231	0.030776514	0.047775014
$\Delta\omega_2$	0.064086143	0.140812028	0.038427024	0.016588274	0.143525383	0.062200411	0.134187788	0.056451518	0.071918852	0.013307239
$\Delta E'_{q2}$	0.140812028	0.064086143	0.016588274	0.038427024	0.062200411	0.143525383	0.056451518	0.134187788	0.013307239	0.071918852
$\Delta E'_{d2}$	0.067237947	0.024694792	0.017100461	0.071309112	0.069503523	0.085008543	0.0613602	0.091652824	0.023272988	0.095877975
$\Delta E'_{fd2}$	0.024694792	0.067237949	0.071309112	0.017100459	0.085008543	0.069503522	0.091652825	0.061360199	0.095877975	0.023272988
$\Delta\delta_3$	0.138849334	0.050657388	0.022641575	0.052496491	0.054527333	0.124766222	0.048722258	0.133691682	0.008880072	0.085190368
$\Delta\omega_3$	0.050657388	0.138849334	0.052496491	0.022641575	0.124766222	0.054527333	0.133691682	0.048722258	0.085190368	0.008880072
$\Delta E'_{q3}$	0.145203971	0.156397867	0.055151508	0.023656827	0.072211953	0.176151076	0.065018165	0.165595592	0.005727819	0.076092874
$\Delta E'_{d3}$	0.156397866	0.145203971	0.023656827	0.055151508	0.176151076	0.072211953	0.165595592	0.065018165	0.076092874	0.005727819
$\Delta E'_{fd3}$	65535	65535	65535	65535	65535	65535	65535	65535	65535	65535
$\Delta\delta_4$	6.586801956	0.573664225	4.904358403	8.282464531	1.445971655	5.768349274	1.029189037	6.2966841	2.497546946	11.31199053
$\Delta\omega_4$	0.573664229	6.586801952	8.282464527	4.904358406	5.768349276	1.445971653	6.296684102	1.029189036	11.31199053	2.497546947
$\Delta E'_{q4}$	65535	65535	65535	65535	65535	65535	65535	65535	65535	65535
$\Delta E'_{d4}$	2628583.784	976030.9811	692500.7035	3024005.04	3851432.472	189165.759	3645551.364	304501.1144	1237635.446	2300226.673
$\Delta E'_{fd4}$	976030.9811	2628583.784	3024005.04	692500.7035	189165.759	3851432.472	304501.1144	3645551.364	2300226.673	1237635.446

I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix.

Table 53: Matrix τ_i to design a PSS whose inputs are the speed variations of machines 1 and machine 3 that belong to different areas while its output acts on the AVR ofmachine 3 in one area.:part B

SV	$\Delta\delta_3$	$\Delta\omega_3$	$\Delta E'_{q3}$	$\Delta E'_{d3}$	$\Delta E'_{fd3}$	$\Delta\delta_4$	$\Delta\omega_4$	$\Delta E'_{q4}$	$\Delta E'_{d4}$	$\Delta E'_{fd4}$
$\Delta\delta_1$	0.023937159	0.012373576	0.041780345	0.027669724	0.049344546	0.022963327	0.036495586	0.049365356	0.005267957	0.005267957
$\Delta\omega_1$	0.012373576	0.023937159	0.027669724	0.041780345	0.049344546	0.036495586	0.022963327	0.049365356	0.005267957	0.005267957
$\Delta E'_{q1}$	0.035401699	0.059457891	0.013651521	0.033559499	0.009285353	0.039787909	0.01908797	0.00928412	0.069618563	0.069618563
$\Delta E'_{d1}$	0.059457891	0.035401699	0.033559499	0.013651521	0.009285353	0.01908797	0.039787909	0.00928412	0.069618563	0.069618563
$\Delta E'_{fd1}$	0.050365517	0.033197804	0.090423239	0.069430007	0.122469988	0.058629325	0.078345382	0.122477512	0.007852197	0.007852197
$\Delta\delta_2$	0.033197804	0.050365517	0.069430007	0.090423239	0.122469988	0.078345382	0.058629325	0.122477512	0.007852197	0.007852197
$\Delta\omega_2$	0.074935774	0.015717502	0.123692003	0.050774575	0.097655924	0.040145093	0.10889025	0.097699033	0.026964241	0.026964241
$\Delta E'_{q2}$	0.015717502	0.074935774	0.050774575	0.123692003	0.097655924	0.10889025	0.040145093	0.097699033	0.026964241	0.026964241
$\Delta E'_{d2}$	0.023228714	0.098304876	0.04923952	0.119622585	0.065306019	0.133057375	0.017836738	0.066557435	0.034909531	0.034909532
$\Delta E'_{fd2}$	0.098304876	0.023228714	0.119622586	0.049239519	0.06530602	0.017836738	0.133057376	0.066557435	0.034909532	0.034909531
$\Delta\delta_3$	0.01103543	0.088289332	0.045756841	0.140964644	0.086084183	0.126501588	0.030845931	0.086419671	0.034669215	0.034669215
$\Delta\omega_3$	0.088289332	0.01103543	0.140964644	0.045756841	0.086084183	0.030845931	0.126501588	0.086419671	0.034669215	0.034669215
$\Delta E'_{q3}$	9.16791E-05	0.0826746	0.057281548	0.156945053	0.149202206	0.162627088	0.062199621	0.149133384	0.040680267	0.040680265
$\Delta E'_{d3}$	0.082674616	9.16791E-05	0.156945053	0.057281549	0.149202206	0.062199621	0.162627088	0.149133384	0.040680265	0.040680267
$\Delta E'_{fd3}$	65535	65535	65535	65535	65535	65535	65535	65535		
$\Delta\delta_4$	2.312845743	10.99143275	0.358403491	6.834966591	3.505434775	8.320642916	0.446065748	3.494202843	6.596879453	6.596879479
$\Delta\omega_4$	10.99143275	2.312845744	6.834966593	0.35840349	3.505434775	0.446065748	8.320642916	3.494202843	6.596879479	6.596879453
$\Delta E'_{q4}$	65535	65535	65535	65535	65535	65535	65535	65535		
$\Delta E'_{d4}$	1323705.648	2198977.473	2311487.842	1228227.585	1745292.39	308866.6295	3637964.898	1745292.39	1745291.666	1745293.109
$\Delta E'_{fd4}$	2198977.473	1323705.648	1228227.585	2311487.842	1745292.39	3637964.898	308866.6295	1745292.39	1745293.109	1745291.666

