Quantum Field Theory

Problem Sheet 8

1. *One-loop vertex*

   Write the one-loop contribution to the three-point function for the ϕ^3 theory in D dimensions using the Feynman rules for correlators. Check the computation of the diagram discussed in the lectures.

2. *ϕ^4 theory*

   The ϕ^4 theory is defined by the interaction term

   $$\frac{\lambda}{4!}\int d^D x\, \phi(x)^4\,.$$

   Compute the one-loop contribution to the scalar propagator at $O(\lambda)$ in $D = 4$. Identify the relevant diagrams, compute the symmetry factors, and write them as integrals in momentum space. Regularise the integrals using DimReg, and discuss the divergences.

   Write the renormalization conditions for the field and the mass in the on-shell renormalization scheme, and determine the corresponding counter terms at $O(\lambda)$.

3. *Superficial degree of divergence*

   Find the superficial degree of divergence of a diagram with E external legs in D dimensions for a scalar ϕ^n theory, and for the Yukawa theory discussed in PS6.

4. *Four-point 1PI vertex*

   Check the computation of $V^{(4)}$ in ϕ^3 in $D = 6$.

5. *Renormalizable interactions in $D = 4$*

   Consider fermionic interactions of the form $g_n(\bar\psi\psi)^n$, for $n \geq 2$. Find the mass dimension of g_n in D dimensions.

   Consider interactions of the form $g_{nm}\phi^m(\bar\psi\psi)^n$. Find the mass dimension of g_{nm} in D dimensions.

   List the renormalizable interactions for $D = 4$. Do they look familiar?

6. *Two-particle amplitude at one-loop*

   Using the skeleton expansion compute the one-loop contribution to the scattering process:

$$p_1 p_2 \to p_1' p_2' \,.$$

   The tree-level contribution was already computed in PS4, and the four-point vertex $V^{(4)}$ has been discussed in lectures.