1. Yukawa Theory.

$$Z_0[\eta, \bar{\eta}, J] = \exp\left[\frac{i}{2}\int d^Dx\, d^Dy\; J(x)\,\Delta(x-y)\,J(y)\right] \times$$

$$\times \exp\left[i\int d^Dx\, d^Dy\; \bar{\eta}(x)\, S(x-y)\, \eta(y)\right]$$

$$V(\psi, \bar{\psi}, \phi) = g\,\phi(x)\,\bar{\psi}(x)\,\psi(x) \quad \leftarrow \text{Lorentz invariant.}$$

$$Z[\eta, \bar{\eta}, J] = \exp\left[i\int d^Dx\, V\left(\frac{1}{i}\frac{\delta}{\delta\bar{\eta}(x)}, i\frac{\delta}{\delta\eta(x)}, \frac{1}{i}\frac{\delta}{\delta J(x)}\right)\right] Z_0[\eta, \bar{\eta}, J]$$

$$= \sum_V \frac{1}{V!}\left[ig\int d^Dx\left(\frac{1}{i}\frac{\delta}{\delta J(x)}\right)\left(i\frac{\delta}{\delta\eta(x)}\right)\left(\frac{1}{i}\frac{\delta}{\delta\bar{\eta}(x)}\right)\right] \times$$

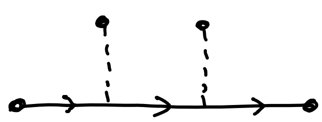
$$\times \sum_B \frac{1}{B!}\left[\frac{i}{2}\int d^Dx\, d^Dy\; J(x)\,\Delta(x-y)\,J(y)\right]^B \times$$

$$\times \sum_F \frac{1}{F!}\left[i\int d^Dx\, d^Dy\; \bar{\eta}(x)\, S(x-y)\, \eta(y)\right]^F.$$

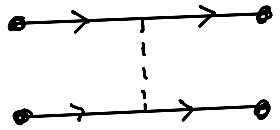
$$E_B = 2B - V, \qquad E_F = 2(F - V)$$

(a) $V = 2$, $B = 2$, $F = 3$

$$(iW)\Big|_a =$$



(b) $V = 2$, $B = 1$, $F = 4$

$$(iW)\big|_b = \frac{1}{2}$$ 

Take the functional derivatives.

$$(iW)\big|_a = i^7 \quad \frac{1}{i^2}\,(ig)^2 \int d^Dx'\, d^Dy'\, d^Dz_1'\, d^Dz_2'\, d^Du_1\, d^Du_2$$

$$\times\ \bar{\eta}(x')\, S(x'-u_1)\, S(u_1-u_2)\, S(u_2-y')\, \eta(y')\ \times$$

$$\times\ \Delta(u_1-z_1')\, J(z_1')\quad \Delta(u_2-z_2')\, J(z_2')\,.$$

$$\langle T\psi(x)\bar{\psi}(y)\phi(z_1)\phi(z_2)\rangle_c = \frac{1}{i}\frac{\delta}{\delta\bar{\eta}(x)}\ i\frac{\delta}{\delta\eta(y)}\ \frac{1}{i}\frac{\delta}{\delta J(z_1)}\ \frac{1}{i}\frac{\delta}{\delta J(z_2)}\ (iW)\big|_a$$

$$= i^5(-)(ig)^2\int d^Du_1\, d^Du_2\ S(x-u_1)\, S(u_1-u_2)\, S(u_2-y)\, \Delta(u_1-z_1)\Delta(u_2-z_2) +$$

$$+\ (z_1 \rightleftarrows z_2)\,.$$

$$\langle T\psi(x_1)\bar{\psi}(y_1)\psi(x_2)\bar{\psi}(y_2)\rangle_c = \frac{1}{i}\frac{\delta}{\delta\bar{\eta}(x_1)}\ i\frac{\delta}{\delta\eta(y_1)}\ \frac{1}{i}\frac{\delta}{\delta\bar{\eta}(x_2)}\ i\frac{\delta}{\delta\eta(y_2)}\ (iW)\big|_b$$

$$(iW)\big|_b = i^5\,\frac{1}{2}\,(ig)^2\int d^Dx_1'\, d^Dx_2'\, d^Dy_1'\, d^Dy_2'\, d^Du_1\, d^Du_2\ \times$$

$$\times\ \bar{\eta}(x_1')\, S(x_1'-u_1)\, S(u_1-y_1')\, \eta(y_1')\, \bar{\eta}(x_2')\, S(x_2'-u_2)\, S(u_2-y_2')\, \eta(y_2')\, \Delta(u_1-u_2)\,.$$

