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HOME WORK - 0

I

1) $y \cdot z = (1 \ \ 3)\begin{pmatrix}2\\3\end{pmatrix} = 11$

2) $Xy = \begin{pmatrix}2 & 4\\1 & 3\end{pmatrix}\begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}14\\10\end{pmatrix}$

3) $\cancel{X^{-1} = \frac{1}{2}(A)}$ $|X| = \begin{vmatrix}2 & 4\\1 & 3\end{vmatrix} = 2$

So, yes

$X^{-1} = \frac{1}{2}\begin{pmatrix}3 & -4\\-1 & 1\end{pmatrix}$

4) $|X| = 2 \neq 0$ & $|X| \leq 2$

So, rank is 2

II Calculus

1) $\frac{dy}{dx} = 3x + 1$

2) $\cancel{\nabla \cdot f = \frac{\partial f}{\partial x_1}\hat{x}_1 + \frac{\partial f}{\partial x_2}\hat{x}_2}$

$\cancel{= \sin 2x_2 (e}$

2) $\nabla f = \begin{pmatrix} \partial_{x_1} f \\ \partial_{x_2} f \end{pmatrix} = \begin{pmatrix} e^{-x_1} \sin x_2 (1 - x_1) \\ x_1 \cos(x_2) e^{-x_1} \end{pmatrix}$

III Prob & Stat.

1)

$$\text{mean} = \frac{1+1+0+1+0}{5} = \frac{3}{5}$$

2) Variance $= \sqrt{\langle S^2\rangle - \langle S\rangle^2} = \sqrt{\frac{1^2+1^2+1^2}{5} - \frac{1^2+1^2+1^2}{5}}$

$= 0$

3)

$$P(S) = \frac{P(S)}{32} = \frac{1}{25}$$

4) let $P(x=1) = p$ & $P(x=0) = 1-p$

so, $P(S) = p \cdot p \cdot (1-p)\, p(1-p)$

$= p^3(1-p)^2$

for maxim,

$$\frac{dP(x)}{dp} = 0 \Rightarrow 3p^2(1-p)^2 - 2p^3(1-p) = 0$$

$\Rightarrow p^2(1-p)^2[\ldots$

$3p^2(1-p)^2 - 2p^3(1-p) = 0$

$p^2(1-p)[3 - 2p] = 0$

for max $\Rightarrow$ $3 - 2p = 0$

$p = \frac{2}{3}$

5) • $P(z=T \text{ And } y=b) = P(z=T) \cdot P(y=b)$

$= 0.1$

• ~~$P(z=T \mid y=b) = P(z=T) + P(y=b)$~~

~~$= (0.2 + 0.1 + 0.2) + (0.05 + 0.15 + 0.3)$~~

~~$+ (0.1 + 0.15)$~~

~~$= 0.5 + 0.25 = 0.75$~~

$= P(z=T \text{ \& } y=b) / P(y=b) = \frac{0.1}{0.1+0.15} = \frac{0.1}{0.25} = \frac{2}{5}$

II) Big - O

1) both true, both functions are same.

2) ~~neither~~ $g(n) = O(f(n))$ as 3^n increases faster than 100^n for large n.

3) $g(n) = O f(n)$, since $3 > 2$ grows faster

4) $f(n) = O(g(n))$. $O(n)$ grows faster for larger

Median Background test

Algo:

```
def dq(A):        # A is the array.
    m = int(len(A)/2)
    if A[m] == 1:
        if A[m-1] == 0:
            return m-1
        else:
            dq(A[:m-1])
    else:
        if A[m+1] == 1
            return m
        else:
            dq(A[m+1:])
```

So, by this following recursion Algo, A is divided to half and selected as Algo and so on. So the other half is neglected and so on. So run time is $O(\log n)$.

i.e., $T(n) = a + T(n/2)$

Prob

I

a) true

b) true

c) False

d) False

e) True

II

Multivariate Gaussian $-\frac{1}{\sqrt{(2\pi)^d|\Sigma|}}\exp\left(-\frac{1}{2}-(x-\mu)^T\Sigma^{-1}(x-\mu)\right)$

Bernoulli $-$ $\binom{n}{x} p^x(1-p)^{n-x}$

Uniform $-$ $\frac{1}{b-a}$ where $a \le x \le b$.

