Resource Allocation in OFDMA-based Wireless Networks

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Table of contents

System Model and Notations

Uplink Transmission

Consider A Multi-cell OFDMA-based Wireless Network consisting of

- In OFDMA, the available bandwidth W is subdivided into $\mid \mathcal{C} \mid$ multiple sub-channels
- Thus Bandwidth of each sub-channel k is given by $W^k = \dfrac{W}{\mid \mathcal{C} \mid}$
- A set of C sub-channel (SC) is denoted by $\mathcal{C} = \{1, 2, \cdots, C\}$ (also so called resource block, sub-carrier, channel, or carrier)

System Model and Notations

Uplink Transmission

Consider A Multi-cell OFDMA-based Wireless Network consisting of

- A set of B Base Stations (BSs) is denoted by $\mathcal{B} = \{1, 2, \cdots, B\}$
- A set of U users is denoted by $\mathcal{U} = \{1, 2, \cdots, U\}$
- A set of C sub-channel (SC) is denoted by $\mathcal{C} = \{1, 2, \cdots, C\}$ (also so called resource block, sub-carrier, channel, or carrier)
- The Base station assigned to user i is denoted by b_i where $b_i \in \mathcal{B}$
- The set of users served by the BS j where $j \in \mathcal{B}$ denoted by $\mathcal{U}_j$.
- The path gain from user i toward BS j where $j \in \mathcal{B}$ on sub-channel k is denoted by $h^k_{j,i}$. Thus $h^k_{b_i,i}$ and $h^k_{b_l,i}$ are path-gain from user i toward BSs associated to user i and l on sub-channel k, respectively.

System Model and Notations

Uplink Transmission

- The transmit power of user i ($i \in \mathcal{U}$) on sub-channel k ($k \in \mathcal{C}$) is denoted by p_i^k where $p_i^k \geq 0$

$$\boldsymbol{P} = \begin{bmatrix} p_1^1 & p_1^2 & \cdots & p_1^C \\ p_2^1 & p_2^2 & \cdots & p_2^C \\ \vdots & \vdots & \ddots & \vdots \\ p_U^1 & p_U^2 & \cdots & p_U^C \end{bmatrix}$$

where $\mathbf{p}^k = \left[p_i^k\right]_{i \in \mathcal{U}}$ and $\mathbf{p}_i = \left[p_i^k\right]_{k \in \mathcal{C}}$

System Model and Notations

Uplink Transmission

- The binary variable a_i^k denotes the sub-channel allocation index:

$$a_i^k = \begin{cases} 1, & \text{if sub-channel } k \text{ is allocated to user } i \\ 0, & \text{otherwise} \end{cases}$$

$$\boldsymbol{A} = \begin{bmatrix} a_1^1 & a_1^2 & \dots & a_1^C \\ a_2^1 & a_2^2 & \dots & a_2^C \\ \vdots & \vdots & \ddots & \vdots \\ a_U^1 & a_U^2 & \dots & a_U^C \end{bmatrix}$$

  where $\mathbf{a}^k = \left[a_i^k\right]_{i \in \mathcal{U}}$ and $\mathbf{a}_i = \left[a_i^k\right]_{k \in \mathcal{C}}$
- A set of users that transmit on sub-channel k is denoted by $\mathcal{U}^k$ such that $\mathcal{U}^k = \{\forall i \in \mathcal{U} | a_i^k = 1\}$

System Model and Notations

Uplink Transmission

- A set of sub-channels which is allocated to user i denoted by $\mathcal{C}_i$, where $\mathcal{C}_i = \{\forall k \in \mathcal{C} \mid a_i^k = 1\}$.
- The received SINR of user i on sub-channel k is:

$$\gamma_i^k(\mathbf{a}^k, \mathbf{p}^k, b_i) = \frac{a_i^k p_i^k h_{b_i,i}^k}{\sum_{l \in \mathcal{U}^k,\ l \neq i} a_l^k p_l^k h_{b_i,l}^k + N_{b_i}^k}$$

  where $N_{b_i}^k = N_0 W^k$ in which N_0 is the Noise power density (Noise power per Hz).
- For a single-cell network, the received SINR of user i on sub-channel k is given by:

$$\gamma_i^k(\mathbf{a}^k, \mathbf{p}^k, b_i) = \frac{a_i^k p_i^k h_{b_i,i}^k}{N_{b_i}^k}$$

  In this situation, each sub-channel is allocated to at most one user i.e., $\mid \mathcal{U}^k \mid \leq 1, \forall k \in \mathcal{C}$

System Model and Notations

Uplink Transmission

- The achieved data rate of user i on sub-channel k based on Shanon's formula is:

$$R_i^k(\mathbf{a}^k, \mathbf{p}^k, b_i) = W^k \log_2(1 + \gamma_i^k(\mathbf{a}^k, \mathbf{p}^k, b_i)) \qquad \text{bps}$$

  where W^k is the Bandwidth of sub-channel k which is given by $W^k = \dfrac{W}{\mid \mathcal{C} \mid}$

