### AL\_week6\_lec6TUgame1-20241014

说话人1 00:00  
Just now,

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actually,

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we already have a method to generate a stable outcome was certain confiscating, right? We just all the players months before and right, and then give them a clear marginal contribution on top of the previous survival agency. Ok and they can also see that actually, you have a lot of possible state outcome for this kind of conversations, because they're altogether n factorial permutations, right? Each communication, you can have one possible payment, a lot of stable outcomes, right? In the game. Okay? Now, this outcome can be computed as long as you have another time oral to the characteristic function. As long as you can ask, the oracle, what's the value of this subset? What's the value of this subset? Then the payments, right?

As far as the oracle is very easy for you to compute this stable outcome. Now.

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But you see,

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in reality, the subset, the possible success, it is, exponential excise, right? So that also means that even though you need to when you represent the game, you need the exponential number of parameters. But when you compute the same outcome, you do not need that much time. Okay? So much about conversing core. Now this is a simple game. What is the core in simple game? In a simple game? First of all, you can always assume that you form the grand coalition. Why? And it's not very obvious.

Although the grand coalition, a it's one, it can be one, right? But sometimes I think that there will be some complicate issue. If you assume what I am a religion forms, because there might be some possibility that if it's not brand coalition, there is a call, but maybe not, because earlier we have seen that, sometimes simple game that's not at all, right?

Now, let's simply decide before even grand polish is useless. But it would be the case that in simple game has for then recollection, always has a stable money distribution ok so let's see about the grand college.

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Right?

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The property says, if you want a simple game to be have a call, then there must be a veto play. Okay? Simple game has support if and only if it has a veto play.

Now, there are two directions about this. Right? Now, let's see one direction. First. If it has been clear, then it is a call right now. When the ground condition is zero, it's very easy. Why? Ground going to zero? That means every quality is zero, right? I just form the grand coalition, give everyone $0. If it's very stable, nobody is waiting for us. Still. Right? That's why we only need to look at the case of grand coalition. That is one. All right. Now, because we assume there's a legal player, suppose I is a legal player. Then what can we say? I as we complain. Now we need to design way to distribute the money, right? So because we don't really know it's very powerful without the legal player, no way, right? So very natural education is that giving people there a lot. Every now everyone else is zero. Is this good?

Now let's see whether some coalition will have a chance to deviate. Right? There are two cases for this deviation. One case, this coalition includes this video player I right? If this coalition includes people player I then this code is already has $1, because we don't get $1. So if the coalition includes little player, I then this coalition already gets $1 important. Then it has no incentive to deviate, because after deviated, the best you can get is $1, right? So no chance to give it no incentive to deviate. It's okay. If this college, so you drive somebody and say, please, we together, we move out, right? And unfortunately, you try, you included one little player. Then he tell you I already get $1. Right? That means our group already gets $1, and then you go out, you also get only $1.

Then why do we go out? Right? The other case, if your coalition does not include people there, that does, what does it mean? It means you can get nothing right? Because we said without little player, nobody can win, right? If you dress up a little, please, let's go out. They found out that. Let me do this. My group. That means we do this outside is voting against you, right? So you cannot wait. Right? She also have no incentive to go out, no matter whether it could be to play, the coalition has no incentive to go out. That means this is our company as well single, right? The single. Okay, is there any question here? I it will be a bit fast now. So let me know if you have a question. And one, we are going to prove the other direction. This direction will finish it, right? If you have a bigger player, then there is a single outcome, which means there is a call. The calls are empty. Right now. The other direction, as well as you have a call, right? Your course is not entry. And then you need to argue there is a beetle player, right?

Ok now, again, if you graduate, that is zero is very easy. How about the grand collision that is one? Now, why it is very easy? Because in this case, what is true? If vn is zero? What can you say about equal player? If vn is zero? That means what? Lovely love it again. No 13, no one mean. And what does it mean? No one mean. We won't suspect about become clear, right? What can you say about? We complain. If vn is zero, what can you say about? We complain? They are nominal. What's the definition of little player? There is some risk here, but it's actually very easy to understand it by carrying the answer.

