### Algorithm\_week7\_lec01-20241021\_14:07

说话人1 00:00  
Now, if I sharply value, here is a message. What does it say? It says that, first of all, I add up all the limitations in all these limitations, eyes minus contribution. So you do not look at this. The right hand side is the summation of marginal contribution, right? You have many minor contributions, because i'm trying it over all the pile. What is pile highest permutation, right? For each presentation, I confuse i's, marginal contribution. And then I add them up together, right? Okay? And then divide by one, one divided by n factorial, because altogether, you have n factorial permutations. This is exactly the average marginal contribution of the player I and then we call it phi it five g is eyes sharply better, ok so any questions?

It's okay. We are going to move on to the real thing in the previous example, the sharply valued player one and the two. Both of them are ten, right? $10. Okay? Now, actually something that also has a probabilistic understanding, what is that? Now, just how to say the shopping value is the average marginal contribution. This is the old definition. Average manual tradition of clear line. Competitive is predecessor right now. We can also say, if you randomly take a presentation, what's the expected game? What's the expected margin of traditional player? I she is a profession, right? Because average and expectation has some similarity, right? Because all of the mutations here have the same chance, same probability showing up. If you do random selection, right? You randomly say that reputation, and then they the expected marginal gain would be the same as this average marginal gain.

Sharply. I can also tell you, now this is the thing you can get expectation. We are not talking about worst case. We are talking some about something in the expectation, right? Okay. Let's see some examples. The first one is ice cream game, right? So this ultimately, ii changed the numbers again. It becomes 642 ice cream size. It didn't change still $7, $9, and $11.

Now, let's look at marginal contribution of c now focus on this person, right? Marginal contribution of c on top of what's this? Cmp means c arrived first. Then enemy that was c's marginal value, zero, because $6 you cannot buy anything. Zero. This one is a zero, because the array again, arrives at the first. How about this? Mcp now mcp means $4 come first, and then $6 next, right? After c join, you have $10. You can buy this one, right? But $4 can find us nothing. So before c joins nothing, and then after c joins, you buy 750 grams. This is my contribution.

And then look at pcm so again, c arrives the second right? Second place. And pos too. Two also can buy nothing. And then we have c you have $8, and you can buy this right. Now you have 500 marginal contribution. And then how about c arrival late last? When c arrives last, you already have $6 before c but $6 can buy nothing. Again, there are zero, but after the c joins, you can buy this. Right? The biggest ask me. Now you can buy 1,000 grants, right? Okay, so both of these, there are 1,000, because people see arrived nothing. After see arrived, you can buy the biggest right. Now, how do you calculate the sharp devaluing? Average of these values? Right? Good. Now you have this one, 3,255 or 6, right? And they have a sharp value of 542 grams, right? This is the way to calculate sharpening value. Now comes the very important moment. We have defined shopping value, but we want to know what properties this kind of definition has.

Now we are going to talk about full properties. All of them are very natural, except the last one is not that natural, but the first three are very natural. Okay? Let's see. The first one is very, very easy. The first one says, using sharper value, you can give all the money out. That means shopping value of 1 plus 2 plus until the last player shopping value. They are adding up to value of grand college. Right? Okay, so why is this true? This is actually quite easy. That's, although I say quite easy, you still needs one trick to do the proof. You have the permutation high. And then suppose the high I is the position of I into position higher.

Sorry, it's a player in position I the ice player isolated sequence. Okay, so sharply done, how do you calculate? We just use the original definition, phi I equals what? Phi I equals this. Every phi I is equal to one over n factorial times some sum. Let's forget about this one, because this one is equal to this one, right? So you're adding up all the people shopping value. So this submission is for all the people, right? Or the player. Now, for each player, you do this. What's this? This is exactly the definition of shock value, because you are doing calculation over all the permutations pi in pm a every permutation. And for every permutation you do, what you compute, I is marginal values. Remember, as time, I is people before, I implementation time, right? This one means I are together with this predecessor, right? Their value minus I predecessors back. This is the i's marginal contribution in the sequence, pi ok this is how you do it, because s pipeline is before I right?

