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Now, what is optimal strategy? I have an optimal strategy also is always a style. So the stylist interview some people first, right? Do not see that. And then you try to see that first one in the second group who is better than the first group. Say the different thing is just what is the threshold? Now, the threshold is actually n over e people, maybe is 2 . 73 something that is essential number, mentor law base.

Now, this is threshold. You do not see adam. And after this, you see how the first one that is better than all the group, one, aids, group one, carrots. And then doing this strategy, you have the probability one over e selecting the best candidate. Earlier is 1 / 4. Right? Now. It comes already, which is better, right? This analysis is a bit tricky, but I hope that I will spend a sufficient amount time. I think this process. Okay. Now there are some observation of the so called optimal algorithm. First observation is very important. The algorithm will never pick an element that is not best. So far. What does it mean? It means that you need to do somebody, right? And then come here as well. This might not be good time. Let me wait for a bit. And then after some time you see, maybe the best is really on. Then how about the current one, which is not the best so far? There is one person better than this one. But I see that is not the best part was, would that be good?

Now, in terms of the current measure, it will not be good because you are selecting someone who is not best up to now. That means this person cannot be the best global. All right? So it will never be such a choice. You always say that's the best up to now candidate. All right. Now we analyze the best so far, candidates ok so we define a function fi this fi means when the eyes element is best so far, what's the probability that the eyes item is global best? That's the probability that at position, I you see the best so far candidate, and then it becomes the global best. What's the chance? Right? How to calculate this? This eg is a conditional probability. Probability of condition a or probability of a conditional b is just the probability of a divided by probability of b probability that I is out of this global best divided by probability of I I gonna be the best so far.

All right. The top one is easy. You only calculate it, right? Any position that element is best globally is one over n right? In this one. How about this? I item is best so far? Same, right? 1234 until I you only have high elements. And this last element, highest element is the best. In these high elements. The probability is one of the higher. It is the best in the position. One is it? Basically? I one over n divided by one over I that is I over n so that means what? That means the later you need to do candidates. If you manage to see that one to be the best so far, then the chance that this person is the global best becomes higher and higher, right? Good. This is some existing trucks, right? You choose in position I right? What's the chance it becomes global best?

Now this mi as we already denies, it is increasing, right? Now this is defining a second country called gi what is gi gi is the probability picking a global desk, using optimal strategy from I I on it. What does it mean? It means that on acquisition, I okay.

Now, suppose I have an oracle, a very smart tool, smart agent, telling me, if you do not pick anyone before, what's your best chance in the future? What's your best chance in the future?

Now, this function is a non increasing function. The best chance in the future will be become smaller and smaller. All right? Why? Because if you throw away I minus one elements, your best in the future have more candidates than, if you stand one step on the right, you say after only first high elements, one more, right? What's the best future? So you have this less choices. Right? Now you want your chances becomes smaller. So when you have more candidates, you have better chance. That's why gi gi will be, at least as good as gi gi plus one, because you can simply throw away the current ii then you adopt a strategy for gi plus one, then you can see something as good as gi plus one, right?

Okay. So having more item will always benefit in terms of choosing the best. Here you see, gi plus one is a not increasing function. Now we have two functions in each order. Where is the one are in. So now every state she is different. Right? So after increasing, she is increasing, as long as your f is smaller, g smaller than this, as long as the fi is smaller than gi plus one, then it is not so wise to see that item I because then you have a smaller probability to the best. You would rather throw away at mine and go for gi plus one. That will give you a better chance.

You see that in this picture, in this picture, if fi is less than gi plus one, you will not say I I at position I it's best so far. Can you say, do I see that design? Then if I see that my chance of winning is this, this is hard. But if I do not see, ii go for gi plus one, right? Again, I higher probability I you will never see that fi at that moment. So your best strategy is to do the following as long as fi is less than gi plus one scale. Okay, I do not see that and go on to next time. And then, for these things, you will treat them to be in the first group. You introduce them, but you do not see that, right? Okay? And then after that, you will pick the first element that is best so far. Okay. Now you see, I throw away something, right? This group I throw away, and then you continue to the second step. You see something. And now this person is the best so far, so far, ok the first enemy best so far, I was the editor.

Now, it is actually the same as well as the first element better than this group. You have some groups which you interview. You do not pick, right? And after that, you interview.

Now, if somebody is better than the first group, it must be, then the first one you see better than the first group must be best so far. Right? Because those people in the middle, they didn't beat this route, so they cannot let me guess so far. This one beat this rule.

