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We have hospitals where you assign medical students to them, and then one hospital usually taking multiple students, right? It's one of many machines, and also when you assign children to schools. Again, bus school takes in multiple children, right? In this case, what is stable or what is the unstable pairing? Basically, it's a pair xy where x is a school, y is a student. And then this student is not admitted to the school, and the school likes y to one of his currently assigned students. School likes y better than one of his current assigned students. And then the student likes x school, x more than its current school.

Then basically x has incentive to take. One in white house has incentive to join x right? This is the unstable pair, right? Similar.

Then, again, here, using a sharply different sentence algorithm, after slightly adjusting the algorithm can be used to produce a statement. This is one extension, and then sometimes there may be the so called unacceptable pairs. What is that? Now, unstable pairs, unstable pairs means some hospital students, they find each other unacceptable. So even if you take me, I will not go to your place. All right. Now this one will extend to the problem with unequal thoughts on both sides. So because just now we say you have the same number of men and women, right? But if you do not have the same number of men and women, you can purposely add some salt on one side to make them equal. And this artificial salt is unacceptable for any people on the other side. Right? The same. It's just, for example, man is more than women, and they add some sort on the woman side, right? But those thoughts are virtual, right? Artificial. So then every any man with that slot is unacceptable, right? Because they are virtual, they are not real. Woman is just a virtual thing, right? So this unacceptable pair can help us to solve the problem when the man and woman do not have the same size, right?

And then again, you have a unstable pair like this. If the student y is not admitted to school x then y if y and x find each other as sensible. At the same time, x has a free spot, or prefer why to one of his current students. And then why is unmatched or like x better than pencil? If all conditions are satisfied, we say it's an unstable pair, right? And then using deal shopping algorithm, slightly changing it. It will still work for the second, right?

Now, next is about indifference. What is the difference? Be some the preference as times, right? I like one or two equally, maybe two means the same to me.

All right, one example here, a it's not a local example. It's probably a special example. We can say that there are some schools which belongs to so called coe all right? And they will condition the students into 8 groups according to their preferences. The first one, whether they live in the so called attachment area to consider whether they live in a and expensive area, maybe not be that.

So basically special area, right? Whether they live in special area. Second, whether they have a sibling in the school. All right, sir, whether they belong to the same system as acod all right? Students in the same category, which means they're ca value same, that's value same. And in coe now, everyone, every student belong to the first group, the school has no other preference among them. He will not look at who has a better grade, okay? I don't care about these three factors as long as two children have the same value for these three factors. Then I treat them the same.

I consider them the same, no preference among them. This is so called indifference. In this case, we use the so called weak stability. What is weak stability? Whenever we say some concept is weak, that means we require the deviated pair to deviate a lot, because now we have indifference, so we require unstable pair to strictly like each other, compared to the current partner. It is not because there is a difference. It could be possible that one party likes this one strictly better, but this one consider that side same, right? This pair, we do not consider the mass blocking as a unstable.

We require both directions to strictly like each other over their current partner. All right? You see that when you require the dbat pair to to be more strict, right? And this concept usually is weaker. This concepts we get, if you recall, when we define the peripheral dominance, you will see this. We have, we can create a lot in a free model. If we break out more, we require the dominating area strictly dominant in every dimension. Permanent. Okay, so that's the one thing. So this scenario, can you sharply? Everything will still work after some adjustment, right? Even with the difference is still okay. You see that the algorithm itself is quite strong, right? It's quite strong. Okay. Now let's come to the question of this matching. The resulting matching, is it good for men or women? Right? But before we answer that question, let's proceed some other properties of that result match.

Now, one thing in general, if I give you an instance, there could be multiple stable action, right? This is very likely. Okay? For example, here you have one thing, one another two tables. And actually, in this instance, there are two stable matchings. One is enab john, bella, kyle, claire. The other one is em bella, john amy tower clad. All right. The reason you have to stable matching, you see, even though these two men get the second choice, but for these two women, they got their first choice. Right? So it's tuesday. Right? When you have multiple stable matching, you will hire people under game shopping. Algorithm. Will return us. So management right?