According to the table the amount of a_i related to the machine 3 is equal to 6.86163826122444 while the amount of τ_i for this machine is equal to 0.0826746160616562. Therefore:
$T_1 = T_3 = a_i \tau_i = 0.569$
$T_2 = T_4 = \tau_i = 0.083$
As it is clear, all amount of T_i are the discussed range.

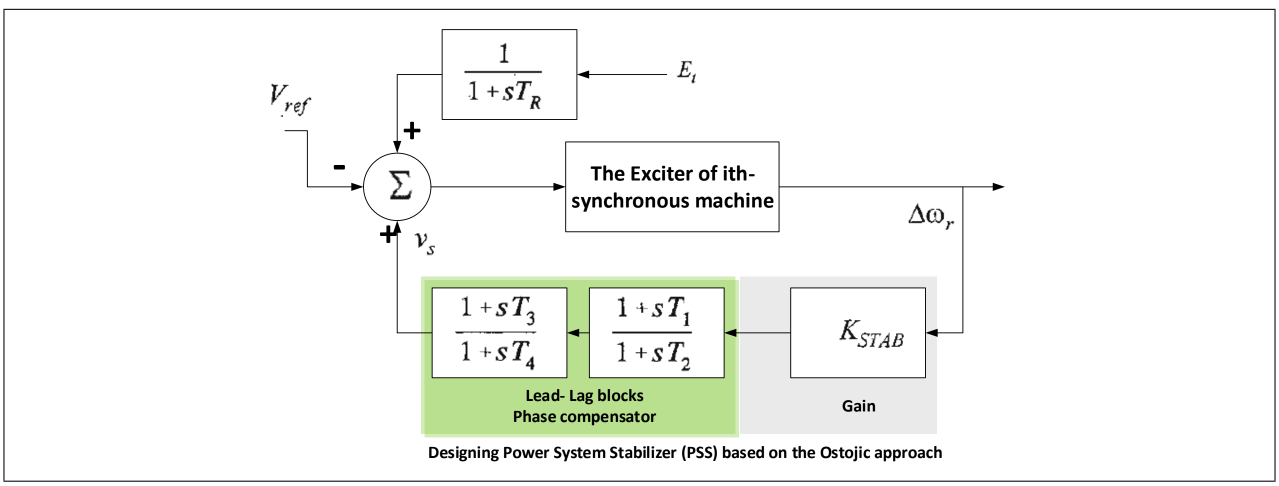

Figure 8: Designing Power System Stabilizer without washout

We have : $T_1 = T_3$ and $T_2 = T_4$
A a result:
$\frac{\Delta V_{si}}{\Delta w_{ri}} = K_{PSS}(\frac{1+sT_1}{1+sT_2})^2$ (79)
$\frac{\Delta V_{si}}{\Delta w_{ri}} = K_{PSS}\frac{T_1^2 s^2+2T_1 s+1}{T_2^2 s^2+2T_2 s+1}$ (80)

We assume that:
$X_1 = \Delta w_{ri} \frac{1}{T_2^2 s^2 + 2T_2 s + 1}$ (81)

According to this assumption, we have:

$T_2^2 s^2 X_1 + 2T_2 s X_1 + X_1 = \Delta w_{ri}$ (82)