So what can you say about me to the

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people in my room?

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So how many people do you have in this game? 120, or something else, especially for those who still didn't get bonus? It's very different. In this game, vn is zero. How many little players do you have in the game with hand and say his hand who said that? That's the answer. All of them are little players, because no coercion can win, right? Yeah. According to the definition of people player, without this player, nobody can win as a group. The remaining group can win, cannot win, no matter which player you choose. The women group, they are zero, right? Ok so everyone has been prepared. So this is the one that they also want to hear from. Vn is zero. Everyone is v four player. So it's automatically approved, right? Because what we want to prove is there is one people there, ok so it seems that this easy case is not necessary for, because if you think opposite and the opposite, there are two jumps in the arguments.

Anyway, listen to me. Now general case, if vn is one, then what happens? Vn is one. Then we prove that contradiction. Suppose there will be compare. Can we arrive at some contradiction? Right? So let's see. No people clear.

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And then we

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look at an arbitrary payoff factor x we know that all the x they add up to one, because grand coalition that is one, so they add up to one. Then that means there is some xi larger than zero. See, this argument very familiar, right? If all the people together get $1, there must be one person who gets something, right? Who gets some, because there's no people there, right? What does it mean? It means that no matter which person you get rid of the remaining group can win, is it right? No matter which one you remove, the remain is set, the value is not. Because otherwise, this person we compare, again, one place

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we told them is

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removing this per person remains that there is zero, right?

Now, we are saying, no, be complete, no, be told there is no matter which person I move, the remaining set value is one, is it?

Now that means if I remove this, I the remaining set generator of anyone, what's the consequence? The consequences? How much money does revenue set get? Because I gets larger than zero, right? The remaining set get an amount of money less than 1 gram. But we just know that the remaining set, if they work alone, they deviate, go out, they can get anyone right? Okay. So will they deviate? Yes, because currently they get less than $1. If they go out, they throw away this I person, right? They can get $1. So why not? Without division happens, right? That means this outcome is not stable. But

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why this is

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not stable? Because remember, we assume this direction, right? If you tell me there's a noise before, right? How can this one not be stable? The reason as well? The reason is we assume there is no people there. That's the key issue here, right? Because you made this assumption, we have a contradiction. Somebody will deviate. So the only thing that can be wrong is your assumption. So if your assumption is wrong, what's correct?

This finishes the proof that you have been okay, must have a bit of it if you have a not empty for. All right. Okay. Yeah, aaa bit. This kind of proof is not so easy to. You think about, because there are multiple jumps in the middle about the definition of legal player. After lecture, you can spend more time to think about it if you still didn't get what it talks about. Okay? This is the whole thing about the single game stability. Now, next one, we already know that there is an equivalent condition about simple

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game,

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have 94 and the legal player. But now let's see whether we can check a single game has another equal. Now, it is the same as checking whether you have a little 5th, right? Okay. Then there's one property. Actually, the property says, in a simple game, a payoff vector x is in the court and only if you give money zero to any non vehicle player. Right? You can, because earlier we say you have a core, you have I I give people pay $1.

Now middle player is zero. It's stable, right? But how about in some cases, you have multiple middle players, two little players, or three little players. How do you give them the money? I actually don't care. I want to make sure that non legal get zero. They have no bargaining powers. They should get $0. Why? Because if you give them more than zero, the remaining people will need it, because the remaining people can win, right? Because this person is nominal, the remaining people, their value is one, right? If you give this non legal more than $0, they have to keep the remaining less than $1. That's not good. Because then the remaining will deviate. Ok that's why you have to give no legal player $0. Right? Now, checking player eyes veto is easy. You just check this one. You asked the horrible whether removing I have a value 10. If value one, it is not equal. If it's not a zero, it's equal. It's very straightforward checking.

