I. RANDOM MIXED STRATEGIES IN QUANTUM HAWK-DOVE GAME

We will first consider a pure bimatrix classical game, one in which Alice and Bob are allowed to play "H" (I) and "D" (X) (representing the strategies corresponding to pure *Hawk(H)* and *Dove(D)*). And then we will add the Mixed strategy Nash equilibrium if there is, (else we will add a random mixed strategy) where, both Alice and Bob plays H with probability p = q = 0.5 and D with 1-p and 1-q respectively. And then we will consider a pure Quantum game with the pure quantum strategies represented as "Q", defined as-

$$Q = U(0, \pi/2) = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix} \tag{1}$$

and the Miracle move defined as-

$$M = U(\pi/2, \pi/2) = \frac{1}{\sqrt{2}} \begin{bmatrix} i & -1 \\ 1 & -i \end{bmatrix} \tag{2}$$

And then we will add a random mixed strategy "R_{QM}" where, Alice and Bob plays Q with p = q = 0.5 and M with 1-p and 1-q respectively. In Hawk-Dove game, we may choose the initial state as $\hat{J}|DD\rangle$

A. Classical Game

The classical payoff matrix for hawk dove game is represented as follows:

		Bob	
		H	D
	H	$\left(\frac{v}{2} - \frac{i}{2}, \frac{v}{2} - \frac{i}{2}\right)$	(v,0)
Alice	D	(0,v)	$(\frac{v}{2} - d, \frac{v}{2} - d)$

(3)

where v and i are the value of resource and cost of injury, respectively. The cost of displaying patience and waiting is d. Let $v = 50$, $i = 100$ and $d = 10$. Then for the said set of values the payoff table when both Alice and Bob pursue pure strategies is given as-

		Bob	
		H	D
	H	(-25,-25)	(50,0)
Alice	D	(0,50)	(15,15)

(4)

The Nash equilibrium for the classical Hawk-Dove game is (H, D) and (D, H). And (D, D) is the Pareto optimal. The mixed strategy nash equilibrium can be found by plotting the fitness of Hawk, W(H) given as,
W(H) = p.\$(H,H) + (1 - p).\$(H,D), and fitness of Dove, W(D) = p.\$(D,H) + (1 - p).\$(D,D), and finding the point of intersection where W(H) = W(D).

Thus, from FIG. 1 the mixed strategy nash equilibrium is when p =7/12. Since the game is symmetric q is also 7/12.

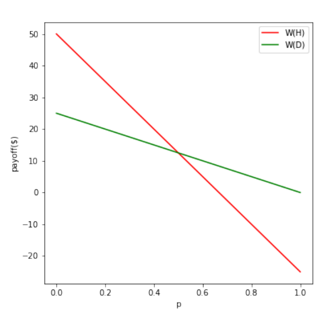

FIG. 1: Plot of W(H) and W(D), where they intersect at p = 7/12

B. Adding Classical mixed Strategy

We will now add the mixed strategy equilibrium, which is also evolutionary stable strategy "ESS" with p = 7/12 and q = 7/12,

		Bob		
		H	D	ESS
Alice	H	(-25,-25)	(50,0)	(6.25, -14.58)
	D	(0,50)	(15,15)	(6.25, 35.42)
	ESS	(-14.58, 6.25)	(35.42, 6.25)	(6.25, 6.25)

(5)

C. When Alice and Bob are restricted to play only the Quantum Strategies

Considering Alice and Bob are restricted to play pure quantum stragies Q and M defined in eq(1-2), we will get the following payoff matrix.

		Bob	
		Q	M
Alice	Q	(15, 15)	(32.5, 7.5)
	M	(7.5, 32.5)	(10, 10)

(6)

In this scenario we can see that (Q, Q) becomes the Nash equilibrium and Pareto optimal too. But from FIG. 2: we can see that there wont be a Mixed strategy nash equilibrium since W(Q) and W(M) don't intersect.

