### Algorithm\_week9\_lec03-20241104

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Now, let's see. Suppose you have 100 males and 100 females in a party. All right? Then you want to achieve 100 pairs using the da algorithm. Aa very logical prediction is that the metric is stable, and then honestly is best for everyone. Right? So this is something you'd expect. But in reality, will this really help? Let's see, the one assumption. The assumption is, male has a pre defined order of preference of the females, right? And then you will decide whether to accept, offer or not based on this practice.

Now assume you are a male. And then initially you prefer alice over becky, only spider ok but let me propose to you first and first round and you look separately for that, right? For the moment you separately. And then after many rounds nobody proposed to you. And then in the round 99 as proposal ok and then you face choice. So whatever place making with us, I will not vote again. I I believe many of you, he will not do this swap, because what does that you see? You are making the choice. Right? Which choice is alice to you. So the artist viewpoint, what's the order you are in? Could be very, very late, right? Close and check, the proposal check and then in the end come to you. Okay. Now, maybe you will just take it better, not change the address, because for future, right? Probably that's not good. But go shopping, da algorithm will ask you to replace becky by understand. Okay. So in reality, sometimes some assumption may not be working, right?

Now we come to the manager one matching, so manage one matching. We already discussed about it in the last lecture, especially when schools are admitting students. Right? We can use an extension of the area mechanism. And then the students correspond to female, school, person correspond to male, and then they were proposed to male, right? Ok so you can still do it.

Now, if the very basic one of them make one machine, you have a bunch of students, a bunch of schools. All right? And school has a student has a fixed preference, strict preference of the students at schools, and each school has a strict preference of this students. And then each school will have a so called maximum quota. All right, all its capacity. When news, actually, we say us is a school where student is matched. You see, is the students that the match to school c right? And then we say i'm actually feasible if the number of the students matched a certain school is not beyond its capacity. These are all very standard definition. Okay? And then the mechanism for this management matching, again, is that you every student apply to school and then school accept the top of qc right? Students reject all the others, right? And then when there are new proposals, just replace, it's a very straightforward extension. I don't know why the sign was so strange. It's what place. Firstly, when I opened another system, it does not have such a problem.

Okay, by the way, this is just kind of you finance the students, apply to schools in the school except several students, and it's very stable, just like this.

Now you have a max hotel of different three shoes. And then at the beginning, the applications like this, they prefer y 33 of them prefer y and one of them prefer t and one of them prefer s they upset these two and upset the top two, the yellow and purple reject the pink.

Sorry. My dad, see. Because, sorry, I it's not the three goals of time to buy as it's these 2 and the 3 months. So that's why they accept it, all too. Right? And then we want to apply to its next preference, which is here s right? When you write s because the mexico does one, and then the school has to prefer everyone to this boy, and then it will be replaced, right? And then the last one, this this one go to the second choice t okay. And then everything is happy because he can accept two people, two students, and now they are ready to. The final solution is just a state. That's the outcome of many former chain and da algorithm.

Okay? Now, again, no matter who propose, right? That sign, no single person will deviate, right? They will not cheat. Okay? So again, students, they cannot get an outcome by changing their parents. And then we, at the same time, the guarantees of things. Okay, so that is, we also mentioned it a bit last time. So there are two. Actually, one is no student has justified any. All right, so what does it mean? So the student, it is rejected across to c and then all the students accepted to school c should have high priority than this student. All right. This way, the students estimate the ending. You will not say, because so iii choose this school, but this school choose somebody below me, right? Below my below his preference. That's not good, right?

