### AL\_week6\_lec6TUgame2-20241014

说话人1 00:00  
What a single game you satisfy

说话人2 00:04  
some

说话人1 00:06  
conditions, because this the value of any coalition is either zero or one. This definitely matches with the vote weight weighted voting game, because the weighted voting game, the value of coalition dose is 01. You don't win or you developing right ok this is first second set. One of them. Right? There are more people in the game, the better chance you are one. Right? In other words, if originally you are one, if you add some more people, right? Some more people from the you can see, and then the value is is still one. So basically, you will not lose because you have more people. All right. I we did motivate, you have a similar concept, right? If you have more support, you only doing better, right? You will never say I have more people support me. I lose, right? That's gonna happen. So it seems that these two definition, these conditions, they are exactly what waiting for the game is thinking

说话人2 01:10  
about, right?

说话人1 01:11  
Then we say a collision c in a single game is winning. If vc is one, otherwise it is.

Now, based on analysis, you may think only be which one engagement simply they are the same thing. What do you say? It is a generalization. The generation is different game, right? But according to what we discussed about these two, it seems that they are the same. Now, let's see whether they're the same. Now, in the simple game, we will further define two concepts about players. What is called now clear, what is now clear. Now clear means, no matter where it goes into, it does not affect the results. I joined this route. If this route earlier wins, it still wins. If earlier loses, it still loses. Right? So basically, this person is just nothing, right? You can consider this now player. It is nothing. No matter which group it joins, it does not affect the result of the group. Right?

Now, there's another extreme player called veto player. What is b to pb to p says? If this person does not support you, you will not be one vote. In other words, it means that without I the value of the coalition is zero. Right? This is the grandfather is in, right?

说话人2 02:41  
Everyone, though,

说话人1 02:43  
I support you are zero. Now is true. This is because by definition, we do clearly just says c right? For any c for the biggest one it wasn't, right without eyes support.

Now, let's see whether these two are indeed the same. Now, firstly, we know wait for thinking, are single game, because we already analyze these two, right? But we did one again. Let's look at it if you try. But when the game is, it's my boss, right? This is true. This is also true. Where is the game? One? Game is simple game. It is a kind of simple game. A simple game. Is it with a boarding game? Or can it be modeled as a way to vote for you? Welcome to see. In waiting for the game, you need to specify n plus one, premise, quota two, then w one, w two until wn right now, the question becomes, if I give you a simple game, can you model it into

说话人2 03:55  
w

说话人1 03:56  
a sequence of q into the q and the sequence of w right? Such that whenever the summation of w in the coalition is at least quota, you win.

Okay? Now your expectation now becomes, it should be wrong, right? This structure should be wrong. Otherwise, simple games will not be important. Generalization of waiting for the game. So why is it wrong? Is there some simple game where you can do nothing? Right? You cannot go all the way a with a golden game.

Now, let me give you one example that this example is also very simple. You will altogether have four players, and then these four players, when can you win? This is a simple game. I define a single game. When can you win? You can win. And only if you have a support, you have a supporter from 31, and 3 to get support from 131 of them. And you also get support from 24, also one of them. All right. I at least you at least get one support from group one and one support from group two, then you will, right? You get the value one. Clear about this simple game. Let's continue. Okay? When can we, for example, if we know our target is to decide as qw one, w two, w three, w four, because we need to decide those parameters such that it comes away the boarding game.

Let's see whether they do it. So where do you win? If you have support from one and two, can you win? 12, can we? Yes, right? Because I said only if this is not empty, this is not empty. They have support from one and two. Those are not empty. Right? So we that means w one plus w two is larger than equal to q good.

Now, how about if I have support from 3 to 4? Can we? Yes, wait. So you have w three plus w four is at least from these two summation of w is at least two

说话人2 06:20  
cube. Right?

说话人1 06:26  
Now you can guess what i'm doing? Talk about next.

说话人2 06:31  
No, can

说话人1 06:32  
you win if you have support from 12? Okay, i'm sorry, my dad. Can you wait if you have to go from one street? No, right? I guess you don't have support from

说话人2 06:46  
the other group, right?

说话人1 06:47  
You cannot wait. W one plus w three is less than q

说话人2 06:51  
right?

