### AL\_week5\_lec5gametheory\_1-20241012

说话人1 00:00  
What other people choose? Then we say sij is a dominant strategy, because strategy sij will never lose to any other strategy sif

说话人2 00:12  
right?

说话人1 00:14  
Okay. Good. Give other people no matter what people they are, right? No matter what they are. So I choose cisich is always

说话人2 00:27  
good

说话人1 00:28  
compared to other

说话人2 00:29  
strategies.

说话人1 00:32  
This is so called dominant

说话人2 00:34  
stretch. All right?

说话人1 00:36  
And then, furthermore, if the

说话人2 00:38  
inequality

说话人1 00:39  
is strict, then we say it is strictly dominant strategy. Strict means what? No equals are, right? It's just larger than if sig always gets more benefit compared as il then it's strictly dominant direction. Right?

说话人2 01:00  
Okay.

说话人1 01:02  
Now, let's define the parental dominance and the peripheral humanity. So this one, I believe that earlier we mentioned a bit, and then I believe many of you, you also know what it means by parental dominance. So basically it means. So I remember last time or the week before I mentioned some kind of a you have some marks for your different subjects, and what's the critical dominance? It means that if you are not dominated by others, that means what it means that if other people they have higher marks in some subjects compared to you, then they must have a lower mark. In one subject compared with it. They cannot have all higher marks than you, then they dominate you. Right? Here, we'll define three levels of critical bonds. What's the first step?

说话人2 01:57  
Now,

说话人1 01:59  
before we talk about creating dominance, we first let s and t be two strategy profiles. From s s and t are two strategy profiles.

Now, we say, as we create peripheral dominance t if gis is larger than or equal to git this is for the pre to dominate relationship for a single player I right?

For a single player, I is gis not equal to git then we say s weakly cradle dominance t now. But this weekly freedom is we know that it has to be applied in multiple dimensions. So this relationship should be correct. I for all the items means for all the players, right? For all the players, certain profile is better than the other profile. Then we say of this certain profile trade for dominance. This second one, right? If this relationship is correct or is true for every player, I then we say it's weekly trade of dominance.

说话人2 03:12  
Okay?

说话人1 03:12  
Now we can make it a bit stronger, right? We can say maybe this is a larger vehicle to becomes larger. No equal. The next level definition says, if for one player this relationship, this is strict, it's for one player.

There is one player j that the quality the 85 equals. There's one person who strictly likes these strategies profile than their strategy profile. Then we say it is a peripheral dominance.

Remember, just

说话人2 03:51  
now, this

说话人1 03:51  
one is called weekly period of dominance, right? So weekly means it could be the case that everything is the same, right? Everything is equal. That is weakly greater than this. But if one of them is strictly better, then it's pretty good, nice.

Now, the next level says, if all of them are strictly better, then it's strongly the parental dominance.

There's a strongly parental dominance relationship where the inequality, the larger than is applied to all the players, cr larger than equal to everything is weaker. One of them strict. It's normal, right? And then everything is strict as well. Strong. Right? Now that's easy. Just three levels of definition. Now, based on these three definitions, we are going to define the so called parental optimal.

说话人2 04:45  
Okay?

说话人1 04:46  
What is the principal? Pretty optimal says. Now, we say t is pareto optimal. If for all the other strategy profile, st is weakly brittle. But if p is weakly rate dominated by s then everything is the same.

That means it when do I say t is pretty often? All ideas is something else. We could create a dominant mean. Then there's somebody else must be the same as me. They should achieve the same effect in every dimension. It cannot be the case that some other people is strictly better than me in one dimension. That's not good. Somebody, freedom dominance b then i'm not afraid of, but if somebody we could result, prefer optimal preferential dominance b it's fine.

This is the definition of pareto optimal. Now we also have a strongly or operate time like that. It's a weekly rate of, okay, what is weekly reform? That is probably there are more strong people before. I say, if somebody weakly created up with me, it must be the same as me, right? Then what is weekly result? It says if somebody, if somebody, we always say the following, but if there is no other profile that can strongly create a dominance.

