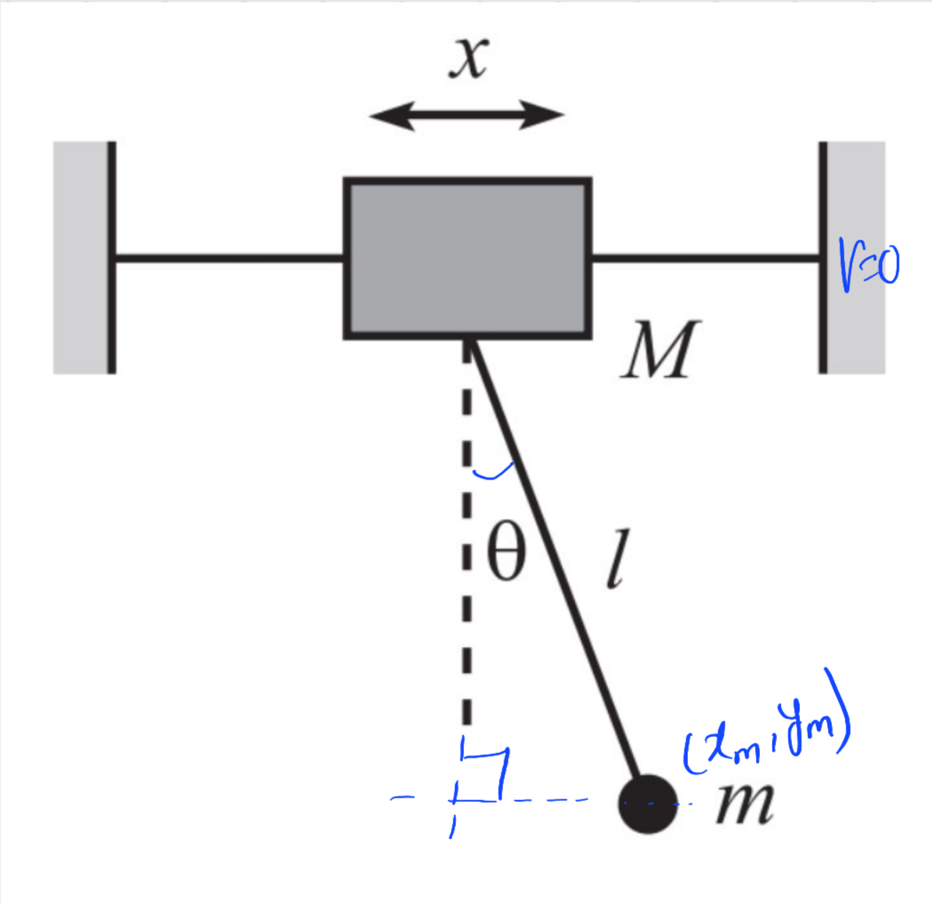
Pendulum with a free support



GK: x and θ

$x_m = x + l \cdot \sin\theta$

$y_m = -l \cdot \cos\theta$

$\Rightarrow \dot{x}_m = \dot{x} + l \cos\theta \cdot \dot{\theta}$

$\dot{y}_m = +l \cdot \sin\theta \cdot \dot{\theta}$

$$v_m^2 = \dot{x}_m^2 + \dot{y}_m^2 = (\dot{x} + l \cdot \cos\theta \cdot \dot{\theta})^2 + (l \cdot \sin\theta \cdot \dot{\theta})^2$$

$$= \dot{x}^2 + 2 \cdot \dot{x} \cdot l \cdot \cos\theta \cdot \dot{\theta} + l^2 \cdot \cos^2\theta \cdot \dot{\theta}^2 + l^2 \cdot \sin^2\theta \cdot \dot{\theta}^2$$

$$\Rightarrow \boxed{v_m^2 = \dot{x}^2 + 2 \cdot l \cdot \dot{x} \cdot \dot{\theta} \cdot \cos\theta + l^2 \cdot \dot{\theta}^2}$$

$$L = T_M + T_m - \cancelto{0}{V_M} - V_m$$

$$= \frac{1}{2} \cdot M \cdot \dot{x}^2 + \frac{1}{2} m (\dot{x}^2 + 2 \cdot l \cdot \dot{x} \cdot \dot{\theta} \cdot \cos\theta + l^2 \cdot \dot{\theta}^2)$$

$$+ (m \cdot g \cdot l \cdot \cos\theta)$$

$$\boxed{L = \frac{1}{2} M \cdot \dot{x}^2 + \frac{1}{2} m \dot{x}^2 + m \cdot l \cdot \dot{x} \cdot \dot{\theta} \cdot \cos\theta + \frac{1}{2} m \cdot l^2 \cdot \dot{\theta}^2 + m \cdot g \cdot l \cdot \cos\theta}$$

<u>Lagrangians</u>

$$\frac{\partial L}{\partial \dot{x}} = M \cdot \dot{x} + m \dot{x} + m \cdot l \cdot \dot{\theta} \cdot \cos\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = M \ddot{x} + m \cdot \ddot{x} + m \cdot l \cdot \ddot{\theta} \cdot \cos\theta - m \cdot l \cdot \dot{\theta} \cdot \sin\theta \cdot \dot{\theta}$$

$$= (M+m) \cdot \ddot{x} + m \cdot l \cdot \ddot{\theta} \cdot \cos\theta - m \cdot l \cdot \dot{\theta}^2 \cdot \sin\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = m \cdot l \cdot \dot{x} \cdot \cos\theta + m \cdot l^2 \cdot \dot{\theta}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = m \cdot l \cdot \ddot{x} \cdot \cos\theta - m \cdot l \cdot \dot{x} \cdot \sin\theta \cdot \dot{\theta} + m \cdot l^2 \cdot \ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = -m \cdot l \cdot \dot{x} \cdot \dot{\theta} \cdot \sin\theta - m \cdot g \cdot l \cdot \sin\theta$$

<u>Equations:</u>

<u>for x</u>

$$\boxed{\underbrace{(M+m)\ddot{x}}_{(i)} \underbrace{+ m \cdot l \cdot \ddot{\theta} \cdot \cos\theta}_{ii} \underbrace{- m \cdot l \cdot \dot{\theta}^2 \cdot \sin\theta}_{iii} = 0}$$

<u>for θ</u>

$$m \cdot l \cdot \ddot{x} \cdot \cos\theta \cancel{- m \cdot l \cdot \dot{x} \cdot \sin\theta \cdot \dot{\theta}} + m \cdot l^2 \cdot \ddot{\theta} \cancel{+ m \cdot l \cdot \dot{x} \cdot \dot{\theta} \cdot \sin\theta} + m \cdot g \cdot l \cdot \sin\theta = 0$$

$$\Rightarrow m \cdot l \cdot \ddot{x} \cdot \cos\theta + m \cdot l^2 \cdot \ddot{\theta} + m \cdot g \cdot l \cdot \sin\theta = 0$$

$$\Rightarrow \ddot{x}\cdot\cos\theta + l\cdot\ddot{\theta} + g\cdot\sin\theta = 0$$

$$\Rightarrow \boxed{\underbrace{l\cdot\ddot{\theta}}_{(iv)} + \underbrace{\ddot{x}\cdot\cos\theta}_{(v)} + \underbrace{g\cdot\sin\theta}_{(vi)} = 0}$$

$$\ddot{x} = \frac{m}{M+m}\cdot l\cdot\dot{\theta}^2\cdot\sin\theta - \frac{m}{M+m}\cdot l\cdot\ddot{\theta}\cdot\cos\theta$$

$$\ddot{\theta} = -\frac{1}{l}\cdot\ddot{x}\cdot\cos\theta - \frac{g}{l}\cdot\sin\theta$$

Unit comparison

(i) $kg\cdot m/s^2 \Rightarrow kg\cdot m\cdot s^{-2}$

(ii) $kg\cdot m\cdot 1/s^2 \Rightarrow kg\cdot m\cdot s^{-2}$

(iii) $kg\cdot m\cdot 1/s^2 \Rightarrow$ "

same

(iv) $m\cdot 1/s^2 = ms^{-2}$

(v) $m/s^2 = ms^{-2}$

(vi) $m/s^2 \Rightarrow ms^{-2}$

same