Now, the first question is, whenever you run the deal sharply, everything, does it always give you the same measure? Yes, right? Because it's a deterministic algorithm, right? Sorry. By that. It's not a domestic. You choose man in arbitrary order, right? Because when you have a 100 man possible to propose, you take some of them, right? To take some of them, anyone is possible. But surprisingly, no matter which one you pick in which round, it always gives you the same measure in the end. That's also some miracle thing that happens in stable matching problem, because although, right, the algorithm seems deterministic, but actually not because it has many choices in the middle rank.

So then why would there is a man not matched? Let him propose, but this game you have multiple choices, right? The elements actually will follow many different parts, but many parts it was. It gives you a saying machine. Then this machine is independent of the order levels. So that's actually very, very surprising. And we are going to show this one mention fixed mention. It belongs to some category right now, which we call. Now, i'm not calling the name now, because if I call the name that the answer, right?

So let's first define related concept called valid partner, right? What is valid partner? We say a woman is a valid partner of a man. If there exists a stable matching where this man and woman is a pair, is it easy? There exists one stable measure where this man is matched to this woman. And then we say this woman is a valid partner of this man, and we also say this man is a valid partner of that woman.

Now you see, because we have multiple stable edges for 1 minute. He can have multiple that comments. Then give sharpie algorithm, will give you a stable machine where this man is matched to some value partner.

Now comes the answer. Some kind of other business among better partners. The man has a preference, right? Like this one better than this one. So chinese name, this man has to do the best one or the worst one is man's, that's the best one. Then that means the final result is very good for the men, right? If men match the worst one, then that means the final result is very bad for the men. Right? Here. Again, two stable matches. Right? You see, amy and bella? All see each other are their partners for ian and john, right? And then claire is only a better partner of k right? Because there are only two that matches I is better partner. Abj is better, but also ab case that partner is just c right? Now is the answer. All right, men optimal measures. So this is the matching gave sharp email with them for giving.

All right. In other words, here, shopping algorithm will give you a matching which is very good for men, right? Which is minority. Just now. I I was all one, one I forgot. So one hand here choosing is good for them, right? Because there's some kind of the intuition that the plus you are thinking that man is possible because you see woman's offer is always upgrading. Right? Because we say it can always replace the current partner, but you want right? So you see that the top of women keep improving, but also of men keep upgrading, because although he was accepted for his choice, early, but the first choice will be checking. They don't, right? Because they have better partner. Therefore, the decreases are downgrades. How come the net still optimal? Right? That's very, very constant intuitive. On the other hand, maybe because woman is passive, take this terrorism, right? They cannot voluntarily choose partners, the only passive except courses, but then proposed in the order of preference from top to down. Right? This is nest advantage.

Now let's see mathematically how to prove is still shopping out reasonable to you, man optimal mention.

Now, first of all, let's define what is an optimal mention in. Men optimal mention. Every man receives its best valid partner. Remember, what is that partner? Some partner that they appear in some stable measure. Right? As long as this woman can clearly once they were getting right, who is this man? This woman is a valid cop, right? Among all the better partners. And then it matched the best better time we are going to show this. Now, this is may not imagine it is a it is a perfect matching, and it is also stable because men optimal.

But so first, we need some time effort to prove these two. But I think i'm gonna do it here. All right. Now, at least, should can you show? All right? Can you show now many more people matching em match ab john has developed, right? Then how much prepare? This is the matching. Right? Okay. So next, let's show this man object. Again, we are going to improve it by contradiction. How to do it? Now, computer says, if something is not matched to the best value part, then what happens? Right? I suppose now, if men is not measured to the better partner, that's better partner, it must be in case that this man is rejected. Buy some bad part because men proposes in order, right? When he proposes to somebody who is his best partner, then she must reject this man immediately or as somebody around her. Okay, this must have, otherwise, this man will not matched to that person, right?