In continue, we assume that:

$\frac{dx_1}{dt} = X_2 \Longrightarrow X_2(s) = sX_1(s)$ (83)

by using equation (59) and definitions declare in equations (82) and (83) , we obtain will be:

$T_2^2 \dot{X_2} + 2T_2 X_2 + X_1 = \Delta w_{ri} \Longrightarrow \dot{X_2} = -\frac{2T_2}{T_2^2} X_2 - \frac{1}{T_2^2} X_1 + \frac{1}{T_2^2} \Delta w_{ri}$ (84)

Therefore, we have the following space-state for PSS:

$$\begin{bmatrix} \dot{X_1} \\ \dot{X_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{T_2^2} & -\frac{2T_2}{T_2^2} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ +\frac{1}{T_2^2} \end{bmatrix} \Delta w_{ri} (85)$$

From equations (80) and (81), we have:
$\Delta V_{si} = K_{PSS}(T_1^2 s^2 + 2T_1 s + 1) \frac{\Delta w_{ri}}{T_2^2 s^2 + 2T_2 s + 1}$
$= K_{PSS}(T_1^2 s^2 + 2T_1 s + 1) X_1 = K_{PSS}(T_1^2 X_2 + 2T_1 X_2 + X_1)$ (86)

By using Equation (85), we finally obtain:

$\Delta V_{si} = K_{PSS}(T_1^2[-\frac{2T_2}{T_2^2} X_2 - \frac{1}{T_2^2} X_1 + \frac{1}{T_2^2} \Delta w_{ri}] + 2T_1 X_2 + X_1)$
$= [K_{PSS} - K_{PSS} \frac{T_1^2}{T_2^2}] X_1 + [2K_{PSS} T_1 - 2K_{PSS} T2 \frac{T_1^2}{T_2^2}] X_2 + K_{PSS} \frac{T_1^2}{T_2^2} \Delta w_{ri}$ (86)

$$\Delta V_{si} = \begin{bmatrix} K_{PSS} - K_{PSS} \frac{T_1^2}{T_2^2} & 2K_{PSS} T_1 - 2K_{PSS} T2 \frac{T_1^2}{T_2^2} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + K_{PSS} \frac{T_1^2}{T_2^2} \Delta w_{ri} (87)$$

Accrediting to the PSS and Figure 2 and Equation (66) , we have:
$\frac{d\Delta E_{fd}}{dt} = -\frac{d\Delta E_{fd}}{T_A} + \frac{K_A}{T_A}(\Delta V_{ref} - \Delta V + \Delta V_{si})$
$= -\frac{d\Delta E_{fd}}{T_A} + \frac{K_A}{T_A}(\Delta V_{ref} - \Delta V + [K_{PSS} - K_{PSS} \frac{T_1^2}{T_2^2}] X_1 + [2K_{PSS} T_1 - 2K_{PSS} T2 \frac{T_1^2}{T_2^2}] X_2 + K_{PSS} \frac{T_1^2}{T_2^2} \Delta w_{ri})$
(88)

As a result, according to the equations (85) and (88) the matrix A_1 for the Machine with PSS (Bus 12) will be:

A_{SYS} will be:

$$\begin{bmatrix} \Delta\dot{\delta}_i \\ \Delta\dot{\omega}_i \\ \Delta\dot{E}'_{qi} \\ \Delta\dot{E}'_{di} \\ \Delta\dot{E}'_{fdi} \\ \Delta\dot{X}_1 \\ \Delta\dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{D_i}{M_i} & -\frac{I_{qio}}{M_i} & -\frac{I_{dio}}{M_i} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{T'_{doi}} & 0 & \frac{1}{T'_{doi}} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{T'_{qoi}} & 0 & 0 & 0 \\ 0 & \frac{K_A}{T_A}K_{PSS}\frac{T_1^2}{T_2^2}\omega_i & 0 & 0 & \frac{-1}{T_A} & A & B \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & \frac{1}{T_2^2\omega_i} & 0 & 0 & 0 & \frac{-1}{T_2^2} & \frac{-2}{T_2} \end{bmatrix} \begin{bmatrix} \Delta\delta_i \\ \Delta\omega_i \\ \Delta E'_{qi} \\ \Delta E'_{di} \\ \Delta E'_{fdi} \\ \Delta X_1 \\ \Delta X_2 \end{bmatrix}$$

Where:
$A = \frac{K_A}{T_A}[K_{PSS} - K_{PSS}\frac{T_1^2}{T_2^2}]$
$B = \frac{K_A}{T_A}[2K_{PSS}T_1 - 2K_{PSS}T2\frac{T_1^2}{T_2^2}]$
Frequency o f eigenvalues(EV) according to diiferent amount of K_{PSS} and wothoit PSS is provide in the table 54.Henece, I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix.

