# **Simulation Offline-3 Report**

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## 1. Uniformity test

For the uniform test, there is one rejected when n = 20. When the number of total generated numbers is low, it is natural that when the total number of generated number is large, the probability to become IID random variable increases.

n	k	$\chi^2$	$\chi^2_{k-1, 1-\alpha}$	verdict
20	10	10	14.684	Not rejected
	20	30	27.204	rejected
500	10	11.56	14.684	Not rejected
	20	18.96	27.204	Not rejected
4000	10	8.895	14.684	Not rejected
	20	19.1	27.204	Not rejected
10000	10	8.95	14.684	Not rejected
	20	17.052	27.204	Not rejected

## 2. Serial test

The serial test is the chi-square test in higher dimensions. This provides an indirect check on the assumption that the individual U<sub>i</sub>'s are independent. When the dimension is small the hypothesis will not get rejected, when the dimension is larger or the interval is larger, the hypothesis will get rejected.

n	d	k	$\chi^2$	$\chi^2_{k^d-1, 1-\alpha}$	verdict
20	2	4	15.6	22.307	Not rejected
	3	4	100.667	77.745	rejected
	2	8	66.8	77.745	Not rejected
	3	8	506	552.374	Not rejected

500	2	4	10.48	22.307	Not rejected
	3	4	69.952	77.745	Not rejected
	2	8	53.616	77.745	Not rejected
	3	8	549.566	552.374	Not rejected
4000	2	4	14.624	22.307	Not rejected
	3	4	74.568	77.745	Not rejected
	2	8	64.128	77.745	Not rejected
	3	8	491.072	552.374	Not rejected
10000	2	4	10.566	22.307	Not rejected
	3	4	62.034	77.745	Not rejected
	2	8	53.158	77.745	Not rejected
	3	8	509.074	552.374	Not rejected

#### 3. Runs test

We examine the  $U_i$  sequence (or, equivalently, the  $Z_i$  sequence) for unbroken subsequences of maximal length within which the  $U_i$ 's increase monotonically; such a subsequence is called a run-up.

n	R	$\chi^{2}_{6, 1-\alpha}$	verdict
20	6.841	10.645	Not rejected
500	4.609	10.645	Not rejected
4000	2.613	10.645	Not rejected
10000	2.495	10.645	Not rejected

### 4. Correlation test

The final type of empirical test we consider is a direct way to assess whether the generated U<sub>i</sub>'s exhibit discernible correlation: Simply compute an estimate of the correlation at lags j. The test should probably be carried out for several values of j, since it could be, for instance, that there is no appreciable correlation at lags 1 or 2, but there is a dependence between the U<sub>i</sub>'s at lag 3, due to some anomaly of the generator.

n	j	$A_j$	$z_{1-\alpha/2}$	verdict
20	1	0.853	1.645	Not rejected
	3	1.26	1.645	Not rejected
	5	1.009	1.645	Not rejected
500	1	0.274	1.645	Not rejected
	3	0.617	1.645	Not rejected
	5	0.616	1.645	Not rejected
4000	1	0.98	1.645	Not rejected
	3	1.511	1.645	Not rejected
	5	0.896	1.645	Not rejected
10000	1	1.163	1.645	Not rejected
	3	1.769	1.645	rejected
	5	0.527	1.645	Not rejected