# Advanced Logic: Problem Set 02

- Due date: Friday, September 16.
- Your proofs may appeal to any facts stated in the first three lectures.

1. (80%) Suppose we have a function $f : A \to B$. Let $f^* : \mathscr{P}B \to \mathscr{P}A$ be the function such that for any $Y \in \mathscr{P}B, f^*Y = \{x \in A \mid fx \in Y\}$.

   (a) Break down $f : A \to B$, this statement assumes that there is a function(a relation(Cartesian product of two elements from A and from B) that is both serial and functional) from set A to set B

   (b) Break down $f^* : \mathscr{P}B \to \mathscr{P}A$. This statement assumes that there exists a power set of B and a power set of A and assumes that there is a function from the one to the other

   (c) Combining the condition (a) and condition (b), the question asks us to prove that with all these condition above, we should have a causation structure that $Y \in \mathscr{P}B, f^*Y = \{x \in A \mid fx \in Y\}$

   (d) Break down $Y \in \mathscr{P}B, f^*Y = \{x \in A \mid fx \in Y\}$, This Disgusting Notation represents that 2 assumptions. The first assumptions assumes that there is a set Y that is the element of power set of the set B, this $\mathscr{P}B$ is consistent with the above definition. The second assumption(here comes the nasty part) $f^*Y = \{x \in A \mid fx \in Y\}$ assumes that there is a set $\{x \in A \mid fx \in Y\}$, the subset of A (such that) for all elements x in set A, x is a memeber of the $\{x \in A \mid fx \in Y\}$, (if and only if), the subset of A that contains ALL and ONLY those objects x (for which) $fx \in Y$

   (e) Above are my understanding of the question and assumptions, and now I will start my proof. None of the above are my assumptions, thoses are assumptions coming from the question itself.

   (f) I will start with the first assumption, that there is a function from set A to set B. This is a function that is both serial and functional. This means that for any two elements from set A, there is only one element from set B that is the image of the function. This is a function that is both serial and functional. This means that for any two elements from set A, there is only one element from set B that is the image of the function. This is a function that is both serial and functional. This means that for any two elements from set A, there is only one element from set B that is the image of the function. This is a function that is both serial and functional. This means that for any two elements from set A, there is only one element from set B that is the image of the function. This is a function that is both serial and functional. This means that for any two elements from set A, there is only one element from set B that is the image of the function. This is a function that is both serial and functional. This means that for any two elements from set A, there is only one element from set B that is the image of the function. This is a function that is both serial and functional. This means that for any two elements from set A, there is only one element from set B that is the image of the function. This is a function that is both serial and functional. This means that for any two elements from set A, there is only one element from set B that is the image of the function. This is a function that is both serial and functional. This means that for any two elements from set A, there is only one element from set B that is the image of the function. This is a function that is both serial and functional. This means that for any two elements from set A, there is only one element from set B that is the image of the function. This is a function that is both serial and functional. This means that for any two elements from set A, there is only one element from set B that is the image of the function. This is a function that is both serial and functional. This means that for any two elements from set A, there is

(a) (50%) Show that $f$ is injective iff $f^*$ is surjective.

(b) (30%) Show that $f$ is surjective iff $f^*$ is injective.

3. (10%) Using the Axiom of Separation to show that there is no set that contains all sets. (Hint: adapt the reasoning in Russell's Paradox.)

4. (10%) Show that for any set $A$, there is no injective function from $\mathscr{P}A$ to $A$.

5. (10%) Suppose that $f : A \to B$ and $g : B \to A$ are functions such that for any $x \in A, x = g(fx)$, and for any $y \in B, y = f(gy)$. Show that $g = f - 1$.