- The spectral efficiency of user i on sub-channel k is obtained by $\dfrac{R_i^k(\mathbf{a}^k, \mathbf{p}^k, b_i)}{W^k}$ i.e.,:

$$\frac{R_i^k(\mathbf{a}^k, \mathbf{p}^k, b_i)}{W^k} = \log_2(1 + \gamma_i^k(\mathbf{a}^k, \mathbf{p}^k, b_i)) \qquad \text{bps/ Hz}$$

System Model and Notations

Uplink Transmission

- Also, the spectral efficiency of user i on all of its allocated sub-channels is :

$$R_i(\mathbf{a}, \mathbf{p}, b_i) = \sum_{k \in \mathcal{C}} R_i^k(\mathbf{a}^k, \mathbf{p}^k, b_i) \qquad \text{bps}$$

- As seen, in contrast to CDMA, in OFDMA, there is no one-to-one relationship between the SINR vector and total rate (data rate) for each user. In other words, for a given user i, there exist many SINR vectors for that user ($\boldsymbol{\gamma}_i = \left[\gamma_i^k\right]_{k \in \mathcal{C}}$) which may result in the same rate (R_i).
- So, a constraint on rate (data rate) for each user does not correspond to a single- constraint on its SINR vector

OFDMA-based Wireless Networks

Resource Allocation Problems

- Power control problems
- Scheduling (sub-channel allocation)
- Base Station assignment problems

OFDMA-based Wireless Networks

Resource Allocation Problems

- Power control problems
- Scheduling (sub-channel allocation)
- Base Station assignment problems

Decision Variables

- Transmit power level
- Sub-channel (SC)
- Base Station

Constraints

Uplink Transmission

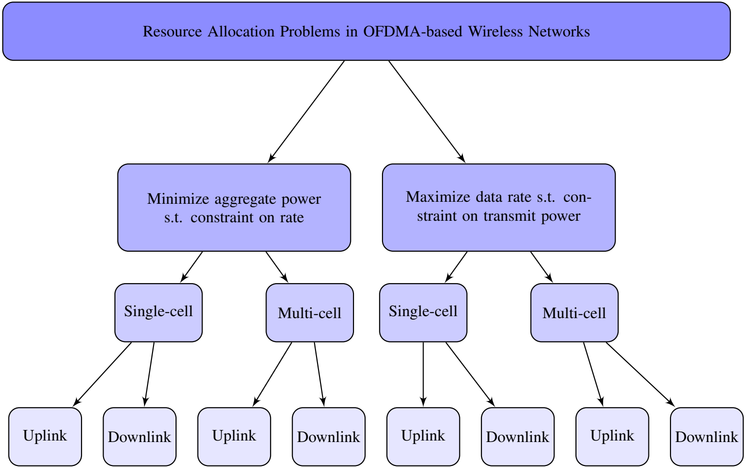
- Transmit power level constraints
  - Maximum transmit power of user i i.e., $\sum_{k \in \mathcal{C}} a_i^k p_i^k \leq p_i^{\max}$
  - The transmit power on each allocated sub-channel is lower than a mask power i.e., $p_i^k \leq p_i^{\max,k}$
- Rate constraints
  - Minimum target rate for each user i i.e., $\sum_{k \in \mathcal{C}} a_i^k R_i^k \geq R_i^{\min}$
  - Target rate for user i on each allocated sub-channel k i.e., $R_i^k \geq \widehat{R}_i^k$
- SINR constraints
  - Target-SINR for user i on each allocated sub-channel k i.e., $\gamma_i^k \geq \widehat{\gamma}_i^k$

Constraints

Uplink Transmission

- Sub-channel constraints
  - each sub-channel is allocated to at most one user in each cell i.e., $\sum_{i \in \mathcal{U}_j} a_i^k \leq 1, \qquad \forall k \in \mathcal{C}, j \in \mathcal{B}$
  - The number of allocated sub-channels to user i is less than a given threshold i.e., $\mid \mathcal{C}_i \mid \leq X_i$
  - The number of allocated sub-channels to user i is equal to a given threshold i.e., $\mid \mathcal{C}_i \mid = X_i$

Resource Allocation Problems in OFDMA-based Wireless Networks



Problem 1.1: Minimizing the Aggregate Transmit Power Subject to Minimum Data Rate Constraint for a Single-cell Single-user Network

Minimizing the aggregate power

- Given a sub-channel allocation for a single user

$$
\begin{aligned}
P1.1: \min_{\boldsymbol{p}} \quad & \sum_{k \in \mathcal{C}} p^k \\
\text{subject to.} \quad & C1: \sum_{k \in \mathcal{C}} \log_2\Big(1 + \boxed{\frac{a^k p^k h^k}{N^k}}\Big) \geq R^{\min}, \\
& C2: p^k \geq 0, \qquad \forall k \in \mathcal{C}.
\end{aligned}
$$

- Problem P1.1 is a convex problem and same in UL and DL transmissions. This problem is optimally solved by water-filling method in [1]* and [2]†.