说话人1 06:52  
Similarly, if you have score from two and four, can you win? I know. W two plus w four is also less than qm and then a you have this is a contradiction. Because you, on one hand, you say submissions at least too cute. On the other hand, they say this is, at most too cute. So there is no way to find such w one, w two, ws beta four. But this is a simple game. We know, not every simple game can be represented as weighted voting game, right?

Again, it's only as a slow trip, right? Now it's time to define the stable search. Now, we have to come back to the last example. Now, first questions is this example? The same as the one, the example I gave you earlier? No, right? Because the one I gave you earlier, $6, $4, $3 now is $4, $3, $3, right? They're different. We define or write down the value of different coalitions. Everyone's individual cannot find nothing. If you have cnn together, they buy 500, c and p together, 500. Tp together is zero, right? Because they only have $6 not enough to buy the cheapest ice cream, which is seven, right? So this is a coalition with two people, two children. There are about three for money, three together. It becomes $10. They can buy the $9 one, which is 750, right?

This game, we already know it is super additive game, right?

说话人2 08:52  
Now,

说话人1 08:56  
the question is how to share that? Ask,

for example, can I share this? Is it good? No. Why not

说话人2 09:15  
cnncnn more.

说话人1 09:19  
Very good. So cnn right? We can get 500 grams, but here you will pay 400. They are very unhappy. So give me two lessons too little. They go out, right? We go out by ourselves,

说话人2 09:34  
right?

说话人1 09:35  
It's not good. And the reason is cnn they can deviate together, ask me.

Now we define this stability. We define the core of the see you game as a set of all the stable outcomes.

What's stable outcome? Stable outcome is the outcome where no coalition wants to deviate. No subtle for people. They go out, they can get more compared to what they get under the current structure. You are getting more. It's nothing. If any successful ultimately cannot get more. That is stable, right? As far as it's stable, we say it is in the core.

This is a definition I apology is, could you actually is a pair of cs and x because this outcome, we are talking about whether the outcome is stable or not. Right? If you want to say outcome, you have to specify the coalition structure. How do you divide all the people into groups? Right? The division already needed to be decided. And then, so what does the division do? Actually? The division restricts you about the possible distribution of money to players.

说话人2 11:09  
Right? And

说话人1 11:10  
also their collaboration pattern. When you have certain division, how this will generate this money, the school remedy is not money, and this money will be divided among group member friends. Division together with the payoff factor will be the outcome, and the call is all the stable outcome. What is stable outcome? Stable outcome says no matter for which coalition c the submission of the money you get in this coalition is at least the value this coalition alone can generate.

So that means this any coalition, if they go out, they can only get less or equal, then compared to the money they get.

Now, right? Okay? Now, here, this g the graph, the game g we do not assume that to be super added.

For example, when you have this one. So the red route, $9, green group, $4. And I have another group, which is crossing these two groups, two, player two, and player four. They can generate $7. In this case, you see that this one is not in the core. Now this one is the one we give you earlier, right? 31, right? But in this one, you see player two and player four, they get altogether $6. If two of the four, they work by themselves, they get $7, right? This is not stable. The outcome I gave you earlier is not stable if we have this extra

说话人2 12:57  
possible.

All

说话人1 13:01  
right. So again, ask me again. I will just repeat the setting. Just now. We already know that this one is not in the core, because b is lambda n the money they get now, right?

说话人2 13:22  
Okay.

说话人1 13:23  
Now this one is in the goal in the core. The reason is, you see any two people get $500, sorry, three grams, right? And then three together get 750. So see any two together, they can almost get 500. So if you divide it in such a way, nobody will debate, right? Because they cannot get more. Okay, so this one is good. This one is also in the call. You get all the ask me to the first person.

说话人2 13:54  
Right?

说话人1 13:55  
Again, we have an argument, and this argument is a bit more persuasive. It's okay. If you choose to go outside, right? They are not satisfied zeros. They are not satisfied. You choose to go outside, you can buy nothing. Only $6. I give you nothing. It's okay.

说话人2 14:12  
Right.

说话人1 14:14  
Again, i'm a bad guy here, but this plus this time, this this vector is different from the vector. I introduced you earlier. Remember another vector, something like the 0 800, 200 that one.