So if the school admitted students are lower and higher positions compared to this s then it is okay, right? Then there is another phase one. Another stable consideration is called non wasteless. Now, let's call this basically says, if a student is rejected from school c then c is filled to its maximum capacity. So in other words, c no do not have extra capacity to assess me, upset the student. If c still has capacity, syc then that's not good, right? Because s will be cleaning and receipt. It has school, right? Ok so a solution, we say it is stable if both conditions are satisfied. So the student will not add me someplace or claiming an empty seat from school. And then a blocking pair would be a pair where a student have and the oil claim emphasis. So this is only happening in many more mention. Okay? The da mechanism will obtain the automatic for students, as usual, as who propose, who will get the best outcome, right? But then today will be the same days when your school has a minimum quota or regional maximum quota, then the algorithm can no longer satisfy the good property.

So this is also something to talk about today.

Now, especially when there are these constraints, there could be no statement. That's the most serious consequence, right? Because last time in the last lecture, we say many variants of matching. We can use da to get to generate stimulation. But when you have minimum code or regional maximum code, then these things no longer.

You can imagine why do you need to have a minimum open? Because you want to operate. You need to have a minimum size of students, for example, for this master class, right? I cannot say for cs we only recruit five students. That's not good, right? You have to guarantee that, for example, at least 70, right? So you need to have a safe amount in order to run the class, right? Little things, right? We already mentioned why students who need to maintain a certain minimum number of students. And also, some residents must be allocated to a certain and not city area, rural area. All right.

Now, and on the other hand, all right, we usually require this school can only include up to 400 students, for example. But this 400, it's not, you cannot say a hard limit. For example, if I have been 401, maybe it's still okay, right? 402. Maybe it's still so something extremely thought, absolute mass. Sometimes it's not so reasonable. That's why you will. Well, you will allow the so called regional maximum instead of individual school, mexico. You have a new school maximum, but regional maximum is also very important. Here, the maximum quota sometimes can be addressed by adding some extra resource for. It can be refunded in the regional as important this one.

What's the maximum problem? Original maximum order, you have students, you have schools and their regions. Each region contains some schools, for example, in the shenzhen, there are high schools. In guangzhou, there are also high schools. They don't want their high schools, right? So each region had a bunch of high schools. And these high schools brought together, they will admit students up to a certain rate. For example, into one, maybe there are to make a smaller. Maybe there are ten ten high schools, right? Too small, too few. Anyway, let's assume ten. So each so there are ten high schools, which maybe one each one of them can admit. 200 students was 300 students every year. All right, 300, but as the region, as a whole, may not have the ability to host 3,000. Maybe the regional maximum is 250. So that means some schools, even though they didn't reach the maximum, they cannot recruit more students because of the region maximum constraint.

Okay, so say the first eight schools already amid 300. All right? So that already, maybe Sorry, 2,400, but my global maximum as well. The region maximum is 200. That means the last two schools they together can only admit one 100 students ok in order not to buy it, even though individually they still have capacity, but they can only be on that. Right? So that's a very natural constraint, which is useful in reality as well. Okay? And then you can see each school, right? You have a maximum folder. You see each region has a maximum folder. If you are. They so actually the paper is written by japanese scholars, and that also is because of the special situation in japan. All right. Okay, so basically, when you reduce the number of, well, number of residents in tokyo, then enough residents will be assigned in rural areas.

Ok I restrict the region maximum, then you can say already, I can do a balance application, and other regions can also have application, not all to the yes, right? Ok so this is their motivation. Then here, in this region, maximum scenario, we also have a so called claim and receipt. What does it mean? Now, claim a disease usually as well is that the soon as calling by apc in school c if school c didn't reach its individual maximum, right? And also visualize it, didn't reach the product sheet yet. Okay. This is just a simple claim, empty seat, because the student going to that school is feasible, right? Not only in the school perspective, but also in the region sector, right? The whole region, the capacity also not over the region maximum. There's another more how to say to be, well, stronger claim slowly, or maybe we it's a weaker claim. Ss is s is regionally claiming an accuracy of c if c is still have some capacity. But regionalize is already full.

So that's why I say it's a weaker version, because if the student really goes to sushi, that application will hide in the weather. Region maximum, because if you go to school see, the region amount goes younger, right? That's not good. That's not easy, right?

