Quantum Field Theory

Problem Sheet 6

1. *Yukawa theory*

   Consider a Dirac field ψ, and a real scalar ϕ. Write the path integral for the free theory with these two fields as the product of a path integral over the fermionic degrees of freedom, and a path integral of the bosonic ones.

   Consider the interaction lagrangian

   $$V(\psi, \bar{\psi}, \phi) = g\phi(x)\bar{\psi}(x)\psi(x)\,,$$

   and define the path integral for the interacting theory.

   Compute the contribution to the generator of connected correlators $iW[\eta, \bar{\eta}, J]$ in a perturbative expansion at order g^2 for the following cases:

   (a) Two external currents J, one external current η, one external current $\bar{\eta}$.

   (b) Two external currents η, two external currents $\bar{\eta}$.

   Compute the connected correlators $\langle 0|T\psi_\alpha(x)\bar{\psi}_\beta(y)\phi(z_1)\phi(z_2)|0\rangle_c$, and $\langle 0|T\psi_{\alpha_1}(x_1)\bar{\psi}_{\beta_1}(y_1)\psi_{\alpha_2}(x_2)\bar{\psi}_{\beta_2}(y_2)|0\rangle_c$.

2. *Translation Ward identity - 2*

   Find the variation of the action for the free scalar field under the field transformation

   $$\psi(x) \mapsto \psi'(x) = \psi(x) + a(x)\partial_\mu\psi(x)\,,$$
   $$\bar{\psi}(x) \mapsto \bar{\psi}'(x) = \bar{\psi}(x) + a(x)\partial_\mu\bar{\psi}(x)\,.$$

   Deduce the Ward identities generated by translation invariance.

3. *Transverse projector*

   Check that

   $$\Pi^{\mu\nu}(k) = g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}$$

   is a transverse projector, *i.e.*

   $$k_\mu\Pi^{\mu\nu}(k) = 0\,,$$
   $$\Pi^\mu_\sigma(k)\Pi^{\sigma\nu}(k) = \Pi^{\mu\nu}(k)\,.$$

4. *One-particle irreducible*

   When calculating the full propagator for the real scalar field

   $$G^{(2)}(x, y) = \langle T\phi(x)\phi(y)\rangle\,,$$

it is convenient to first compute the sum, $i\Pi^*$, of all the one-particle irreducible diagrams, *i.e.* all diagrams that cannot be disconnected by cutting one single internal line.

Show that

$$\tilde{G}^{(2)}(p,p') = (2\pi)^D \delta(p+p') \frac{1}{p^2 - m^2 - \Pi^*(p^2)} \,.$$

NB: the quantity Π^* in this question is *not* the projector on the transverse modes discussed above!!