So there could be some people political dominance. That's fine. Ii have 90 points for every subject. Somebody else has a 90 51 project and 94 other subjects. That's still fine for me. I okay, that as long as somebody do not strongly break dominance me, then i'm happy. So in that case, i'm called weekly parental. Okay? Because there could be somebody who peripheral dominance be in the normal sense. If somebody could peripheral dominance be in the normal sense, i'm no longer perito optimal, but I can still be weekly peridot. As long as nobody beats me in every subject, then it's

说话人2 07:11  
all

说话人1 07:15  
right. So now, we will define the national program. What is the national program? So basically, the national program is the concept saying that if there is a state of a strategy, everyone choose my strategy, this player choose very one, this player choose strategy, et cetera, with this combination. If nobody is willing to change its strategy to another, so means what willing means changing will bring them more benefits, right?

So in other words, you fix one player, you say, I give you a second chance. Would you like to change your strategy? And then this person says, no, my current strategy is best for me, so I will not change. If this is the case for every player, then it is a nash.

So in that case, strategy, sij is the so called best response strategy to the other people's profile. So when other people fix their strategies, the person you are looking at, right? Its current choice is the so called best response. Best response before this strategy can generate the largest benefit to this player. This player will not change its strategy

ok mathematically, it just says this

说话人2 08:48  
the

说话人1 08:48  
gain of I on ij is better than or equal to the gain of I under strategy il

说话人2 08:57  
right?

说话人1 08:59  
Now, you also have a stronger concept of national premium saying that actually this strategy, it's the best. And at the same time, no other strategy can be the same as this one. This one is the only best strategy, because sometimes you can have two or three best strategy. All of them give you the same benefit. But this time it says, if this is unique, right? This best is unique, then it's called what? Strong. Again, you have a strong. So, all right, so it's here, right? So it's everyone. So it's exactly one such strategy. That is the best response. Then this is a strictly best response strategy. Right? Now. We have a strict here. Right? And then if this is strict, so if everyone has a strict strategy, be the best response, then the whole profile, we call it strict national program.

说话人2 09:59  
All right.

说话人1 10:00  
Basically, strict means equals science part, right? You have no equal sign,

说话人2 10:06  
smart.

说话人1 10:06  
So now we come to the picture about relationships of different concepts. For example, here we know that a great optimal strategy will imply a weekly rate off if the strategy is great optimal this week, right? As well. This is something that can guarantee. And on the other hand, that if you have a strictly dominant strategy that it implies, you have dominant strategy, right? It also implies you have strict national freedom if everybody choose one strategy. So that is a pure strategy case. Pure strategist means everyone may choose one strategy. Right? And then both of them, again, to imply that you can have a nash for the real impure strategies. These are the relationships between different concepts of the national prison and also dominant strategy and great wall.

Okay, now, at least not only for you to understand that how to define certain strategy profile is good or not, right? So whether people will deviate from these kind of strategies. Okay. Now let's briefly cover the topics we are going to talk about in the game theory part. In this lecture, we'll talk about the inefficiency of equilibrium. That means how difficult it is to compute an equilibrium. Then the complexity of finding equilibrium. Actually, the one i'm talking about is complexity. Inefficiency means the equilibrium. Although it's stable, its social efficiency is very bad, and which means compared to the optimal solution. It is very bad. This is two topics we are going to talk about today.

Now, other than these, we also talk about other type of space in this lecture. And then after that, before this one, we'll talk about the missing component here. There's a so called cooperative gains, which means people work together to make some efforts. Right? This will be the lecture on monday, which is 2 days

说话人2 12:19  
away, 2

说话人1 12:21  
days later. And then after that, we'll talk about market program, multi agent systems. The market problem is, we'll talk about the market matching and option setting. All right? Matching the original pair of different people. And also option means you sell things to the users and how users will bid for the item. Right? So it's an option theory. Then we'll talk about multi agent systems, including the fair division subject and also scared subject. Then we'll talk about the so called mechanism design approach, and we'll use the facilitation games as an example to explain them. Ok so this is one of my research focus as well.

And finally, there is also a broader scope called computational social choice, ok so this is a even broader field, basically about you have some candidates, and you want to sit on some of them and how to use a reasonable rule to decide who will be the winners. Right? So these are the subjects that will be covered here.

Now, we start with this inefficiency of equipment, so that the natural problem is actually very good, because nobody will deviate from its current choice. Right? Everyone thinks of my current decision on my current strategy. It's very good for me. I'm not willing to change to another one, right? But national program also has its drawback. What's the drawback now? Basically, for one game, there could be multiple equilibrium, multiple national career. Now, some of them could be very bad in terms of what. So let's see this damage. There's a definition called poa price of energy. So energy means no governments, nobody graduates, the behavior of people. Everyone just act according to their own best interest. Right? So this means the energy. So in this case, in this case, you will arrive at a certain national program, right? No central control, a certain approach. Now, this equivalent could be very bad.

