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If, sorry, basically, I is a pivotal for the people before it in pi, right? And then you say I is pivotal for implementation. So basically, again, it will lead to the people before it turning from the user set in going set, right? Ok this is I usage. Now, take a simple game. This player I sharply done, basically, is equal to the probability is pivotal for a random permutation. An invitation to give it to me. And then let's check when I will be pivotal in this case. Right? If then I say this, I will make one contribution. And then I add up all the problem, all the cases. It becomes a probability, for example, for ten cases out of 100, I is difficult, right? Now it becomes 10 to 100 becomes 0 . 1. This is the probability I will make. People devote become what he said. Okay.

Now, this one usually is used to measure the voting power of a certain player, right? And then people will say, does this player is very useful, right? And then probably they will try to this way this player more in order to make their own group win the game. Okay? And also because of this, right? Because of this shop together is used to measure the power in different voting points, right? Ok this is about weighted voting game by defining this kind of pivotal player, right? Just now, what we talk about is the so called tu gain, transferable utility gain. In other words, you can the one coalition, they produce some money, right? Some value, and they can distribute value on among the members in an arbitrary way, right? But sometimes you cannot do that, right? So sometimes the coalition, the very generator, it has a fixed pattern to be allocated to members in the college. Instead of instead of the appealing to the coalition, the shared right instead of shared among the players.

Now, the coalition value generated by it can only be given to individuals. They cannot be transferred, right? Okay. In other words, we are using one example here. Suppose you have two people, x and y they can perform action a or action b the one thing to call action a you will, x will get a model, I would guess these dollars. If they perform x should be, then x will get $4 while getting $2.

Now remember that in the last lecture, actually, the farmer scheme, right? In the farmer scheme, what we say, you can either grow orange or grow apple. And then when it is a transferable utility game, then basically you don't need care a bit about which strategy team, whether they were orange or whatever it doesn't matter. If the only thing that matters is how much money they can get, right? They always go for the best action, right? Getting the most amount of money because that money can be divided among the members. So the more money, the best, right? But here is different. We still have two strategies here. We cannot say a dominant b or b dominant b because we've been up in the day, because you see these two strategies strip a for why it's better, because why do you get six words better than two? But for x b is better, because in bx can get $4, but it made it only one.

So basically different people. They will consider different strategies to be best, right? Instead of one single strategy. All right. First, we have an even better example on the next page about writing academic papers. Right? So you can see that there are many scholars in the world, and sometimes they will collaborate with each other. So when people collaborate on some some different papers and what happens? Now, you can consider each paper. It hasn't that true, right? For example, this paper is a we need paper also award, et cetera. And then this paper will add value to members for sure, right? But this value, can it be given to the members in an arbitrary way? That's a question, right? You can say all the paper generates $1,000, and I can give arbitrary amount of dollars to the co authors of the paper. Can you do that? Actually? No, let's see. Now, basically, each authors pay off. You can consider it to be the reform if he or she received from the university. It will also right, for example, there's somebody from pops or somebody from zambia, et cetera.

Each individual university will have some remote system to the faculty, right? So write one paper, and then that's okay. Ii I will give you promotion, right? If you have several surveyors, right? Good, one thing. Another thing is maybe because you produce a lot of papers, then you can teach one less course every semester, right? So much, it becomes zero payment. There will be some implication, right? Promotion, as I said, and some university bonus, one paper, one on some dollars, but this is to do that.

So I think that and there will be some money possibly in the some universities, right? And there could also be teaching the reduction, right? As I mentioned, to be originally is 1 year three courses and outcome, 1 year two courses, et cetera. Right? This individual benefits came on versus says, can come up and say to my courser. All right, I got a promotion. Ii did half the promotion to. Can you see that? No. It cannot affect the decision of another university, right?

So this kind of state is not transferable, even though it's real benefit, right? It's not transfer. Also not to say teachers or reduction, right? I I have one that's supposed to teach and say ten requests for half course less, and then the other course and also half course. That if you don't have this kind of negotiate power, negotiation, power with another university, right? And that's definitely not going to happen. All right? So you see that the nontransferable game, not just global utility gain, it's also widely existing in our daily life, right? Okay. In this case, we will introduce a special kind of game called kingdom game.

Now, actually, it is very interesting, basically, including every player. It will just tell you which coalition it likes best, and which coalition second and which coalition serve. So every player will rank the coalitions, because whenever the coalition fix the fixes, then this players gain is also fixed, for example, coalition 123. And then it's fixed that one player one will get, say, $10 from this coalition. This case. So basically, the data knows exactly which coalition is best for this for himself herself, because he exactly know how much money, how much value you can get from that position. So then you can hold it up. This one I like this one second, this 1/3. All right? So more formally, every coalition only has one action, right? Because it only has one action exactly how much you fail objects. Because if you have two actions, then it's likely that actually one, they won't get the $2, actually, two, they won't get $4, even if you fix the coalition, play with money, get fixed.

But if you only have one action for each coalition, then every players game is fixed for any coalition, right? And then the player can order these coalitions from best to worst. Right? Now, in this case, this kind of game, we called, he dont a game. And then here's one example. All right. Each player will just rush all the coalitions, for example, player what says, i'd like one too, better than 13, and 13, better than all three people together and all three people together better than myself.

This is what we'll do. And then two will do similar things, right? To also rank all the colleges. He or she can get out. Right, three also the same. All right. So in coalition, in the human game, sometimes because we want to say stability exams, but for human game, the stability, it's not guaranteed. In other words, it's possible that you could be does not have a lot call. Very good ok does not have a call. So press the wrong button.

Now, again, we can define more, right? What is it for me? Is no group of players as can go out and form one coalition such that every member of this group x likes is s better than their current assign coach, right? Similar, right? Currently, everyone is on the coalition, right? If there's a group of s they go out together before what? S right? And then everyone likes s better than they're already assigned coalition. And then we say s will go for deviates, right? But if there's no such as possible, let me say it is alone. It is look as possible, look at which subset you drag out. It's not possible that everyone thinks this is better than before. It's not possible. Then we say the whole thing is stable is a call. Now, what he doing again, unfortunately, coal can be empty. Right? Here is one example. You can look at an example for a moment, and then we'll discuss, we'll explain very soon why this one has no call.

Now, in order to say, there's no core, safe way is to say, no matter which coalition structure is not adopt, it is not stable, right? Somebody is only is going to deviate, right? Let's see. For example, if if you put everything together, is this stable? No, right? Because you see everyone pays to be altogether, right? No matter how you need, it is better than altogether. How about everything as an individuals do not collaborate with each other. Is it good? Again, it's not good because you see 12, one legs, one, 22 legs, 12 better than individual, right? So they too will go out and form a group, right?

Again, it's not stable. Now, the only remaining coalition structure as well, 123, 132, and 231, the only three remaining, let's see whether they are stable. For example, 123, right? 12 together. Three individual. Is it good? For one? Is very good, right? 12 is on top, right? One is not going to be. How about two? You say, two? It's okay. We want two, but it likes 23 more. Ok there's three also like 23 more than being alone. So two and three go out, right? They'll both benefit. It is 23 and one individual. Is it good?

Now again, for two is very good, but for three, he says, I like 31 better than 32. Right? So she would rather join with one and one, also like 13 better than one, one, and three we got. Now, that would say 13, and 2. Is it okay? But for three is good, because 131 is top, right? But for one, cr one 2 is better than 13, one says, now i'm going to join two. We are better, right? Ok you see, there's a cycle, right? In this 12 or deviate. And this 23 would deviate. This. 31 deviate. Right? Ok this is the triangle relationship, which will never end, right? Keep improving now. Nothing is stable, right? So you see, it's not so good, it's not stable, right? Then what can we do? Now, we are going to talk about something where the stable solution can exist.

Now, this is a very big class of games. It's the first introduce the game. So in this game, we have a set of men and women. We have an man and women. And then every member they were ranked the members of the opposite sex, men who ran for the women. And then one woman will rank all the men ok so basically, it's a fuel ranking, right? People here, rank people there, but there are people here ok so again, very standard scenario. Okay? One example here, we have three men here on a tile was the woman. They be better, claire. So the men's table idea and ranked 81st better next class thirds. We have so ranking like this. And for women side is similar. Amy ran john first, ian x and kyle last, et cetera. You see that these two tables, two people are very unlucky.

