BANGLADESH UNIVERSITY OF ENGINEERING & TECHNOLOGY
DEPARTMENT OF COMPUTER SCIENCE & ENGINEERING

Testing Random Number Generators (Assignment #3)

Student name: *Md. Shahedul Haque*
Student ID: *1505046*

Course: *Simulation and Modeling (CSE 411)*
Course Teacher: *Dr. Mostofa Akbar*
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Question 1

In this assignment, you will create a linear congruential generator and perform four empirical tests on the generated random numbers. You will calculate the values for n = 20, 500, 4000 and 10000. For each n, perform 10 tests in total:

(1) Uniformity test at $k = 10$ and $k = 20$

(2) Serial test using $d = 2; 3$ and $k = 4; 8$

(3) Run length test

(4) Correlation test with j =1, 3 and 5.

The value of α for all tests will be 0.1 .

Uniformity Test

n	**k**	χ^2	$\chi^2_{(k-1,1-\alpha)}$	**Decision About Null Hypothesis**
20	10	19.0	14.68	Rejected
20	20	22.0	27.20	Not Rejected
500	10	9.88	14.68	Not Rejected
500	20	19.2	27.20	Not Rejected
4000	10	6.28	14.68	Not Rejected
4000	20	19.42	27.20	Not Rejected
10000	10	3.67	14.68	Not Rejected
10000	20	14.84	27.20	Not Rejected

Table 1: Uniformity Test

Reasoning: In this test, we will check this hypothesis,

$$H_0 = U_i's \text{ are IID random variables.}$$

So we check if the generated random numbers are uniformly distributed. So with smaller n, the null Hypothesis is rejected. As the number of generated pseudo number increases, it becomes more probable of them to be IID random variables.

Serial Test

n	d	k	$\chi^2(d)$	$\chi^2_{(k^d-1,1-\alpha)}$	Decision About Null Hypothesis
20	2	4	18.8	22.31	Not Rejected
20	2	8	66.8	77.75	Not Rejected
20	3	4	58.0	77.75	Not Rejected
20	3	8	506.0	552.37	Not Rejected
500	2	4	9.328	22.31	Not Rejected
500	2	8	65.904	77.75	Not Rejected
500	3	4	58.386	77.75	Not Rejected
500	3	8	518.722	552.37	Not Rejected
4000	2	4	18.4	22.31	Not Rejected
4000	2	8	61.568	77.75	Not Rejected
4000	3	4	52.866	77.75	Not Rejected
4000	3	8	464.185	552.37	Not Rejected
10000	2	4	19.104	22.31	Not Rejected
10000	2	8	54.5152	77.75	Not Rejected
10000	3	4	59.499	77.75	Not Rejected
10000	3	8	505.694	552.37	Not Rejected

Table 2: Serial Test

Reasoning: We will check whether d-tuples are uniformly distributed on the d-dimensional unit hypercube $[0;1]^d$. In this test, we will check this hypothesis,

$$H_0 = U_i's \text{ are IID random variables.}$$

So with smaller dimension,d and interval,k, the null Hypothesis is not rejected. But if we increase the dimension and intervals, that hypothesis will get rejected (This does not depend on the number of generated random variables).

Runs Test

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n	**R**	$\chi^2_{(6,1-\alpha)}$	**Decision About Null Hypothesis**
20	233.536	10.645	Rejected
500	10.627	10.645	Not Rejected
4000	6.492	10.645	Not Rejected
10000	2.422	10.645	Not Rejected

Table 3: Runs Test

Reasoning: We will evaluate the independence assumption of the random number generator in this test and we will check this hypothesis,

$$H_0 = U_i's \text{ are IID random variables.}$$

So we check if the generated random numbers are independent or not. So with smaller n, the null Hypothesis is rejected. As the number of generated pseudo number increases, it becomes more probable of them to be IID random variables.

Correlation Test

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n	**j**	$\lvert A_j \rvert$	$z_{1-\alpha/2}$	**Decision About Null Hypothesis**
20	1	0.032	1.645	Not Rejected
20	3	0.157	1.645	Not Rejected
20	5	0.366	1.645	Not Rejected
500	1	0.0009	1.645	Not Rejected
500	3	0.0042	1.645	Not Rejected
500	5	0.0013	1.645	Not Rejected
4000	1	0.0003	1.645	Not Rejected
4000	3	0.0001	1.645	Not Rejected
4000	5	0.0001	1.645	Not Rejected
10000	1	0.00007	1.645	Not Rejected
10000	3	0.00001	1.645	Not Rejected
10000	5	0.00054	1.645	Not Rejected

Table 4: Correlation Test

Reasoning: In this test you will directly assess whether the generated U_i's exhibit discernible correlation at lag j.In this test, we will check this hypothesis,

$H_0 = U_i's$ have zero lag j correlation.

As there is less correlation between at lag j, the hypothesis is never rejected. Also with bigger n, the correlation becomes smaller and at bigger lag j, the correlation gets bigger.