* J. Cioffi,“ Ee379c course reader, chapter 4,” http://www.stanford.edu/group/cioffi/ee379c, frequently updated

† P. He and L. Zhao,“Solving a class of sum-power minimization problems by generalized water-filling”, *IEEE Transactions on wireless Communication*, vol. 14, no. , pp. 6792-6804, 2015.

Problem 1.1: Minimizing the Aggregate Transmit Power Subject to Minimum Data Rate Constraint for a Single-cell Single-user Network

Minimizing the aggregate power

- Water-filling method to address problem P1.1:
  - Problem P1.1 is convex and therefore its local solution is the unique optimal solution. Using Lagrangian method, the optimal solution to problem P1.1 can be obtained in polynomial time. The optimal solution is
$$p^k = [\lambda - \frac{1}{H^k}]^+, \tag{1}$$
where $[x]^+ = \max(x, 0)$, $H^k = \frac{h^k}{N^k}$ is channel gain for user on sub-channel k and $\{H^k\}_{k=1}^{|\mathcal{C}|}$ is a sorted sequence with monotonically decreasing, and λ is the water-level chosen to satisfy the minimum target rate constraint C1 in P1.1.

Problem 1.1: Minimizing the Aggregate Transmit Power Subject to Minimum Data Rate Constraint for a Single-cell Single-user Network

Minimizing the aggregate power

- Based on Lagrangian function, we have

$$\mathcal{L}(p^k,\mu) = \sum_{k=1}^{|\mathcal{C}|} p^k - \mu\Big(\sum_{k=1}^{|\mathcal{C}|} \log_2(1+p^kH^k) - R^{\min}\Big)$$

$$\bigtriangledown_{p^k}\mathcal{L}(p^k,\mu) = 1 - \mu\Big[\frac{H^k}{\ln 2(1+p^kH^k)}\Big] = 0$$

$$p^k = \frac{\frac{\mu H^k}{\ln 2} - 1}{H^k} \quad \longrightarrow \quad p^k = \Big[\lambda - \frac{1}{H^k}\Big]^+$$

where $\lambda = \frac{\mu}{\ln 2}$.

Problem 1.1: Minimizing the Aggregate Transmit Power Subject to Minimum Data Rate Constraint for a Single-cell Single-user Network

Minimizing the aggregate power

- The value of λ is water level chosen to satisfy minimum target rate constraint with equality $\sum_{k=1}^{|\mathcal{C}|} \log_2(1 + p^k H^k) = R^{\min}$.
- According to equation $p^k = (\lambda - \frac{1}{H^k})$, we have

$$R^{\min} = \sum_{k=1}^{|\mathcal{C}|} \log_2(1 + (\lambda - \frac{1}{H^k})H^k)$$

$$= \sum_{k=1}^{|\mathcal{C}|} \log_2(1 + (\lambda H^k - 1))$$

Problem 1.1: Minimizing the Aggregate Transmit Power Subject to Minimum Data Rate Constraint for a Single-cell Single-user Network

Minimizing the aggregate power

- The rate constraint becomes

$$R^{\min} = \sum_{k=1}^{|\mathcal{C}|} \log_2(\lambda H^k) = \log_2\left(\prod_{k=1}^{|\mathcal{C}|} \lambda H^k\right) = \log_2\left(\lambda^{|\mathcal{C}|} \prod_{k=1}^{|\mathcal{C}|} H^k\right).$$

So, the value of λ is obtained by

$$\lambda = \left(\frac{2^{R^{\min}}}{\prod_{k=1}^{|\mathcal{C}|} H^k}\right)^{\frac{1}{|\mathcal{C}|}} \tag{2}$$

Problem 1.1: Minimizing the Aggregate Transmit Power Subject to Minimum Data Rate Constraint for a Single-cell Single-user Network

Minimizing the aggregate power

- $p^k = \left[\lambda - \frac{1}{H^k}\right]^+$ implies that there exists k^* (called the optimal number of sub-channels with positive power) so that

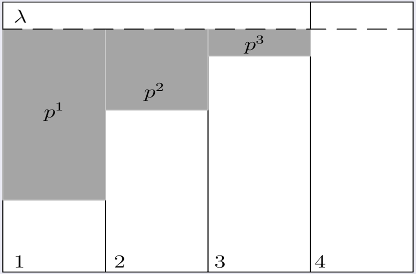
$$p^k = \begin{cases} \lambda - \frac{1}{H^k} & k \leq k^*, \\ 0 & k > k^*. \end{cases} \tag{3}$$

- For obtaining p^k in (3), we need to calculate the value of λ and k^*. Note that the method of obtaining these values affects the computational complexity.