Table 54: Frequency o f eigenvalues(EV) according to diiferent amount of K_{PSS} and wothoit PSS

EV	No PSS	K=0.1	K=0.2	K=0.3	K=0.4	K=0.5	K=0.6	K=0.6318	K=0.7	K=1	K=20
1	2.684761039560422	2.686935575445946	2.689198037919239	2.691540965028546	2.693955384170947	2.696430921002590	2.698955955858272	2.699767263321526	2.701517818649311	0	0
2	2.684761039560422	2.686935575445946	2.689198037919239	2.691540965028546	2.693955384170947	2.696430921002590	2.698955955858272	2.699767263321526	2.701517818649311	2.709286657986249	4.129810908329747
3	1.842903235229960	0	0	0	0	0	0	0	0	2.709286657986249	4.129810908329747
4	1.842903235229960	1.833771847065767	1.822820714282446	1.812668599730314	1.811986796662932	1.827937126513637	1.854384114484332	1.863737265829971	1.884358361777466	1.973652447565900	2.667500975928165
5	1.158167674467106	1.833771847065767	1.822820714282446	1.812668599730314	1.811986796662932	1.827937126513637	1.854384114484332	1.863737265829971	1.884358361777466	1.973652447565900	2.667500975928165
6	1.158167674467106	0	1.344261257194341	1.423890816602209	1.488713903082572	1.531310031071369	1.558146060296468	1.564733725556376	1.576824267480306	1.612060628404869	1.729204315175189
7	1.108341071774107	1.258565206287523	1.344261257194341	1.423890816602209	1.488713903082572	1.531310031071369	1.558146060296468	1.564733725556376	1.576824267480306	1.612060628404869	1.729204315175189
8	1.108341071774107	1.258565206287523	0	1.080836046797962	1.079666867831783	1.078945834209954	1.078463508450266	1.078342603178848	1.174919203835555	1.169401767770982	1.055841784500865
9	1.195815847215360	1.087821184431476	1.082977353544322	1.080836046797962	1.079666867831783	1.078945834209954	1.078463508450266	1.078342603178848	1.174919203835555	1.169401767770982	1.055841784500865
10	1.195815847215360	1.087821184431476	1.082977353544322	1.189466735302713	1.185013826816166	1.181002048136676	1.177649091429976	1.176718118685217	0.998846504540615	1.003250954838572	1.156083206201388
11	1.088396743169725	1.196026296058607	1.193599277559881	1.189466735302713	1.185013826816166	1.181002048136676	1.177649091429976	1.176718118685217	0.998846504540615	1.003250954838572	1.156083206201388
12	1.088396743169725	1.196026296058607	1.193599277559881	1.014784028543331	1.003734818091550	0.999313601665686	0.998305923395776	0.998364671595612	1.078121421694706	1.077519804124203	1.076356607395988
13	0.734909794593742	1.060808931240217	1.034637545279718	1.014784028543331	1.003734818091550	0.999313601665686	0.998305923395776	0.998364671595612	1.078121421694706	1.077519804124203	1.076356607395988
14	0.734909794593742	1.060808931240217	1.034637545279718	0	0	0.699936159401031	0.680314122672886	0.674117082151958	0.661108073514551	0.610166412123615	0
15	0	0.738126565655305	0.738129270703520	0.731918825254419	0.718250497967338	0.699936159401031	0.680314122672886	0.674117082151958	0.661108073514551	0.610166412123615	0.156403963317799
16	0.013433666873753	0.738126565655305	0.738129270703520	0.731918825254419	0.718250497967338	0	0	0	0	0	0.156403963317799
17	0.013433666873753	0	0	0	0	0	0	0	0	0	0.211644205957712
18	0	0	0	0	0	0	0	0	0	0	0.211644205957712
19	0.000000037772545	0	0	0	0	0	0	0	0	0	0.129117355495316
20	0.000000037772545	0	0	0	0	0	0	0	0	0	0.129117355495316
21		0	0	0	0	0	0	0	0	0	0
22		0	0	0	0	0	0	0	0	0	0

The Damp ration o f eigenvalues(EV) according to diiferent amount of K_{PSS} and wothoit PSS is provide in the table 55. Henece,I run the MATLAB code for introduced equations of the two-area system. The Matlab code is provided in the appendix.

