CS270 Digital Image processing

Homework 1

Due data: October 9, 2018

Name: chaizhenjie

Student ID: 2018233123

1. Suppose that a flat area is illuminated by a light source with intensity distribution.

$$i(x, y) = K * 10^{-(x^2+y^2)\frac{1}{2}}$$

If the reflectance characteristic of the area is

$$r(x, y) = \frac{1}{x^2 + y^2}$$

What is the value of K that would yield an image intensity of 1 at (x=6, y=8)?

Answer of question 1:
At $(x = 6, y = 8)$, $r(x, y) = r(6, 8) = \frac{1}{100}$, and $i(x, y) = i(6, 8) = \frac{K}{10^{10}}$,
so we can conclude that the value of K is 10^{12}

2. There are one thousand 1024*768 images with 256 gray levels.

(1) calculate the capacity (megabyte) that is required to store these images

(2) A common measure of transmission for digital data is the baud rate, defined as the number of bits transmitted per second. Generally, transmission is accomplished in packets consisting of a start bit, a byte of information, and a stop bit. Calculate the time that is required to transmit these 1000 images at 9600 baud.

Note: 1byte = 8 bits

Answer of question 2.1:
We know that 256 gray levels need 8 bits to represent,now we calculate it:

$$(1000 * 1024 * 768 * 8)/(8 * 1024 * 1024) = 750M$$

so 750MB is required to store these images.

Answer of question 2.2:
We can calculate the whole capacity including start bit and end bit:

$$1000 * 1024 * 768 * 8 * \frac{10}{8} = 7864320000bits$$

then calculate the time is required to transmit:

$$7864320000/9600 = 8192000sec$$

so now we know that the required time is 8192000sec.

3.A CCD camera chip of dimensions 14*14mm,and having 2048*2048 elements, is focused on a square flat area, located 0.5m away. What is the spatial resolution that this camera will be able to resolve? The camera is equipped with a 35-mm lens(Hint: Model the image process as in Fig 2.3).

Answer of question 3:

Step 1:

We calculate value of x by $\frac{14}{35} = \frac{x}{500}$, we know that x = 200mm.

Step 2: Let's calculate the spatial resolution: $\frac{2048}{200 * 2} = 5$

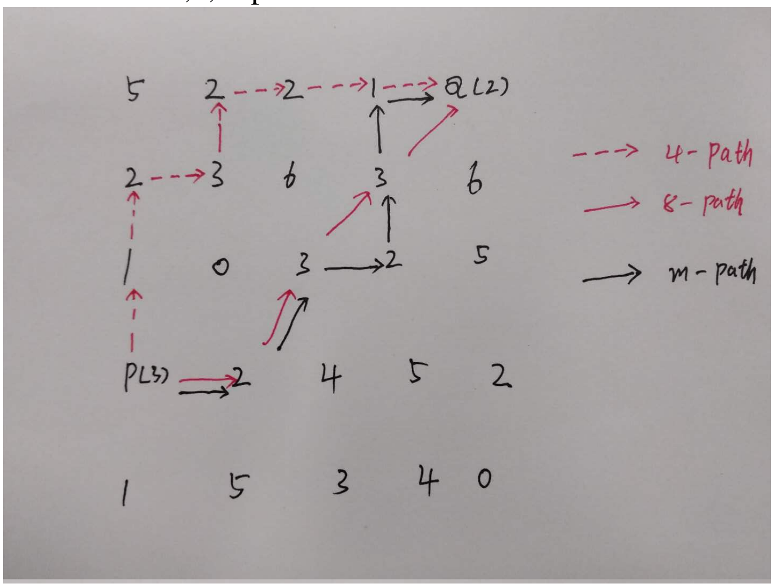
So the spatial resolution is 5 elements/mm

4. V =1,2,3, mark the shortest 4-, 8-, m-path between P and Q, and calculate D 4 , D 8 , D m from P to Q respectively.

5	2	2	1	$Q(2)$
2	3	6	3	6
1	0	3	6	5
$P(3)$	2	4	5	2
1	3	3	4	0

Answer of question 4:

The shortest 4,8,m-path:

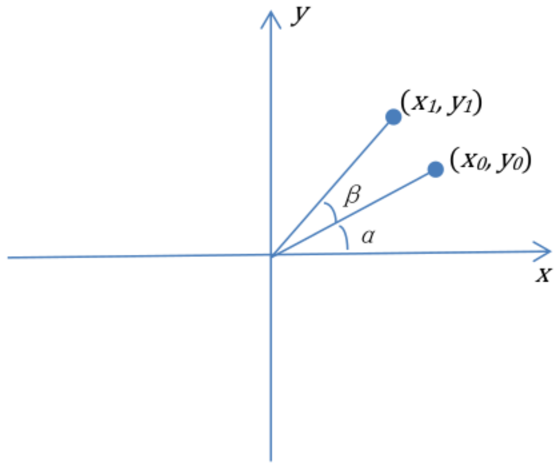


so $D_4 = 7, D_8 = 4, D_m = 6$ from P to Q respectively.