Now there is a very important trick. And actually, I believe many of you have seen it. What's that trick? The trick said, i'm going to swap these two sums. So that becomes this. I will do summation of all the players list inside, and then do the submission on permutation. That's the same. You swap the sum. It just group the terms in a different way, right? Originally moved it in a horizontal way, right? For each permutation. And then now I improve it vertically, right? In this way. I do every players, right? And then some overall limitations. It's just a sum in a different direction, right? Originally, real wise, and now column wise. But the total sum is still the same. Right? Now this one is easier, right? Because these two index set they are independent. But if these two indexes are dependent on each other, then when you saw these two, you need to be careful.

Right? Now, although in this class, i'm not going to do that trick, but I hope you bear in mind, right? When you swap the two sums, you need to be careful. All right? Only when they are independent. You can directly swap them if they have some relationship. Probably not so easy, right? You have to change the index. After swapping, what happens you see inside is what? Eyes multiple contribution in permutation time, and then this eyes from 1 to 2 to 3.

That means in competition, I I am all the peoples, all the players are contribution. But the order I am is a bit strange, because I am player once marginal contribution, and then player two and then player three. You see, i'm jumping back and forth, right? I'm jumping back and forth. This way is not so friendly. I'm going to change the way to the following. I'm going to change the adding order to be from part one to part two. Part three still remember what is pipe. Pipe means that pipe one is the first player in that sequence. Pipe two is the second player in that sequence. Then pi three, pi four. It's again, the same. These two are the same. The reason is it's just another order of addition right previously. And there was much more. There is two is much more. And then there's three marginal players, 4th marginal.

Now I change the sequence, change the order. I say I had the first players, money, and the second players money, and then third one, right? If I do this, the sum is still the same, but the summation becomes easier to understand, because now the summation is what? You first have an empty set, then add one small contribution, then the two smaller contribution, three time contribution, four. Right? Now, i'm talking about order or an order of the player. 123. For this order. When you do this, you can cancel the terms because this one is the pi one minus the empty set. What's the second term? It's v pi one, pi two minus v v pi one. Go on, right? Disappear.

Then I I can cancel a lot of terms. And in the end, I only have one, the end minus vmp seven, right? Ok so vm minus vmp vmp seven is nothing, right? They only have the end left in the end, not at the end, right? Actually, this is the end for every permutation. No matter which permutation give me, it is always the end, right? Then in the end, how many of the ends do you add? It depends on how many permutations you have, right? How many editorial, right? And then this editorial will cancel with this editorial and becomes the end. Right? As I said, it's easy, but it's really some trick to show. Indeed, I give out all the money, right? For sharply value. It does not store or hide some money from the players. Basically, all the values generated by the players will be given to the players. Right? This is the first property. Any questions here? Hold on. Right?

Now let's look at the second one. Second properties one is about now trade. Remember that if you recall, I mentioned now there's some time earlier in this course, if then where the end of encouraging which game or what game simple, then he called me, I forgot whether I further mention that in simple game, but we didn't mention it in the voting game, right?

So what is that called voting game? There's another word before it definitely weighted, very good weighted voting game, right? Weighted voting game.

Now, that game we mentioned now, player basically means that you add a player in, no matter which group you add in is useless. And then we have an opposite concept of equal player, right? Without this person. Everybody else is nothing right. Now, the idea is the same. A player, I you say, is now player. If you add this player to any coalition, the value of the collision itself is the same as medical collision together with player line. Basically, I thesis. So if I is now clear, what do you think it's marginal or sorry, it's sharper than it is. If I is down there, what is eyes shock? 11? Did what? Depends why there is an action here. I ask you, what is the sharply value of this player? Right? Zero. This is what we are going to show. Shall we then of, now there is zero.

How to prove it? It's again, very easy, because remember, what do we do with a sharply value? Sharply value is add up all the minor contribution player line for these possible permutations, right? For each permutation, I do marginal contribution, but because I is marginal contribution, no matter what predecessive enhance a lot of people, which group of people are before it, this multiplication is always zero, right? Every term here is zero. Then no matter how many you add, it is still zero. That's a ii think it's the easiest to prove. You gotta see, right? Because any every term is zero, the whole is zero. After you divide by n factorial is also zero. Now, this is the one direction of things. It says. If I is now clear, then sharper delta zero. Then can you do it? Reverse? Said, if shopping level I is zero, can I say I is not there? Now, what is my hint here? The only if condition, this condition is only true when the game is small town. Now, still remember what is small town?

Market Not decrease, right? So when you have more people, you better become speaker, right? You will not say only because that one more person, the value becomes that, right? You will not see this. Now, one example of the normal game is the following. You see, in this game, you have two players, and then player one and player two. Each of them all along has a value of one. But if you put them together, that is zero. That means these two people, they probably are enemies, right? They refuse to work. If the enemy is together with that person, put them together, it becomes zero.