Now, here we define the cost of a solution. In this way, ok we have a so called optimal solution. This optimal solution has the minimum cost, smallest cost. And then your national program can also have a cost.

说话人2 14:56  
Right?

说话人1 14:56  
So what is the president? He says, then among all the national career, the worst one, which means the equilibrium with the largest cost and take that one divided by the cost of our solution. Let's see how big that value can be. For example, the optimal solution, it could be cost 100, and then the worst natural problem is cost to be 1 billion, right? So 1 million ÷ 100. It is

说话人2 15:28  
10,000.

说话人1 15:29  
Right? It's a huge number, right? So that means in order for people to arrive at some stable status. All right? Without the control from government, then the equivalent probably could incur a large cost, right? Compared to the centralized control. So if you have centralized control, the overall cost is only 100, right?

But if it's decentralized, Every people go for their best interests, then the cost could be huge. Right? So if this ratio is very big, then this game probably is not quite ideal, right? Because if you give the distance enough freedom, then the social efficiency becomes very, very low, right? Or the social cost because becomes huge. That's not what we hope for, right? Ok this value will character will characterize how good your game is.

说话人2 16:23  
Right? No.

说话人1 16:24  
Because which game you use to let people play is the designers choice, right? You need to design some game where the poa is not too big.

说话人2 16:35  
Right?

说话人1 16:37  
Now, another thing, there's a symmetric concept called price of stability. What is this? Now, here is stability, right? Price of stability. Now this stable stability just means nash equilibrium.

The price of stability means if you want the society to achieve stable status, how much do you need to sacrifice? At least? Remember, there's a word at least, because just now, this is how much you sacrifice at most, right? Then this one says he wanted to achieve stability. How much do you need to sacrifice? At least? So that means now you are looking at the so called best national program. Right? Because you have multiple equivalent, right? Among them, there is this best one should have a very small cost, but not as small as possible ok it is still not an optimal, but it could be close enough. So here, pos is the minimum value of the cost of certain national program divided by the possible. So

说话人2 17:50  
it's the best mesh

说话人1 17:52  
divided by optimization. This one is called price stability.

All right. Now we'll cover some very complex topic.

After we talk about natural problem, now we define the so called zero sum games. What is zero sum game? The zero sum game says the following. It says that, for example, we have two players. One player, if it gets $10, then the other player will lose $10.

说话人2 18:33  
All right?

说话人1 18:34  
So their game end up to be, but zero, zero always zero, no matter which string combination is always zero.

Now, for zero sum case, people prove that the national program, the computational method program can be transformed into a minute max strategy problem. And this one can be modeled as a linear program. And we know media for program can be solved in polynomial time ok zero sum game. This is easy. The computation of the equivalent is easy. Now, how does general form gain? General form game is a general form game. It means it's not zero sum. It could be the case that I gain $10. You lose only $5. It's okay. Now, for this kind of games, people show that it is ppad complete. This is a very interesting term. What is pta to complete?

Firstly, let me tell you the name of the problem. The name is called polynomial parity argument for directed graphs is a there are five keywords, and then they put the two p a and one d into this abbreviation, right?

And then actually this concept was designed was discovered from it. So it should be designed by the scientists called habitat ritual.

And people usually say that because habitat ritual, the name had many p and a and d so the so that's why he made the name tpad so somehow it represents name. But anyway, it's just a story. But this problem is also called end of the line problem. What is the end of the line?

Now, here I tell you what it means. Now, suppose you have a huge graph

in this huge graph. Every node in the graph only has, at most, one edge coming in, also, at most, one edge going out.

说话人2 20:55  
It's very simple graph.

说话人1 20:57  
Only one edge coming in and one edge going out panels. Now you can say this one is very special. What

说话人2 21:17  
can this graph be?

说话人1 21:21  
Can the graph be a tree? A circle case cycle, or we see cycle, right? So it's one thing went out, right? One in, one out, and you go back to the original point that becomes a cycle, right? Or it can be one. The line or pass, right? So you stop here and go on, hold on. I never come back. That's also okay. Let's stop here. That's also basically this graph is just a collection of cycles and pass right.