So some men must be rejected by a better part. And then there is a very interesting trick here. We consider the first time in the algorithm. When this happens. This is very important. And also in the last hour, there will be some bonus. Questions? This lecture, one of these questions I am as last session, so pay full attention to the proof. Okay. Now we use why to represent the man rejected at this point of time. Why is it unlucky person? First, a lucky person rejected by a valid comment. All right? Okay. And that really is the woman who rejected at this one. Now we have two people, one, a and then a reject y because she sees a better man, c all right? Okay.

Let's use s to represent the stable matching with ya in s You should figure it. So we use this figure is that, so because a is a valid partner of y there must be one stable matching where a and y they match together, right? We call s such a stable matching. And then we assume z its partner in s is b this is the picture. And now you have one information here, which is a likes that better than y that's why a rejects y and takes that offer. Right? Good. Now the question is we want to, somehow on the show, z likes a better, and then it will be an unstable pair. And it will contradict with the stable mention, because this is still mentioned, right? Right? This is emerging. And then we can show c prefer a better than b and then c and a they form announced it. Okay? This invention is no longer stable, then contradiction, right? So can you show them on first stage?

First one last question is this. So we claim time tz has not been rejected by b why is that? This kind seems to be very strange coming, because we even never say anything about when the z or when does b reject c or when does c proposed to be? The only thing we know is now c is proposing to a and then make a rejected, reject why? Right? At the current moment, c proposed a and then we also actually, at this time, c c has not yet be reject, believe, which means which can give us a conclusion.

Actually see, they don't supposed to be yet why z has not been rejected as b think about it. We have time for this bonus for. Sure. So use your time. So you have a slide with your hand, right? Produce one in a heat that will help you to answer this question. Who's that? See, could it be? But I didn't say that. I didn't say that yet. Right? That's the conclusion we want to draw. Is somewhere here, ok so you prefer it to be somewhere here. All right, not yet provided so much. S is a stable matching. But as is emerging, why is he has not been rejected, but he is a very partner. So what then that not yet, you not yet touching the critical component. Anyone would like to try? Yeah. First time present is why you get all very good.

Remember, in the previous page, what do we say? First time in the execution of the algorithm, when this happens, what's this? What's this? Is some man is rejected by a bad partner, right? Some man is rejected by a bad partner. Remember, we assume this is the first rejection of this type, right? This b because you just mentioned b is also a very kind of z if b rejects z that's a similar type of rejection now. Because this is the first time this type of rejection happens. So this rejection cannot have happened by now. It's going to be must be some future rejection right now, not before ok never before a rejects why?

This is why z cannot be checked before the time t b didn't we just see? What does it mean? Remember, at time tz proposed to a right? Why can they propose to a why can be equals eight or I can c equals eight? Because c is one, c is unmatched, right? Because c is unmatched. But why is these unmatched? Because either he didn't propose to anyone, all right? Or he's rejected by someone, right? Okay? If he didn't propose to anyone, then, of course, when he proposed a he considered a to be better than me, right? If he was rejected by someone, but not be shared by me, right? Because you see, not rejected either. He rejected by someone, but not by b what does it mean? He didn't suppose to be yet, right? Because he proposed, and he must be rejected in order to propose again. Right?

So now the is available, right? He is rejected by somebody, but he didn't reject him. Is it possible that he ever proposed to be? No, right? Because he never supposed to be, in order to be available again, he has to be rejected by me, right? The reject, but it didn't happen. So can propose we have. No, right? You cannot supposed to be before them. That's why she has not yet propose to be. Right? Okay. It didn't propose to be, but proposed to a what does it mean? It means in these view, a is better than b because c proposing order of preference, z composed to a first. Right?