Problem 1.1: Minimizing the Aggregate Transmit Power Subject to Minimum Data Rate Constraint for a Single-cell Single-user Network

Minimizing the aggregate power

- In water filling method, the dashed horizontal line, which is the water level λ, needs to be determined and then the powers levels at each sub-channel (water volume above the step) are obtained.



Problem 1.1: Minimizing the Aggregate Transmit Power Subject to Minimum Data Rate Constraint for a Single-cell Single-user Network

Minimizing the aggregate power

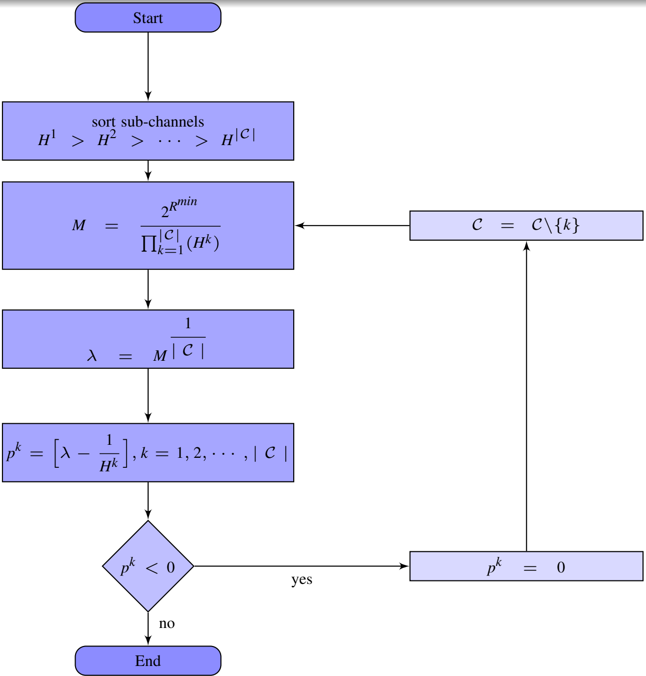
- Using water filling method, in each iteration, the water-level λ is obtained by

$$\lambda=\left(\frac{2^{R^{\min}}}{\prod_{k=1}^{|\mathcal{C}|} H^k}\right)^{\frac{1}{|\mathcal{C}|}}, \tag{4}$$

- Based on λ, the power level on every sub-channel k is calculated by $p^k=\lambda-\dfrac{1}{H^k}$.
- If this obtained power violates the constraint C2 in P1.1 for a given sub-channel k, this sub-channel is removed from the set $\mathcal{C}$ i.e., $\mathcal{C}=\mathcal{C}\setminus\{k\}$.
- This procedure recurs iteratively until positive power levels are obtained for all the remaining sub-channels. In the final iteration, λ is given by

$$\lambda = \left(\frac{2^{R^{\min}}}{\prod_{k=1}^{k^*} H^k}\right)^{\frac{1}{k^*}}, \tag{5}$$

where k^* is the optimal number of sub-channels with positive power.


Start
sort sub-channels
$H^1 > H^2 > \cdots > H^{|\mathcal{C}|}$
$M = \frac{2^{R^{min}}}{\prod_{k=1}^{|\mathcal{C}|}(H^k)}$
$\lambda = M^{\frac{1}{|\mathcal{C}|}}$
$p^k = \left[\lambda - \frac{1}{H^k}\right], k = 1, 2, \cdots, |\mathcal{C}|$
$p^k < 0$
yes
$p^k = 0$
$\mathcal{C} = \mathcal{C} \setminus \{k\}$
no
End

Problem 1.2: Minimizing the Aggregate Transmit Power Subject to Minimum Data Rate and Mask Power Constraints for a Single-cell Single-user Network

Minimizing the aggregate power

- Given a sub-channel allocation

$$
\begin{aligned}
P1.2: \min_{\boldsymbol{P}} \quad & \sum_{k \in \mathcal{C}} p^k \\
\text{subject to.} \quad & C1: \sum_{k \in \mathcal{C}} \log_2\left(1 + \boxed{\frac{a^k p^k h^k}{\sigma^2}}\right) \geq R^{min}, \\
& C2: \ 0 \leq p^k \leq p^{\max,k}, \qquad \forall k \in \mathcal{C}.
\end{aligned}
$$

- The problem P1.2 is optimally solved by water-filling method in [2] * and [3] †.

* P. He and L. Zhao, "Solving a class of sum-power minimization problem by generalized water-filling," *IEEE Transactions on wireless Communication*, vol. 14, no. , pp. 6792-6804, 2015.

† S. Khakurel, C. Leung, and T. Le-Ngoc, "A generalized water-filling algorithm with linear complexity and finite convergence time," *IEEE Wireless Communications Letters*, vol. 3, no. 2, 2014.

