POISSON DISTRIBUTION

A small business receives, on average, 8 calls per hour. (a) What is the probability that the business will receive exactly 7 calls in 1 hour? (b) What is the probability that the business will receive atmost 5 calls in one hour? What is the probability that the business will receive more than 6 calls in one hour?

b) $\mu = 8 \qquad P(X \leq 5)$

$P(X \leq 5) = P(X=0) + P(1) + P(2) + P(3) + P(4) + P(5)$

$$P(X=x) = \frac{\mu^x e^{-\mu}}{x!} = e^{-\mu}\left[\frac{\mu^x}{x!}\right]$$

$$P(X \leq 5) = e^{-8}\left[\frac{8^0}{0!} + \frac{8^1}{1!} + \frac{8^2}{2!} + \frac{8^3}{3!} + \frac{8^4}{4!} + \frac{8^5}{5!}\right]$$

$= 0.191236$

$\boxed{= 19.12\%}$

Scanned with CamScanner

(6) $\mu = 8$ $P(X>6)$

$P(X>6) = 1 - P(X \leq 6)$

$= 1 - [P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6)]$

$= 1 - e^{-8}\left[\frac{8^0}{0!} + \frac{8^1}{1!} + \frac{8^2}{2!} + \frac{8^3}{3!} + \frac{8^4}{4!} + \frac{8^5}{5!} + \frac{8^6}{6!}\right]$

$= 0.6866626$

$\boxed{= 68.7\%}$

A bank observes that on average, 10 customers arrive per hour. The bank wants to understand the likelihood of different numbers of customers arriving in a given hour to optimize staff allocation.

$$P(X=k) = \frac{e^{-\lambda} \lambda^k}{k!}$$

$$P(X=7) = \frac{e^{-10} \cdot 10^7}{7!}$$

$e^{-10} = 4.539993 \times 10^{-5}$

$10^7 = 10\,000\,000$

$7! = 5040$

$$P(X=7) = \frac{(4.539993 \times 10^{-5}) \cdot 10\,000\,000}{5040}$$

$$= \frac{453.9993}{5040}$$

$P(X=7) = 0.0901$

$\boxed{= 9.01\%}$