Z concerns a that's a contradiction, because in this stable matching, a likes, c better, z likes a better. This is an unstable pair. This makes the stable, mention unstable. It's a conjugation. Why do we have such a conjugation? Because we assume somebody is rejected by that economy. Right? Some of them is rejected by that economy. You just pursue this. So there's a combination. Then what's the opposite? What can we know that no man is rejected by any valid partner, right? Is that right? So if no man is rejected by any man partner, that doesn't mean everything is matched to the top, then come in the preferences. Right? So every man guess the best that part about all the parts, right? Now we finish the proof of men or people mention. I'll give you 2 minutes to discuss. So that is what I just we can discuss.

Now we come back to the two questions here, right? Previously, I say these are very difficult to prove that I can think about it. Actually. How do you prove that game shopping algorithm? Give you a man of matching? Then these two questions already solved, right? On the one hand, it is a perfect matching. On the other hand, it is stable because we showed him shop here with a stable matching. At the same time to prove it is a man of matching, right? So everything perfectly interest, right? Now, next, he would say and ultimately good student men, but commercial is their medical women. It may not be true, right? Because it could be the case that both men were not very good, right? But for this case, all right, the stimulation problem, if you allow men, posters, and there will be women have seen guarantee.

So basically, every woman will get her worst valid part. Right? Remember, then it's best better partner and opposite way. Every woman gets is worse than time. And now we are going to show why it is correct. The proof is easy given our results of mammal convention.

Now, we suppose under this algorithm, a woman a is matched to man c but c is not the worst bad partner. Assume woman a gets lucky, all right, get some better partner. Then let me show the figure. All right, should it be? So assume that a guess y not y is worst, right? C is not worst in the gale shopping on what we can see. The worst part is why. All right? So definitely because y is worse, so a prefers c to yokc is better than a right? And then because why is better partner? Again, there is one stable matching where a is matched y then this z must be matched to somebody. Again. We assume the z is matched to b matched to b okay?

Now we know a prefer c to y we only need to show z also prefer a to b then it's unstable and then contradicting the fact that it is a stimulation.

How do we show z equal to a to b the reason is very simple, because it's a man optimal matching. All right? No. In this matching, a is matched to z because it's men multiple. So in the eye of za must be the best. Among all the stable partners. They see that's a better than b for sure. Now you see a contribution. Right? Just use the result that for z its partners best, right? That's better partner. Z likes a more, right? Z likes a better than b now, this is the contradiction. That means, that's right. That's the end. That means it is the women pessimistic solution, right? Okay, now this part seems easy, right? Okay. We have another bonus question. This bonus question is very difficult to get that we still have 17, 16 minutes before we finish the session. Am I going to let you? Please.

Now this is an anonymous question. Sorry, this is a yes or no question for sure. You'll also be known because if I allow it to be early, it's illegal, right? Okay? Why I take you so long? For this question, what am I going to talk about?

Now? Remember, it's not talking about the, because earlier we have one question which I already explained a bit about the tree, the cold blocks, the past, right? So I already explained that problem. Just then I didn't explain some solutions yet, but that's not important. I probably would not degrade to that. But there's been some other thing i'm going to explain what is that? I can also determine myself like, this is not the technical component i'm going to talk about. I'm going to just trying to clarify that the next week we move the monday lecture to thursday, to return monday. We will have no class. Good afternoon. But that's not technical component. We are going to talk about technical component. I'm going to do something here. And if I show you the same, if you tell me the answer, you still get bonus. Although I already downloaded still. No, I teach 8:52. But that does not mean that I i'm going to talk about that topic. I'm just changing the label in order to fit the weekend to be determined, right?

Okay. Still, no. I remember if you want to ask a question, raise your hand.

First. No, still, no, you are about to lose the bonus. If I create this folder to lose the bonus, do you know you update the sites? I didn't understand, I know, I didn't update the material of the size. The option mention, no option question is nine. It will be explained in nine. I cannot steal materials from nine. Wait, it's a different. Yeah. I really want to give this bonus very anxiously want to give bonus. From this page. You didn't see anything strange. 5147. But I don't have 5 . 2, three. Only 13 years ago. This is special thing, but that's not me what i'm going to talk about, right?