Example games, checking on emphasis of the core or checking if a given outcome is pretty easy. Right? As long as you can carry it. But if you have some structure about collision, then it's no longer easy. Right? Now, because here we assume ground coalition, remember, right? We assume in simple game, we use the ground coalition. And then if you allow the coalition structure, then some argument may not hold. Right?

So now that's move on to something else. Earlier, in the last lecture, we talked about national program. Also, we talk about the so called approximate national equilibrium. Approximate national equilibrium is what? It means that although changing my strategy can give me a bit more money, but not too much, you can only give me excellent war, right? And then i'm not going to move, because my move also consume me some effort. Moving my strategy.

If you want to give me xx epsilon, so little money, It's not worth it. That's the definition of epsilon nash equilibrium. Now, here, for call, we have a similar concept. We're trying to define the so called approximately stable solution. Wus

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core

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to be the following excellent core. So remembering core the definition is, for any coalition, the money you get is at least the money you can generate. Then in that case, you will not deviate, no group will deviate. But sometimes it's too harsh. So we add an episode here. You say if the money gets is not too little, it's larger than equal to the money can generate minus epsilon. So which means we are okay to sacrifice extra money. That's fine. Because anyway, going out to do alone, right? Do by yourself, is also an effort, right? If this effort is more than epsilon, i'm not going to go out. Right? All right. Even though i'm losing some money, right? I'm losing some money compared to my working alone. I don't care as long as this amount is not too much.

Right? Absolutely. If it's 0 . 01 of that, very happy a right, i'm not a lot. This is called absolute core. If deviation does not bring more money, sorry, does not bring too much money. Okay? Of actually not say bring will not. So pda will not bring more too much money, which means the value generation will not do p to be too big. It is less than equal to the money gets plus epsilon. Basically, these d mediating will not bring you money more than episode.

Then it's okay. It's episode. All right. Epsilon core is usually defined for super additive games. And then we are giving one example here. In this example, we have three players. The value of coalition is one. If you have at least two people in the coalition, two people together. Very well. One person, 03 people possible, right? This is the game. And then let's see the following payoff method. I I give every person 1/3,

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right?

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By doing this, it is a 1/3 call.

How to bargain that? If I show the next thing, this is a little about. So how about do that? You see that? Two people together, they can generate value one, right? But here, every two people, they get two third dollars, right? But they go out to get $1, so they will get how much more? Only 1/3 more. Right.

Now I get 2/3. If I go out, I get $1, I get 1/3 more, right? So no matter which two people, they can only get 1/3 more compared to the current money they have. So it is a 1/3 corporate. And the one circle says, no matter how you deviate, right? You get, at most, excellent amount of money. And now this epsilon is 1/3, right? Okay? Nice. This is 1/3 more. And then on the other hand, we can show that epsilon four will be empty. If epsilon is less than 1/3, what does it mean? If I want to make this 1/3, 1/4? It's okay. So no, it's not okay. Any better less than 1/3 will make the core empty, will make the absolute core empty. Let's see the reason, again, is very similar as before among the three people, right? Because the three people divide $1, so you divide $1 to three people. There must be one person who gets at least 1/3 dollar, right? Okay, that's good. That's easy. So suppose there's one xi it gets at least 1/3 dollar.

Now, for the remaining

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two people,

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they get, at most, 2/3 dollar, right? One person gets at least 1/3. So the remaining two people get almost 2/3. So these two people, they go out, how much money do they get? $1. So they will get at least 1/3 more, right? Is it right? So they get at least 1/3 more. How can you make your absolute less than 1/3? If you make the absolute last month, then 1/3, these two people go out, right? Ok so this is why any value less than 1/3 will make the excellent call and not single. You see, this 1/3 is the best thing to do, right? For the x report. And then also brings the next definition of least call.

Okay? Just now we say outcome is excellent for this is that most episode, right? This is that most episode. Then we can't define the minimum epsilon such that epsilon core of g is not empty, because this is the one we want. Most. If absolute can be zero, the core, right? In the absolute 000 core is just for, right? We want this absolutely as much as possible. If there's no core, no zero core, we want a 0 . 1, 0 . 0014, not, then slightly bigger.