So going to that school is not a feasible solution, but the students still will end the less empty seat. He said your school still have some seats there. If you shift somebody, right? If you persuade somebody else to drive. I can go in, right? Not in your school. You persuade somebody from other school job, right? I can go to the school. This time, the student is need to make a stronger case, right? He says what he wanted to, let me in, you have to sacrifice somebody else, not like the first one you do not need to sacrifice. But the second one you need to send us. Right? But still, this student has incentive to any sense to claim that a disease, right? Okay. Now, when do you have international? There are indeed exists instances where no stable metric exists. Ok no simulation is one. They satisfy both name, fixed and no wasteless. And this includes love, regional, non business, which means there should be no students regionally cleaning an active seat. So later on, I will show you one example where this the unstable thing happens.

This is something that you can try. For some time. I just 1 to 2 minutes. You try to show this fence and non wasteless are compatible with regional maximum code. When you have regional maximum code, then there is an example where no signal mention. Okay. Now, this is only a very small example, two student. So I it is in the next page, two students, two schools, very small example. Try to see whether you can find an example, 12, a two, sa now maybe we slowly look at the example here. We have two students, a and b and two schools, one and two. Then these two schools are the same region and the region a minimum quota, a maximum quota is one. And school, the individual maximum order is also one, which means only one student and the students, right? Only once we can admit one suit. And then preference suit a likes one better than two, student really like school two, better than one. And for school one and two, they also have a reverse, right? Symmetric preference.

Now, let's see, in a matching, right? Because anyway, you need to have matched one person, one student, one school. If you do not have anyone, every student will claim, as you see, right? That's not good. One student matched to one school. Suppose, because all these things are senators symmetry, so we just assume a is matched.

Suppose b is match. Then there are two cases, a match to school one, a match school two, right? So if they match the school one, you see, it is very happy. Right? They are happy, but b is not happy. He says, i'm not matched, but sorry, but school one like me better than a right? They have anywhere have to chase my hand. But that's not good, right? If a match is so too, what else they mentioned school too, and then for a he had one better, okay, but he's mentioned too. Then he says, can go to one. Thank you. Iii I went to one. I I didn't hurt anyone, right? I didn't hurt anyone. I improve my own progress. Canada, but it is able regionally.

And now actually then probably let's go back to the place where regionally claim is what regional companies. So regionally claim mpcc it seems that the solution, regional claim cannot hurt somebody else. This might be because earlier, I say that you can hurt somebody else, right? But it seems that maybe, according to this definition, it does not say it can hurt somebody else. Right? With the this is s this s yeah, so that's why I really want to stay here. Actually see the school see that this student s lives better and the school the student is allocated. Now they belong to the same region. Right? In other words, this kind of shift is to shift the student itself. The students are not here, right? And then he says, I want to go to one school in the same region, which is better for me. So my biggest it will not change. It will not violate the regional maximum quota, because this region still has so many students in a change.

My dad, I explained something wrong, right? Okay. So that's why you see later on. So later on, you see, in this case, right? If it is special to, then he says I would rather go to one, because one to me is better and in the same region. So going to one does not hurt the overall visibility. So able regionally claim an empty seat in another school, right? So that's the another unstable seat, right? Ok so you see no matter which school you match a two, it's not safe. Right? Then there's no single search, because the ideas happen. Right? Any questions here?

Okay, so now, let's see what do they do in the last regional setting? Now they will do the following. So they play round on the ordering of schools in the region ok without 12120 k because this is one region, 1212 ground running order. Okay? And then according to this order, every school will be further upset. One student, as long as doing this will not buy it, either individual maximum quota or regional maximum quota. Okay? Otherwise just reject the students. So here in this example, you see, these two people, they prefer school one to school three to school two. These three prefer school two to school one to school three.

