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Now, let's see the option. It seems nice, but still, there is some drawback. All right. So what's the drawback? Now, first of all, hard to understand that's also why some of you come to ask me to the cheating thing is it seems not so easy to comprehend, right? So why some cheating will not help them, some cheating will help them. The the more important thing here is that, in fact, right? Sometimes you do not know the truth valuation of yourself. That's a very travelling part, right? We know it might be that but very good, right? But if you do not know, then how can you report the true values? That's something. Can I do it? Right?

The second thing is, remember, I said everything is announced by seller seller, says this person highest. This person is second when he says this person is highest, maybe he can achieve. Because if he cheats, then somebody else will say you're wrong. I'm the highest, this person is not highest.

But sometimes even this is difficult to persuade, because when he says somebody tires, probably he does not know need to say how much money this person is willing to pay. This is some hidden information. If this is hidden, then the setup can still play some trick, right? You can say this person, right? This one is higher, right? Then even the real high, highest one cannot argue, et cetera. Right? Now, another thing is, who is the second? Right? Who is the second is also very important. But the good thing for second is the value of second.

For example, the second price is $100, right? If somebody, well, if somebody is true second, but the seller didn't announce his or her price as a second, then that was only complain with the seller. But usually I think the second probably is not so easy when they do it. All right. But anyway, it depends on what information the seller announced to the public. All right? If you announce the values of the 2 bits, then people will somehow be persuaded that they are the facts. Right? But if they do not say the value, then the whole thing is will be a chaos, right? Okay? And then sometimes some people are not willing to reveal their own private information. For example, how much money i'm going to pay for certain things, right? This information I do not want to say no. Then if I write something on the paper and give it to the seven instead of knowing my personal information, right? That is not so good.

That doesn't mean that even though in theory, it works very well, right? But in practice, there are various issues, then we say theory uses. Actually remember that this very famous example of the search engine, right? You see that whenever you search for some keyword, there were be some entries on the right, which are the advertisements related to those keywords, right?

And when people order whether companies bid for those slots, they were also bid for price ok so if you put me in this place, how much money i'm willing to pay? Right? Ok so in old times, usually, whenever somebody search, what's your name? So they ask and advertisements to the top candidates will be shown to use. All right? And then when people click on the link, then the advertiser will pay the search engine.

Now, so then how much money advertiser is willing to pay for a certain stock to so that company can show up in that link.

In a very early system, they usually do well. They send dummy search request, send a dummy search request to the search engine. After and based on the search result, you see, i'm not here. I'm not yet in the top ten list. Then I will raise my price a bit. So too, so that in my next try, i'll show up in the axis. So I can at least get some user click. Right? But this kind of mechanism, the price will keep changing. It's not stable. It's not very good. So basically, because there is some use one used as so forth, first price option. Whoever pays the highest will pay that price. So the advertiser is not willing to be the very high price, because they try to see whether lower the price can still maintain them on the list. Right? So that's why in recent systems, they just use the so called generalized second price option. So because here we are not saying one item. We are setting k slots, right? K slots. So basically who are winning the top k prices for weeks, but they will pay the k plus one's price.

Okay, so that's why it's called generalized second price. The winners pay the first person who didn't win that price, okay? And then the big price become more stable. So that becomes the most frequently used option mechanisms in the world. Okay? So still, people will like those mechanisms which have theoretical guarantee, not only practical verification, theoretical thing. It's not very, very good, very nice. Okay. Now we come to the combinatorial, because the first hour we talk about single item motion. This is common at all on material. There are many interesting combinations, for example, p cpu we should not write pc here. So this is cpu and this is memory. You have cpu memory. If you combine them together, they are useful. But if you only have memory, is this right? Unless really, it's only memory in memory, it's useless.

So basically these 2 type of products, we call them complement. Then another thing is the so called substitute. You have 2 types of no notebooks, right? One is a thinkpad, the other is this friend. And then if you have one notebook, it's good enough. You have two. Maybe sometimes good, but sometimes it's just redundant. Having two is not better than one, because at any time only is one, unless some of you can use two hands on two computers, one of them calling the others. Taxi.