Now, this game, how do you company sharply values? Sharply value, one, smile contribution on top of empty. Seven is one, but then to smile to control one, smile contribution on top of two is what? - 1? Because two itself can have one. But then when you get one in, it becomes zero. It's - 1, 1 + - 1. Sum is zero. That means sharply value of both players. They are zero here. They are not nowadays. Actually. They do have some negative effects on top of the other person, right? Not now. In fact, it's negative effect. You see all in this game, because it is monotone. It's possible that the shock value being zero does not mean it is a loud player. Okay? This is still very easy, although a slightly more difficult than this. Because you need to find one example, but it's right, it's reverse direction.

Okay.

But now, before I show you the next one, I would like to ask you something. What do you think this next property should be? Recall the motivational example we give you to introduce sharply better. They should expect. Shall we get a sense that satisfy that condition? Right? What's the example I gave you in the beginning of sharper than section? Remember that example? How many players do we have? Two, right? Two players. On this alone, $5 to people together, $20, right? That's the example which make us think what value should give to each player, right? Eventually, we give how much to each pair $10, right?

Then what does it mean? It is a sharp variable satisfy some criteria, some property. What's that property? Same treatment for what? Same treatment for people making same contribution. How do you define people making same contribution? That's the the definition we are going to introduce two players, ioj we call them symmetric. What does it mean? Symmetric means if I have a base set, basic collision c you bear binding. It makes a minor condition right? At trading. It also makes a minor contribution. And these minor contributions are the same, right? So they basically say they are symmetric.

Now, remember this property for this condition, hopes for any coalition c right? You cannot say for a specific collision and ij in same effect, not enough, it should be the same, no, no matter which base you use your colleges, you can do anything.

And then based on this anything, you have ijn they make the same marginal contribution. In that case, we say inj are symmetric. And for symmetric players, what do you expect to see for the shopping values of them? They are equal very good, okay? Expect them to have equal shopping value. This is very natural expectation, right? This is also echo back to the example I gave you show you at the beginning, right? So how to prove it? This time, again, will be some trick, but not too difficult. Basically, the trick says you couldn't be one permutation pi. I will give you a corresponding permutation pi prime, which swaps ioj you see, implementation pi is something like the I and then something mj and something. And that's what imj and j is here. I is there, and then comes this, right? This kind of permutation is 1 to 1 mapping, right? You have one condition here. It maps to some of it. Is there all right? Now? What can you say about this? In g's round marginal contribution.

Now? Remember, you see that i's modular contribution in this permutation should be the same as jays modular contribution in this permutation. Why? Because these people are the same, right? I didn't move them. The people before I in this limitation and the people before j in this limitation. They are the same group of people, right? Because they are the same group of people. According to definition of symmetric, I is my contribution in one side is the same as james minor contribution on the other side, right? Your delta are the same. Delta I is the same as delta j for these two sequences.

Now, you see, you can find a 1 to 1 mapping for I and j there might be because I is minor contribution. You have a lot, right? You have a lot. J is monetary contribution, also a lot.

Now you find a way to link one marginal contribution on this side to another marginal contribution. Traditional this side, these two chairs are same. Then second one is the same as this one. Third one is the same as this one, right? You can always find a mapping. Then in the end, if you add both sides together, what do you see the summation that should be? What? Should be the same? Right? Good. Now you have the same summation and the same submission divided by the oil is still the same. That's why the shopping value of I and j they are the same. Right?

The third property, I think it's major property. We hope to see more sharply valued, but it's not last. Probably. We still have one more, but that one is not so not so straightforward that one open. On the other hand, that one more is more useful when you try to analyze the property for games. Let's see. The last one. The last one is about the combination of games is that I see, we define the addition games. There are two games, g one and g two. Each of them has a value function. G one has a value function u and g two has a value function d now we define a new game itself, having these two games together, g one plus g two, g one plus g two. And then what's the value of the new game? The value of the new game is the value of the first game plus the value of the second game.

Okay? All right. So now, what can we say about the sharp development of these games? Right? Because anybody want to build up the relationship? Among the shopping values? You have first game and shopping value, second game and shopping value in the big game one plus game two also has company value. What's your expectation? For example, for player I it's shopping value game one, shopping value game two, and shopping value is a big game. What's the relationship? The bigger one is equal to the sum of the small are very good.

That's the very natural expectation myself. Then for the iron in the beginning, it's sharply that should be. It's sharply that in a small game, one plus the sharply, there is a small game, too. Right? Now, how to show this? Again, we can simply prove it by definition, right? What is phi ig one plus three two? Marginal contribution, right? So all the marginal confusion has 100 and sectorial. So keep it here. And then among all the presentations, right? Among all the time, what do you add? What's this? These first two terms is i's marginal contribution in the big game, right? Because big gain value is one, big gain value is u plus v right? So I just need to u people before I was I and v people before I was I right? Ok these two add together is the value of s pi. I is I in the beginning, and then minus the similar term in the a similar term without line. Right? So these are only people be blind.

I is wrong. That's the marginal contribution on line in the big game, but then we just shuffle these two terms. I say I move it. Ii combine this first term with the third term, right? First minus third becomes this, second minus 4th becomes this. Right? You just reshuffle the trends. And then what's this? This is something about value come to you. And is that means it's only about game one, rank g one game. This is only about v so it is only about g two game. Then this one is the same as five ig one, and this one is the same as five ig two. Then it's initiative proof, right? Okay? I also see it is, again, a very simple trick about shifting the terms in the summation, right? Okay? Then it may be within your expectation that these properties, they seem very natural, right? And many of them, they are very easy to prove. That's a good thing. But on the other hand, it's a more challenging thing to show that if you want to satisfy all four properties, what do we call it? Efficiency? Give out all the money.

Now player, right? The value is zero symmetry activity. These four properties, if you want to satisfy them all, can you have optical possibilities to define the money to be given to every player? The answer is no. There's only one way the only payoff banker that satisfy multiple properties. It's shocking. That's a more surprising result. Right? If they give you some value, I ask you to prove some properties. That's easy, right? That's at the time. We don't need to even bring it to the master level, right? Or even underground. We don't need to bring that in, because that's just a straightforward must report that it seems to prove.

But if I want to show, I want you to show that if you for to define a certain payoff factor to satisfy for, there is a unique option. Right? You need choice. That's something more challenge, right? And that's also the more important contribution of sharply value. Sharply value is not only giving you one option or one possibility to satisfy the properties. But it tells you there's only one, this is the one and only one possibility that satisfy possible. That makes shock ability extremely important.

All right. So next, sharply value, in order to compute shopping value of one specific player, the computation time. And it's about why? Because there is a one over editorial. That means your summation. You need to add up n factorial terms, right? The quantum terms, there's a huge computation cost, right? Because usually we say on the algorithm complete in for the new time, this is not even for the new time. It's not even two to the end. It's n factorial, right? It's almost impossible to compute m when ten is already almost impossible to compute, not to say even large numbers, even larger numbers now. But as they can cheating a bit, this can probably still cave, right? But certainly definitely not cave. All right? It's already beyond your power to compute.

So that's why if you can bring down the computation to a faster time, that will be very attractive. Now we are going to analyze how to bring down the complexity. Now, remember, sharp 11 phi ig the competition is like this, right? So it is talking about permutations. For each fermentation. I add i's marginal contribution, right? Okay? Is the second way to understand this one is not good, is editorial terms too big. The second one to be computed is the fault. What's this? The first one is talking about. Second one says I do not look at imitation and look at coalition. You see, what's the difference? That's a big difference. I what do you mean? Right? What do I by coalition? So this is a player, right? Play a nice part in contribution. Does it depend on how you order these terms? No. Right? So basically, as far as the group of people before, I is the sexy, then I sponsor contribution is the same for them. No matter before that is 12331312, it doesn't matter, right?

By this observation, you are able to break down the complexity to the number of Coalitions, the size of coalition. Let me basically, you only need to enumerate what are possible c before I how many terms do you have. Then I just need to add up how many values of highest management contribution. Okay. So now the question is, how many different c do you have? How many different c at - 1? Democracy, few. Yeah. So basically, it's a another exercise to, well, in order to divide c you need to decide whether certain elements are in c right? How many elements do you have? Or how many players do you have? Altogether, we have n players, but remember, this guy, I cannot be c right? Because I is outside of c for sure, right? Because you are calculating, I is minus inclusion, right? So I is not in c if you remove I how many players do you have? Yeah, very good. And much so much players for each of them, they can either BC or not easy, right? So every player has two choices, right? Yeah. So that would be two to the power of, you are very good, n minus one.