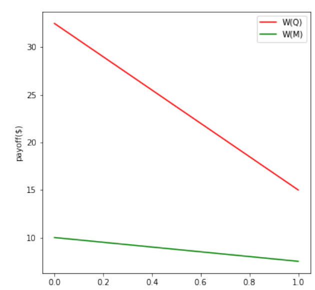

FIG. 2: Plot of W(Q) and W(M) doesn't intersect. Thus, there is no mixed strategy Nash equilibrium

D. Adding Quantum random mixed Strategy

We will now add the random mixed strategy "R_{QM}" p = 1/2 and q = 1/2,

		Bob		
		Q	M	R_{QM}
Alice	Q	(15, 15)	(32.5, 7.5)	(23.75, 11.25)
	M	(7.5, 32.5)	(10, 10)	(8.75, 21.25)
	R_{QM}	(11.25, 23.75)	(21.25, 8.75)	(16.25, 16.25)

(7)

(Q,Q) is still the Nash equillibrium after adding R_{QM} to the payoff table. But we can see that the game is totally different when played with only Quantum strategies with Nash equilibrium being (15, 15) and new Pareto optimal (R_{QM}, R_{QM}) which is slightly greater than (D, D) of classical game. And this cannot be replicated in a Purely classical game. Thereby, negating the Van Enk-Pikes assertion.

II. RANDOM MIXED STRATEGIES IN QUANTUM PRISONER'S DILEMMA

We will first consider a pure bimatrix classical game, one in which Alice and Bob are allowed to play "C" (I) and "D" (X) (representing the strategies corresponding to pure *Confess(C)* and *Defect(D)*) and a random strategy "R_{CD}" where, both Alice and Bob plays C with probability p = q = 0.5 and D with 1-p and 1-q respectively. And then we will consider a pure Quantum game with the pure quantum strategies represented as "Q" and the Miracle move defined in eq(1,2) and a random mixed Strategy "R_{QM}" where, Alice and Bob plays Q with p = q = 0.5 and M with 1-p and 1-q respectively .

A. Classical Game

The classical payoff table for pure strategies in PD is represented as-

		Bob	
		C	D
	C	$(3,3)$	$(0,5)$
Alice	D	$(5,0)$	$(1,1)$

(8)

The Nash equilibrium is (D, D) here.

B. Adding Classical random mixed Strategy

Since, (D, D) is the only one dominant strategy in PD, there is no Mixed strategy Nash equilibrium. But we may add a Random mixed strategy with p = q = 1/2

		Bob		
		C	D	R_{CD}
	C	(3,3)	(0,5)	(1.5, 4)
Alice	D	(5,0)	(1,1)	(3, 0.5)
	RCD	(4, 1.5)	(0.5, 3)	(2.25, 2.25)

(9)

It's clear that (D, D) is still the Nash equilibrium after a adding mixed strategy to the payoff table.

C. When Alice and Bob are restricted to play only the Quantum Strategies

		Bob	
		Q	M
	Q	(3,3)	(4, 1.5)
Alice	M	(1.5, 4)	(2.25, 2.25)

(10)

In this scenario we can see that (Q, Q) becomes the Nash equilibrium and Pareto optimal too. But from FIG. 3: we can see that there wont be a Mixed strategy nash equilibrium since W(Q) and W(M) don't intersect.

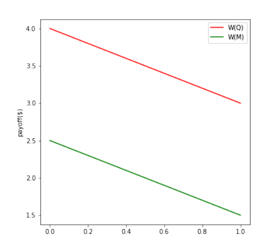

FIG. 3: Plot of W(Q) and W(M) for PD

D. Adding Quantum random mixed Strategy

		Bob		
		Q	M	R_{QM}
Alice	Q	(3,3)	(4, 1.5)	(3.5, 2.25)
	M	(1.5, 4)	(2.25, 2.25)	(1.88, 3.12)
	RQM	(2.25, 3.5)	(3.12, 1.88)	(2.69, 2.69)

(11)

In this case, clearly (Q, Q) is NE with an added Mixed Quantum strategy in the payoff table. But its clear the that payoff obtained here cannot be replicated in a pure classical game. Thereby, negating the Van Enk-Pikes assertion.