Binomial $-$ $\binom{n}{x} p^x(1-p)^{1-x}$

III

a) $E[(X-EX)^2] = E[X^2-2X(EX)+(EX)^2]$

$\cancel{= E[X^2 - 2E}$

$= EX^2 - 2E(X(EX)) + (EX)^2$

$= EX^2 - 2(EX)^2 + (EX)^2$

$= EX^2 - (EX)^2$ //

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5) ~~$p(1-p)$~~

5) p is mean $p(1-p)$ is variance,

entropy is $-(1-p)\log(1-p) - p\log(p)$

IV Law of large numbers.

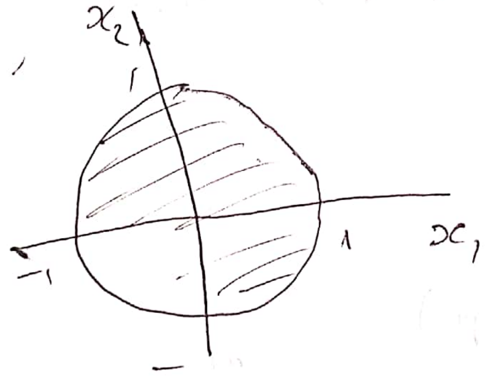
a)

a) by law of large numbers, since probability of 3 is $1/6 \Rightarrow \frac{1}{6} 6000 = 1000$

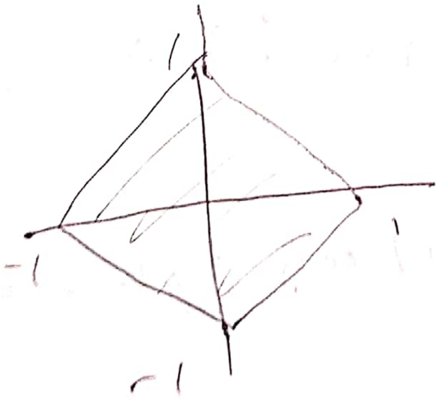
b) by central limit theorem, as $n \to \infty$ L.H.S should tend to R.H.S.

Linear Algebra

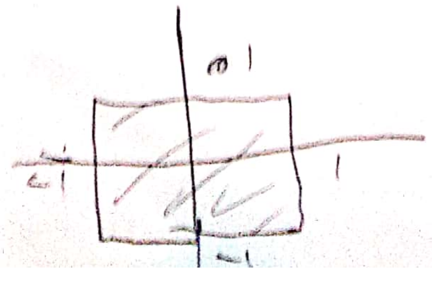
a)



b)



c)



9 Geometry

a) Consider two point on line x_1 & x_2.

so $x_1 - x_2$ is $\parallel$ ~~to~~ vector to the line.

from line eqn,

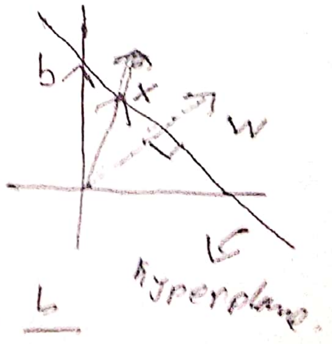
$$w^T x_1 + b = w^T x_2 + b = 0$$

$$\Rightarrow w^T (x_1 - x_2) = 0$$

So w is orthonormal as $x_1 - x_2$ is parallel to the line.

b) We showed w is orthonormal to the line.

so if we project ~~x from~~ x from the origin to w we get the distance.



So distance, $d = \cancel{h}\cos\theta$ hyperangle.

~~$\Rightarrow d = b \times T$~~

$$\Rightarrow d = \frac{|w^T x|}{\|w\|} = \frac{|-b|}{\|w\|} = \frac{b}{\|w\|}$$