Now, it also beats these, because these people didn't beat the first route, right? And this was the first one beating this group, and then it's also larger than these things in middle. So this one is the best so far, right? That's exactly the structure about. Now, critical thing is, where is it? Sis and gf what's the average? Let's calculate this. Now, if we reject the first tau items, what's the probability we succeed taking the global test? Let's see. What probability is the summation of the following? What's this? T starting from tau plus one to n because it's really the first tau items. From the next item on, I will check whether it is a global asset. Second, the best of the first t minus one items is in the first tau positions, ok what does it mean? This is top right here. There's at position, right? The t position.

And then now the chance that you see that this item, and at the same time, this item is global best as well. Split two parts. One, it is global best, right? Two of two, you really select this item, right? Ok so global best is easy, right? Cp global best is one. It's just one of the camera. That's easy. How about the case that you do? Cft if you want to cft you have to make sure because t t's progress. And then the one among all these items, the second best should be second mess should be here, because if the second mess is here before t after tau, then you will see the second best, right? So you have to make sure this time second best need to be the first group. Only these habits, you can see a team as the global best and also best enough, right?

What's the chance that the second best is here? You will check the lens, right? How long is this? Right? According to proportion, right? You have tau over t chance, then the second best is over there on the left. But my writing is not correct, because I included the last elements, right? You should not include this, because this one cannot be second best, right?

So I need to - 1. I remove this team, right? You only have t minus one positions now, right? I hope my second best in is the first composition. Right? Now top that's top over the total source, t minus one. That's exactly what i'm going here. Right? This is the chance that in the first t numbers, the second best is in the first top positions, right? Ok so now with this, actually, you can see, I just take out tau over n then inside, it will be one over t plus t minus one, one over t minus one.

Now it becomes what? One over t minus one, and one over some other - 1. This t is starting from tau to n it start on one over tau, one of the top, last one, et cetera. Let me see whether i'm correct. Absolutely. T starts from tau plus one. You start from tau plus one, top one over the top, and then one over the top, one plus one, and the one over the top + 2, et cetera. And what does it remind you? This is what kind of series one over something, right? What is this? Harmonic, very good harmonic series, right? The harmonic series, ok one, tau plus one over tau plus one, et cetera. Then until one over n is it n possible? Sorry, go back. N minus one. Right? So now you care about this summation, right? But what's this submission? I don't know what it is, but I know if I add one plus one over two plus one over tau minus one, then the whole thing becomes this harmonic series sum up to n minus one.

Now, I believe that there is a small type of here. It should be, it should be hm minus one is not hmhm minus one, adding from them, one over two one over three, until one over n minus one.

You use this hn minus one, subtracting this part. It will be the one you want. It's ahn h one, h tau minus one is exactly here. Right? You subtract a small piece from a big piece. You get the middle part. And the middle part is exactly the party. All right. So now, actually, this is the definition of g tau plus one. Why? Because we rejected the first tau items, what's the probability we can pick the global best in the future? That's exactly the definition of g right? G throw away the first titles, right? What's the chance I think the global best in the future? That's the probability that you pick the world best in the future. Every possible position I calculated, this is the definition of g tau plus one. Then we calculate f tau. F tau is tau over ni want this to be larger than or equal to g tau plus one. This is the moment I should start taking my items.

All the previous moments I just throw away, but starting from this ., I will Seriously consider my candidates. And then if you solve this inequality, you will exactly get tau is roughly equal to n over e for a very large n this is how you get the threshold up to this moment. N over e candidates you do not take anyone. They are just some people who should be thrown away. All right. Let's start from this point. Whenever you see somebody better than the previous group, you take it because you can make sure you can guarantee this take is best so far, right? It is better than the first group. It's best so far. Right? Now you have this final probability of selecting the best candidate, which is exactly the value of g how plus one? G plus one is the best probable, the probability seeing the best candidate in the future, right? If you throw in the first half hands ok so that finishes the whole time.

Now I will give you 2 minutes. You can discuss to see whether you have any questions about analysis. All right. 2 minutes up to china. That's so any question, ok so maybe we should move on to the last problem. Okay? The last problem is called profit inequality. To tell you the truth. When I first see this title, I don't know what it is talking about. Right? Although I know those words, I don't know what he's talking about.

Now, anyway, it is about equality, right? Okay? Any part? So it has something to do with profit and what is profit? Probably something to somebody knows everything. Let's see. Basically, the second says you have ten random barriers. These ten random variables come one by one. It's an online problem. Now, we know the distribution of direct values, but we do not know the realization. We do not know the value before we open the box. So we need open box here, but here you can open every box. Right? I will cost one and a little better next one. For the lawsuit, I know that last suit. All right. And then you need to decide at the time you open the boss, whether you take that item or not. Okay, you do not take it. You go to the next item, you cannot come back. All right? So that's the same. And australia, all right, is to decide when we see the value of xi whether we choose xi or not. Right? Goal is to maximize the so called expected reward expected.

All right. Now, what is the offline album? Just supposed to know all that is, right? What would you do to investment? Largest area? This is what we do. So often, offline, optimal is just like you have a profit, who knows everything. All right? Who knows the realization of every day of these very random variables? And it's very easy for the profit to select the desk, because it simply select the largest value among the n random variables. Sometimes x two is largest, xx two, sometimes x four is not just set up x four.

Anyway, the profit is selecting x max. But remember, x max is also a random variable. Sometimes it's x one, sometimes x two, sometimes x three, right? The best the profit you get is expectation of the random variable, a variable x max. This is best we can do. Now, what's your errors of doing? Whoever them is doing something? But you cannot beat the ex max. We try to be as close as possible to this ex max. Right? How close do you get? Now, before we see, how close can we get? We first see, you cannot go back to lessons, okay? Right? The first one to have some found in our mind. That's all you can never read that. So I will never try to read that down, which found is this. Can you guess? This is a maximizing problem, right? The bar must be something like 01, something, 01, something, right? What about you guess? It's a very easy guessing problem.

What value do you guess? Even though the answer 051, although the fact, what do do you say five? So why do you guess is that my father tell us the secret? How is it important? Why is it important? It must be some reason, right? You see, very good. We see this some time ago when I see it. Okay. Right. Anyway, what is security? Is 0 . 5. So very naturally, we also guess this is 0 . 5. Is it correct? Very lucky. It is correct.

So let's see the example we get here. In this example, the first variable will take a value one with probability one. Ok second variable will take a value very large value one of the epsilon with probability epsilon. Then with the probability one minus epsilon, the value is zero. What does it mean? It means that x one is very stable. It's always one, x two. It's not stable. It is somewhat capricious. So it will be very big for a very small chance. But it will be zero for the digits. If I draw axis here, this is the xx one minus two. A is x now, basically, x one will be something like this. X two will be something like this. This is x two. The rate, let's pretend this is one, okay? With probability one, x one is always one, right? Ok about this, probability space is always one. X two is very high. A small window x it's a zero and remaining for this kind of distribution. What can you ever do? You ever want to see as one first, right? Is one.

Now would you take it? You have to make a decision whether you take it or not, right? If you take it, you cannot guess you cannot see it anyway anymore, right? You just take one. Right? If you do not take it, then you have to take it two. Right? In that case, you're expecting expected value is still one only. Even x two, x two, the expectation for x two is also one.

There is no matter which variable you take, your expectations one, right? Only get one. But what can profit? The prophet says, I already know the value x two for this part, I take x two, then for this part I take x one. This is what the prophet will do, right? So you can see what x two is, right? The pcx two is super large. Then you'll skip at one, right? That's one point. And I take it, right? That's true. How big is that expectation? It's one of the epsilon is the epsilon probability + 1 is what probability? Very good, one minus excellent. Altogether, this is two minus t right? Ok so that means profit can get expected value or expected reward to minus x but your algorithm can only get one. What's the ratio? Roughly half, 1 / 2, rather than 1 / 2, it's half.

Okay. So now, how difficult this problem is? Right? What's next? They are trying to design something which can achieve is half. All right? So how the chip are. Actually, the strategy is easy. The strategy says, now we focus on their random variable x maps. Let's see where x maps strikes a balance in the probability. What does it mean? Let's see, this one. We want to find a special way in tau, such that the random variable x max larger than we talk, this probability is exactly half.

Okay? It's a guess. All right. I don't know where our colleagues, anyway, there existed to them, because altogether the probability is one, right? When tau is very small, then the chance x max, london, tau is very big, right? It could be 0 . 99, right? And then increased habit. The chance drops a bit and increase further. It drops further until it drops to half. I stop. This is the place of car. All right? Now you we assume something. We assume that at the traditional part, there is no so called mass. Sorry? Yeah, there's no so called . mass, which means probability that x max equal to top is not positive. A it's a bit not so easy to understand. Let me just explain this case. We have a probability distribution. You will cut. The distribution is half, this part is half, this part is half, and this is half. In other words, less than equal to tau is half a chance. Greater than equal to tau. I'm sorry, this also is less than equal to. Greater than equal to tau is also half. Both sides half. That is at the point of time, no pointless, but don't worry too much about the term.

Anyway, it tells you there is a magic value card success on either task. The probability is exactly half. Okay? I use this time as one important factor in our what's the strategy? The strategy says, I will take the first xi which exceeds this time. As long as I see them very large and top, I pick it. This is strategy. Now let's see how good this strategy is. Now, because we assume the distribution of random variable, it, right? Even though it's online algorithm, you still know the distribution tau, right? Although it's a magic number that it time ok that's why you can adopt this strategy. Otherwise you cannot do this. Right? Now let's see how good it is. In order to analyze how good it is, we need to first start the expected reward of the optimal error, right? How to found the expected reward optimal algorithm.

Now lets do an imaginary experiments. You say, now, if your x max is less than tau, I still give optimal solution, the benefit of top, because optimal solutions, usually, we assume it can only get x max. Because after solution to everything, every time you get x max, but in this excellence investment power, you can only get less than power value, right?

But now let's just give the optimal solution more benefit. You say, in that case, I also give you top. Then in these optimal solution, you can't figure it out. Because then the original, I make a larger. Okay. Now, if the x max is at least how, then I just keep the optimist game to be x max. So doing this, it's kind of relaxation of the or increase of the optimal solution. I'm increasing optimal solution. Doing this. How to calculate this? E max? The e max is less than we go to tau plus this. What does it mean? It means that at least i'll get top, right? Because ii said, even this is smaller than tau, I was giving contract optimal solution will always get tau, at least, but sometimes it will get more than tau, right? Okay? It's more than tau. How much more? X minus minus tau plus? What is this? This means if x max is larger than power, I just take this value.

If it's smaller than tau, I take zero. I it will never be negative that. At least you you see, tau plus x max minus tau is x max. This is the corresponding to the second case. For the first case, this is negative, but I take a plus, which becomes zero. For the first case, it becomes tau of zero. Anyway, if I do this, it's an upper bound of the optimal solution.

This one, I can further write it into the following one. Tau plus the summation of exi minus tau plus. Why is that? Because we said x max sometimes will be x one, sometimes will be x two, right? Sometimes be x three. I just expand this one to be the summation of x one minus help us x two minus tau plus, x three minus tau plus. If I do this, I will just make the sum even bigger, because x max must be one of them. Right? And this one of them I already had. On top of that, I had a few more things, right? A few more terms. And this can only be given. Therefore, this is correct. Right?

Now this is the upper bound for optimal solution. Now let's see the area. What were the element for the algorithm? You see our strategy as well. Take the first element, which is larger than how already at this time. Okay? Yeah. At this time, i'll pick somebody at this time. So that means what? Whenever your x max is at least half, I can always get a profit top, right? That is your x masses at this top, then the algorithm must be able to choose somebody, right? Because somebody's at the start up ok so the first term is what the algorithm can guarantee and must be able to get so much. Right? But other than this, I can get a bit more.

So now I analyze item by item ok let's look at this second question. What does this one mean? This one i'm looking at item I so i'm looking at a case when item I has a chance to be chosen. Okay. All right. When will I can I be chosen? Has you as I I has a chance to be chosen? Okay? There is a special condition here. What's this is all logical. So that means all the items before I it's excellent, is at most top. So before item I all the earlier items, their value is less than half, right? In other words, they will not be chosen, right? Because that's the part I will choose it. So this is the air condition, which means all the items are less than tao.

So in this case, I I have some chance to be chosen. If I don't mind is larger than how or at least how it is chosen, right? If it's still smaller than how it's not true. Okay. So this is the chance the item I can be chosen. And what's this is? Item I standing on top of top or beyond power, right? Beyond how much more I I will contribute. If this one is zero, that is, I didn't choose item either because itemize that is less than how I want to choose it. So it doesn't hurt, it's zero anyway, right? But if I don't hire is at least taobao higher than taobao. This extra part should be attributed to the algorithm, right? Because now this is the chance that item I is chosen, and then the extra benefit contributed by item I right? On top of time, tau plus xi minus tau is xi the algorithm choose xi it should get xi these are the two components.

With this. We will further write it in a falling way. All the earlier items less than top, this probability will be at least the probability that x max less than top. Why? Because xx is less than power, automatically implying. This is true, right? Because max is less than power than everything, is less than power. This is true. Therefore, the probability here must be smaller than this probability. This is smaller set, right? If it is true, we must be true, then the probability of a must be smaller than probability of b I change it to this. It's a larger than relationship, right? Larger than relationship.

Now, what's next? These two are very special. We said this is tough, right? Although this is less than tao, according to our analysis, this is also half, right? Because we said tao splits the probability to half, this is half. This is also half. So you can just change this one to be less than tau. It is the same. It's both tough. And then you put this tau inside, and you bring down the sea here.

And then what do you see? What's this? This is exactly same as this. Right? This one is the same as this one. This can be replaced by larger than equal to ex max, right? Basically done, managed to change the two forms to be the same. And then the inequality naturally goes of. And then it becomes larger than equal to half max ehs now, I believe you need a bit more time to digest all that is happening here.

All right. But that one, I guess you can do it. After the lecture, I I want to introduce one more bonus point, so they can get one now. You see, look at the clock. So 7 minutes, finish everything today, but actually we have something more to discuss what is that if I believe anything, which I say I will talk in the future. What is that? It sounds like ii go to some problem. I say this problem you kind of do not talk about today. We talk about in the future. What is that? Which one on the response? What about have a boss? No, that's too close, although it's also something, but ii didn't promise I will talk about all kinds of boss. That's another extension of this cost, not in the store.

There's some time earlier, not today and not have memorizing. Thanks. Is that for these gaps, which I live open earlier, you need to pay attention to that because there will be potential bonus points in the future ok just like f and gok all right.

Anyone can give us more heat is in the first lecture, not the second first lecture only have three methods or not four methods and immersion addp and development, which section is it in this path development? Not bad. No. Second page is it dynamic problem?

So what did I miss? Or which part I didn't explain? We play some benefit methods, right? And then we also trying comfortable group, which you mentioned a treatment. What we didn't mention is what we didn't mention is also a tree. What is that? What kind of tree? But it's a binary tree. But what does tree represent? If you can answer that? You can also get bonus? What does this mean? Who said concrete, comfortable, and comfortable, every maximum, comfortable, maximum, what comfortable, maximum, comfortable, but that won't be explained in the lecture. Maximum comfort rule. We explain that what does history represent? It has a meaning, right? What does it represent? It's very tough. Days, a little. Recall, recalling some earlier things are always tough.

By the way, it's okay. There are four words about the history. The cs right? Because coal call, what does this mean? What does it mean? It means if what does it mean? Means the left is smaller than the right is bigger. You mean, compare the finance industry. Because we are talking about cold, it is not a purely value. Every node is a cold block, a piece of code, then the code doesn't. Branch means, for example, if else state change the switch or something, right? It's like that. What's the problem? Actually? I didn't tell you the problem at that time. The problem is if you have cash, it is a cash, which can store one whole lot, one of them. And after you store the whole block in cash, you don't need any time to ask you this part, because it's all don't any time to read this part, because it's very fast in cash is very fast.

So I want to know which whole blog I will put into cash so that the longest path is minimize. Okay? What is on this class here? For four notes, ok suppose very old, is time one. Now, in order to reduce the longest path or make a longest path, smallest, which node, which code block would you put in the cache? Which one we test? John one. Very good, ok why? One is on every part. Right? So you put one in, you release the length of every past a month. That's the best possible, right? So you always put the roots into cash. Right? But after now, we didn't know which method it is, because earlier you said that it's adpdp session. So the direct dynamic programming section, it seems, has nothing to do with dynamic programming. Let's change the problem with it. It's no longer a single spot. You can put two nodes in cache. What do you put? Put in 2 million cash, which two, one, and three, very good ok so why one and three?

So basically, here we are using some method. What is that method called? Not the last word, dns is to the last. When we talk about one and three, we are using some method which belongs to one of these four big categories. And which method are these? This can be regardless for this original values, advanced ok we don't have four in the region, three dpp divide and conquer which one will be using, which one who separately? It's greedy. Why? Easy? Right? First choice is rude, right? After you remove root, you have two sub trees, which sub tree is a bottleneck? This one is because this is long, right? This is short. So you can put the route into the cache. So this is a greedy. It's a very beautiful meat match. Right?

Now, the next, I i'm not going to spend too much time. The next is dp right? How to extend the dp but now you can say, how about if we put the size on every node earlier, we say every node takes size one. Right now, how about if this size is 90? This size is 70. This is a 20 ok every node as a size. All right? And then you can also exercise and then how to put something in to reduce the longest parts, right?

Or make the longest path as soon as possible. And this time we also apply kind of time difference. For example, you can say no to the original time is 20 seconds, new time after being cashed is a pencils, right? You can make everything different. Then the dp what he says, otherwise, we don't need to use dp just in screening station, right? Then you can introduce those different dimensions of parameters, and then you can do it in solutions. So there will be one goal. Possible exercise, not a concept as if some exercise you can do when you have time. Right now, as I said earlier, this is one of my first research I did after join. Said to you. So that's very interesting. How about cash multi person? Okay, so I guess today we just stop here. All right? I will stay behind for some time in case I have questions.