So this is round one, right? When they proposed to the region, the first school upset some students, right? And then second school, because these are proposed to second school, right? The second school is a second one student. Then you will accept the second one here and also the one here. Then you want it. Now, remember, the regional x here is four. You weigh yourself in this order, 1212. Then the last one will be rejected, right? Because it goes beyond the region maximum. This purple one would be rejected.

And then the purple one next will apply to school work, right? Then again, you will 11 ~ 1, too, because the school also have a preference. 1212. This is a preference. This is school's purpose, 12345 when the purple one joins, according to 1212, and this one will be checked, right? The yellow pair rejected, then you propose to school three, right? This one, this is rejected for the three. Yellow one will go to three. Right? Ok so that's the whole story about when the students apply in different schools, how the school do, regular school will follow the ground robbing order, which is pretty different, pre defined, right? According to this order to accept students in their preference list. Okay, done. All right. So your question, before you go on, now, this revised the algorithm, they can show it is very true and also fair, and then meaning no student will claim an empty seat. But still, there's a chance that students were regionally playing at this. So that will still happen because we already said there could be no stable solution. There could be no stable solution when you have regional product, but then you cannot avoid that.

Okay? But this one is more flexible than you give an artificial care of every school, just maintain a regional maximum will be good enough. Right? And then this way you will allow more students to be assigned to popular schools. Okay? The same matching is optimal. In some sense, this opportunity, I think I will not go deeper into it. It's very complicated. Okay? And then let's look at the more general constraints. So again, we have students, schools, right? And then students who have preference. These are all the same, right? And then suppose you have a new, the schools have an allocation factor that describes the students assigned to different schools.

Now, a you could have more distribution of constraints, not only maximum and regional maximum. You can allocate more general constraints to this factor so that you have to follow it, for example, in this one, you have a maximum photo three, then if you only have action for the three, then 32 is good, right? This one is okay.

Now, basically, all the feasible vectors are in this square, right? Because it only says maximum three, right? Inside square, anything is good. But then you could have another some other more complicated constraints, like hera, heracity, and complex.

For example, this one, this looks like convex. How about right? So this is aa aa region satisfying heredity and convex. This one known, it satisfies reality, a humanity, but it does not satisfy any complex. You see some hollow here, right? So that's probably not good. And then I think that I will skip these days. But what about the definition of these and complex set? But the good news is that the regional maximum and minimum order all these days, they satisfied and complexity. Okay? And sometimes the some soft minimum order, it goes, there are also some distance constraints. Right? For example, the metrics are close enough, et cetera. And these are also satisfying this and complexity requirement. Okay. Then there's one example about violation of the any complexity. This is the example shown here. Basically, this example, we have 2 groups. You can consider the schools, right? 22 groups, and then you have two rooms. One room has size, which can host the one person. The other room has size, which can host two people.

Here, you see that the application vector can not be something like 33, because this room bias, this room is more accountable to students. This room a big room can host four students. But you can never assign the students into groups such as they are 33, because 13 can go to the room with four people, but the other three cannot fit into the room with two capacity. Right? So when you have a room together with schools or groups right here, we treat the schools as rules, right? Project group. So your project group can only be almost four people here, almost two people here, or almost four people there, almost two people here. That's okay. That's possible, but you can never have 32. Right? Dividing students into 33 will be invisible, because you don't have two rooms, which can both host three students in the room. Right? So that's why the application factor goes like this. All of these parts are not possible, and we have l shape, and it's better. And this sense is not ever complex.

And then they show that the generalized da can handle the set with the satisfied, heroic tree and complex property. Okay? You want to handle more broader constraints. Then you need to use the so called adapter in the air. And then this one will not talk about in detail in this class.

Now, the last thing we are going to talk about is a top trading cycle mechanism. This is a very interesting mechanism which is used in many places.

