Flop and Complexity

Linear Algebra

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Complexity of vector computations

- Computers store (real) numbers in floating–point format
- Floating point= 64 bits or 8 bytes
  - How many possible sequences of bits?
  - How many bytes to store n–vector?
- Current memory and storage devices, with capacities measured in many gigabytes (109 bytes), can easily store vectors with dimensions in the millions or billions.
- Sparse vectors are stored in a more efficient way that keeps track of indices and values of the nonzero entries.
- Note about floating point operations and round–off error.

FLOP (Floating Point Operations)

- ❑ The unit of complexity when comparing vector and matrix algorithms
- ❑ 1 flop = one basic arithmetic operation in R or C $(+,-,*,\sqrt{\ },\cdots)$
- ❑ Estimate the time of computation= counting the total number of Floating Point Operations (FLOP)s.

Notes:

- ❑ We don't distinguish between the different types of arithmetic operations.
- ❑ We don't distinguish between real and complex arithmetic.
- ❑ We ignore integer operations (indexing, loop counters, ⋯)
- ❑ We ignore cost of memory access.

Complexity of vector computations

- Operation count (flop count)
  - Total number of operations in an algorithm
  - $runtime \approx \dfrac{number\ of\ operations\ (flops)}{computer\ speed\ (flops\ per\ second)}$

- Dominant term: the highest-order term in the flop count

$$\frac{1}{3}n^3 + 100n^2 + 10n + 5 \approx \frac{1}{3}n^3$$

- Order: the power in the dominant term:

$$\frac{1}{3}n^3 + 10n^2 + 100 = \text{order } n^3$$

Complexity of vector computations

- How quickly the vector operations can be carried out by a computer depends very much on the computer hardware and software, and the size of the vector.
- The complexity of an operation is the number of flops required to carry it out, as a function of the size or sizes of the input to the operation.
- Crude approximation of time to execute: (flopsneeded)/(computer speed)
- current computers are around 1Gflop/sec (10^9 flops/sec)

- Zero vector 0_n
- Ones vector 1_n
- Unit vector e_i (e_i is the entry with 1 value)
- Sparse vector: a vector if many of its entries are zero
  - Can be stored and manipulated efficiently on a computer
  - nnz(x): number of entries of vector x that are nonzero

Example

- Write all unit vectors with length of 3.
- What is the most sparsest vector?

Complexity of vector computations for $v \in R^n$

	#FLOPS		Complexity	
Operation	**General**	**Sparse**	**General**	**Sparse**
Scalar-Vector product				
Vector-Vector sum				
Inner product				
Outer product (vectors with sizes “n” and “m”				
Hadamard product				

References

- Introduction to Applied Linear Algebra Vectors, Matrices, and Least Squares
- Linear Algebra and its applications
- Linear algebra A Modern Introduction David Poole
- Floating Point Operations in Matrix–Vector Calculus, Raphael Hunger