You have two to the power of n minus one curve. Different c because each c it only contributes to one variable. Right? Then whenever you fix c you need to multiplacent coefficient next to the Delta IC this coefficient is what this coefficient tells you, but then you finally see, actually, you can permit elegance in c right? And you can also produce elegance here. Right? No matter how you shuffle, it's a valid permutation.

So how many permutations here? It's c factorial, the size of c factorial. How many conditions here? This, right? Total number of players removing players in c and then removing player I right? So you have to take for use, multiply together be the coefficient for I smile the contribution on Taobao c right? Okay. This is a easier way to compute usan factorial two to the n minus one. Both of them are exponential, right? It's a difference. All the differences are big, especially when you can try. For example, we can try n equal to ten, ten here, right? This one is still only $1,000, but not more than 1,000, but this one is already rich. For practical consideration, two to the n minus one is much better than entrepreneurial.

All right. Any questions here? The question is, countries some examples. All right. The example we are going to talk about here is induced subgraph game. Right? For this game, remember, players are vertices, right? And then valuable coalition is the total weight of the internal edges, right? Okay? Like this. So this is induced subgraphing. Now, you can do some grounding. What is shocking that every individual player, which is that we need to know here, right? What is there sharply that have this problem set up? How do I solve it? Right? Because I need to enumerate all the permutations of vertices. That seems too much for me, right?

For example, even if you only have five notes, the number of competition is already 120, right? Do you expect me to compute all the permutations, each individual vertices on the value, and then add them up and divide by the number of pages? Probably they need one Alpha, do it, at least ok if you do fast. All right. It's not a practical way. Then it reminds you of something that probably can use. All right? We not only have one way you shall be done, actually have two ways. If I say two ways, what's the second way? Now you can compute chapter value by definition. Now this is one way, right? Always work, but it's just too. So right. Now we say we can reduce from the number of competitions to the number of coalitions, which slightly decrease the amount of time, but it's still too much. All right. Okay. Now the question is, can we use some properties before to? Yeah, activity very good. So we can use the property activity to compute the shop within a big game from the shop, a little small games, right?

So let's see how to do it. Suppose you are trying, and then I can split this game into three small games, first game, this edge, second game, this section, third game, this section I am split the ground into individual edges, e one, e two, ek and then first, we need to show the value of big game.

So the value function of big game is the additional value function for smaller games. Is that right? Should be correct, right? Because that's all the edges. If the big game has some kind of internal edges, then this intuitive edge must show up in one of the small game. Right? So definitely, I will not miss anything when I compute the weight of the internal edges, the summation of game, the big game can be split smaller game. This one should be correct. Okay? What's the next thing we'll do? We need to analyze? The next thing is we just wanna gain, right? What's their shopping that? This is much easier because now, although you still have many players, but the permutations, many of them belong to the same category, right? Because now, for example, for this person, this person has no value at all. Right? There's no edge connect this node. So no matter where they are, it cannot bring extra value to the community, right? When only who can bring extra value to the community, only this person and this person, right? Because only if you can bring in on the action, then you can bring in marginal value, right?

For each smaller game, only two notes. They could bring in marginal value. That means the sharper value of these players. Only these two are not zero, all the others are zero. Those with no edges, they are zero. Then for the edge, for this edge, the two end points, what's their marginal value? Yeah, either what? Right? Sorry, not bad, because there's a weight, right? There's a weight on edge, either the weight of edge or zero, right? And these two choices, it's half, right? Because in random competition, 50 % chance, this notice before that.

All right? And 50 pages. After it is before, marginal is zero, it is after marginal is weight of the edge, right? Of 50, 50. Therefore, you see, marginal, the sharp limit of smoking, j is just half of the weight j of ej for the nodes adjacent to that action, right? For these students, half of it, right? Half of it. Okay? Each of them are half w let's get it together. What's the sharper term? Is just half the average that connects to this one. I have a mold. I have three edges going on and just add these three ways together. Give up, too. That's my sharpie letter, right? Because you are, in the addition, right? Addition of these small engine. Right? So I guess we'll take a break. Now, this break could be a standing order, because I have some more to to pick up the start of it later. But meanwhile, please digest what we just talked about the in use of graph game, and also start thinking about waiting for the game. What is sharply done? Yeah. One in 10 minutes break. Okay.