So that's a slightly different because that one probably is not saying, but this one is saying you give all the money to the first player player with the most amount of money. Right? Any question here? For the basic definition of call, if then we move on, let's see. Core actually is a very attractive concept, right? If you have four, then the outcome is stable. Everyone is happy, right? Nobody will be when you go out, but sometimes some games do not have four.

说话人2 15:13  
Now,

说话人1 15:16  
let's see. We come back to the way we vote again. Where is the boarding game? Now I set the quarter between then everyone has a waiting month, which means you have three people as far as I get support from two people. I wait. Right? Now, this is just an old outcome to see whether they are safe. The first one, everyone was alone, everyone walks alone, nobody will find. Everyone is only generating value zero. They also get 000, right?

But then grand coalition, You put everyone together, they can generate one point. Their outcome is one. So you have $1 to divide among three people. All of them are happy, right? This one is not stable. You let everyone play by their home is not stable, right?

说话人2 16:18  
Okay.

说话人1 16:20  
Second one, if you have one or two together, three individual. Is this stable? Now, here you will be seeing a very interesting argument. These are the steps, one and two work together in the way, right? Because two people win, right? The way they get $1. So one and two, they would divide this $1. Now. When they develop $1, there must be someone who gets positive amount of money, one or two, one of them, one two. This is one lesson two. One of them gets more than one or more than $0. The simple symbol gets more than $0. Then. What does it mean? It means this one gets less than $1, right?

This one gets less than $1. He says I will join player three. We too, go out. Remember, at this moment, psb gets nothing right. This one gets less than one. If this one joins players b they can get how much dollar? $1, right? Because any two people will get $1, right? So they get explained strictly more dollar, more money compared to before. So this person will go out to anyone in the winning group. If it gets less than $1, it was willing to go out with the one who was alone.

Now. Is that right? 123 is not simple. A similar reason, 231, or 312. Any of them not stable because in the group with two people, there's also always one person who didn't get all the money, right? This person will deviate with the one who didn't get any money, right? Okay. Now, any questions here? It's not very straightforward, but I hope that everyone can follow the argument here. Any questions? It's? Okay. Now we still have one more left, right? The

说话人2 18:47  
last one monthly

说话人1 18:50  
graduation. Is it stick?

Now? Again, we are using similar argument, grand coalition, right? The grand coalition that means there must be somebody who gets some money, right? That person gets some money. What means the remaining two people gets less than $1, right? This person gets some money. These two people get less than $1, and then these two people think, why not we go out? Right? We go out. We get $1 exactly, right? But we are with you. We get less than one, right? Okay? We are willing to go out. That's the argument. Somebody gets strictly larger than zero. And then people, other than this person, they are going to go out because they get less than $1. And if they go out, you get $1, which is strictly better than not. Right? You see grand coalition also not stable. Okay. Very interesting. It's a very simple game, but

说话人2 20:23  
the code is

说话人1 20:24  
ends.

说话人2 20:26  
Right?

说话人1 20:28  
Okay, although we say this is very easy to understand, but ii know it's not so easy to understand right away.

If you didn't understand yet, you can spend some time during the break to see whether you can catch up with this. And also we'll have similarly long about about this kind of thing. Somebody has you have to do 101.

说话人2 20:54  
Good.

说话人1 20:55  
Empty core is not good. And then very naturally, people will think, is it possible to find a certain class of games

说话人2 21:07  
where all

说话人1 21:07  
always exists, right? Because in general, in this game, right? In general, games may not have coal, right? They have empty coal, but is it possible that there's a subclass of games where core is always not empty? You can always find some stable outcome, in the sense.

说话人2 21:30  
Right?

说话人1 21:32  
So that's why we introduce the next concept. Next class of games or complex games. What is complex series? Now? Here is the third in class exercise. After this, you are ready to the submit that assignment on campus. Okay? The conversation is what it is that now, before we define conversation, because we define how to say, because they find the so called super modular function.

Now, when you look at this in quality, actually, ii don't know like it. It's very, very complicated. I don't even know what it is talking about. All right. So it says if this division is true for any subset a and b then you say, is super model. I believe, no idea. I just do not say no one. And I say few of you will find this one intuitive, the meaning of this one is very unclear. I don't even know what it's talking about. Right?

So that's why usually we use this one.

说话人2 22:50  
Instead,

说话人1 22:52  
you want to know this one is much easier to understand what is this one? This one says I have two sets. These two sets, p is smaller, s is bigger, s contain c s contain c and then I find some elements I not in s so I will use my pen here.