Anyway, ii don't think many of you can do that. So you see that for items, right? Sometimes they have to be sold together and sometimes you sell them to the same person. It's useless, right? Now, for commentary options, the value of the goods they will be specified, right? On a combination use. To say these two things give it to me together. I i'm willing to pay $100. These three gives me, I i'm going to pay $200. I will specify the value on combinations of items. All right. There are many applications of this on the corruption. Im not going to name and the boss.

So basically, you have scenarios where you have to specify your value of a bunch of items. Sometimes you can also consider the reverse scenarios, so called procurement motion. Ok so in normal option, you have one set up. You sell the item to the buyer, right? And then in the procurement setting, the seller no longer sell items, so you switch the role of seller and buyer. So now this person becomes buyer. I'm buyer, i'm buying service. All of you. All right. And you're offering, if you want to dynamite service, how much I will charge or charge you. Okay? Not paying charge. So it's a reverse concept, right? Procurement is the reverse of the normal option.

Now, but the trouble is, if you allow the valuation on a bunch of items or a group of items, then a user need to give a lot of values, because number of combinations will be many. When you have 100 items, there are 2 to the 100 different of set of sub items, ok you have to the user have to specify a lot of value, a lot of values. And this value is exponential in size of the number of goods. That is not very efficient, right? Even if you do it, it's very hard to. Right? Okay. This is the problem itself. We call them window determination problem. So basically, you have end leaders, end leaders. And then the possible combination of the bidders will be two to the end, right? And then if it's a one instance of the so called way to set back any problem, and then this problem is a bit hard. Let me come to this. I'm skipping this, because this is just having you how much time it needs when you scale up the problem.

Now, here, i'm just trying to tell you something about the heuristic. Now this one is not theoretical heuristic, so it's called branch and cut. Now, what is this? You see that there are 44 players. A says I want one and two, right? Ok he says, I I want two is, and this one is c says I want two. He said that once now, you will decide what I can do. I give to whom. All right? This tree tells you what she says. If I give 12 ~ 8, which means in, right? It means I give 12 ~ 8. And then next, can I give something to me? No. Right? Because they also want two, but you already give 2 to 8. So you cannot give to, indeed.

So when consider all you count a in, you can only the next thing you can count in the c next one, I get c basically, you are trying to see that substance such that the total value of the subset to the respective players is as hard as possible for this branch and bound or branch and cut algorithm, you can get out of the solution by checking branches. And they including the running time.

So let's see one example. Suppose the four players, the value towards different bonds bundle here, 12, at least one was forty three four two three and fifty four six two, thirty eight one three and 60. And actually, we are talking about 1 specific kind of commentary option called single minded option. Single minded means every person is only interested in one, one minus ok because just now we mentioned if one person he is interested in many different combinations of the items, then even specifying these values will be to. It's time consuming right now. We restrict the players to be single minded, which means every one of them is only interested in one specific one. For example, here a is only interested in 12. C is only interested in c even in this very simple case, it is still in the heart, but then you can do rational bargain with them to find the best solution. Right? So here you see, when you do a and cm 17, right? 6, 17, a in c out, 14, and then a out, when you say a out, remember, bcd all of them on three, that means bcd only one of them can get the model, right?

If one of them can get bundled, then the best, this one, the best of these, even the best, is worse than this 70. Then that means this branch you don't need to consider at all, because no matter what solution you find out that much, it will not be as good as 70, right?

So you can just stop searching. This is so called branch and cut or branch and bound, which means you maintain a bond, which is currently best ok and then for branch whose best solution cannot beat your power best, then that branch you don't need to search. You just cut it. You don't need to go down that tree. It'll save your time, right? Okay? Now, here, you can use the linear programming to solve or to estimate the best amount in the later branch, if that best bound, and that was the best solution in the later branch, is worse than your current best. Then that branch you don't need to consider at. All. Right. Okay. Now, here, basically, what you need to follow is try to find a good solution as early as possible, because find a good solution earlier will help you cut a lot of useless branches later. Now, that's why you need to set all of this, for example, 81. But here, you also say, be equal or c equal. Is it on the root? Can be any figure, right? Which figure do you select first?

