Quantum Field Theory

Problem Sheet 4

1. *Feynman rules - 1*

   Use the Feynman rules in momentum space to compute $G_b^{(2,2)}$. Check that you get the same result by performing a Fourier transform of the result in position space.

2. *Scattering amplitude*

   Compute the amplitude for the scattering process
   $$p_1 p_2 \longrightarrow p_1' p_2'$$
   at order g^2 in the ϕ^3 scalar theory.

3. *LSZ reduction for 2 to 2 processes*

   A particle localised in momentum space near $\mathbf{k}_1$ is created in $D = 4$ dimensions by the operator
   $$a_1^\dagger = \int d^3k f_1(\mathbf{k}) a^\dagger(\mathbf{k}) \,, \tag{1}$$
   where f is some function peaked at $\mathbf{k}_1$, and $a^\dagger(\mathbf{k})$ is the creation operator in the free theory. In the interacting theory, we shall assume that a time-dependent creation operator is defined as
   $$a^\dagger(\mathbf{k}, t) = -i \int d^3x \, e^{-ik\cdot x} \overleftrightarrow{\partial_0} \phi(x) \,. \tag{2}$$
   Show that
   $$a_1^\dagger(+\infty) - a_1^\dagger(-\infty) = -i \int d^3k \, f_1(\mathbf{k}) \int d^4x \, e^{-ik\cdot x} (\partial^2 + m^2) \phi(x). \tag{3}$$
   The scattering amplitude for a $2 \to 2$ process can be written as:
   $$\langle k_1' k_2'; \mathrm{out} | k_1 k_2; \mathrm{in} \rangle = \langle 0 | T \left(a_{1'}(+\infty) a_{2'}(+\infty) a_1^\dagger(-\infty) a_2^\dagger(-\infty) \right) | 0 \rangle \,. \tag{4}$$
   Show that
   $$\begin{aligned} \langle k_1' k_2'; \mathrm{out} | k_1 k_2; \mathrm{in} \rangle = & i^{2+2} \int d^4x_1 \, e^{-ik_1\cdot x_1} \left(\partial_1^2 + m^2\right) \int d^4x_2 \, e^{-ik_2\cdot x_2} \left(\partial_2^2 + m^2\right) \\ & \times \int d^4x_1' \, e^{ik_1'\cdot x_1'} \left(\partial_{1'}^2 + m^2\right) \int d^4x_2' \, e^{ik_2'\cdot x_2'} \left(\partial_{2'}^2 + m^2\right) \\ & \times \langle 0 | T \left(\phi(x_1) \phi(x_2) \phi(x_1') \phi(x_2') \right) | 0 \rangle \,. \end{aligned} \tag{5}$$

4. *Generalised LSZ*

   Generalise the LSZ reduction for arbitrary numbers of particles in the initial and final state.

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