Anyway, what this estimate is, as much as possible, we define absolute star to be the smallest epsilon such as epsilon cosine. And we call this epsilon star called the least more ok epsilon star called is called least code of g which means that's the minimum, right? The closest ever solution to a stable solution. Then we say this epsilon star g is the value of the score. And then in the example I gave you earlier, what's your call? Right? This 1/3 is the best you can do, because you cannot for any epsilon less than 1/3, there is no epsilon core. 1/3 is exactly the value of epsilon star. And then the least call is just as wonderful. This is the best, right? The smallest excellent you can have to put here, such that epsilon core exists. Right? This is about the

called leasehold. All right. I think that I will give the remaining time to my ta in case she needs a bit set up

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time.

Let's finish our tomorrow, one, the three, three questions. You can finish the last scheduling weeks test. Then let's together with this question and then give you some time to think about themselves. So about this scheduling, this testing, we have several jobs that are need to be processed on a single machine. They have any jobs is job set, 212. And they need to be possessed on a single machine. Associated with each job. We have two parameters. One is the new test, and the other is the check. One is the check together, the other one is the new test. So the these jobs processing time, but different from the normal setting. Now in the normal setting, normally we only have one person time for each job. Indicate a lot of time that you but that you need to finish that job.

But now in this question, we have two parameters, two processing time. One is a check, which is smaller one, and one is the bigger one, which is in hat. And we assume they are all the negative. And we can do a special operation of if we test a job j then it's taking time will be the lower one. Do you check? If we do not test the job, then it will be the pej hat, just in time will be a bigger one. We can check, we can choose, at most k jobs. Here we can only test, at most k jobs to test, test them. And our objective is to minimize the main span. In other words, here, the mixed bank is just the summation of all the jobs to take time. Let's see this example. If we have three jobs, we have three jobs, and so one, three, and is to check this is the cut.

Wait, for example, I will answer this. Write three numbers. For example, this is three or five. This is 678. So for example, we have three jobs here, and these are the check and check them to help. If we passed the job one, then the same time will be for 43, for job one. If we do not have the job two and three, then their presenting time will be the seven and eight. Totally, that makes that our objective value for section has arrangement will be 3 + 7 + 8, is 18. So ii think it is, I do not interact with another situation. So now our second and objective is just like this. We have three questions. First, if the author can check, all these numbers are zero, what do we do to minimize the summation of persistent time? And the second question is, if we do not have the assumption, which is all the contracts are zero, then what should we do to test the choose which key jobs to test to make the expanding math?

The third question is, if we do not know the value at all, this second question also assume that we know the to check and keep we have.

But the third question is, if the algorithm does not know the value of check for each job, and we know that he had, then we are asked to design an algorithm such that the algorithm is two approximation. And we need to prove this. Now, the below time, 10 minutes, it's for you to think about ourselves. I think this is the first be trying to do.

So. The first question, suppose all the can change. The lower processing time are zeros for all class, which in the jobs should be tested such that makes sense, minimize. So. Do you think that is that right? Good as the patients in jobs with the largest

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42. So

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the first largest p jobs, those jobs is first in that is the he had, right? Two thousand and half. The second question. You remove the assumption of the return of zero. And we know we can check in the past, which you guys should be testing something that will expand the scene. Yeah. Two jobs is the lattice in the end.

What's the third question? It says, if we do not know the value of the check for each job, in other words, we only know the p hat. So what kind of values that we can design? And such that the average do it. So the same with the looks, like we have not many, many like the possibilities to our we only know the impact. Let's see if we use the same algorithm as the pd as the first one. What's the estimation ratio on that?

Let's first introduce some notations to describe our objective. Let's be the probably the job site is j right? I use this set is right. Educate all the jobs. I want to use this circle a means so jobs in this area, I use the ja denote those set of jobs tested by algorithm. This jja is the set of key jobs. You just tested algorithm. Our algorithm is to the pay jobs with the numbers of pj test. And is jar is the outside area, which is those jobs are tested by a challenge.