One is, Sorry, wrong button. Again, one is clear, actually, the third choice for every man, right? And then there's a similar person here. Is the last choice for everyone, right? Okay? Now, somehow you will have a feeling. These two probably will be together, so you'll see very soon. Now, how to solve this problem? Right? How do you do the matching such as a stable purpose? What we want is stable, right? Because you need a scenario. We already see. It's not stable. Sometimes there is a cycle, no stable solution. But how about stable the natural problem? Can we find stable solution?

Now, then what has solution is finding? We are finding some solution called perfect measure. What is perfect mention we have and men and women, right? Perfect measure says you match every woman and man. So basically, you hate this man with this woman, this man with this woman, this man with this woman. Anyway, in the end, every man is nice. Every woman is also next. So nobody is left in the final solution if everyone is okay, but maybe anyone has a partner, right? So no one is very unhappy. Okay? This is the definition. As every man appears in your matching, every woman also appears in one matching in your matching. In appearing in matching means what it means that it appears in animals in 1 pair are. You cannot say I measure one man is two women that's not allowed. It's a 1 to 1 match, 1 to 1 matching, where nobody is left unmatched.

Okay? So thats the definition of perfect matching. You mention, says you mention size, the number of pairs you find is equal to n in no question, no problem. Right?

Now, here, again, I give you one example where john ian is measured clear. John is measured better, colors measured 80, right? So on the other table, you can see similar things, same partners as one. All right? Okay. Now here is the metric, here is ici with c right? J with bk with a right? This is one perfect match. This metric is not good. One, which question? I can choose ad right? Ab can choose in which is better than before, right? Both new pattern is better than before. And then you see iron ab in this matching, they are not happy, because they have a better partner who likes each other more. Right?

So this is also the intention to study. The unstable is in the sub matching ok in some matching. It's unstable because there are two people, at least. They like each other better than their current partner, right? It can happen, right? It can happen. So this is exactly what we are talking about. If you have one man and a woman, right? This man prefers w to its current partner as this woman refers to m to her current partner, I guess. Then this mw pair, we call it unstable gain, right? This one unstable gain. Then they launched in the next lecture. After the term. We also talk about national war. And there we call it the so called blocking pair, because this pair is blocking the solution to be saved is blocking.

Right? Other than enab red line, enen bella is also a unstable pair. En can choose bella cen better than john. There are multiple unstable pairs in this solution in this matching. What is daily match? Then? Very easy. It's a matching where there is no unstable gas, right? The mission which can take out. It cannot be the case that they like each other better. It can only be the case that this person likes this one better, but this one say, no, I would love about, right? My current partner is better. Okay? So a perfect matching with no unstable pairs is called a stable matching. Okay. Now, our stable matching problem says a genius preference of an men and women. Please find a stable edge. Okay? Now here there's one stable matching. I matches, and I here next match the baby, john matches bella and tyler matches claire.

You see that in the stable matching, it's very, very safe that college declare because they can, people will not like any of them is better than their current partner. So that's the least wanted, at least wanted. Well, that's very stable pair, right? That was a pair, and they're stable matching.

This is one possible stable matching. And then you can take some time to check whether it is indeed stable, for example, because they are. Is it possible that was actually no matter in group? Because en is assigned to ab right? En will not be h right? En can not belong to any unstable care, because he already has the best partner, right? Okay? John also will not deviate, right? Similar. Now. Will kyle deviate a horse? Because now is the last choice, right? So kyle says, I can I choose ab right? It's better. Ab says, no. You are my last choice, right? My program is better than you. So basically, no man can be involved in any unlocked and unstable pair, right? Either they already got the best choice, or their father will not choose them, right? It is the person is the last choice of the time, right? Here, stable, right? Okay.

Now, actually, you can also understand stable game as key role in game. Then you can feel a bit surprise. Okay? Can you begin is talking about ranking one ranking coalition, right? We have coalition here. It seems not, but actually we can. All right. People thought, say, a model, this stable mentioned problem into a human game, where each player has certain preference over different collision. For example, each man they prefer being a lot to any coalition, not of this moment. You have many questions, many people or men or et cetera, or many men. No, these it's not as good as being alone. All right? And then if the coalition is all the format, man and woman, then it's always better than being alone.

Then among all these pairs, which one goes first, which one goes next? It depends on this man's preference, right? Because remember, man has a preference list. I like this best, this next, and this next one, then they just order. Right? All these pairs, all these two person coalition, according to the preference. Right? I don't like woman one better than woman two, better than woman three, and then this man was one would appear first, right? And this man was woman two next, the men's member of the three next, right?

So you see that then you can specify an order for the coalitions. This manager belongs. Now it's just a special case of coalition gain, right? Okay. That's nice, right? Ok what is stable mention is just finding a core is what he donating. Now remember, earlier, we say he donated in general may not have a core, right? But for this special setting, it has a call. All right. Now. We will go very soon. But before we show, stable matching really exists, we'll first show something which is even more challenging to deal with. This is something that's also has an interesting name, is called stable roommate. What's this problem? Stable roommate. We have two n people. You have n of the choice.

And then I want to assign room depends two people, one room just like here, right? I remember here is also two students, one room, right? I have a chair in my room, so now the promise can assign the roommate in such a way that the solution is stable.

Right now, what's the difference between this problem and the stable mentioned problem? What's the main difference? You have no restriction about which two people you can assign, right in the stable matching. Because I you can only do one man possible, right? But here you can do two men, two women. Everything is possible.

So that's the difference. Then here i'll show you one example in this example. Adam like the four people, adam or chris, and then adam like me more the most, and then chris and then, right? Everyone has a preference order. Now my answer is just next to it, but I hope you still can spend some time when you want to 2 minutes, figure it out. Whether you have a stable solution, you have try something.

Now, the first question, how many different assignments can you have for this problem? Three, very good ok because you have four people, the group, two together, right? 12 together, 13 together, or 14 together. All these three choices, right? The other thing would be another room. So basically only three choices. And for these three choices, just do it again. The first one, dabcdabc is good. Daba is very good for d right? D the c is very good for me. They likely they like the option a lot, right? Dabc they are good. How about these two? Dabca is assigned class choice. C is assigned this choice, right? Then a and c says, why not we move together to create? The c has a which is top choice. A has seen the second last choice better, right? This is a and c they are not happy. They are going to go out to to another room, right?

Now, how about the dbac so dbacdbac you see, b is the last choice, d right? And ac a is assigned to c then a and b said, if a is matched to be better, right? B is matched to a also better. In this case, ab will be unstable, right? Because they have this big inventor partner compared to the current assignment.

And then finally, dcab dcab here, right? D with c last twice, they can guess d will deviate. Okay? You probably abcabab ab now, who can deviate?

Here it's going to bc can deviate, right? So can deviate because I cd is matched to c seems that he is very unhappy, right? Can be drag some body to deviate. Can be deviate with b no, because b is as I made, but be hasty, right? No, no way. Can d deviate with a again, no, because they also does not like the right. Although b is assigned last choice, we will not debate, but bc will debate why. So bc you see, b is assigned to a now. If you choose c is better, right? C is come to assign to d and then if you choose b is better, right? You see no matter which combination you choose. There is always 1 pair, at least, who will give it right? They can find each other at the top compared to the current assignment. Therefore, stable ruling problem may not have a stable solution. Right?

Now we see stable, probably still magic problem. They are very similar. Then the only difference is still magic has a constraint of what constraint required to say. You can only match people on the left with people on the right. You cannot match people inside, right? So with this restriction, we are able to find the stable solution for stable matching. Then what's their solution? The solution is called gave sharply deferred acceptance errors. Right now, what's this sharply? The same sharply as sharply value ok so that's why actually, that's not only these two cases actually sharply has more contribution in economics. Can he win the nobel prize in economics? What's this algorithm? This area was actually, it's quite easy. All right. Basically, that will collect inputs, evidence, preference. I collect all these things, and then every time, I find one man who is not yet matched to someone. All right?

