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$$S \rightarrow aAB \mid aABB \mid aab$$

$$A \rightarrow aBc \mid aA \mid \epsilon$$

$$B \rightarrow bBA \mid bA \mid \epsilon$$

$$c \rightarrow Ab \mid ba \mid bb \mid aa$$

$$D \rightarrow a \mid b \mid \epsilon$$

Null-production: $\omega_1 = \{A, B, D\}$

$$\omega_2 = \omega_1$$

$$S \rightarrow aAb \mid ab \mid aABB \mid aBB \mid aAb \mid aab$$

$$A \rightarrow aBc \mid \epsilon \mid aA \mid a$$

$$B \rightarrow bBA \mid bB \mid bA \mid b$$

$$c \rightarrow Ab \mid b \mid ba \mid bb \mid aa$$

$$D \rightarrow a \mid b \text{ — not reachable}$$

CNF:

$$S \rightarrow aAb$$

$$S \rightarrow XP \text{ — (i)}$$

$$X \rightarrow a \text{ — (ii)}$$

$$P \rightarrow AB$$

$$P \rightarrow AY \text{ — (iii)}$$

$$Y \rightarrow b \text{ — (iv)}$$

$$S \rightarrow aABB$$

$$S \rightarrow XAYB$$

$$S \rightarrow QR \text{ — (v)}$$

$$Q \rightarrow XA \text{ — (vi)}$$

$$R \rightarrow YB \text{ — (vii)}$$

$$S \rightarrow aBB$$

$$S \rightarrow XYB$$

$$S \rightarrow XR \text{ — (viii)}$$

$$B \rightarrow aAB \text{ — (ix)}$$

$$B \rightarrow XAY$$

$$B \rightarrow QY \text{ — (x)}$$

$$\begin{array}{l}
 S \rightarrow ab \\
 S \rightarrow XY - (11) \\
 S \rightarrow aab \\
 S \rightarrow XXY \\
 S \rightarrow TY - (12) \\
 T \rightarrow XX - (13)
 \end{array}$$

$$\begin{array}{l}
 A \rightarrow aBC \\
 A \rightarrow XBC \\
 A \rightarrow UC - (14) \\
 U \rightarrow XB - (15)
 \end{array}$$

$$\begin{array}{l}
 A \rightarrow aC \\
 A \rightarrow XC - (16) \\
 A \rightarrow aA \\
 A \rightarrow XA - (17)
 \end{array}$$

$$\begin{array}{l}
 B \rightarrow bbA \\
 B \rightarrow YYA \\
 B \rightarrow VA - (18) \\
 V \rightarrow YY - (19)
 \end{array}$$

$$\begin{array}{l}
 B \rightarrow bb \\
 B \rightarrow YY - (20) \\
 B \rightarrow bA \\
 B \rightarrow YA - (21)
 \end{array}$$

$$\begin{array}{l}
 C \rightarrow Ab \\
 C \rightarrow AY - (22) \\
 C \rightarrow bA \\
 C \rightarrow YX - (23)
 \end{array}
 \quad
 \begin{array}{l}
 C \rightarrow bb \\
 C \rightarrow YY - (24) \\
 C \rightarrow aA \\
 C \rightarrow XX - (25)
 \end{array}$$

Reduced CNF:

$$S \rightarrow XP | QR | XR | QY | XY | TY$$

$$P \rightarrow AY$$

$$Q \rightarrow \dot{X}A$$

$$R \rightarrow YB$$

$$T \rightarrow XX$$

$$A \rightarrow UC | XC | XA | a$$

$$U \rightarrow XB$$

$$B \rightarrow VA | YY | YA | b$$

$$V \rightarrow YY$$

$$C \rightarrow AY | YX | YY | X$$

$$X \rightarrow a$$

$$Y \rightarrow b$$

String check

\emptyset	Rejected	
$\{S, U, P\}$	$\{O, e\}$	
$\{X, A\}$	$\{Y, B\}$	$\{X, A\}$
a	b	a

$\{S, P\}$	accepted	
$\{O, T, A, e\}$	$\{S, U\}$	
$\{X, A\}$	$\{X, A\}$	$\{Y, B\}$
a	a	b

②

$S \rightarrow AB|BA|AC|AB|a|b$

$A \rightarrow DB|BC|a$

$B \rightarrow BD|BB|b$

$C \rightarrow c|c|c$

$D \rightarrow d$

$E \rightarrow \epsilon$

$F \rightarrow \epsilon$

$G \rightarrow \epsilon$

Null production: $\omega_1 = \{E, F, G\}$
 $\omega_2 = \omega_1$

Unit production: $\omega_1 = \{\emptyset\}$

Non-Generating: $\omega_1 = \{A, B, C, D, S\}$

Non-Reachable: $\omega_1 = \{S\}$

$\omega_2 = \{S, A, B, C, D\} \cup \{a, b\}$

$\omega_2 = \{S, A, B, C, D, a, b\} \cup \{c, d\}$

$\omega_3 = \{S, A, B, C, D, a, b, c, d\}$

$\omega_n = \omega_3$

Reduced: CNF

$S \rightarrow AB \mid BA \mid AC \mid AD \mid a \mid b$

$A \rightarrow DB \mid BC \mid a$

$B \rightarrow BD \mid BB \mid b$

$C \rightarrow CC \mid c$

$D \rightarrow d$

String check:

accepted

$\{S, A\}$		
$\{A, \}$	$\{B\}$	
$\{D\}$	$\{S, B\}$	$\{S, B\}$
d	b	b

rejected

\emptyset		
\emptyset	\emptyset	
$\{S, A\}$	$\{S, A\}$	$\{S, A\}$
a	a	a

③

$S \rightarrow AS | BS | SS | SC | a | bS | SD | b$

$A \rightarrow AA | AB | a$

$B \rightarrow BB | BC | b$

$C \rightarrow CC | c$

$E \rightarrow E$

$F \rightarrow E$

$G \rightarrow E$

Null production: $w_1 = \{E, F, G\}$
 $w_1 = w_2$

Unit production: $w(S) = \{S\}$
 $w(A) = \{A\}$
 $w(B) = \{B\}$
 $w(C) = \{C\}$

Non-generatable: $w_1 = \{S, A, B, C\}$

Non-reachable: $w_1 = \{S\}$
 $w_2 = \{S, A, B, C\} \cup \{a, b\}$
 $w_3 = \{S, A, B, C\} \cup \{a, b, c\}$
 $w_4 = w_3$

Reduced CNF:

$S \rightarrow AS \mid BS \mid SS \mid Sc \mid a \mid b$

$A \rightarrow AA \mid AB \mid a$

$B \rightarrow BB \mid Bc \mid b$

$c \rightarrow c \mid cc$

String check:

