Bangladesh University of Engineering and Technology

Simulation and Modeling : Assignment 3

Testing Random Number Generators

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seed = 1505118
alpha = 0.1

1 Distribution of generated Random numbers

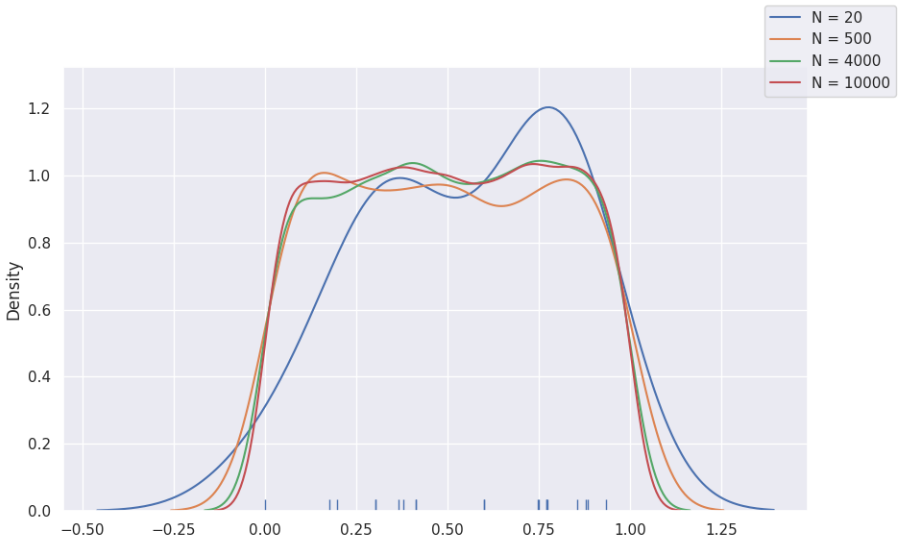

Figure 1: Distribution of generated Random numbers

2 Uniformity Test

A Uniformity Statistic is a means of measuring the extent to which a sample conforms to a uniform distribution.

The null hypothesis i.e U_i's are IID random variables is rejected when $\chi^2 > \chi^2_{k-1,1-\alpha}$ at level α where $\chi^2_{k-1,1-\alpha}$ is the upper $1-\alpha$ critical point of the chi-square distribution with $k-1$ degrees of freedom.

n	k	χ^2	$\chi^2_{k-1,1-\alpha}$	Rejected
20	10	12.0	14.68366	NO
	20	28.0	27.20357	YES
500	10	3.16	14.68366	NO
	20	8.4	27.20357	NO
4000	10	4.705	14.68366	NO
	20	12.09	27.20357	NO
10000	10	5.794	14.68366	NO
	20	11.044	27.20357	NO

3 Serial Test

The serial test is a generalization of the chi-square test to higher dimensions. This uniformity in higher dimensions matters because if the individual U_i 's are correlated, the distribution of the $d-$vectors U_i will deviate from the $d-$dimensional uniformity; thus, the serial test provides an indirect check on the assumption that the individual U_i 's are independent.

Here a matrix of size k^d is generated which is why we keep values of d smaller i.e 2 and 3.

n	d	k	χ^2	$\chi^2_{k^d-1,1-\alpha}$	Rejected?
20	2	4	15.6	22.30713	NO
	2	8	66.8	77.74538	NO
	3	4	58.0	77.74538	NO
	3	8	506.0	552.37393	NO
500	2	4	13.68	22.30713	NO
	2	8	62.32	77.74538	NO
	3	4	50.67470	77.74538	NO
	3	8	487.87952	552.37393	NO
4000	2	4	17.31200	22.30713	NO
	2	8	74.04800	77.74538	NO
	3	4	52.29032	77.74538	NO
	3	8	494.14479	552.37393	NO
10000	2	4	13.65120	22.30713	NO
	2	8	76.63360	77.74538	NO
	3	4	62.60996	77.74538	NO
	3	8	517.67627	552.37393	NO

4 Runs Test

The third empirical test - the runs (or runs-up) test, is a direct test of the independence assumption. We do not test the uniformity here.

Run-up : unbroken subsequences of maximal length within which the U_i 's increase monotonically

Run up array:

$$\begin{array}{ll} n & r \\ 20 & [2,1,2,0,2,0] \\ 500 & [80,103,44,13,6,0] \\ 4000 & [651,818,385,100,22,8] \\ 10000 & [1627,2088,937,247,59,17] \end{array}$$

n	R	$\chi^2_{6,1-\alpha}$	Rejected
20	33.68708	10.64464	YES
500	4.18046	10.64464	NO
4000	6.23016	10.64464	NO
10000	6.41907	10.64464	NO

5 Correlation Test

In this test we directly assess whether the generated U_i 's exhibit discernible correlation at lag j.

Under the assumption that there is no correlation and assuming n is large A_j should have an approximate standard normal distribution. We test the null hypothesis i.e U_i 's have zero lag j correlation at level α and reject the hypothesis if $|A_j| > z_{1-\alpha/2}$.

n	j	A_j	$z_{1-\alpha/2}$	Rejected
20	1	0.93451	1.64485	NO
	3	1.00042	1.64485	NO
	5	0.56386	1.64485	NO
500	1	0.50991	1.64485	NO
	3	0.01297	1.64485	NO
	5	1.32214	1.64485	NO
4000	1	0.61936	1.64485	NO
	3	0.51350	1.64485	NO
	5	0.32983	1.64485	NO
10000	1	0.14678	1.64485	NO
	3	0.26231	1.64485	NO
	5	0.01205	1.64485	NO