So I guess that there's a 5 . 3. Why 5 . 3? Suppose they should happen after. We find, I never touched that before. All right. I'm gonna touch it now. That's the only thing that does not belong to normal weeks, but it has a label, 5 point something, which means it should follow, right? After the game theory, introductory sign, because that slide is for five, right? 5 . 3. All right. But i'm not going to finish that everything in the site introduce briefly the problem. Okay? Now the title is defending with shared resource on the network. Ok so it's a how to say, isn't my schemes? Here? It was. The power systems tell me about this. It means that although it looks like a game, but actually, it's an optimization problem. All right? Why is the game? Because we have so called attacker and defender? All right? So they have strategies. And they will play with each other if I don't want to prevent attacker to get a lot of, then from the network attack want to get more.

So basically, it's again. But at the same time, it's an optimization problem. That's why ii would like to explain this right after we have some knowledge about game theory, but also we have knowledge about about the paralysis, right? Prosecution algorithm or some. And then it really out with them, right? And when we deal with it, so what's this problem? This problem actually is very interesting. Every note you in the graph has a treasure, the value.

And then every note, You can give it certain resource called defending resource. No, you among the defending resource. After now, that is special, right? The value you have resource available. You can advocate. An attacker can attack notes, but can only attack one note. All right. Now, the interesting thing here is the form. So let me go to the next page first. The oldest interesting thing here is I need to introduce this one first. So you have some way to compute the defending power of a note. If any power is equal to this node resource, plus neighbors share more resource. So basically, when certain note is attacked, that is neighbors can help this. But they cannot give all the resources to this node. They only give partial resources to this node. For example, I think I will give 50 % resource to this number, right? This person says I'm going to give 10 % of my resource to this person to this note.

So basically, every note with this whole resource, together with the neighbors contribution, right? Now he has aggregate. Now this is my so called the ending power. All right? So now, between attacker and defender, they will compare, right? If a tamper attacks somebody whose defending power is high, higher than upper bound, then attacker gets zero. Right? Very next time, if a defender, then the node is very strong, right? Attack attacks, then it takes catch it right. Now, if it attacks a very weak, no, whose depending power is below lower bound, and then the attacker will just get the full amount of value from that. If the node has $1 million attack, again, $1 million, these two streams, they are quite easy to understand.

Then the only thing that makes the problem very interesting is the middle range came to the middle range here. If the attacker attacks a law, who's the main power is between lower and other democrats?

First of all, you can imagine the attacker cannot get all the better. That's not possible, right? Because you have relatively good defending, right? So he cannot get everything. He doesn't have enough time to get everything. Right? So the attendant will only get part of the value and try to escape. All right? Okay. Now, when this attack escapes, because defendant is also relatively strong, so attack the defender, they will fight each other. The attack must do some power for sure, right? No matter what kind of power. All right? Anyway, the attack will be weaker after the combats, right? And then attacks will try to escape to some neighborhoods in the park. If the attacker manages to find one neighbor, which is very weak, okay, then the fact that they stayed to level and escape via a helicopter. All right? Get it. If this attacker cannot find any neighbor, which is very weak, then the attacker will be captured, is one of them.

What do you mean by very, very weak? Means? If any power below the slow down, if one neighbor below this doorbell, then the attacker will lose everything. He will not get anything. That is 00, if sorry. Ii bet if one of the neighbors is very weak, right? And then attacker will get gu prime, partial part of the value of no deal.

Now, if every neighbor is somewhere above the lower, and then the attacker cannot escape anywhere. Is it escape? It also will be captured because he does not have sufficient power to attack the event of anymore. All right. So in that case, you will guess you no neighbor. All right. If no neighbor is very big, then attacker gets you. All right? This is the problem set up. Any question?