说话人2 23:18  
Byebye.

说话人1 23:22  
T is s right? This is s content ci find a clear eye outside.

This is there. And then what does this one say? This one says, if I add I into t this is my marginal gain on top of t because ft is what t can generate, what can get, right? The amount of money t can generate, right? Then if I add this I into t this is a lot of money in a new set of generous, then I do the subtraction. What does it mean? It means that this is i's marginal contribution. On top of t right? In this result, he can earn $100 I and t together can earn $200. Then rs market contribution is $100. It's 200 - 100. It's extra $100. Right now. On the right hand side, what's this? It means every I is s right? I am I to si can generate this amount of money, subtracting the money as to generate. This is I marginal contribution to set

说话人2 24:35  
s right? Then.

说话人1 24:37  
What does this inequality be? It means that the marginal value or marginal contribution I can have increases. When the basis set becomes bigger, you join a bigger group. I can generate more marginal contribution, right? This is the meaning of super model.

Now, I believe that this one is much more intuitive than this one. And also in class exercise, it costs exercises to prove these to us. Actually, the same thing. This can imply this, and this can imply this. Now clear about my third a class exercise or pass on. Now write it down, right? Because after the lecture, nowhere to be found, I will not post the question on campus. Okay? Now you see super model module, it just means if any new player you add into a bigger set, the contribution is even more right compared to any smallest. Then similarly, you have super modular, you have somebody. The sub modular means you add into a smaller set. The contribution is more.

Now. This is already ok let's see some examples for complex things, right?

Now it's time to be 100 % focus, because on this page, there could be multiple bonus points.

说话人2 26:31  
Okay?

说话人1 26:31  
This game, c squared. This is a complex thing. Now, earlier, we know it is a super additive game. Now, we are going to show this is a complex game. Why? For example, you have two sets, ens t size is small, ts size is small st is less than s it means t is smaller, right? P is smaller. Now i'd say there's one player I not in s I add you on top of s what happens? Now, because you add normal elements, now the value becomes s plus one squared minus s squared. It's two s plus one. Then you do know t what's the value? Say, t plus one squared minus t squared is two t right? Two t plus one two, t plus one. Which one is bigger? + 1 is bigger, which means adding the bigger set, your contribution is bigger, right? Margin ok so this is super modular, good, very good.

So now first of all, this question, the ice cream games,

说话人2 27:40  
topics, a child,

说话人1 27:59  
bringing some money. He joins a bigger group. Can you always contribute more answer? Competitive chinese smaller in real life, I can become what do you mean it? Because when you have more money by

说话人2 28:34  
more

说话人1 28:35  
buy more, you can get more marginal, but it doesn't mean I have more money, I have the same amount of money. Actually, my figures will apply to my smaller group. My money is fixed. I always have been $10 for them.

Now, if we counted ones, I cannot use the exam because the original i'm thinking of maybe this group is have any more. I have to say this present, maybe this group has slightly more money. I'm sorry. No. This group has aaa certain amount of money, and then the first role also has certain amount of money. If I just joined, first of all, how much extra asset I can get, right? If I join the big group here, how much extra money I can get? Answer the question, not always. Can you give one example? For

说话人2 29:33  
example, for this group and like $4, 10 or 5 months, another three, you are in number five grade that

说话人1 29:43  
would be Who has $4.

说话人2 29:47  
The first css $4, another charge joint, right? Another child is $3

说话人1 29:54  
ok

说话人2 29:56  
by paragraph ok 0 ~ 5. So I the vs

说话人1 30:03  
so this one already know, right? So we can, this market contribution is 500 grams, right?

说话人2 30:10  
For a group that already is for three, they powered by 54 cs two.

说话人1 30:17  
Already what? Cs

说话人2 30:19  
four three,

说话人1 30:20  
43 up this way, right?

Then you have another three coming in

说话人2 30:27  
this. It changed from To 705

说话人1 30:33  
ok the last contribution is some 250. This is one possible example, right? So basically, it's about, if you join a larger group, it's a larger group earlier does not have any remaining money for you to do to buy high street zero and they join, you find out all.