5. Derive the rotation operation equation

$$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} cos\beta & -sin\beta & 0 \\ sin\beta & cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ 1 \end{bmatrix}$$

according to the figure as below



Answer of question 5:

The rotation operation equation is

$$\begin{cases} x_1 = x_0 \cos\beta - y_0 \sin\beta \\ y_1 = x_0 \sin\beta + y_0 \cos\beta \end{cases} \tag{1}$$

6. Let the original image is f(x,y), the transformed image is g(x,y). If rotate f counter-clockwise 30 degrees around the pixel f(2,2) to get g , then
Forward mapping is g = T * f
Inverse mapping is f = T^c * g
Derive the transformation matrix T and T c

Answer of question 6:

suppose (x_0, y_0) ~~on~~ in $f(x, y)$ and (x_1, y_1) in $g(x, y)$

$x_0 = r\cos\alpha$

$y_0 = r\sin\alpha$

$x_1 = (r-2\sqrt{2})\cos(\alpha+\beta) + 2 = \frac{\sqrt{3}}{2}x_0 - \frac{1}{2}y_0 + 3 - \sqrt{3}$

$y_1 = (r-2\sqrt{2})\sin(\alpha+\beta) + 2 = \frac{1}{2}x_0 + \frac{\sqrt{3}}{2}y_0 + 1 - \sqrt{3}$

$$\therefore \quad T = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 3-\sqrt{3} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 1-\sqrt{3} \\ 0 & 0 & 1 \end{bmatrix}$$

$$[T\,|\,E] = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 3-\sqrt{3} & 1 & 0 & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 1-\sqrt{3} & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & 1 & 0 & \sqrt{3}-3 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 & 1 & \sqrt{3}-1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} \frac{2}{3}\sqrt{3} & 0 & 0 & 1 & \frac{\sqrt{3}}{3} & \frac{2}{3}\sqrt{3}-2 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 & 1 & \sqrt{3}-1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

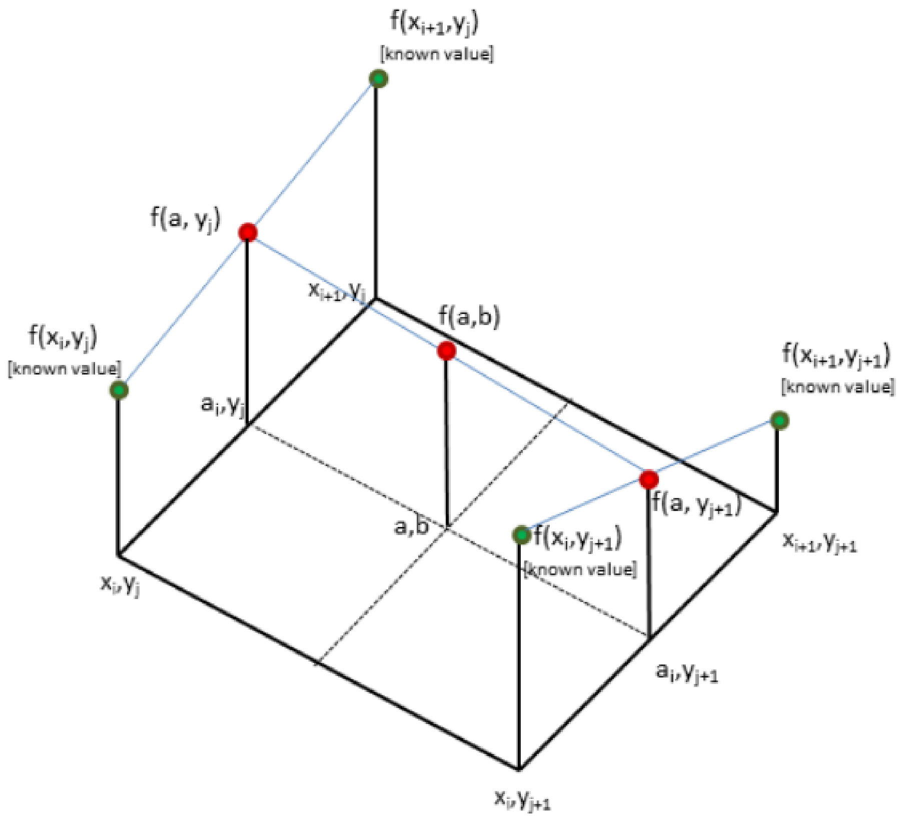
$$\sim \begin{bmatrix} 1 & 0 & 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} & 1-\sqrt{3} \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & 3-\sqrt{3} \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \quad T^c = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 1-\sqrt{3} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 3-\sqrt{3} \\ 0 & 0 & 1 \end{bmatrix}$$

