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Lecture 7

Divergences

7.1 A first look at divergences

In this lecture we will aim to develop a self-consistent treatment of divergences in QFT. This is a vast topic, which can hardly be addressed exhaustively in the time that we have. Therefore we will follow the following steps.

1. Compute the scalar two-point function beyond the first order in perturbation theory. As we try to perform this calculation we will encounter our first divergent integral.

2. Discuss the regularization of divergencies; *i.e.* a procedure that allows us to manipulate well-defined mathematical expressions, and to identify the structure of the divergencies.

3. Discuss the renormalization of divergencies; *i.e.* the conditions that are necessary for a quantum field theory to produce finite, unambiguous predictions.

7.1.1 Scalar two-point function

Working in perturbation theory, we compute the two-point function

$$\tilde{G}^{(2)}\left(p, p'\right) = (2\pi)^D \delta\left(p + p'\right) \frac{1}{i} \tilde{\Delta}_F(p)\,, \tag{7.1}$$

as a Taylor expansion in powers of the coupling constant

$$\tilde{G}^{(2)}\left(p, p'\right) = \sum_k g^k \tilde{G}^{(2,k)}\left(p, p'\right)\,. \tag{7.2}$$

As discussed before, the delta function in Eq. 7.1 ensures momentum conservation. For all practical purposes, we should remember that it is there, and work on the perturbative expansion of the full propagator $\tilde{\Delta}_F(p)$:

$$\tilde{\Delta}_F(p) = \sum_k g^k \tilde{\Delta}_F^{(k)}(p)\,. \tag{7.3}$$

From our previous computations

$$g^2 \frac{1}{i} \tilde{\Delta}_F^{(2)}(p) = -\frac{g^2}{2} \frac{1}{p^2 - m^2 + i\epsilon} \tag{7.4}$$