Actually, I I cannot buy any extra, because according to this composition, ii cannot care with any remaining money in the first group to buy more ice cream. So basically, in the extreme case, the increase here, the increase here can be as small as zero. Let me just change the example a bit, and they will see it's zero. For example,

说话人2 31:16  
let

说话人1 31:19  
me see. So we can say one, $1, and that is probably six. And this is a 61. Sorry, this is one. Right? So this six months, they can already buy a $7 last week, right? You have one more dollar. No help. A dollars cannot buy anything. But $67 I spent, right? This one actually contribution. The market is zero. Right? Ok so it's not always convex, right? So actually, on the slide, there are some examples.

说话人2 31:57  
Okay.

说话人1 31:58  
First example about nothing to

说话人2 32:01  
the

说话人1 32:01  
share the examples is it's just too easy, but now there's a second one is question. Just now, we said that the obviously, the characteristic function is super model. Then we also mentioned the definition of sub, modular, right?

Now, any questions about some of them? Because i'm going to ask a question about some module. Characteristic function. Super model means joining a bigger group generates more margin, right? So

说话人2 32:33  
someone will

说话人1 32:33  
need what? The only a smaller group generates more marginal contribution. Now, my question is it's ice cream game. Always sub module. It is

说话人2 32:46  
characteristic function.

说话人1 32:51  
Thus joining a smaller loop, always give you more or more I contribution. Why not?

说话人2 32:59  
Because the big enjoy a small uk is 1,000, also thousand enough money for you to buy, ask them. And maybe when you look at it as the

说话人1 33:11  
no more money for you by so very good. So that's the point, right? So if you join a small group, maybe after joining, you still cannot imagine. Remember the $4, $3, without example, your $3 joint $3, is it useful? This is zero.

If one is zero, then how about if you have a 0331, you have a group of 31, And then another three counts. And this three will give you marginal contribution of 500 grams, right? To see the mastering name, not only, is not complex, but also its characteristic function is not some model, not super and also more stuff. It is just what I speaking is just what kind of game. They are super additive game, right? It's just as relatively meaning you group two people together. It's always better, right? More ice cream by talking. So this is about the complex eight. And then I hope that you now have a clearer picture about what complex games are.

Now, let's look at the other example we introduced earlier. Induce a graph game. So players are notes. Then valuation value of the coalition is the weight of internal edges, summation of weight, internal edges. Now, again, if all the edges, there are no negative weights, this scheme is complex, which means you add a node into a bigger sense. Your value increase. It should be larger than equal to you add into a smaller set.

Now, this is actually very intuitive, because you add into a smaller set, you may only contribute a few edges connecting to the small set. But if there is a large set, you can have more connections from

说话人2 35:18  
just like

说话人1 35:19  
this. You added a team. You only contribute to the edges, right? But if you add s you interview what three a is, right? I think the bigger step contribute more, right? You can add more extra edges into the collection. So you are heavier. It could be more marginal contribution, right? But there is a condition here. The weights should be non negative. Otherwise, you have more edges. It does not mean better because the value is small, and that is negative. It's not good. Right? So any questions here? Okay. Now it's okay. Now we are going to and ice, because the reason we introduce how fast being as well is that we want to find a class of gates where coal is always along embassy. Right. Now, we are going to show core is not empty for complex things.

Now, before I show you this, maybe we can do something first, right? What do I do? Well, the thing I do is we define and because early on the same marginal contribution, right? Because for complexity, what we are talking about is what adding a player in a bigger sense?

说话人2 37:06  
The marginal

说话人1 37:07  
contribution is bigger, right? Ok now we define marginal contribution according to a certain order of players, 1234, and 10,

说话人2 37:17  
right?

说话人1 37:18  
To say x one is the margin contribution of one on Top of MPC then x two is the marginal contribution of player two on Top of one. You add 2 to 1. What's its marginal contribution? And then add 3 to 2, three, x 312. What's the marginal contribution? I define Xi to be adding I into the set of one, 1234, and two, I minus one. I is marginal contribution.

But define these x lines in such a way.

说话人2 37:58  
Right?

说话人1 38:00  
Now, after the defining exercise, we have a payment back. I pay player r the amount of xri just pay it its marginal

说话人2 38:16  
contribution.

说话人1 38:17  
Now, we are going to show this kind of payment is a stable action for the brand.