Now, what's the goal here? The goal has here says, if I tell you one starting point, which means once one source, I know, I tell you 11 node who only has outgoing labor, no incoming. I tell you the storage, a source. And then the question ask you for another source or a sink signature. You stop here, you never go out, right? This is sink.

The question is just, I I give you the information of my source. Please tell me the information of a sink or the information of another source, which means the starting point of another task, right? That's the goal of the problem. They would say it was very easy, right? Why? Because you already have this source, right? I just go on and go to the end. I I I never move on again, and that's the thing you're looking for, right? It seems very easy, but the tricky thing here is the number of nodes in the graph. It's not polynomial. It is exponential.

说话人2 23:07  
Okay?

说话人1 23:09  
And then can you find it? So basically you cannot carry like this. Right now. You can ask everyone, what's your next note? What's your next note? You can ask a question like this, because you can ask a question like this. You will spend exponential time. That's too. So

that's why cr this problem, right? It is not polynomial. They define that the difficulty of the problem to be ppad complete, which means this is the most difficult problem in the class, ppad okay?

And then this one, this problem, if you can solve it, you can solve every problem in this class in the class of ppad now, ppad complete and np complete. There are two different things. They are not equivalents. They also do not have some clear indication, because ppa complete, this one is defined on another school on the game theory part. All right. So now here, the thing you can do is just a inquiry to oracle rising before every node. You can ask, what's its national and what's what is your previous level? So these are the questions you can ask. All

说话人2 24:28  
right?

说话人1 24:29  
This is the so called end of the line trial.

Now, today, i'm not going to show, for example, what kind of problems are getting complete or so, what kind of the games it can describe. So today i'm going to show the relationship between this one and the so called sperm and lemma. So this one is actually more interesting.

说话人2 24:56  
Okay.

说话人1 24:57  
This will, well, this will be used to prove the hardness of the international program or the hardest of the end of the line problem.

Now, i'm not going to link this problem to computing computation of national program, but i'm going to link this problem with end of the line problem. So what's this problem? This problem is following. It says I have a triangle, this triangle, I will divide it into many small triangles in this way. This kind of three direction of lines to divide a big triangle into smaller triangles. Now, let's play a game. What's the game? The game i'm talking about is different from game series, this game. Okay? Let's label all the vertices of the triangles in the following way, and label this node as zero. Label is known as one. And they will know this known as true, three, big vertices, 012, then along the edge, 01, I labeled the middle vertices as either zero or one, as either 01. Then along the edge zero, and two, I label them either as zero or two on the edge 12. I think of them as either one or two. So basically, along which cache people can only label the notes in the middle by the two end points, the two labels in the end months.

So up to now, any questions?

说话人2 26:30  
Is it okay? All right.

说话人1 26:32  
And then inside the triangle, for example, this one, right? These vertices just label, as you wish, because they weren't anything, 012, as you wish.

说话人2 26:44  
All right?

说话人1 26:46  
So now in this problem, we are trying to show something. The smallest number says, if you play the game like this, you can always find a triangle, small triangle whose three verses

012 respective. No matter how you label it, as long as you follow my rule along the edge, no matter how you label inside, you can always find a triangle whose three vertices are still want to. That seems a interesting problem, but very

说话人2 27:25  
hard

说话人1 27:25  
to prove if you do not know the tools, right?

说话人2 27:30  
Okay?

说话人1 27:32  
But when you see my picture here, you can already have some guess, right? Because i'm drawing some edges here.

Now, can you guess you, according to what rule i'm drawing these edges? Very good. 01 edge. Basically, when your triangle is 01,

I draw the edge.

说话人2 28:22  
You

说话人1 28:23  
are 01. I draw the edge.

I draw those edges whenever I see a zero one segment, but you still need to decide the direction of the edge, right? What's the direction of the edge? What do I do like this?

说话人2 28:46  
Sorry. Yeah.

说话人1 28:48  
What's the rule to decide the direction of the action that zero is one? Bonus,

说话人2 29:05  
second one.

说话人1 29:06  
That's the direction of edge. Greedy, ok what kind of greedy? Choose the zero attention? You want? Yes, but ii draw edge for every 01, right? For every zero, want to have an edge, for every zero want to have an edge. I will not miss any of them. Very good on the left of the arrow. So if you go up like this and zero is on the left, one is on the right.