Problem 1.2: Minimizing the Aggregate Transmit Power Subject to Minimum Data Rate and Mask Power Constraints for a Single-cell Single-user Network

Minimizing the aggregate power

- In the water-filling method, transmit power on each sub-channel k is obtained by

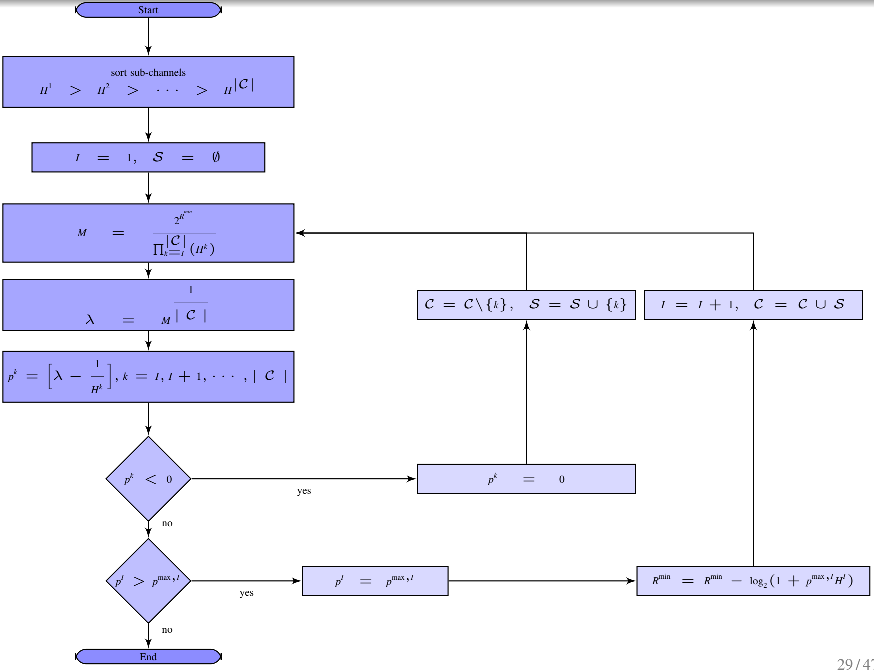
$$p^k = \left[\lambda - \frac{1}{H^k}\right]^+_{p^{max,k}} \qquad (6)$$

  where $[x]^+ = \max(x, 0)$, $[x]_y = \min(x, y)$, and λ is given by (4).
- Obtaining transmit power on each sub-channel k is same as with solution of problem P1.1, but to satisfy the mask power constraint on each sub-channel k, following algorithm is employed.

Problem 1.2: Minimizing the Aggregate Transmit Power Subject to Minimum Data Rate and Mask Power Constraints for a Single-cell Single-user Network

Minimizing the aggregate power

1: **Initialization**:
2: $\{H^k\}_{k=1}^{|\mathcal{C}|}$, for $k = 1, 2, \cdots, |\,\mathcal{C}\,|$ and $p^{\max,k}$.
3: Compute λ by (4).
4: If $p^k < 0$, $p^k = 0$, $\mathcal{C} = \mathcal{C} \backslash \{k\}$
5: Else if $p^k > p^{\max,k}$, $p^k = p^{\max,k}$, $\mathcal{C} = \mathcal{C} \backslash \{k\}$
Then return to step 2.


Start
sort sub-channels
$H^1 > H^2 > \cdots > H^{|\mathcal{C}|}$
$I = 1, \mathcal{S} = \emptyset$
$M = \frac{2^{R^{min}}}{\prod_{k=I}^{|\mathcal{C}|}(H^k)}$
$\lambda = \frac{1}{M|\mathcal{C}|}$
$p^k = \left[\lambda - \frac{1}{H^k}\right], k = I, I+1, \cdots, |\mathcal{C}|$
$p^k < 0$
yes
$p^k = 0$
$\mathcal{C} = \mathcal{C} \setminus \{k\}, \mathcal{S} = \mathcal{S} \cup \{k\}$
$I = I + 1, \mathcal{C} = \mathcal{C} \cup \mathcal{S}$
no
$p^I > p^{\max, I}$
yes
$p^I = p^{\max, I}$
$R^{\min} = R^{\min} - \log_2(1 + p^{\max, I} H^I)$
no
End

Summery of Power Control Problems to Minimize Aggregate Transmit Power

$$\min_{\boldsymbol{P}} \sum_{k \in \mathcal{C}} p^k$$

subject to.

$$C1: \quad \sum_{k \in \mathcal{C}} \log_2\left(1 + \frac{a^k p^k h^k}{\sigma^2}\right) \geq R^{\min}$$

$C2:$

$p^k \geq 0, \ \forall k \in \mathcal{C},$ Problem P1.1

$p^k \leq p^{\max,k}, \ \forall k \in \mathcal{C},$ Problem P1.2

$\sum_{k \in \mathcal{C}} p^k \leq p^{\max},$ If the aggregate transmit power achieved by P1.1 is largaer than $p^{\max}$, problem is infeasible.