Now, I think I will give you some time to think about it before I show you the result, because it's a quite interesting exercise. No easy part as well. Easy part is because if you want to prove x is a payoff vector, you need to show all the acts act together becomes the value of rank coalition, right? Is it true? This one is true because v one plus v one two minus v one plus v one two three minus v one two, so many things are cancelled. In the end, only the end will remain. Because when it comes to marginal in the end, ever together, it becomes valuable for the whole coalition. This part and all the exercise together becomes one is pretty easy. The

说话人2 39:28  
challenging part

说话人1 39:29  
is to

说话人2 39:29  
prove one

说话人1 39:31  
to prove that this solution is stable.

Now, in order to show it a stable, you need to prove if I get out, or if I get a subset of players, their money, their exile, act together, is at least the value they can generate by themselves. Right? Now, I will give you performance to think about how to prove it, how to prove that, no matter which subject, I think, right? And they can generate that, at most, their exile added together.

Now, during the group, you need to use the property of complex gain, which means super modular characteristic

说话人2 40:19  
function,

说话人1 40:21  
ok thinking you can discuss as well. All right. And I hope that by the end of discussion, some of you will have some idea about how to do that.

说话人2 43:51  
So

说话人1 43:51  
any idea, we will go slowly,

说话人2 44:04  
some

说话人1 44:05  
sparks to show up.

Okay. Now, what we said is now is that we want to show for any coalition, right? Cij something s these indexes I less than j less than something less than they are not continuous. They are not contiguous. It could be 134, 10, right? They're gonna skip some numbers in between. Totally fine.

说话人2 44:29  
Okay?

说话人1 44:30  
Like this.

说话人2 44:31  
Right? And

说话人1 44:32  
then will show the value of this collision can generate is, at most, Xi plus xj plus two plus xs this is our target. How to show that? Well, firstly, we split the value of c in two marginal contributions, which means it is you add not the items, which means you add an element I player I in the empty set. This is not. Then you add j onto ivij minus VI it's j is minus the division on Top of I and then something marked and conclusion on Top of ij and then s is marginal contribution on Top of c removing s this is the last time, right?

Lastly, just as the marginal contribution of s on Top of the remaining of c now, for each of them, I'm going to bound it using some quantity.

Okay? First, VI what is VI visvi minus vm is set, right? It is atmosphere. V 1234, and two I minus v 1234, and two I minus one, because this one is, according to definition, is Xi because Xi I is amount of contribution on Top of one to I minus one. But now we have the VI minus v empty set, right? It's also hard contribution, but this part of the tradition is also for MD seven, right? Right? Which one is not? This one is not why?

说话人2 46:17  
Convex.

说话人1 46:18  
Because it's convex or is super modular, right? And then I is a bigger sense. Give you more money. Right? So this one is bigger.

So that means your first term is, at most,

说话人2 46:33  
exact.

说话人1 46:37  
Then similarly into what? Vij VI vij minus VI it is a host v j v one to j minus one. This is James marginal contribution on Top of one to j minus one. And this is James marginal contribution on Top of I right? I is much smaller, right? One to j minus one is bigger. If you add j to bigger set, the marginal contribution is higher, right? According to the super modularity. Therefore, this one, again, is smaller than this one, right?

说话人2 47:19  
They

说话人1 47:21  
just keep doing this until last s right? And all the elements or players before su this model intuition must be very big, bigger than this one.

This, according to definition, is SS

说话人2 47:46  
right?

说话人1 47:48  
In the end, you add all the things together on the left side. And on the right side. On the left side, they are colored

说话人2 47:59  
terms,

说话人1 48:00  
which is equal to what they see, right? On the right side. They add together Xi right? X things. They are.

说话人2 48:09  
Not that is.

说话人1 48:14  
Now you see, the value of c will never be more than Xi plus xj plus xs these ijs they are the evidence or elegant players of the coalition c right? Then the reason the whole thing holds is that the game is false. Marginal contribution of a new player is always bigger when you have a bigger base set.

说话人2 48:42  
Right?

说话人1 48:44  
So I think that will just stop here for a moment with 10 minutes break. And then we come back to have another half session lecture materials. And then Mita will finish the unfinished exercise for quiz question.

说话人2 49:06  
Right?

说话人1 49:07  
She uphold the grade.

说话人2 49:09  
Right?

说话人1 49:10  
10 minutes break.