1. $V = 2 \quad P = 3 \quad \Rightarrow \quad \mathcal{E} = 0$

$$\frac{i^2}{2!}\frac{1}{(3!)^2}\int d^Dx_1\, d^Dx_2 \left(\frac{1}{i}\frac{\delta}{\delta J(x_1)}\right)^3\left(\frac{1}{i}\frac{\delta}{\delta J(x_2)}\right)^3 \times$$

$$\times \frac{1}{3!}\,\frac{i}{2}\int d^Dy_1\, d^Dz_1\; J(y_1)\,\Delta(y_1-z_1)\,J(z_1)\;\times$$

$$\times\,\frac{i}{2}\int d^Dy_2\, d^Dz_2\; J(y_2)\,\Delta(y_2-z_2)\,J(z_2)$$

$$\times\,\frac{i}{2}\int d^Dy_3\, d^Dz_3\; J(y_3)\,\Delta(y_3-z_3)\,J(z_3) \;=$$

$$= -i\,\frac{1}{2!\,(3!)^2\,2^3\,3!}\int d^Dx_1\, d^Dx_2\, d^Dy_1\, d^Dz_1\, d^Dy_2\, d^Dz_2\, d^Dy_3\, d^Dz_3$$

$$\times\;\Delta(y_1-z_1)\;\Delta(y_2-z_2)\;\Delta(y_3-z_3)$$

$$\left(\frac{\delta}{\delta J(x_1)}\right)^3\left(\frac{\delta}{\delta J(x_2)}\right)^3 J(y_1)\,J(z_1)\,J(y_2)\,J(z_2)\,J(y_3)\,J(z_3)\,.$$

# of contractions : $\quad 6\cdot 3\cdot 4\cdot 2\cdot 2$

$$\frac{1}{3!}\;\frac{1}{2}\;\frac{1}{\cancel{(3!)^2}}\;\frac{1}{\cancel{2^3}}\;\cancel{6}\cdot\cancel{3}\cdot\cancel{4\cdot 2}\cdot\cancel{2} = \frac{1}{2\cdot 3!}$$

S : perm. of prop. | perm. of derivatives : $3!$

perm. of ends of props | perm. of vertices : 2

$\Rightarrow \quad S = 2\cdot 3!$

2. $\langle T\phi(x)\phi(y)\rangle$ $\qquad E = 2$

$$= \left(\frac{1}{i}\frac{\delta}{\delta J(x)}\right)\left(\frac{1}{i}\frac{\delta}{\delta J(y)}\right) Z[J]\Big|_{J=0} .$$

$O(g^0)$:

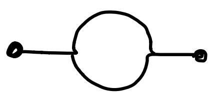
$S = 2$

$$= \frac{i}{2}\int d^D y_1\, d^D z_1\, J(y_1)\,\Delta(y_1 - z_1)\,J(z_1)$$

$$\Rightarrow \langle T\phi(x)\phi(y)\rangle = \frac{1}{i}\Delta(x-y)$$

$O(g^2)$: $\quad V = 2 \qquad E = 2P - 6 = 2 \qquad P = 4.$

(a)



$S = 2\cdot 2$

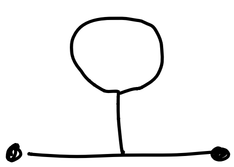
Combinatorial factor:

$$\frac{1}{2}\,\frac{1}{3!}\,\frac{1}{3!}\,\frac{1}{4!}\,\frac{1}{2^4}\,\frac{4!}{2!\,2!}\;\underbrace{2\cdot 2}_{\text{choose ext. current}}\cdot 6\cdot 3\cdot 4\cdot 2\cdot 2 = \frac{1}{2^2}$$

$$= \frac{1}{4}\int d^D x_1\, d^D x_2\, d^D z_1\, d^D z_2\; J(x_1)\,\Delta(x_1 - z_1)\,\Delta(z_1 - z_2)^2\,\Delta(z_2 - x_2)\,J(x_2)$$

$$\Rightarrow \quad = \frac{1}{2}\int d^D z_1\, d^D z_2\; \Delta(x - z_1)\,\Delta(z_1 - z_2)^2\,\Delta(z_2 - y) .$$

(b)



$S = 2\cdot 2$

combinatorial factor:

$$\frac{1}{2}\ \frac{1}{3!}\ \frac{1}{3!}\ \frac{1}{4!}\ \frac{1}{2^4}\ \frac{4!}{2!2!}\ 2\cdot 2\cdot 6\cdot 2\cdot 2\cdot 2\cdot 3\cdot 2 = \frac{1}{2^2}.$$

choose prop in the middle

$$= \frac{1}{4}\int d^Dx_1\, d^Dx_2\, d^Dz_1\, d^Dz_2\ J(x_1)\,\Delta(x_1-z_1)\,\Delta(z_1-z_2)\,\Delta(0)\ \times$$
$$\times\ \Delta(z_1-x_2)\,J(x_2)$$

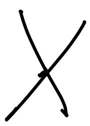
$$\Rightarrow\quad = \frac{1}{2}\int d^Dz_1\, d^Dz_2\ \Delta(x-z_1)\,\Delta(z_1-z_2)\,\Delta(0)\,\Delta(z_1-y)$$

3. ϕ^4 theory

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi(x)\,\partial^\mu \phi(x) - \frac{1}{2} m^2 \phi(x)^2 - \frac{\lambda}{4!}\phi(x)^4 .$$

$$Z[J] = \exp\left\{ -i\frac{\lambda}{4!}\int d^D x \left(\frac{1}{i}\frac{\delta}{\delta J(x)}\right)^4 \right\} Z_0[J]$$

$$[\phi] = \frac{D-2}{2} = 1 \qquad [\phi^4] = 4 \ , \qquad [\lambda] = 0$$

 $\lambda \int d^D x$

$$Z[J] = \sum_V \frac{1}{V!}\left[\frac{-\lambda}{4!}\int d^D x \left(\frac{1}{i}\frac{\delta}{\delta J(x)}\right)^4\right]^V \sum_P \frac{1}{P!}\left[\frac{i}{2}\int d^D y\, d^D z\, J(y)\right.$$

$$\left. \times\ \Delta(y-z)\, J(z)\right]^P .$$

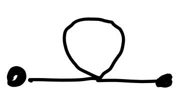
$$E = 2P - 4V$$

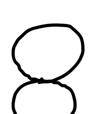
$E = 2$ $\qquad P = 1,\ V = 0 \qquad \lambda^0$

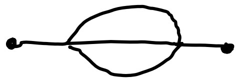
$\qquad\qquad P = 3,\ V = 1 \qquad \lambda^1$

$\qquad\qquad P = 5,\ V = 2 \qquad \lambda^2$

$\lambda^0 =$  $S = 2$

$\lambda^1 =$  $S = 2^2$

$\lambda^2 :$  $S = 2^3$



$S = 2 \cdot 3!$

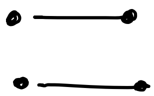
$E = 4$

$P = 2 \quad V = 0 \quad \lambda^0$

$P = 4 \quad V = 1 \quad \lambda^1$

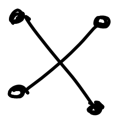
$P = 6 \quad V = 2 \quad \lambda^2$

λ^0:



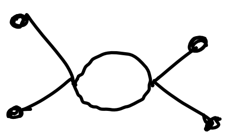
disconnected !

λ^1:

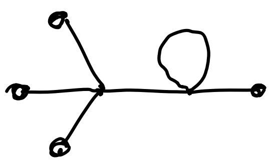


$S = 4!$

λ^2:



$S = 2^4$



$S = 3! \cdot 2$