7. Find the bilinear interpolation value of f(a,b) using 4 nearest neighbours as shown in the figure. Denote:

$$f_{00} = f(x_i, y_j)\ f_{10} = f(x_{i+1}, y_j)\ f_{01} = f(x_i, y_{j+1})\ f_{11} = f(x_{i+1}, y_{j+1}),$$
$$x_{i+1} = x_1\ x_i = x_0\ y_{j+1} = y_1\ y_j = y_0$$

Use $f_{00}, f_{10}, f_{01}, f_{11}, x_1, x_0, y_1, y_0$ to describe $f(a, b)$



Answer of question 7:

首先在 x 方向进行线性插值:

$$f(x, y_i) = \frac{x_{i+1} - x}{x_{i+1} - x_i} f_{00} + \frac{x - x_i}{x_{i+1} - x_i} f_{10}$$

$$= (x_1 - x) f_{00} + (x - x_0) f_{10}$$

$$f(x, y_{i+1}) = \frac{x_{i+1} - x}{x_{i+1} - x_i} f_{01} + \frac{x - x_i}{x_{i+1} - x_i} f_{11}$$

$$= (x_1 - x) f_{01} + (x - x_0) f_{11}$$

然后在 y 方向上进行插值.

$$f(x, y) \approx \frac{y_{i+1} - y}{y_{i+1} - y_i} f(x, y_1) + \frac{y - y_i}{y_{i+1} - y_i} f(x, y_2)$$

$$= f_{00}(x_1 - x)(y_1 - y) + f_{10}(x - x_0)(y_1 - y) + f_{01}(x_1 - x)(y - y_0) + f_{11}(x - x_0)(y - y_0)$$

$$\therefore f(a, b) = (f_{11} - f_{10} - f_{01} + f_{00})ab + (-f_{11} + f_{10} + f_{01} - f_{00})a + (-f_{11} + f_{10} + f_{01} - f_{00})b$$
$$+ f_{11} x_0 y_0 - f_{10} x_0 y_1 - f_{01} x_1 y_0 + f_{00} x_1 y_1$$

8. If the image in Problem 6,

$$f(x,y)=\begin{bmatrix} 0 & 8 & 10 & 5 & 8 & 7 \\ 1 & 5 & 7 & 8 & 10 & 6 \\ 5 & 4 & 2 & 11 & 9 & 8 \\ 3 & 6 & 2 & 3 & 5 & 9 \\ 2 & 3 & 6 & 9 & 12 & 11 \\ 1 & 4 & 0 & 15 & 13 & 14 \end{bmatrix} \quad (0 \le x, y \le 5),$$

use the method in Problem 6 & 7, calculate the gray value of g(3,3).

Answer of question 8:

先找出 $g(3,3)$ 对应f下的 a, b 值.

$$\begin{bmatrix} a \\ b \\ 1 \end{bmatrix} = T^c \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 1-\sqrt{3} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 3-\sqrt{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix}$$

$$\therefore (a, b) = \left(\frac{5-\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}\right)$$

根据第7题图

$$\begin{cases} x_{i+1} - x_i = 1 \\ x_{i+1} + x_i = 5-\sqrt{3} \end{cases} \qquad \begin{cases} y_{i+1} - y_i = 1 \\ y_{i+1} + y_i = 3-\sqrt{3} \end{cases}$$

$$x_1 = x_{i+1} = 3 - \frac{\sqrt{3}}{2} \approx 2 \qquad y_1 = y_{i+1} = 2 - \frac{\sqrt{3}}{2} \approx 1$$

$$x_0 = x_i = 2 - \frac{\sqrt{3}}{2} \approx 1 \qquad y_0 = y_i = 1 - \frac{\sqrt{3}}{2} \approx 0$$

$$f_{00} = 8 \quad f_{01} = 5 \quad f_{10} = 10 \quad f_{11} = 7$$

$$\therefore f(a, b) = 0 + 0 + 0 + 0 - 10 + 0 + 8 \times 2 = 6$$

$$\therefore g(3,3) = 6$$

故 the gray value of $g(3,3)$ is 6