Problem 2.1: Maximizing the Aggregate Data Rate Subject to Maximum Power Constraint for a Single-cell Single-user Network

Maximizing the aggregate data rate

- Given a sub-channel allocation

$$
\begin{aligned}
P2.1: \max_{\boldsymbol{P}} \quad & \sum_{k\in\mathcal{C}} \log_2\left(1+\boxed{\frac{a^k p^k h^k}{N^k}}\right) \\
\text{subject to.} \quad & C1: \sum_{k\in\mathcal{C}} a^k p^k \le p^{\max}, \\
& C2: p^k \ge 0, \qquad \forall k \in \mathcal{C}.
\end{aligned}
$$

- Problem P2.1 is a convex problem and same in UL and DL transmissions. This problem is optimally solved by water-filling method in [2]* and [4]†.

* P. He, L. Zhao, S. Zhou, and Z. Niu, "Water-Filling: A geometric approach and its application to solve generalized radio resource allocation problems," *IEEE Transaction on Wireless Communication.*, vol. 12, no. 7, pp. 3637-3647, 2013

† D. P. Palomar and J. R. Fonollosa, "Practical algorithms for a family of waterfilling Solutions," *IEEE Transactions on Signal Processing*, vol. 53, no. 2, pp. 686-695, Feb 2005

Problem 2.1: Maximizing the Aggregate Data Rate Subject to Maximum Power Constraint for a Single-cell Single-user Network

Maximizing the aggregate data rate

- Water-filling method to address problem P2.1:
  - Problem P2.1 is convex and therefore its local solution is the unique optimal solution. Using Lagrangian method, the optimal solution to problem P2.1 can be obtained in polynomial time. The optimal solution is

$$p^k = [\lambda - \frac{1}{H^k}]^+, \tag{7}$$

where $[x]^+ = \max(x, 0)$, $H^k = \dfrac{h^k}{N^k}$ is channel gain for user on sub-channel k and $\{H^k\}_{k=1}^{|\mathcal{C}|}$ is a sorted sequence with monotonically decreasing, and λ is the water-level chosen to satisfy the maximum transmit power constraint C1 in P2.1.

Problem 2.1: Maximizing the Aggregate Data Rate Subject to Maximum Power Constraint for a Single-cell Single-user Network

Maximizing the aggregate data rate

- Based on Lagrangian function, we have

$$\mathcal{L}(p^k,\mu) = (\sum_{k=1}^{|\mathcal{C}|} \log_2(1+p^k H^k)) + \mu(p^{\max} - \sum_{k=1}^{|\mathcal{C}|} p^k)$$

$$\bigtriangledown_{p^k}\mathcal{L}(p^k,\mu) = (\frac{1}{\ln 2})(\frac{H^k}{1+p^k H^k}) - \mu = 0$$

$$p^k = (\frac{1}{\ln 2})(\frac{1}{\mu}) - \frac{1}{H^k} \quad \longrightarrow \quad p^k = \left[\lambda - \frac{1}{H^k}\right]^+$$

where $\lambda = \dfrac{1}{\mu \ln 2}$.

Problem 2.1: Maximizing the Aggregate Data Rate Subject to Maximum Power Constraint for a Single-cell Single-user Network

Maximizing the aggregate data rate

- The value of λ is water level chosen to satisfy maximum transmit power constraint with equality $\sum_{k=1}^{|\mathcal{C}|} p^k = p^{\max}$.
- According to equation $p^k = (\lambda - \frac{1}{H^k})$, we have

$$p^{\max} = \sum_{k=1}^{|\mathcal{C}|} (\lambda - \frac{1}{H^k})$$

$$= | \mathcal{C} | \lambda - \sum_{k=1}^{|\mathcal{C}|} \frac{1}{H^k}$$

Problem 2.1: Maximizing the Aggregate Data Rate Subject to Maximum Power Constraint for a Single-cell Single-user Network

Maximizing the aggregate data rate

- The maximum transmit power constraint becomes

$$p^{\max} = \mid \mathcal{C} \mid \lambda - \sum_{k=1}^{|\mathcal{C}|} \frac{1}{H^k}.$$

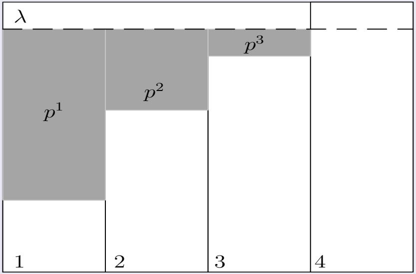
So, the value of λ is obtained by

$$\lambda = \frac{1}{\mid \mathcal{C} \mid}\left(p^{\max} + \sum_{k=1}^{|\mathcal{C}|} \frac{1}{H^k}\right). \tag{8}$$

Problem 2.1: Maximizing the Aggregate Data Rate Subject to Maximum Power Constraint for a Single-cell Single-user Network

Maximizing the aggregate data rate

- In the water-filling method, the dashed horizontal line, which is the water level λ, needs to be determined first and then the powers levels (water volume above the step) are obtained.