Usually it is a larger ones. Okay? Because you see a larger one, when it is in, you have little choice to select other people, right? So that will help you to make the first branch during faster you can do it. Right? Okay. Now, another thing, when you face whether you search in or search out first, it's also very important, right? Because, for example, if you do not go this route, you go out for us when you go out the best solution to find a 60. But then again, you can search for this branch. So most of the time you do not know which branch, which would be more out which one will give you a better solution. You don't know. This cannot be decided according to your real experience or past experience about which site may give you a better solution, right? But you want to find the cursors early, all right? And then decide be outside which one is gold earlier.

Okay. Now I think that I will not go deeper into that, because that will boil stamp. We're going down to using the commercial, healthy solver, but that is not our focus. Now, it's time for us to look at the bcg mechanism for bcg mechanism. The scenario is, every agents will declare violation towards subset of goods.

Now, this is a traditional combinatorial option, right? They will tell you, if you give me decide that how much money and money I i'm going to pay, give me that how much money i'm going to pay, right? Okay? And then the goods will be allocated to maximize social service. The seconds. Now this payment is very interesting. A the payment, for example, for asian one, the payment is equal to the how to say how much hurt you bring to other people? What does it mean? Suppose i'm gonna play it. All right? And you are here. Without me, you can get a social surplus of 100. All right.

Now ii so when I come in the social surplus maximization solution, you can only get 90, which means iii grab something, right? You only get 90 with my existence. Then how much should I pay? I pay ten, because that's a loss at break to the remaining community. All right. Many you alone, 100. Because of my existence, you become 90, right in the best season. And then I pick him. This is the classic interpretation for bcgk later on, i'm going to explain a better interpretation that better interpretation is more constructive. This one is more how to say a compensation base, right? How much benefit or how much loss I cause because of my existence to you, right?

And then I pay much. So that's some kind of compensation. I pay $10 to compensate for your loss, right? So there is another indication where and ii the new comer will be rewarded by something. You will see it very soon. Okay. Now, then this is a mechanism, the payment together with allocation. It satisfies incentive, compatibility or truthfulness, and also at the same time period issues. So basically, the solution is best. Nobody will cheat two things. Let's look at one example. In this example, we have two items, coffee and cake, then three pillars bitter. One says you can eat coffee, i'm going to pay $6. You give me take $0, you give me both also only $6, then player two says you can be coffee only $0. Cake only $0, give me both $8.

Last one, symmetric three, give me coffee zero, give me cake, 15, give me both also five, which means bigger one has no interest in cake, because he has no interest in coffee. Beta two has to get both to have into it. Okay. Here, according to the vct definition, what's the allocation? You want to maximize what? Social service? What's it? I don't want to get coffee, very good. So play one, get coffee, let's see, get tea. Then the social surplus as well. No. Right? It's the best. Now payment. Payment is interesting. So player one is without anyone how much money or how much value they expect, without one, how much they get? Eight, right? Which sale? And how much do they get? There is an investment. 500 with their one, they get eight without their one they get also with without they want to get things, which they only get 58, - 5, see us.

Then there's three. Without this seat, they get $80, right? With the s three, they get six a minus $62. Okay? It's always without this player, what's the social service maximization? Right? With player one up with this player, but you need to remove the players contribution, right? With player one, how much other people get, right? And then subtracting that part. That's the price. All right. Now let's go to the other interpretation. There's a bonus . here. Okay, so remember, we say without their one, other people get $8, right? And then with real one, other people get $5. This is three one's value, which is $6. What's the payment? 10 minutes, $3. With this comparison from a positive, you are, why should they want to pay $3? $3 from one perspective, is three minus the other. This is our first interpretation. How much other campaign without the one? How much other campaign with the other one? Right? This is a bit standard definition. Can you tell me another story about why they want me to pay $3? If everyone didn't pay the $3, the other, two, the other people will know the participate in the.

