Parallel and Grid Computing - Introduction to Mandelbrot set-based generation of fractals*

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Mandelbrot' set is a particular fractal object discovered by B. Mandelbrot in 1980 and, although presenting a infinite complexity, this subset of $\mathbb{C}$ is quite simple to generate thanks to a small recurrence formula in $\mathbb{C}$.

Definition 1 *Mandelbrot' set is defined as the complex numbers $c \in \mathbb{C}$ such that the sequence $(z_n)_{n\in\mathbb{N}} \subset \mathbb{C}$ defined by*

$$z_0 = 0, \qquad z_{n+1} = z_n^2 + c,$$

remains bounded in $\mathbb{C}$.

In order to represent this fractal in a subset $[x_{min}, x_{max}] \times [y_{min}, y_{max}] \subset \Omega$, we start from each pixel (a, b) of the picture, we define $c = a + ib \in \mathbb{C}$ and the sequence $(z_n)_{n\in\mathbb{N}}$, and we finally define $I(a, b)$ as the luminous intensity with the smaller integer n required for observing the divergence of $(z_n)_{n\in\mathbb{N}}$. In practical cases, we consider an integer $N \in \mathbb{N}^*$ fixed for the whole picture and $I(a, b)$ is defined as follows:

$$I(a, b) = \frac{1}{N} \min_{n=0,\ldots,N} \{n : |z_n| > 2\}.$$

In order to produce high quality fractal pictures, it is necessary to consider a very high number of pixels along with a high value of N. As a consequence, a good graphical representation of Mandelbrot' set can require huge CPU resources because of a high number of pixels and/or a high value of N. This constitutes the main motivation of implementing this graphical representation by using parallel computing.

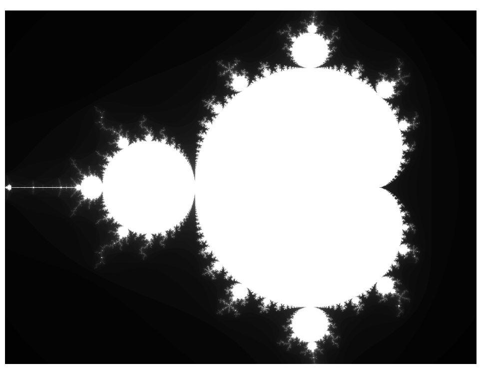

Figure 1: Mandelbrot' set on $\Omega = [-1.78, 0.78] \times [-0.961, 0.961]$ with 1024×768 resolution and a convergence threshold $N = 100$.

*Taken from a Master project by N. Melab, A. Mouton and J. Gmys (Université de Lille).