What is that? Now, again, we use the 1 to 1 matching as an example. Now, we put the current female in the ok so we first randomly or arbitrary assignment opinions. All right. We already for some pairs. That's fine. These pairs are now in the system. And then we use female as notes in the graph. Basically, we have a graph in this graph. Every node represents a female. And then female will point to another female. Only when what is true. Only when that females partner is this females, top parents. Okay? Now, everything I try to where is the top offers here? This is my top premise and just in the lake to that position ok so everything new does this re pointing? I like that person best? I know that was the best. So then your graph becomes, well, every goal has one outgoing edge, right? Every goal only has passed and only has one outgoing edge. Right? But it possibly could have many cutting edge. That's fine, only one monthly.

And then and said there must be at least 1 second. If it's not stable, if it's already stable, then the algorithm stops. If it's not yet stable, then there must be one side. Can you make it? Now need some proof? What kind of proof do you do? Now, in what case, there is no set content. In the graph o this one is not the same as the end of line, probably talk about earlier. On a single path. It's not back, right? But remember, here, there can be love. There can be a self loop, which means this female already match the best man, right? Best male. This is also a cycle, right? Itself, cycle, right? If there is such a person, then it's also a cycle. In that case, we agree that as always a cycle, because if there's no sales cycle, that means terribly . out, right? Every point out, point out at some moment, it will come back to some point, right?

Okay, for sure, is everybody's . out, right? And then go to the middle and maybe stop in the middle. Then there is a second ok so n now you somehow is persuaded by me that there must be a cycle of the graph, either a cell cycle or a real cycle, going to get a meeting. Now, whenever you see a cycle, you just let every video get the feed the mail in the direction of the error to grab that mail, to be this for this female partner and grab that male to this female partner, you just reassign the nails along the cycle. In this way, every female in the cycle, guess its best partner, right? Is that right? Every female in the cycle gets the best part because you do this shipping, right? Okay, they're very happy. Entered into the market, at least. Now, in the case of cell cycle, it's even easier, right? You just, that's the CEO in the market, because he already get the best part, right? Okay. Go out. Shit, go out.

Now, because when somebody go out in the mini graph, it's just a problem with a smaller size. Right? This just means that I can redo this procedure, right? You again, try to let the female point to the remaining best way, right? And then do this check again, find another cycle. Do you assign? Go out. Eventually, the brass will be empty. Nobody left. Now, this solution is stable. But let's look at one example. Here's Alice, Becky Carroll, right? And currently, Alice is assigned to be. Becky is assignment, can carry assignment job. Right? So for Alice, because he likes John best. So they're terrible pointing to Carol discouragement, John, right? Ok so the lights on best. So . 2 and Carol, like Lee best. Now Lee is currently matched to Alice. Right? So that's why you point to Alice. And then Becky, like ten best, sorry, actually is assigned to him, but he also like John best. So he also point to Carol, because Carol is mentioned join now have.

This is the graph for these three people, and then you see, there is a cycle in it. Right? This is a second. What is that? Just swap their two partners, right? You give John to us, give me to Carol. They are very happy, right? Then they leave the market, and then only one person remain and okay, nothing to do, right? Okay. There is a very nice property for the Top trading cycle algorithm. Okay? First of all, it is trying to prove for female cycle, the relevant only prove a female cycle. What? Who are notes in the rough, right? Okay? Not so rigorous. Proof is that if there is exist a path pointing to you, then you can obtain any object in the past. What does it mean? There's a past pointing to you. Somebody wants your part, right? Somebody wants your partner that they can always obtain any mail in the past. Why? Because it's a past, right? So you want somebody in the past, maybe just a link to that. That's a note. And there is a second. Right? It was a passing, right?

And then you say, I want this person, right? And then you can say, all right. And then probably the algorithm you can get this person ok so basically, as long as the past year to you, they are eligible to get anyone in the past, including the Top purpose. Right? This is true, right? Because that will form a several. This part actually will remain in the system as long as you stay in the market. Why? As long as you stay, this password name, Bessie is not so straightforward. If you didn't say you have found your that's a pair. If you didn't say final question, you are right.