Problem 2.1: Maximizing the Aggregate Data Rate Subject to Maximum Power Constraint for a Single-cell Single-user Network

Maximizing the aggregate data rate

- Using water-filling method, in each iteration, the water-level λ is obtained by

$$\lambda = \frac{1}{\mid \mathcal{C} \mid}\left(p^{\max} + \sum_{k=1}^{|\mathcal{C}|} \frac{1}{H^k}\right), \tag{9}$$

- Based on λ, the power level on every sub-channel k is calculated by $p^k = \lambda - \frac{1}{H^k}$.
- If this obtained power violates the constraint C2 in P2.1 for a given sub-channel k, this sub-channel is removed from the set $\mathcal{C}$ i.e., $\mathcal{C} = \mathcal{C} \setminus \{k\}$.

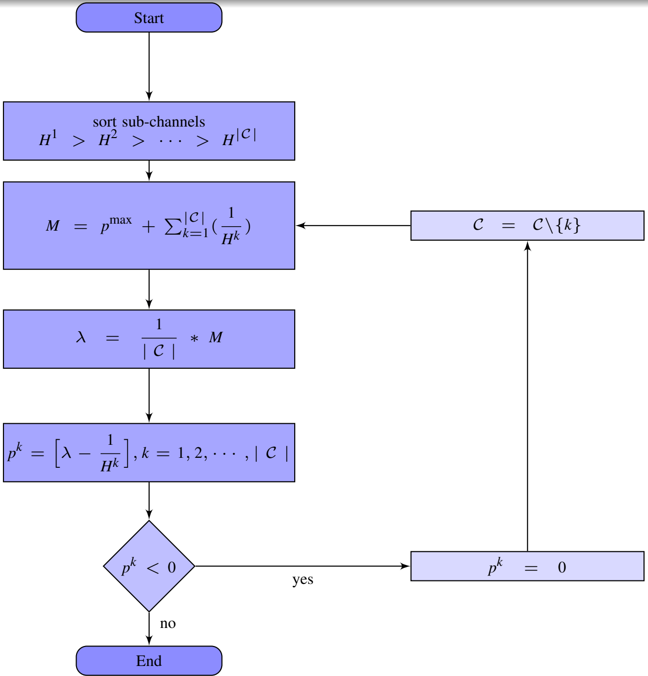
Problem 2.1: Maximizing the Aggregate Data Rate Subject to Maximum Power Constraint for a Single-cell Single-user Network

Maximizing the aggregate data rate

- This procedure recurs iteratively until positive power levels are obtained for all the remaining sub-channels. In the final iteration, λ is given by

$$\lambda = \frac{1}{k^*}\left(p^{\max} + \sum_{k=1}^{k^*} \frac{1}{H^k}\right), \tag{10}$$

where k^* is the optimal number of sub-channels with positive power.


Start
sort sub-channels
$H^1 > H^2 > \cdots > H^{|\mathcal{C}|}$
$M = p^{\max} + \sum_{k=1}^{|\mathcal{C}|}(\frac{1}{H^k})$
$\lambda = \frac{1}{|\mathcal{C}|} * M$
$p^k = \left[\lambda - \frac{1}{H^k}\right], k = 1, 2, \cdots, |\mathcal{C}|$
$p^k < 0$
yes
no
$p^k = 0$
$\mathcal{C} = \mathcal{C} \setminus \{k\}$
End

Problem 2.2: Maximizing the Aggregate Data Rate Subject to Maximum Power and Mask Power Constraints for a Single-cell Single-user Network

Maximizing the aggregate data rate

- Given a sub-channel allocation

$$P2.2: \max_{\boldsymbol{P}} \quad \sum_{k \in \mathcal{C}} \log_2\left(1 + \frac{a^k p^k h^k}{N^k}\right)$$

$$\text{subject to.} \quad C1: \sum_{k \in \mathcal{C}} a^k p^k \leq p^{\max},$$

$$C2: \quad 0 \leq p^k \leq p^{\max,k}, \qquad \forall k \in \mathcal{C}.$$

- The problem P2.2 is optimally solved by water-filling method in [2]*.