I find my such name and ask him, could you please propose to the top choice Who you never proposed before? What does it mean? Is that that's a list, right? And the first class to ask him, can you propose to the first choice? The man said, okay, no, because I already proposed her before. She rejects me. All right? And then I was saying no problem, right? Because you already proposed the first class, and can you hold second choice this time? He said, yes, I didn't propose the second choice yet. I supposed to be a choice. All right. So men will just propose to the first woman on the list. He didn't propose yet in the previous rounds. Okay? It's easy to understand, right? Every ground I picked one man who is not matched and asked him to propose to the top choice in the list who he didn't propose yet.

And then this is men side how the woman side, both sides even easier, right? Whenever a woman receive a proposal, she will see whether she currently has some potential, then who she already from what my privacy happening. I cannot use promise who she already tentatively accepted. If not, then this moment will accept also for a moment. Okay? Because currently I didn't commit to anyone. So as long as there's a new proposal to me, I will accept the proposal for the moment, remembers for the moment. That's also why we have a word defer a sentence. The purchase is when they almost accept a proposal is only tentatively accepted. Later on it can be changed. Okay? This is when the woman is not matched. Okay, before also, sorry, this is why the woman didn't have any tentatively accepted men yet, right? But it could also be the case that this woman already tended to accept some men in the previous rounds.

And then when she received a new proposal, what would she do? This will say, let me compare. All right. Is this even better than a previously? If not, check it, right? Why bother dealing with this? Because the new one is worse than the previous one, then we check the proposal, right? But if the new one is better than the previous one, then it says I will accept a new one, and then tell the previous one you are rejected.

Okay, so this is a different rejects, right? Rejected earlier one, which is not as good as the new one. All right? So the woman replaces her tentatively accepted man.

This is the whole heirloom. It's very easy, two sides, right? One side, whenever there's a man on next and has not proposed to every woman, you just ask him to propose at the top of the list of his list, right? Then this man has not yet imposed ok then choose the top candidate. And if woman is unmatched, you just match this man woman if the woman is already matched. And then the woman just compare the new person and to this old person. Right? If you better know, removed over there and you can, right? This is exactly what I tell you before, right? If you want to find a new partner better than the old partner, replace, right, ok otherwise you just reject the man, right? Reject the proposal. This is the whole about game shop, the equity.

So sounds very easy, right? The only thing that is not so trivial is that women will defer, accept will postpone the final decision to the end, right? Anything happening in the middle is only intended to be accepted, right? Only after everything finishes, then the woman finalize the power, right? But also, what we say is some kind of trick that this trick is not so unexpected. It's very natural policy that will can be adopted by a woman, right? So as long as you allow the woman to only commit after the actress, right? And then very natural for the woman to upgrade the offer of the money. Gradually.

Now, let's analyze the running time of these areas. How long does it mean to run? Now there are some of these issues. One is men will propose to women in the decreasing order of preference. This one has nothing to do or does not help us with the running time. They still have a bit. Okay. Now, a second position, once a woman is matched, she will never become unmatched. Right? What does it mean? As long as woman is matched to some man? The only thing that they happen in the future is she upgrades the partner. So that's all right. Only upgrades. Never he would never say. At some point, I decide to give up this partner. All right? Without new proposal. That's true. Risky. I never do that. So the proposition one will help you to prove that the algorithm will terminate, at most, n squared iterations.

After so many equations like, see, then propose a woman, right? How many proposals this man will average or embrace? Only any woman, right? So then the only proposal and times now, then also we do not allow the men to propose to the same form again. So only few proposals are allowed, right? Every man can propose and times how many men? M right altogether as where possible proposals, right? Every proposal it's very easy to handle, right? Every proposal, so many proposal. And at every proposal, what do you do? Woman decide, right? Whether she accept this proposal or not, right? How does she decide checking whether this new one is better than old one? Right? It's just one comparison. That's very easy. So altogether you just need so that time, right? And so that time done, what is mention? All right. This is about running time. So that means this number is understood, right? It can be finished in polynomial time. Right? Yeah. Very nice. Now, what can this deal shall be dealt with them? Give, well, it will give you a perfect match of how to prove it is a demand for the men and women are matched, because as long as every man and woman are matched, there is a perfect match.