Table 55: Damp ratio o f eigenvalues(EV) according to diiferent amount of K_{PSS} and wothoit PSS

EV	NO PSS	K=0.1	K=0.2	K=0.3	K=0.4	K=0.5	K=0.6	K=0.6318	K=0.7	K=1	K=20
1	0.258759297277793	0.258725667408021	0.258646753853820	0.258519633272786	0.258341761356119	0.258111040882301	0.257825875571112	0.257723596783027	0.257485207181365	1.000000000000000	1.000000000000000
2	0.258759297277793	0.258725667408021	0.258646753853820	0.258519633272786	0.258341761356119	0.258111040882301	0.257825875571112	0.257723596783027	0.257485207181365	0.256127952950811	-0.193083425590341
3	0.404131553308683	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	0.256127952950811	-0.193083425590341
4	0.404131553308683	0.394308923003354	0.381794438832853	0.364251114340697	0.339245530125830	0.310321446373609	0.283521488345370	0.275783554570461	0.260430139776907	0.208202469069793	0.235333776961094
5	0.587684573346446	0.394308923003354	0.381794438832853	0.364251114340697	0.339245530125830	0.310321446373609	0.283521488345370	0.275783554570461	0.260430139776907	0.208202469069793	0.235333776961094
6	0.587684573346446	1.000000000000000	0.521122320240411	0.502767392942120	0.497132612848687	0.499846081812795	0.504177123685715	0.505438365252637	0.507830217982921	0.514223997073489	0.514176274102008
7	0.609281365148132	0.550066615277131	0.521122320240411	0.502767392942120	0.497132612848687	0.499846081812795	0.504177123685715	0.505438365252637	0.507830217982921	0.514223997073489	0.514176274102008
8	0.609281365148132	0.550066615277131	1.000000000000000	0.610038784501189	0.609687724767139	0.609406404785331	0.609180566801236	0.609118140104925	0.150703578893727	0.152964935673263	-0.125389145801004
9	0.100801184211647	0.610791751905059	0.610456626247289	0.610038784501189	0.609687724767139	0.609406404785331	0.609180566801236	0.609118140104925	0.150703578893727	0.152964935673263	-0.125389145801004
10	0.100801184211647	0.610791751905059	0.610456626247289	0.138010777292745	0.143434265561666	0.146932735010036	0.149200815297101	0.149745100434709	-0.008846634571168	-0.036769869832794	0.154897527938901
11	0.112593898285070	0.117842049030237	0.129787896220823	0.138010777292745	0.143434265561666	0.146932735010036	0.149200815297101	0.149745100434709	-0.008846634571168	-0.036769869832794	0.154897527938901
12	0.112593898285070	0.117842049030237	0.129787896220823	0.064521366383794	0.041606011786289	0.021226966775008	0.004591472710508	0.000009634538301	0.608996941281328	0.608611108103166	0.607423340894699
13	0.003897223159129	0.100804757474145	0.085464994532887	0.064521366383794	0.041606011786289	0.021226966775008	0.004591472710508	0.000009634538301	0.608996941281328	0.608611108103166	0.607423340894699
14	0.003897223159129	0.100804757474145	0.085464994532887	1.000000000000000	1.000000000000000	0.193716884995900	0.223611818421748	0.232060258259143	0.248744225031598	0.305875357898879	1.000000000000000
15	1.000000000000000	0.037228787316403	0.075264537045117	0.117054573869628	0.157982093941038	0.193716884995900	0.223611818421748	0.232060258259143	0.248744225031598	0.305875357898879	0.256456293049143
16	0.999826113970634	0.037228787316403	0.075264537045117	0.117054573869628	0.157982093941038	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	0.256456293049143
17	0.999826113970634	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	0.925391281681181
18	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	0.925391281681181
19	0.000000002853857	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	0.989044220275504
20	0.000000002853857	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	0.989044220275504
21	-	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000
22	-	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000

Figure 9 show the eigenvalues of the test power system in two different cases:1)Without Power System Stabilizer and 2)With Power System Stabilizer. The power system stablizer chractristic is as follows: T1=T3==0.569, T2=T4=i=0.083, and K=0.5. According to the figure, designing a PSS whose inputs are the speed variations of machines 1 and machine 3that belong to different areas while its output acts on the AVR ofmachine 3 in one area damp out the osilation. In fact, Eigenvalues of the system are moved to the left side of plane when we consider PSS.