And then, if you say why other people will not leave the market, because now we are talking about any, I try to use this r I didn't open the light in the middle.

Suppose you are here. So you are here, d then why is it not possible that c points the here? They leave the market. It's actually easy, right? Can c . 2? Come back to ac have pointed, you have c only pointed to d right? Everyone can only have one edge going out, right? There were the people in the middle. They will not fight back. So there could be no cycle, continue these edges. That's not possible. Right? So basically, these people know this path will maintain, they will not leave the market. Right? So as long as you do not leave, because if you leave civil, can go back, right? But if you do not leave, nobody in the past can be the system, can be the market because everyone is come towards you, right? Nobody can have secondly going out because you have that whole purpose.

So that's okay, right? So the past remain as long as you are in the market. So for you, no hurry. You do not need to say I you dont need to change my preference. Do I need to originally my best preference is here? Don't need to say my best preference is b I need to say this no. I just keep doing that. He will not be gone, right? He will not be gone. I just wait to see whether I can get my first purpose, right? I don't need to say, maybe I don't have a chance to get those preference. Ii don't have it first. No, you do not need to go to the second. You just keep your first preference there. All right. No, no worry.

Okay. Basically, this finish the strategy purpose, because this shows you don't need to miss report your Top purpose. Right? Just tell people the truth, right? Your Top preference, you don't change it. Right? This is strategy proves.

Now, the second one is pareto efficient. It's also easy because you see, we pick females according to site, right? So in the first round, everybody gets the best part. Do you have a solution where everyone gets the best partner? And other people strictly better? Well, because you need to make sure they have as harder, right? If you want to find a solution in which political dominated solution, you still need to guarantee the first cycle. All the female get the best part, right? We have a solution here. You say I want to find a better parental or parental dominating solution. They have to guarantee the first round female, they still get the best, right? But if you guarantee the first round female can be the best, it doesn't hurt to remove them, giving them the best. Basically, removing the first cycle does not hurt the existence of parental optimal solution, or does not hurt the current solution to be parental optimal. All right? I there's AA big twist here.

Now let me just redo it. Let me just redo it. The first item you see, you say, after assigning the partners to their females, is this solution critical, efficient, critical, optimal? My answer is at least to let them get the optimal partner is maintaining the existence of political optimal solution. With this first cycle, unchanged, the first cycle, it just give them the best part. This one will not hurt the current solution to be political best solution or political optimal solution. Because more dominated solution, you cannot hurt the benefit of any single one in the first second.

Any single one in the first second still need to get the best partner, but to let them get the best partner, remove the reassigned along the cycle is something you must do, right? So let's just do it, right? Let's just do it for a second. And then in the second cycle, if some female guess, the second guess, not the first guess, then their outcome cannot be proved without hurting the first round units. Yeah, right. The second cycle gets the second best. The reason is its first best for the Top preference is assigned to the first round female, right ok so if the second round female want to catch that, first, the best, there's somebody in the first round has to sacrifice. But if they are sacrificed, the solution is no longer predominated. Our search, because somebody in the first round becomes worse, right? That's not the philosophy of pareto optimality, ok so basically, the second round they cannot improve, not possible. They cannot improve without working the first time. Right? Ok so basically, the whole thing is also political of recognition.

Okay, so I think that I I need to stop here. So basically you can do the school and the student assignments. Similar student point to the best school, a school point to the most precision. And there is a cycle and you can do it on the cycle, right? And then after this cycle leave the market. Basically, if you use a similar Top trading cycle area to do the student at school of science, okay? But I think that's the end to a very brief summary. The basics I think II only repeat of option. You need to know the vcg all right, very important in the second price option is also very standard. And for matching, you knew the different seven 7th. And we talk about the fax 48 and also the finally Top trading cycle. So much about today. And starting from next week, we are going to talk about the fair division subjects, which are even more interesting than action. See you next week.