Okay? Why is it true? Name is cr it's very difficult for me to prove this, because I don't know how to show it. So whenever you feel a bit confused to show something, usually we'll try prove by button, because all contradictions. Right? So assume this is not true. If this is not true, what does it mean? Right? Somebody is unmatched, right? So somebody is unmatched. So let's just assume that some man is not nice. All right. If this man is unmatched, then what can you say? Now, what they're obviously is, if some man is unmatched, there must be some woman was not mentioned, right? Because the same number of men and women, right? All right. So I mentioned what's next? So then you say this man, right? At some point of the algorithm must have proposed to this woman specifically, right? Because men must finish all the proposals before every person can finish. Right?

So this man must have proposed to this woman. At some point, there were. But why this woman is unmatched? Seems very strange, right? So it must be the case this woman rejects the men, right? Because if the law didn't reject men that they are maps. So since there are men, that means this woman rejects this man, but why will this woman reject the man? This is a better partner, right? Better part of this vessel which interacts with them, right?

Okay. Remember, once the woman one is matched, she always keeps matched, right? Is this a competition? Yes, right? Because we say this man proposed to women, but now they are also match that this woman must have rejected the man, right? Then the rejecting the man is this woman at that time has a better partner, right? Has a better partner. So that means this moment at that time is matched, right?

Next to somebody, but we have a position to the position to assess. Once a woman man is matched, she always is matched. This is a competition. Right? That's the end of the proof. Is it good? Very nice. Okay. Now this is the first thing, right? This is about the perfect match. And now we are going to show this album gives you a stable match. That's a bit more challenge. How do you show it is stable? Now, let's see. Again, right? We proved by conversation, say, let's assume the solution is not stable. Then there must be one unstable parent. Mw is not in s but they like each other more, right? So can this happen? There are two cases here. The first case is m never proposes to them and didn't propose to them. It means that this w is very low in the sense preference list. When the government finishes, this man didn't propose to the woman. That means this man is matched to somebody who is better than this woman, right?

So if that's the case, when will this man live? This will be better. No, right? Because he is better to somebody better than this woman. So it's not possible this woman like the person. It's not possible that this man likes this woman better, right? Because it didn't propose to the woman, right?

So in this case, it's not possible that and likes w better than his partner is, right? Then it will not be unstable. This mw is not unstable. How about second case? This is n proposes development, but eventually this pair didn't show up in the solution that one must be the case. Yeah, very good. So then we propose to them, but this w didn't finally choose. And that is, she chooses somebody better, right? So let me see the final matching. This woman is matched to somebody better than this man, this man, right? So it's not possible that the woman likes this man better. They are the current part. That's not possible, right?

So in this case, it's also easy. Right? So this woman is the woman said, is it because woman likes her current partner better than them? Right? Did it be the group? Madam, we show that in most cases, there is no direction which is stable. Then that means this arbitrary pair and double will not be an unstable pair. Right? Nice. Okay.

Now I finish the truth of stability, as I told you earlier, right? This is a nobel prize winning problem. Then the theorem says this error with them will terminate after as variations, and they will find you a stable matching. No matter what problem instance you give me. All right? And then in the history is first proposed by david gale and the boy sharply. Okay? And then it's published about 60 years ago. And then we could never come update so sharply end loss with the global pricing in 2012. Okay? Not both one. Okay? Because sharply has more contributions, right? Not only this one is very important, but not all. Okay. Yeah, so I think that we will leave the extensions to the next session, but I I would like to leave one question to you to think about, right? We don't give sharply another example, give you a stable measure.

Now i'm asking you this stable measure, is it better for men or better for women? And now I will vote who thinks better than men, better for men is a man. One to come who says better for women, raise your hand. I see more hands. So take some time to think about it, and then we'll announced the results in a certain power. So we spend a big question of time analyzing how do you search case for both parties? All right. So 7 minutes break.