$$\langle T\psi(x_1)\bar\psi(y_1)\psi(x_2)\bar\psi(y_2)\rangle_c = i^5 (ig)^2 \int d^D u_1\, d^D u_2 \times$$

$$\Big\{ \big[S(x_1-u_1)\, S(u_1-y_1) \big]\, \Delta(u_1-u_2)\, \big[S(x_2-u_2)\, S(u_2-y_2) \big] -$$

$$- (y_1 \rightleftarrows y_2) \Big\} .$$

2. Translation Ward id. for fermions.

$$\begin{cases} \psi(x) \mapsto \psi'(x) = \psi(x) - a(x)\,\partial_\rho \psi(x) \\ \bar\psi(x) \mapsto \bar\psi'(x) = \bar\psi(x) - a(x)\,\partial_\rho \psi(x) \end{cases}$$

$$S[\psi',\bar\psi'] - S[\psi,\bar\psi] = -\int d^D x \Big[a(x)\,\partial_\rho \bar\psi(x)\,(i\not\partial - m)\,\psi(x) +$$

$$+ \bar\psi(x)\,(i\not\partial - m)\big(a(x)\,\partial_\rho \psi(x)\big) \Big] .$$

$$= -\int d^D x \Big\{ a(x) \big[\partial_\rho \bar\psi(x)\,(i\not\partial - m)\,\psi(x) + \bar\psi(x)\,(i\not\partial - m)\,\partial_\rho \psi(x) \big] +$$

$$+ \bar\psi(x)\,\big(i\gamma^\mu \partial_\mu a(x)\big)\,(\partial_\rho \psi) \Big\} =$$

$$= \int d^D x\; a(x)\; \partial_\mu \Big[\bar\psi\,(i\gamma^\mu)\,(\partial_\rho \psi) - g^\mu_\rho\, \bar\psi\,(i\not\partial - m)\,\psi \Big] .$$

$$T^{\mu\rho}(x) = \bar\psi\,(i\gamma^\mu \partial^\rho)\,\psi - g^{\mu\rho}\,\bar\psi\,(i\not\partial - m)\,\psi .$$

3. Transverse projector

$$\Pi^{\mu\nu}(k) = g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}$$

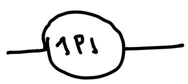
$$\Pi^{\mu\nu}(k)\, k_\nu = \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right) k_\nu = k^\mu - \frac{k^\mu \cancel{k^2}}{\cancel{k^2}} = 0$$

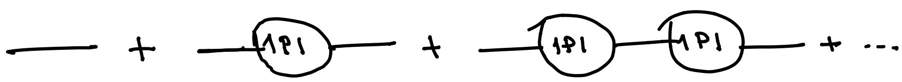
$$\Pi^\mu{}_\sigma(k)\, \Pi^{\sigma\nu}(k) = \left(g^\mu{}_\sigma - \frac{k^\mu k_\sigma}{k^2}\right)\left(g^{\sigma\nu} - \frac{k^\sigma k^\nu}{k^2}\right)$$

$$= g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} - \cancel{\frac{k^\mu k^\nu}{k^2}} + \cancel{\frac{k^\mu k_\sigma k^\sigma k^\nu}{(k^2)^2}} = \Pi^{\mu\nu}(k)$$

4. 1PI diagrams

$$G^{(2)}(p,p') = (2\pi)^D \delta(p+p') \frac{1}{i} \Delta_F(p^2)$$

$i\Pi^*(p^2) =$ 

$\frac{1}{i}\Delta_F(p^2) =$ 

$$= \frac{1}{i}\Delta(p^2) + \frac{1}{i}\Delta(p^2)\left[i\Pi^*(p^2)\right]\frac{1}{i}\Delta(p^2) +$$

$$+ \frac{1}{i}\Delta(p^2)\left[i\Pi^*(p^2)\right]\frac{1}{i}\Delta(p^2)\left[i\Pi^*(p^2)\right]\frac{1}{i}\Delta(p^2) + \dots$$

$$= \frac{1}{i}\Delta(p^2)\left\{1 + \left[i\Pi^*(p^2)\right]\frac{1}{i}\Delta(p^2) + \left(\left[i\Pi^*(p^2)\right]\frac{1}{i}\Delta(p^2)\right)^2 + \dots\right\}.$$

$$= \frac{1}{i} \Delta(p^2) \frac{1}{1 - [i \Pi^*(p^2)] \frac{1}{i} \Delta(p^2)}$$

$$= \frac{1}{p^2 - m^2 - \Pi^*(p^2)}$$