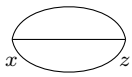
Quantum Field Theory

Problem Sheet 3

1. *Combinatorial factors - 1*

   The term with $V = 2$ and $P = 3$ in the expansion of $Z[J]$ includes contractions that give rise to the following topology:

   (1)

   Compute the symmetry factor for this diagram.

2. *Two-point function*

   Write down the expression for the two-point function

   $$\langle T\phi(x)\phi(y)\rangle\,,$$

   in the ϕ^3 theory up to order g^2. Find the representation of these contributions in terms of Feynman diagrams, and compute their symmetry factors.

3. ϕ^4 *theory*

   Consider the ϕ^4 theory in $D = 4$, defined by the Lagrangian:

   $$\mathcal{L} = \frac{1}{2}\partial_\mu\phi(x)\partial^\mu\phi(x) - \frac{1}{2}m^2\phi(x)^2 - \frac{1}{4!}\lambda\phi(x)^4\,.$$

   Compute the mass dimension of the fields, and of the coupling λ. Write down the generating functional for this theory, and deduce the Feynman rules.

   Draw all connected diagrams with two and four external legs, up to order λ^2. Compute the symmetry factors for each diagram.

   Using the relation between $Z[J]$ and $W[J]$, write the connected two-point function as a function of ordinary correlators.