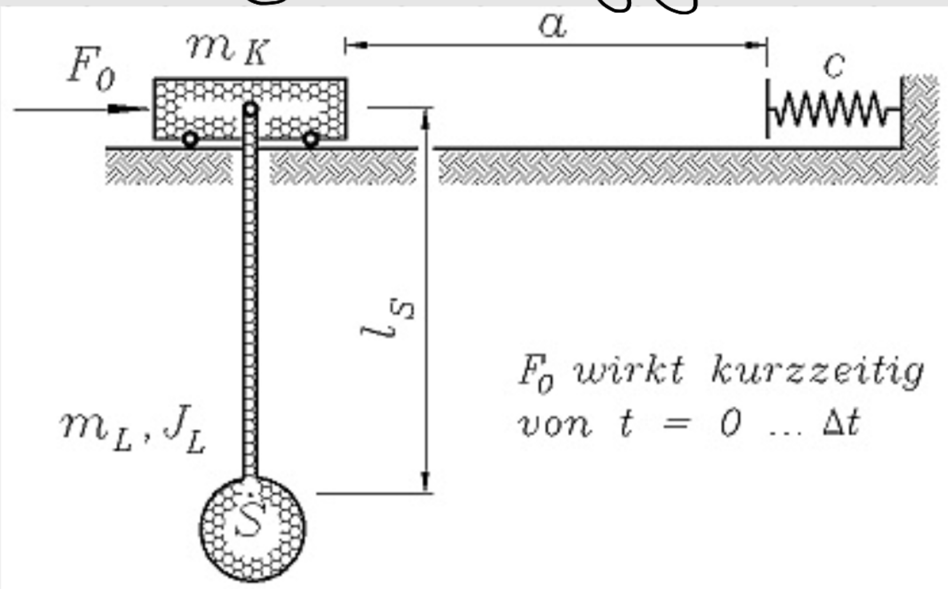
Trolley with swinging load



given:

$m_K = 100\ kg$

$m_L = 500\ kg$

$J_L = 400\ kgm^2$

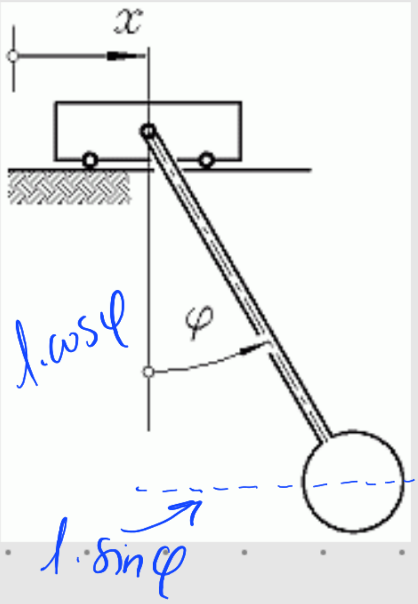
$l_s = 4m$

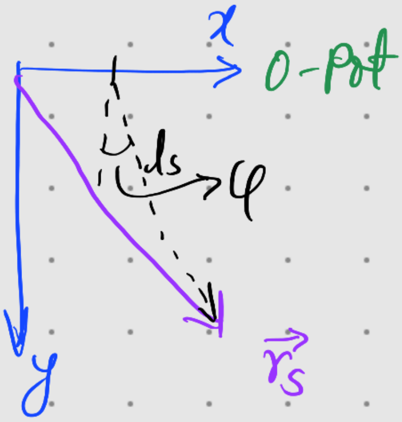
$F_0 = 2000N$

$c = 200000\ N/m$

$\Delta t = 1s$

$a = 5m$





$$F_t = \begin{cases} F_0 & \text{for } t < \Delta t \\ 0 & \text{for } t \geq \Delta t \end{cases}$$

$$c_t = \begin{cases} 0 & \text{for } x < a \\ c & \text{for } x \geq a \end{cases}$$

Soln.:

$$\vec{r}_S = \begin{bmatrix} x + l_s \cdot \sin\varphi \\ l_s \cdot \cos\varphi \end{bmatrix}$$

$$\Rightarrow \vec{V_s} = \frac{d\vec{r_s}}{dt} = \begin{bmatrix} V_{sx} \\ V_{sy} \end{bmatrix} = \begin{bmatrix} \dot{x} + l_s \cdot \dot{\varphi} \cdot \cos\varphi \\ -l_s \cdot \dot{\varphi} \cdot \sin\varphi \end{bmatrix}$$

$$V_s^2 = V_{sx}^2 + V_{sy}^2 = \dot{x}^2 + 2 \cdot l_s \cdot \dot{x} \cdot \dot{\varphi} \cdot \cos\varphi + l_s^2 \cdot \dot{\varphi}^2 \cdot \cos^2\varphi$$

$$+ l_s^2 \cdot \dot{\varphi}^2 \cdot \sin^2\varphi$$

$$= \dot{x}^2 + l_s^2 \cdot \dot{\varphi}^2 + 2 \cdot l_s \cdot \dot{x} \cdot \dot{\varphi} \cdot \cos\varphi \quad \text{—— } ①$$

$$T_{mx} = \frac{1}{2} \cdot m_x \cdot \dot{x}^2$$

$$T_{mL} = \frac{1}{2} \cdot m_L \cdot V_s^2 + \frac{1}{2} \cdot J_L \cdot \dot{\varphi}^2$$