So this is about the bad consequence. Right? And again, but remember, now we are trying encourage people enjoy, right? So if you want to encourage, they want to join, why does player want to pay $3? Because player wants you to pay three and players pay three. Why are they? All they want? Utility history? History is different from history. Ok also, it's different from this, right? Ok this fails you to write three in this payment scheme, right? But that does not explain why you need to pay $3. Why pay one cannot pay $4? $2. What's the reason pay one? Pay $3? This is maybe the most challenging for this ., which need you to use knowledge you learned earlier and basically lectures. So remember my question. Why does player one pay exactly $3, not two, not four solution? Make it stable. In more sense, it equals the that similar surplus back surface, maximum service.

So a good try, but we need to find a series on the side sacred. Who said, let's say what? Shopping, what? Shopping value? Everyone paying $3 and the social support and ok if they I have to now sharply, that is one very related concept to what i'm going to do. Use.

We need to shop a minute here, but we need something here below, not as high as a been lower, marginal control, very good.

You got a bonus, marginal contribution. Remember, we say we want to reward people to join, right? You see, because of three ones, john a dissertation, it brings marginal values compared to without anyone. Player one can bring some marginal contribution, right? Then its utility should be equal to its marginal contribution. Otherwise, player one will be unhappy. He says I bring in extra three, right? Social surplus, and they do not allow me to get utility three. Then why should I come? Right. So you see that another interpretation is, we should guarantee clear ones. Marginal contribution is equal to this utility. In order to make this happen, you have to ask player exactly this amount, right? Because we want to spend one, right? In order to safeguard the marginal integrity of those beyond the green bar, the player wants to play the regular inside of green bar, right? That will make the utility equal to the marginal contribution. So that's a very interesting. Second interpretation of this vcg payment. Right? From the positive . of view, right?

Now we are encouraging people to join us. They will get to be equal to their marginal contribution, right? And then payment will be concluded according. All right. Any questions here? This is actually very interesting. Two interpretation. They converge to the same amount. Although in this literature, people won't usually use how much loss you bring to other people. This one to define payment. But for me, after I discovered this connection, I would rather use this one to explain it, right? The payment is just to make sure utility is margin contribution. Right? All right.

Now one in exercise, we now have one more bit for, then how to allocate goods. And what's that mean? This one is easy. What's the solution? Can you use two words to explain the solution? It's very hard. Two words to explain the solution is very challenging. Memory is correct. You really can use two words to explain the solution. Now, which two words? How many one word? Very good. The same as what? Same as the previous example. Basically, the last bidder does not play any role. Okay? So what think that you should be able to compute that using the social surplus mechanization solution?

And then is if I payment accordingly, and then as side notes, in the final exam, as I promised earlier, we only test game theory subjects. I know pure evidence topics, but still we have five questions, right?

And these five questions, one of them must be bcg education plan. Okay? This one is something safe. All right? You need to know this very well, right? You need to know how to compute the allocation, how to compute payment, but these are very standard. Try not to lose any points. That question. Okay? Now vcg right? Now we talk about c right? Fuck. It's not a superman one, okay? Context. What is context? Let's see. The final scenario. Suppose today we only have 2 hours now. Actually, it's not now. Then i'm asking, so who is willing to extend the lecture to 3 hours instead of 2 hours? Extend lecture by one more hour, right? Then you are bidding for that. Somebody say, I really want to have three hours. I'm willing to pay $30. They say, no, i'm only iii know that ii I ii have to collect $20 from you. If you want to accept it, there are some positive money and negative money for those who are willing to extend the lecture, you're going to right pay the same.