* P. He, L. Zhao, S. Zhou, and Z. Niu, "Water-Filling: A geometric approach and its application to solve generalized radio resource allocation problems," *IEEE Transaction on Wireless Communication.*, vol. 12, no. 7, pp. 3637-3647, 2013

Problem 2.2: Maximizing the Aggregate Data Rate Subject to Maximum Power and Mask Power Constraints for a Single-cell Single-user Network

Maximizing the aggregate data rate

- In the water-filling method, transmit power on each sub-channel k is obtained by

$$p^k = \left[\lambda - \frac{1}{H^k}\right]^{+}_{p^{max,k}} \tag{11}$$

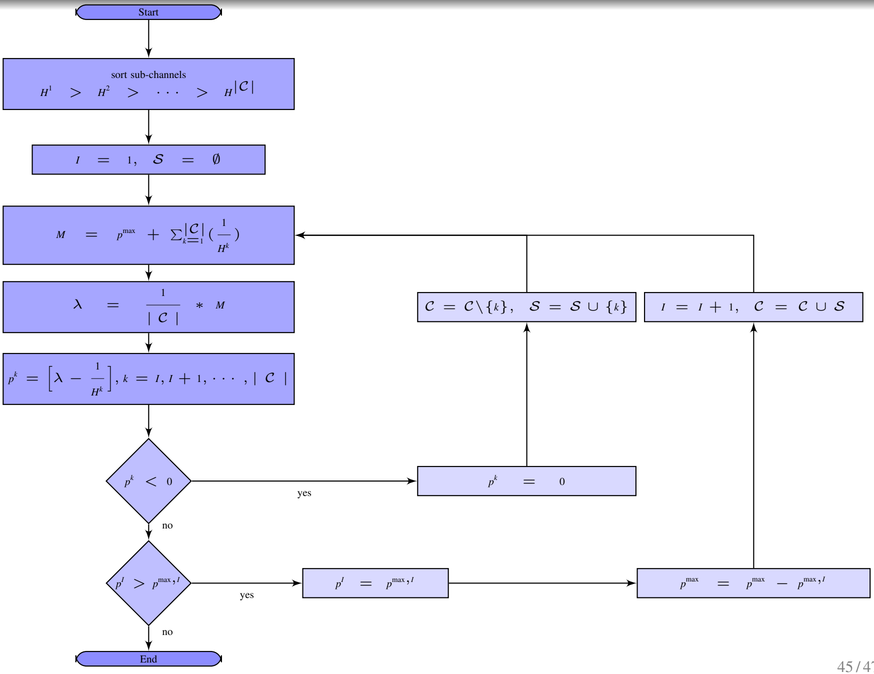
where $[x]^+ = \max(x, 0)$, $[x]_y = \min(x, y)$, and λ is given by (9).

- Obtaining transmit power on each sub-channel k is same as with solution of problem P2.1, but to satisfy the mask power constraint on each sub-channel k, following algorithm is employed.

Problem 2.2: Maximizing the Aggregate Data Rate Subject to Maximum Power and Mask Power Constraints for a Single-cell Single-user Network

Maximizing the aggregate data rate

1: **Initialization**:
2: $\{H^k\}_{k=1}^{|\mathcal{C}|}$, for $k = 1, 2, \cdots, |\,\mathcal{C}\,|$ and $p^{\max,k}$.
3: Compute λ by (9).
4: If $p^k < 0$, $p^k = 0$, $\mathcal{C} = \mathcal{C} \backslash \{k\}$
5: Else if $p^k > p^{\max,k}$, $p^k = p^{\max,k}$, $\mathcal{C} = \mathcal{C} \backslash \{k\}$
Then return to step 2.


Start
sort sub-channels
$H^1 > H^2 > \cdots > H^{|\mathcal{C}|}$
$I = 1, \mathcal{S} = \emptyset$
$M = p^{\max} + \sum_{k=1}^{|\mathcal{C}|}(\frac{1}{H^k})$
$\lambda = \frac{1}{|\mathcal{C}|} * M$
$p^k = \left[\lambda - \frac{1}{H^k}\right], k = I, I+1, \cdots, |\mathcal{C}|$
$p^k < 0$
yes
$p^k = 0$
$\mathcal{C} = \mathcal{C} \setminus \{k\}, \mathcal{S} = \mathcal{S} \cup \{k\}$
no
$p^I > p^{\max,I}$
yes
$p^I = p^{\max,I}$
$p^{\max} = p^{\max} - p^{\max,I}$
$I = I+1, \mathcal{C} = \mathcal{C} \cup \mathcal{S}$
no
End

Summery of Power Control Problems to Maximize Aggregate Data Rate

$$\max_{\boldsymbol{P}} \sum_{k \in \mathcal{C}} \log_2\left(1 + \frac{a^k p^k h^k}{N^k}\right)$$

subject to.

$$C1: \quad p^k \geq 0, \ \forall k \in \mathcal{C}$$

$$C2:$$

$$p^k \leq p^{\max,k}, \ \forall k \in \mathcal{C}, \qquad \text{Problem P2.2}$$

$$\sum_{k \in \mathcal{C}} p^k \leq p^{\max}, \qquad \text{Problem P2.1}$$

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