$$= \frac{1}{2} \cdot m_L \cdot \left(\dot{x}^2 + l_s^2 \cdot \dot{\varphi}^2 + 2 \cdot l_s \cdot \dot{x} \cdot \dot{\varphi} \cdot \cos\varphi \right) + \frac{1}{2} \cdot J_L \cdot \dot{\varphi}^2$$

$$\Rightarrow T = \frac{1}{2} \cdot m_x \cdot \dot{x}^2 + \frac{1}{2} \cdot m_L \cdot \left(\dot{x}^2 + l_s^2 \cdot \dot{\varphi}^2 + 2 \cdot l_s \cdot \dot{x} \cdot \dot{\varphi} \cdot \cos\varphi \right) + \frac{1}{2} \cdot J_L \cdot \dot{\varphi}^2$$

$$\text{—— } ②$$

$$V_{mx} = \frac{1}{2} c_t \cdot (x - a)^2$$

$$V_{mL} = -m_L \cdot g \cdot l_s \cdot \cos\varphi$$

$$\Rightarrow V = -m_L \cdot g \cdot l_s \cdot \cos\varphi + \frac{1}{2} \cdot c_t \cdot (x - a)^2 \quad \text{—— } ③$$

$$\boxed{\begin{aligned} L = {} & \frac{1}{2} \cdot m_x \cdot \dot{x}^2 + \frac{1}{2} \cdot m_L \cdot \left(\dot{x}^2 + l_s^2 \cdot \dot{\varphi}^2 + 2 \cdot l_s \cdot \dot{x} \cdot \dot{\varphi} \cdot \cos\varphi \right) \\ & + \frac{1}{2} \cdot J_L \cdot \dot{\varphi}^2 + m_L \cdot g \cdot l_s \cdot \cos\varphi - \frac{1}{2} \cdot c_t \cdot (x - a)^2 = 0 \end{aligned}}$$

Lagrangians

$$\frac{\partial L}{\partial \dot{x}} = m_k \cdot \dot{x} + m_2 \cdot \dot{x} + m_2 \cdot l_s \cdot \dot{\varphi} \cdot \cos\varphi$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = (m_x + m_2) \cdot \ddot{x} + m_2 \cdot l_s \cdot \ddot{\varphi} \cdot \cos\varphi$$
$$- m_2 \cdot l_s \cdot \dot{\varphi}^2 \cdot \sin\varphi$$

$$\frac{\partial L}{\partial \dot{\varphi}} = m_2 \cdot l_s^2 \cdot \dot{\varphi} + m_2 \cdot l_s \cdot \dot{x} \cdot \cos\varphi + J_2 \cdot \dot{\varphi}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) = m_2 \cdot l_s^2 \cdot \ddot{\varphi} + m_2 \cdot l_s \cdot \ddot{x} \cdot \cos\varphi$$
$$- m_2 \cdot l_s \cdot \dot{x} \cdot \dot{\varphi} \cdot \sin\varphi + J_2 \cdot \ddot{\varphi}$$

$$\frac{\partial L}{\partial x} = \frac{\partial}{\partial x}\left(-\frac{1}{2} c_t \cdot x^2 + c_t \cdot x \cdot a - \frac{1}{2} \cdot c_t \cdot a^2\right)$$

$$= -c_t \cdot x + c_t \cdot a = -c_t \cdot (x - a)$$

$$\frac{\partial L}{\partial \varphi} = -m_2 \cdot l_s \cdot \dot{x} \cdot \dot{\varphi} \cdot \sin\varphi - m_2 \cdot g \cdot l_s \cdot \sin\varphi$$

Equations

for x

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = Q_x$$

$$\boxed{(m_k+m_2)\cdot\ddot{x}+m_2\cdot l_s\cdot\ddot{\varphi}\cdot\cos\varphi-m_2\cdot l_s\cdot\dot{\varphi}^2\cdot\sin\varphi+c_f\cdot(x-a)=F_t}$$

$$\ddot{x}=\frac{1}{m_k+m_2}\left(F_t-m_2\cdot l_s\cdot\cos(\varphi)\cdot\ddot{\varphi}+m_2\cdot l_s\cdot\sin(\varphi)\cdot\dot{\varphi}^2-c_f\cdot(x-a)\right)$$

<u>for φ</u>

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\varphi}}\right)-\frac{\partial L}{\partial\varphi}=Q_\varphi$$

$$m_2\cdot l_s^2\cdot\ddot{\varphi}+m_2\cdot l_s\cdot\ddot{x}\cdot\cos\varphi-\cancel{m_2\cdot l_s\cdot\dot{x}\cdot\dot{\varphi}}\cdot\sin\varphi+J_2\cdot\ddot{\varphi}+\cancel{m_2\cdot l_s\cdot\dot{x}\cdot\dot{\varphi}}\cdot\sin\varphi+m_2\cdot g\cdot l_s\cdot\sin\varphi=0$$

$$\Rightarrow m_2\cdot l_s^2\cdot\ddot{\varphi}+m_2\cdot l_s\cdot\ddot{x}\cdot\cos\varphi+J_2\ddot{\varphi}+m_2\cdot g\, l_s\cdot\sin\varphi=0$$

$$\Rightarrow\boxed{(m_2\cdot l_s^2+J_2)\ddot{\varphi}+m_2\cdot l_s\cdot\ddot{x}\cdot\cos\varphi+m_2\cdot g\cdot l_s\cdot\sin\varphi=0}$$

$$\ddot{\varphi}=\frac{1}{m_2\cdot l_s^2+J_2}\left(-m_2\cdot l_s\cdot\ddot{x}\cdot\cos\varphi-m_2\cdot g\cdot l_s\cdot\sin\varphi\right)$$