That's the the way to decide, right?

For example, why you go down? Because you go down, zero is on the left, right? One is on the right. Anyway, when you face the direction of the arrow, zero is on the left, one is on the right. Okay? So when you draw these arrows, then you see, actually, these edges.

Now the thing below is a bit complicated. Let's ignore them for now. Let's just focus on these edges. Okay? For these edges, there's one very interesting phenomenon. What's the phenomenon? These edges is it possible that one node has two outgoing neighbors? Is it possible? Ii have one. I'm one little here here. Notice triangle. Notice triangle. Is it possible that this one come down and also all there? It's not possible, right? Because you go down, that means this is zero, right? And then how can you come back to me? Because you need to get up to zeros on your left, right? You can never go. So everyone only has one outgoing data, right? A similar reason. Everyone only has one incoming neighbor. You can only come in from one direction. You cannot come in also from here. That's not possible.

说话人2 31:22  
Then what does it remind you?

说话人1 31:25  
What is about one and coming? One? So that's exactly

说话人2 31:31  
which problem

说话人1 31:32  
end of the line problem is one. Only one outgoing when they coming. Right? Let's continue.

Because now the setup becomes very similar. Now. Let's see. What's the nature of a travel whose three vertices are? 012? If a triangle three versus 012?

说话人2 31:57  
What's special about?

说话人1 32:02  
I heard it. So basically, for these, they are the blue channels here, the blue channels. The important feature is they are either source or sink. Because for 012 triangle, there's only one edge, 01. Either somebody come in, stop, all people out, stop. Sorry, not go out, stop it. Go out. No, come in. So basically, these blue triangles are exactly the end points, right? The end of the past or the beginning of the past, right?

说话人2 32:42  
Okay?

说话人1 32:43  
Now this exactly a finding a blue triangle, right?

Finding a blue triangle here Is exactly final finding the one end point, right? Either at the sink or the source. Now basically finding such a triangle. In some cases, in the reduction to national equilibrium, it corresponds to finding a national equilibrium in the general overall games.

说话人2 33:12  
Okay? Now,

说话人1 33:14  
let's see. There's still one thing, which is not very clear. How can you see? How can guarantee in this figure? It's not all cycles. If it's all cycles, you cannot find blue triangles, right? If everything is cycle, right? In f is every important cycle, then you cannot find blue. China. That means this memo is wrong, because this memo says, if you leave it like this, there's always a few China in the system. That means you need to persuade people that you can first find one node, which is a thick, sorry, source. Right? If you are able to find one source, that means one. You at least have one sink. If you have one thing, then that one is blue triangle, right?

Ok now the remaining task is to find a source.

That's why I draw so many interesting curves here. So this is why I draw these, because originally you only have the triangle, nothing below, right?

Now I draw extra edge. I connect this zero with this one

说话人2 34:26  
on edge

说话人1 34:27  
zero with this zero, I connect this zero with everything here. All right? And then I also draw the link according to the same

说话人2 34:39  
principle.

说话人1 34:41  
Guarantee zero is on the left, right? But this zero is on the left, this zero on the left, right? Then this one is also zero on the left. Right? Now I can read the whole picture. Now I have to do this. Then I very safely say I find the source. This is the source, because this is 01. Going right? Can somebody else go this place? No, I didn't find any 01 edge, right? Because the outside area, right? There's nothing that can connect to this one. If you want to connect to this one, you have to use another 01 edge, but there's no other zero edge, which can guarantee that, well, at least this side, there is no 01 edge. This side there is no zero one edge. Only this side has 01 edge.

When you go in this point will be your source. It's a very safe source. Now, since you have a source, then if I follow the path, right? Like this, I can always find one thing for sure.

说话人2 35:47  
Right? So that

说话人1 35:49  
is the truth. Any question? Okay, also, actually, this is very beautiful, although I didn't tell you the whole story about the connection between computing national equivalent. So if you are interested, you may go to related literature to find out more details. The proof of sports minus lemma, it is quite interesting trick. Then actually, this is only a two d version of spin and slammer, because spin and slammer is more general. It's a it is applicable to higher dimension. But in order to explain the concept more easily, I used to be version of spinner cell.

说话人2 36:39  
Right? Okay. Yeah.

说话人1 36:45  
All right. So I think that will take 10 minutes break before we come into different types of games.