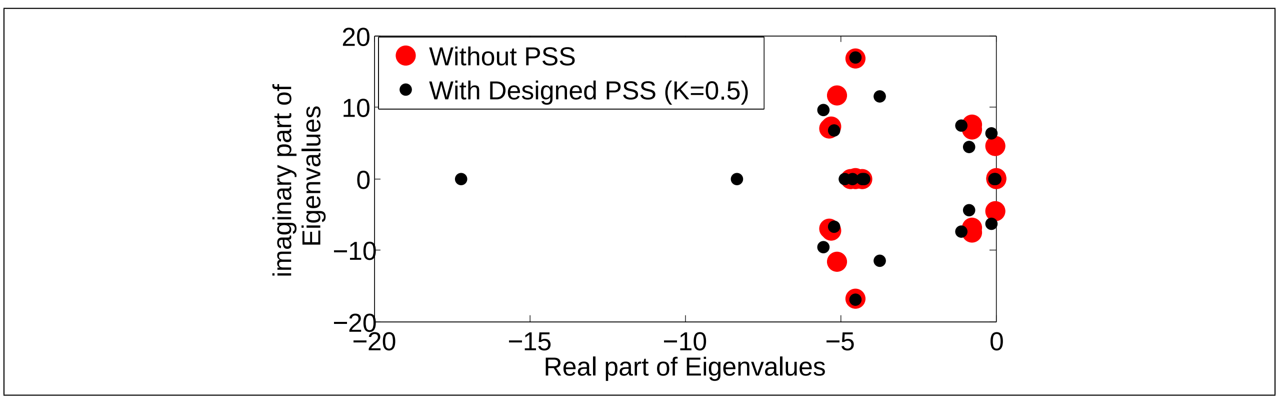

Figure 9: Eigenvalues of the test power system in two different cases:1)Without Power System Stabilizer and 2)With Power System Stabilizer ($K_{PSS} = 0.5$)

Figure 10 show the eigenvalues of the test power system in different cases:1)Without Power System Stabilizer and 2)With Power System Stabilizer with different amount of K_{PSS}. The power system stabilizer characteristics for all PSS is as follows: T1=T3=0.569, and T2=T4=0.083. K_{PSS} includes 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.6318, 0.7, 1, 20. For amount of K_{PSS} =0.1, 0.2, 0.3, 0.4, 0.5, 0.6, the stability of the system is increased compared to the system without the PSS. Hence, designing a PSS with such K_{PSS} whose inputs are the speed variations of machines 1 and machine 3that belong to different areas while its output acts on the AVR ofmachine 3 in one area damp out the osilation. In fact, Eigenvalues of the system are moved to the left side of plane when we consider PSS. On the other hand, the amount of $K_{PSS} = 0.6318$ is assumed to be critical. Because, by increasing the K_{PSS}, the system will be unstable.

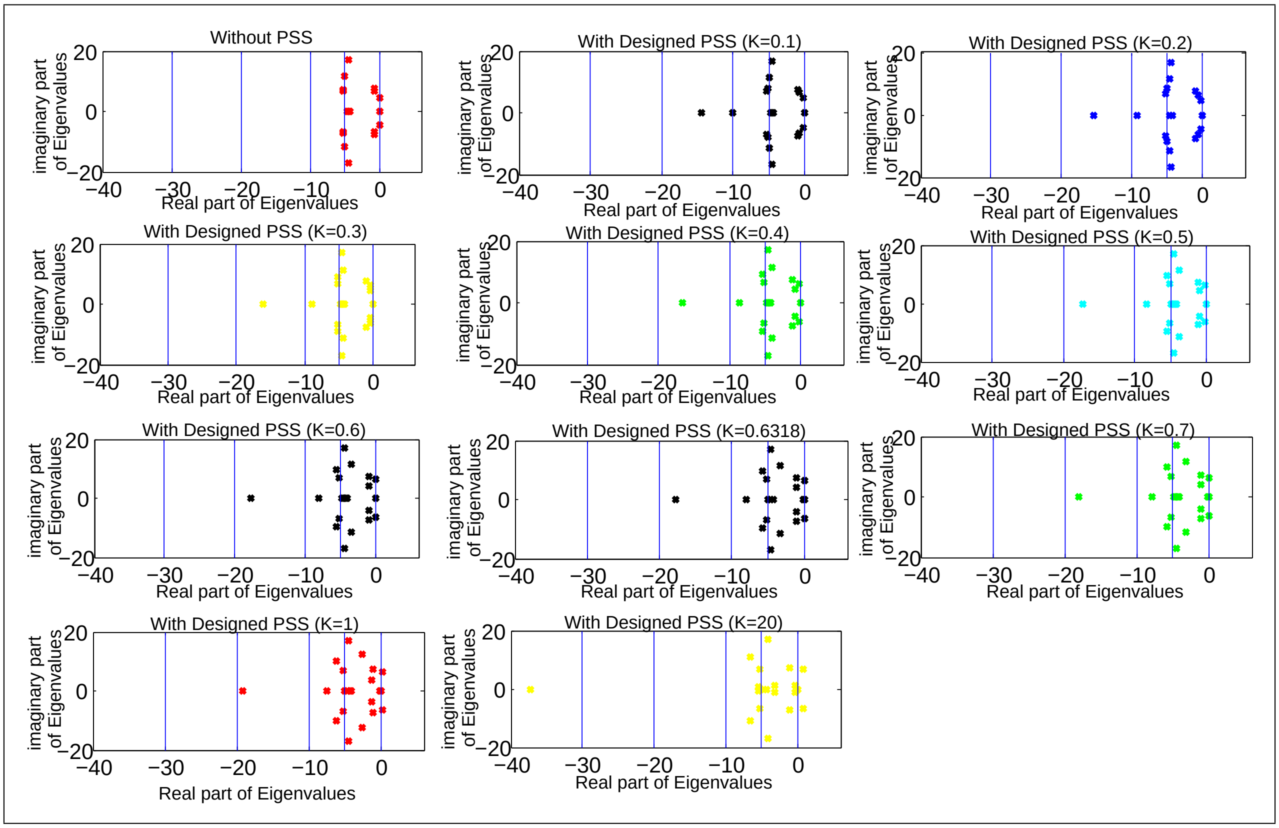

Figure 10: Eigenvalues of the test power system in two different cases:1)Without Power System Stabilizer and 2)With Power System Stabilizer with different amount of K_{PSS}. K_{PSS} includes 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.6318, 0.7, 1, 20.