Now im going to use one example to tell you how we calculate, right? Suppose you have six notes here, and then we have resources 212141, and then suppose the w it's half, which means you can give half the resource to a neighbor to increasing power. Right? But this resource is so shareable. You after you share resource, you still have this resource. Okay? That means you can compute the power like this, for example, for this node, resources to make your resources 1 + 1 + 4, 141. Altogether, 661 × half is 33 + 2 is 5. You can compute the power of this code is 5. Right? And then you use similar way to compute power of these nodes of this, right? Okay. Then I will go on. I suppose this is a robot. And how about this? Four and six, respectively, and treasure is two, partial treasure is one. Then you can see that actually, if you attack node six, because the power is two, be below four. So this belongs to very beautiful, right? So attack this person, you will get value to you, get more than you.

Now. If you attack three, because three powers, five between four and six, so it's a medium level notes. Three can escape. If the attacker attacks this node, you can escape. And then there happened to be a very weak name. We can escape from here and then take the helicopter. Doing this will give the title value one, g prime. That's why attacker attacks three. You can get one. Now, this attacker attack these notes. For example, this node. Now, you see this node is relevant. Four is between four and six, okay? And all its neighbors. They are relatively 57. They are pretty good defend, pretty well defended attacker attacking this. He can, I say, escape here, no use escape here, no use attacker attacking this node, will get zero done.

This is about a real example. Where about a pack of different notes? Protected and different, get different value. And what's the goal of the designer? The defenders goal is to allocate resources in such a way that the largest value among these nodes is as small as possible, because defender want to reduce the loss, right?

So a temper when he sees these figures, definitely were taxed on, right? Because that's all steady against. But for defender, he tried to minimize debt loss, right? So dependent on constructive resource allocation, such that the largest correct number is as small as possible.

Okay. This is the problem. And actually, there are different their versions of the problem, like a single threshold model where no bound itself about. They are the same. Then there are also isolated model where you cannot share the resource. The w is zero. They are not willing to share. All right. If I interest, you can check that paper to see the details. I'm not going to talk about the methodology here. The only thing I'm going to let you know is that this paper I like it a lot. The reason is, first of all, the model looks interesting. Second, the algorithms used to solve this problem. They are very how to say fundamental or put it this way, simple. This is what we use is greedy algorithm. First one, second one, from and from, remember, there's a Mexico terrorism in the network, so we just use the flow, but we do a very nice mapping between the problem and the flow problem ok in the end, we show something behind this proof. There's a reduction showing there's a difficult problem here.

And then is this problem reduced to the defending problem? If any problem is hard. And then in the end, we have some rounding technique ok so we have a linear program for integer linear program population, and then to relax the intermediate program population to linear program and have some solution.

And now we have some solution to do rounding, rounding these when you value 0 . 5 above, I round you up to one. If you are less than 0 . 5, I run you down to zero. If I do this, then I can use Apple's double resource to achieve the same effect as the optimal solution can do. Ok so basically, it's a so called resource augmentation setting where optimal solution can only use our resource. And your solutions are allowed to use 2 hours. I'm asking you, can you do achieve the defending result as good as multiple solution here? So basically, you have reason unfair comparison, right? Optimal solution is using 1 copy of resource, but you are using 2 copies of resource. Okay, so can you do as good as well? Because we said the problem itself is gonna be hard, right? So using the same amount resource, you never get, they cannot do as good as normal solution. That's not possible. So I'm asking all things better as can do as good as possible. It's not allowed to use more resource.

That's why I say, II like this paper a lot, because it integrates a lot of classic. I want to be approaches into one single problem. Okay, so today I think I just stop here, and next week my team will come here. I will not come here. Remember, there are two rooms for the midterm exam. Please sit. Please do not sit next to each other, and you have to leave one seat empty in the. All right. One ca taking care of one group. Also, I will ask my tears to argue with here a bit earlier so that we can do q and a you can ask questions before the quiz, something like after lunch, like 12 o'clock, 12 to 12 ~ 1:30, something like that. I would like my ta come to this room a bit early. All right. Thank you.