But if somebody do not like the expansion, they will claims I don't want $100 if you want me to stay to one hour. Okay? Now, when you collect all these monies, right? And then how can you guarantee you will declare a true purpose? Now, although, as we said earlier, how much money will you pay you yourself? Also do not know, right? Right? Then just let's pretend it you're willing to pay $30. You're going to pay $50 something like that. Okay? But how can we show you tell the truth? We also here is the where clark has coming to play. The first company, the sum of all the money. You bought it, positive, negative s get it. If the sum is positive, then extended class for now. If it's negative, do not extend it. Now, if your bit changed out, what does it mean? I is that, for example, we use the positive side. If you bid for examination, indeed, finally, the lecture is extended, then how much that should you pay? Right? So that's the question, right? Now, we will ask you to pay the minimum amount needed to change the other. Now, here's one example.

$20, - 10, - 2035, Because these values add up to a positive value, the value metric, then let's see how much is the state you want pay? The answer is if you want to pay nothing, why? Because even without anyone, these three also add up to $5 already, positive, right? Agent, agent one does not need to take any money to make the result happen. So you should not ask if you want to pay, if you ask, if you want to pay, if you want to cheat. I'm only going to pay $1. Why do you ask them pay $2? Right? Ok I say, hey, if other people already extend the lecture without anyone who did this, right? How about industry? Eighty four eighty four need to pay some other. Why? Because without 84, these three is - 10.

If anyone 84 does not participate, the result is do not extend, right? And then it's not a good way to fall into this. People says I won't extend, right? You have agent four one will pay $10, offset all the negative, right? Because if you put in $10, then it's zero. Then striking oil will extend, right? Agent for pay ten. He will not complain, right? Because without his ten, the class will not be extended, right? Okay? The line pay will be calculated.

Now this is called threshold payment. So minimum amount you need to pay in order to change our right, change the outcome in favor of you, right? That's a very interesting interpretation as well. Any questions here? About context? So this is a tax. So basically, the money collected will not be returned to people, okay? Not a like a real attacks. So the tax cannot be divided among. Otherwise, they will have an incentive to cheat. Okay? Now in auction, it's easy because this amount is selected by oceania. Right before it will not go back to business. So it's very sexy. Now, with so many interesting positive results for vcg we also need to touch about something bad of vcg okay. What's something bad for this gene?

Good information. What's that? That is computational english ok why? Remember, bcg goes for social surplus maximization and the social surplus maximization solution in many scenarios are empty heart problems. You cannot even compute that solution. How are you going to decide payments, decide allocation, right? That's very difficult, because it's gonna be hard problem. Right? So although theoretically, this method can guarantee growth efficiency, and also no people cheating, right? No agent cheating. But because of the computational issue, it cannot be applied in many scenarios.

That's why you still need to resort to different mechanisms when those optimization problem.

Another thing is about what the peanut just said. If you have a conclusion, right? We said nobody will cheat, nobody will cheat, but it doesn't mean no group will cheat. You need such a mechanism. Sometimes group of people can cheat. Okay? Again, if you want to even more robust mechanism against rule cheating, then can I use vcg you have to use other things? All right. So that also means that you need to design new mechanisms to achieve new new goals. Right? All right. So much about the option. Now, if no problem, we'll go to the matching part. All right. Okay. Now, the matching we already mentioned last time, so there are many time in the second hour. We'll just do a very simple review. What's the real well scenarios like the students to medical schools or workers to companies, et cetera. Right? So there are many scenarios. And then last time we talk about the preferred acceptance rate algorithm, also called give sharply differ acceptance, right? Algorithm.

We also call the matching stable matching problem, the civil marriage problem when the match is 1 to 1.

Now, this part, I think that I do not need to go through it, because they already explained. So basically, the only using an interesting here is, in the last lecture, we use the so called unstable care to represent something. Some pair of people, a like be better, but they are not matched in the solution, right? But usually people call this kind of care, blocking, care, because this hair is blocking the solution from being stable. Right? It's called blocking hair.

Okay. Now, this, I think I have been known to train. This is a standard this is also standard, but the only thing i'm changing today is for fairness issue, right? Because last time we interpret the shopping, I was in the sense that male proposed to phoenix, right? But today you have reversed it. Because last time we know that it's male of men proposed to women, the matching is good for men or good for men, right? So today I reverse ok now today women proposed to men. So today's mention is good for women, right? Ok so the other things are just a standard. All right. So I think that you have to the stable thing, the stable the proof of the stable matching. We already did it last time.

Today we are going to talk about so called incentives.

Now, remember, the main focus today is how to design some mechanisms so that nobody will cheat. Right? For matching similar things. Again, there are some cheating issues, right? Let's see the cheating example before the break. What's this example? In this example, we have a two people, two men, k and j k and j now, the preference we assume for j it likes a more than b more than cokb and c are both here, b and c are both here. For. All right. Okay. He likes being more than him. All right. Now, with this preference, the first round proposal to something like this. You see, because a like a more than a so a impose papers, right? Be like j one and kok be proposed to javas. Right?

C also, j one and k c is also proposed to j now, what will jj receive two proposals? B and c in j view, b is better than c for the purple, j should accept these proposal. Right? K will accept a proposal, right? So k accept a j except b where is cc here. C is rejected, right? And then he will propose the k but for k i'll miss something k in case you be better than a better than c even if c proposal k will change. All right. Done, then done.

Now remember, this is aa special thing where there are more women than men, right? But we said, even if in that case, deferred acceptance algorithm still can generate and stable outcome, right?

In the end of last section, you mentioned that right? When you extend along something, right? Because maybe I did it last time. Did we did I do it last time? Sometimes my memory will mix up. Maybe not, maybe not quite, but we'll see it very soon. It is still okay. So the side of k max dj max d right? It is stable. Why? Now see nobody like it. No worries. C given them is okay. In the remaining pair, j likes a better than b but a like k better than j it will not go together with j he says k better. So j even if you like me, no use, right? This direction, k like b better than a but b does not like a better than jb says j is better than a b does not want to go with k no. Dropping k it is simple and done.

But then j will say, can I improve? I remember tf we can show something that if women propose to men, then women side they have no incentive to cheat. Why? Because women propose according to preference, then proposing to higher preference is always good for them. This you can imagine, although you still need to prove, but i'm not going to prove it here, right? Proposing according to preference. It's always good. But accepting according to preference may not be good. Now, somebody try to deviate from the book.

Now, a little, I already tell you the answer, j or da now what would j do? J can only do one thing, because now b and c are opposed to j according to preference j except e and then j can say no, except c now, if chips, I see what happens and these rejected, right? And what would we do? Propose k bill proposed k then please write to me. He says p is better than a I accept e right? I'm jenny. And then he said so unlucky. So then they go to possible to j and a close to j they say, i'm very happy. I got my first preference, right? See, by accepting some less preferred candidate j in the end, get a better option, right? Get the top preference. Then let's say something that can't be allowed. All right. So any questions about why they can cheap to get better part? So if no question, then we'll come to the result. Basically, the result says, there's no mechanism that can guarantee both men and women were characters not possible. You have to sacrifice 1 second. Either the proposal side, either acceptance side, one side will cheat.

Okay? All right, so now come to the next bonus point, which will happen in the break. What's this next for this time? Now, just how to say the proposing site have no incentive to cheat, but is cheating alone. No single female will cheat alone, but a group of females can cheat together to get better outcome. What do you mean by better outcome? Or there are two dimensions? One is everyone in the group is strict investment. Right? That's a stronger benefits, right? Everyone gains strictly better, high. All right. Good. The second one is some member gets better partner, but some of them same. In that case, they will still deviate, because somebody gets better that this one is weakly better, right? This is strictly better. The bonus question is, can you find example where a group of woman will benefit by cheating together either about his or her case? Could you have a question? And then, for example, 10 minutes, you find the answer you can come to me immediately. Okay? I know some pm ngb for you.