Table 56: eigenvalues of the test system in different cases: 1)Without PSS 2)With Local PSS 3)with designated inter area PSS based on the Ostojic approach for different amount of K_{PSS}

EV	Without PSS	Local PSS	Inter area PSS:K=0.1	K=0.6318 (Critical K)	Inter area PSS: K=20
1	-4.51887+16.8688511i	-47.74929 + 0.000000i	-4.5219016.8825141i	-4.52464 +16.963138i	-37.17533 + 0.000000i
2	-4.51887-16.8688511i	-4.619949+16.870741i	-4.5219-16.882514i	-4.52464 -16.963138i	5.106288 +25.948367i
3	-5.11594+11.5793025i	-4.619949-16.870741i	-14.4048+ 0.000000i	-17.82263 + 0.000000i	5.106288 -25.948367i
4	-5.11594-11.5793025i	-4.999250+11.517418i	-4.9437+11.521928i	-3.35977 +11.710206i	-4.058266 +16.760402i
5	-5.28565+ 7.2769821i	-4.999250-11.517418i	-4.9437-11.521928i	-3.35977 -11.710206i	-4.058266 -16.760402i
6	-5.28565- 7.2769821i	-0.961042+ 8.630452i	-10.0593+ 0.000000i	-5.75899 + 9.831511i	-6.513443 +10.864911i
7	-5.35085+ 6.9639123i	-0.961042- 8.630452i	-5.2086+ 7.907798i	-5.75899 - 9.831511i	-6.513443 -10.864911i
8	-5.35085- 6.9639123i	-0.756584+ 7.480452i	-5.2086- 7.907798i	-5.20381 + 6.775426i	0.838455 + 6.634049i
9	-0.76125+ 7.5135325i	-0.756584- 7.480452i	-5.2725+ 6.834982i	-5.20381 - 6.775426i	0.838455 - 6.634049i
10	-0.7612 - 7.5135329i	-5.352726+ 7.090452i	-5.2725- 6.834982i	-1.11977 + 7.393537i	-1.138903 + 7.263885i
11	-0.7749 + 6.8385989i	-5.352726- 7.090452i	-0.8917+ 7.514854i	-1.11977 - 7.393537i	-1.138903 - 7.263885i
12	-0.7749 - 6.8385989i	-5.257008+ 5.347042i	-0.8917- 7.514854i	-0.00006 + 6.272910i	-5.171305 + 6.762948i
13	-0.0179 + 4.6175748i	-5.257008- 5.347042i	-0.6753+ 6.665259i	-0.00006 - 6.272910i	-5.171305 - 6.762948i
14	-0.0179 - 4.6175748i	-0.086004+ 4.662053i	-0.6753- 6.665259i	-1.01050 + 4.235602i	-0.000000 + 0.000000i
15	-4.6742 + 0.0000000i	-0.086004- 4.662053i	-0.1727+ 4.637785i	-1.01050 - 4.235602i	-0.260743 + 0.982715i
16	-4.5255 + 0.0844069i	-4.513935+ 0.000000i	-0.1727- 4.637785i	-8.03348 + 0.000000i	-0.260743 - 0.982715i
17	-4.5255 - 0.0844069i	-4.436119+ 0.000000i	-0.0000+ 0.000000i	-0.00000 + 0.000000i	-3.246813 + 1.329799i
18	-4.3033 + 0.0000000i	-4.401135+ 0.000000i	-0.0078+ 0.000000i	-0.04980 + 0.000000i	-3.246813 - 1.329799i
19	-0.0000 + 0.0000002i	-4.155296+ 0.000000i	-4.7110+ 0.000000i	-4.89222 + 0.000000i	-5.435460 + 0.811268i
20	-0.0000 - 0.0000002i	-0.205005+ 0.000000i	-4.5779+ 0.000000i	-4.62127 + 0.000000i	-5.435460 - 0.811268i
21	-	-0.0579+0.00000i	-4.30184 + 0.000000i	-4.186424 + 0.000000i	-4.62836+ 0.00000i
22	-	0.000000+0.00000i	-4.43450 + 0.000000i	-4.309912 + 0.000000i	-4.30680+ 0.00000i
23	-	0.00000-0.00000i	-	-	-

Instability that may result can be of two forms. The first form include the steady increase in generator rotor angle due to the lack of synchronise torque. The second form include rotor oscillation of increasing amplitude due to the lack of sufficient damping torque.The small signal stability problem is usually one of insufficient damping of the system Oscillation. Small signal analysis using linear technique provide us a valuable information regarding about the inherent dynamic characteristics of the power system and assists in its design. Here, by installing PSS, damping torque is injected to the system. According to the results, it is clear that the PSS damp out the low-frequency oscillations. Of note, this method may introduce a negative damping on some local mode of oscillation. In other words, although it stabilizes inter-area mode, it may destabilize local modes which is undesirable.

