I. RANDOM MIXED STRATEGIES IN QUANTUM HAWK-DOVE GAME

We will first consider a pure bimatrix classical game, one in which Alice and Bob are allowed to play "H" (I) and "D" (X) (representing the strategies corresponding to pure *Hawk(H)* and *Dove(D)*). And then we will add a random mixed strategy "R_{CD}" where, both Alice and Bob plays C with probability p = q = 0.5 and D with 1-p and 1-q respectively. And then we will consider a pure Quantum game with the pure quantum strategies represented as "Q", defined as-

$$Q = U(0, \pi/2) = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix} \tag{1}$$

and the Miracle move defined as-

$$M = U(\pi/2, \pi/2) = \frac{1}{\sqrt{2}} \begin{bmatrix} i & -1 \\ 1 & -i \end{bmatrix} \tag{2}$$

And then we will add a random mixed strategy "R_{QM}" where, Alice and Bob plays Q with p = q = 0.5 and M with 1-p and 1-q respectively. In Hawk-Dove game, we may choose the initial state as $\hat{J}|DD\rangle$

A. Classical Game

The classical payoff matrix for hawk dove game is represented as follows:

		Bob	
		H	D
	H	$\left(\frac{v}{2} - \frac{i}{2}, \frac{v}{2} - \frac{i}{2}\right)$	(v,0)
Alice	D	(0,v)	$(\frac{v}{2} - d, \frac{v}{2} - d)$

(3)

where v and i are the value of resource and cost of injury, respectively. The cost of displaying patience and waiting is d. Let $v = 50$, $i = 100$ and $d = 10$. Then for the said set of values the payoff table when both Alice and Bob pursue pure strategies is given as-

		Bob	
		H	D
	H	(-25,-25)	(50,0)
Alice	D	(0,50)	(15,15)

(4)

The Nash equilibrium for the classical Hawk-Dove game is (H, D) and (D, H). And (D, D) is the Pareto optimal. The mixed strategy nash equilibrium can be found by plotting the fitness of Hawk, W(H) given as,
W(H) = p.$(H,H) + (1 - p).$(H,D), and fitness of Dove, W(D) = p.$(D,H) + (1 - p).$(D,D), and finding the point of intersection where W(H) = W(D).

Thus, from FIG. 1 the mixed strategy nash equilibrium is when p =7/12, giving a payoff of 6.25.

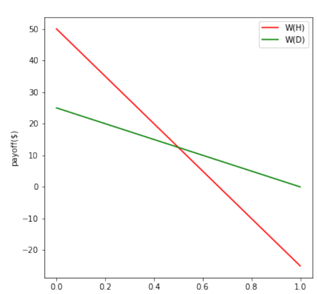

FIG. 1: Plot of W(H) and W(D), where they intersect at p = 7/12

B. Adding Classical random mixed Strategy

We will now add a random mixed strategy "R_{CD}" with p = 1/2 and q = 1/2,

		Bob		
		H	D	R_{HD}
	H	(-25,-25)	(50,0)	(12.5, -12.5)
Alice	D	(0,50)	(15,15)	(7.5, 32.5)
	RHD	(-12.5, 12.5)	(32.5, 7.5)	(10, 10)

(5)

It's clear doesn't Nash equilibrium doesn't change after a adding random mixed strategy to the payoff table.

C. When Alice and Bob are restricted to play only the Quantum Strategies

Considering Alice and Bob are restricted to play pure quantum stragies Q and M defined in eq(1-2), we will get the following payoff matrix.

		Bob	
		Q	M
	Q	(15, 15)	(32.5, 7.5)
Alice	M	(7.5, 32.5)	(10, 10)

(6)

In this scenario we can see that (Q, Q) becomes the Nash equilibrium and Pareto optimal too. But from FIG. 2: we can see that there wont be a Mixed strategy nash equilibrium since W(Q) and W(M) don't intersect.

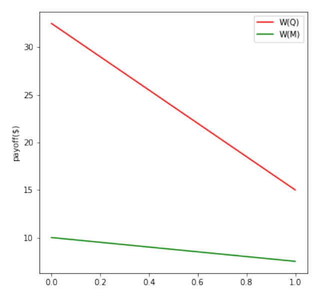

FIG. 2: Plot of W(Q) and W(M) doesn't intersect. Thus, there is no mixed strategy Nash equilibrium

D. Adding Quantum random mixed Strategy

We will now add the random mixed strategy "R_{QM}" p = 1/2 and q = 1/2,

		Bob		
		Q	M	R_{QM}
	Q	(15, 15)	(32.5, 7.5)	(23.75, 11.25)
Alice	M	(7.5, 32.5)	(10, 10)	(8.75, 21.25)
	R_{QM}	(11.25, 23.75)	(21.25, 8.75)	(16.25, 16.25)

(7)

(Q,Q) is still the Nash equillibrium after adding R_{QM} to the payoff table. But we can see that the game is totally different when played with only Quantum strategies with Nash equilibrium being (15, 15), and cannot be replicated in a Purely classical game. Thereby, negating the Van Enk-Pikes assertion.