I also want to use this circle all the job note, the set of jobs passing by old optimal solution. You still o minus the set of jobs not to be back office. They have probability in chinese I so this if we have these notations, then our detriment is to prove. If I use this alg is the next balance of the algorithms have test arrangement.

So basically, to those jobs in the end. So if this jobs are passed by everyone, there is the second time will be check. Plus you may want. They are asked by resentment. There is time will be passed. 1 group is two approximation. Algorithm. Division is our expand. We want to minimize the expand, which means that makes that algorithm is not so much. It can be outbounded by the optimal solution. We won't. We larger than the optimization by doing too much. So is group two times opti use this three operate is character. The new style of operation. Here is optimal solution equals to you, are also is in the, I think, will be jvr and jo this is our objective is to capacity how we prove this. Same. Our group is one.

This student just says very. So do you have some idea about if use this donation ok so further, let's first observe that since our algorithm, this decision criteria, we select the k jobs is the largest PK impact. Right? Now, I want to like build a connection between these jajar and jojor right? We want to outbound into this numbers, and we want to make it bigger and we want to find out about we want to make them bigger, but in the direction, we do not want to make it actually bigger, but in the direction which is connected with this optimization.

So firstly, you can see how we introduce our algorithm behavior. This one, if our algorithm has to, the key jobs is the largest then for those jobs not tested by everyone, it's PJ had compared with this one, which is larger. Or I hope I haven't sent the be answer, answered out.

So a a modest question is how we how we make it bigger and connect with the optimal solution? This one is another one, as long as what and why. And the hint will be consider our algorithms behavior. I'm gonna choose the jobs is any largest. Let's first try to up and down with this. Is something relevant to options of Mexico. Some.

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So I

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think you can say loudly for one sort and selective, but I must probably. Mhm. For pgok I use other words to ask this question. So this one, which is bigger, isn't it? For jobs not tested by Alexander? Those the summation of PG had, before I was, I'm not actually buy up options,

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not as far as

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all of them. Right? And right? Hand is bigger, right? Why?

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Because the job tested by the optimal, everything is smaller.

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Okay?

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Is that the PP

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p check is that is smaller

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than the, we shall find, check them by and

说话人2 51:44  
yes, rough, roughly, because he jumps. The algorithm has the largest. We have, right for the not has been done algorithm. Those and also, these are all contained pay jobs, right? This one can be added on is this one? So oki was thinking you need to. So this is actually a key step for this proof to build connection between algorithm and optimization.

Also, let's expand this to j belongs to, let's use the optimal solution according to the intersection between optimism and average real solution. Let's switch this term up. This is PJ check incidents. This one is exactly equal to these two things, right? Because this is intersect with Jo and this is the intersect with Jo by the Jo union. Jo revise the whole set. This one is exactly this one. And this one is smaller. This one. Some of these three things are, let's compare this one and this optimal solution. If we combine the, let me write this over here. If let's see these two terms together, see, this could come together. This one is exactly equal to this one, right? This one will be a part of this one. If we see these two things together, then they will be smaller or equal to one. Optimal solutions. Makes that, right? This is the development list, and this is a part of this.

The whole thing is that they are marketed by 10 PT and also this one, since we know the paycheck is model equals that we have, right? This can be unbounded by new belongs, a 705 to the new hat. I change it to PJ hat, and then it will be equals to, it will be a part of this one.

This one is a part of this term can also be unbounded by 10 PC this is more this part is smaller or equal to one ot and these two parts is also smaller than one 0 t probably we have 200 t are part is here. Or is there? I'm here correspondence between the they always send it out and

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so don't worry. The notes will be posted on campus. With the earlier session is a beautiful notes, actually have more notes. So 04 we

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and

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also, sorry, for the delay. And then I promise the next lecture I will it should be the finance study and six four fifteen ok and also welcome to say denying a question about this assignment materials. Stay here on time. Is that simple as a sign? And also about, I also stay here to see the last question.

说话人2 56:00  
Hey, guys, that's it.