3 Conclusions

In this report , I analysze the small signal stability by considering a multimachine linearized system. We analyse this system in diffren cases without PSS, and PSS. The basic function of power system stabilizer (PSS) is to add damping to the generator rotoroscillation by controlling its excitation using auxiliary stabilizing signal. To provide damping, thestabilizer must producer a component of electrical torque in phase with the rotor speed deviation.Since the purpose of the PSS is to introduce a damping torque component, a logical signal to usefor for controlling generator excitation is the speed variation.

This report first showed how to calculate the eigenvlaue of the multi machine system. A simple two-area system was chosen to show the process of calculation of eigenvalues by using load flow

data. Eigenvalues of the linearized system was obtained by using the two-axis model for the generator and constant power load representation. We modeled 4 machine considering excitation system,transmission lines. A real eigenvalue correspondes to a non-oscillatory mode. A negative real eigenvalue represents a decaying mode. The larger its magnitude, the faster decay. A positive real eigenvalue represents a aperiodic instability. In addition, complex eigenvalue occur in conjugate pairs, and each pair corresponds to an oscillatory mode. In our study, the two-area system without PSS was stable.
In the next step, we installed a PSS at bus 12 and then we analyze the results of small signal stability. The installed PSS include gain, Washout (High pass filter), and Lead-Lag blocks (Phase compensator). According to the results, the system become more stable when we installed PSS at bus 12.
I determined frequencies and damping ratios of system modes and identified the associated dominant state variables using participation factors. I also Finded the mode shapes of rotor angle modes. I also discussed about controllablity and observabilty of the an inter-area modes. We showed that the inter area mode had a small positive damping (0.0184446).
At the end, I designed a PSS to damp out the low-frequencyos cillations by using Ostojic approach. Electromechanical Oscillatory Modes are the modes associated with the rotor angles of the machines. These can be identified through a participation factor analysis. The installed PSS include gain, and Lead-Lag blocks (Phase compensator). The desined PSS have T1=T3=0.569, and T2=T4=0.083. The eigenvalues of the designed PSS with different amount of K_{PSS} is analysed. K_{PSS} included 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.6318, 0.7, 1, 20. For amount of K_{PSS} =0.1, 0.2, 0.3, 0.4, 0.5, 0.6, the stability of the system was increased compared to the system without the PSS. Hence, designing a PSS with such K_{PSS} whose inputs are the speed variations of machines 1 and machine 3 that belong to different areas while its output acts on the AVR ofmachine 3 in one area damp out the oscilation. In fact, Eigenvalues of the system were moved to the left side of plane when we consider PSS. On the other hand, the amount of $K_{PSS} = 0.6318$ is assumed to be critical. Because, by increasing the K_{PSS}, the system will be unstable.
High penetration of the renewable energy make the system become unstable. Hence, the system need to stabilizer to inject damping torque to the system to become stable. One potential method to make a system stable, is using the demand side management instead of using the PSS. Considering the demand side managment to enhance small signal stability can be done as future research.

4 References

1 Kundur, Prabha, Neal J. Balu, and Mark G. Lauby. Power system stability and control. Vol. 7. New York: McGraw-hill, 1994.

2 Sauer, Peter W., and Mangalore Anantha Pai. Power system dynamics and stability. Vol. 101. Upper Saddle River, NJ: Prentice hall, 1998.

3 Ostojic, D. R. "Identification of optimum site for power system stabiliser applications." In IEE Proceedings C (Generation, Transmission and Distribution), vol. 135, no. 5, pp. 416-419. IET Digital Library, 1988.

4 Ostojic, Dejan R. "Stabilization of multimodal electromechanical oscillations by coordinated application of power system stabilizers." IEEE Transactions on Power Systems 6, no. 4 (1991): 1439-1445.

5 Pagola, F. Luis, Ignacio J. Perez-Arriaga, and George C. Verghese. "On sensitivities, residues and participations: applications to oscillatory stability analysis and control." IEEE Transactions on Power Systems 4, no. 1 (1989): 278-285.

6 Verghese, George C., I. J. Perez-Arriaga, and Fred C. Schweppe. "Selective modal analysis with applications to electric power systems, Part II: The dynamic stability problem." IEEE Transactions on Power Apparatus and Systems 9 (1982): 3126-3134.

5 Appendix: Matlab codes