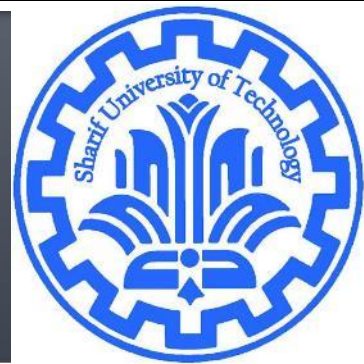


Introduction

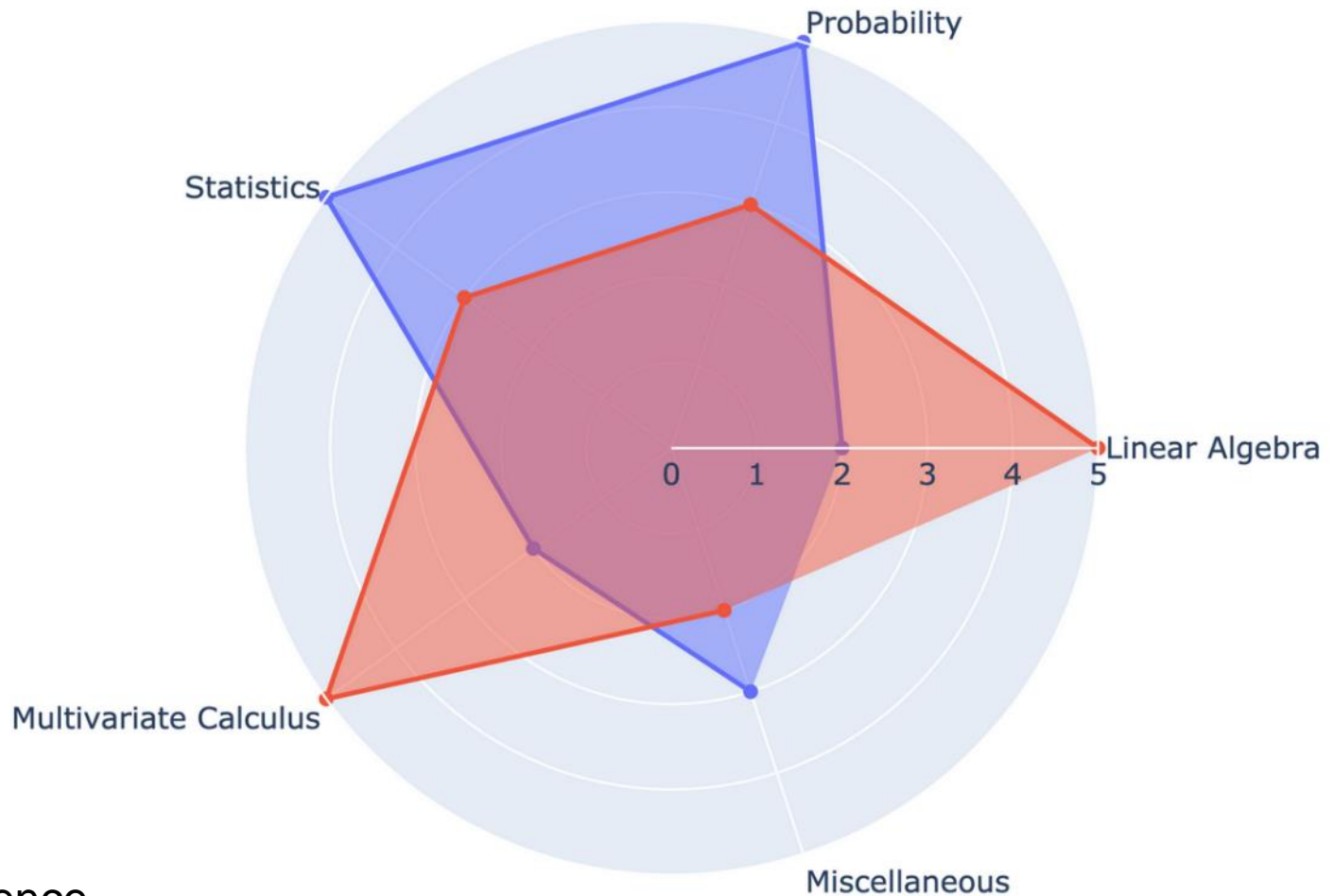
CE40282-1: Linear Algebra
Hamid R. Rabiee and Maryam Ramezani
Sharif University of Technology



Why are we here?

- Linear algebra is a main underlying tool for:
 - Computer Science & Engineering Applications
 - Machine Learning
 - Robotics
 - Operations Research
 - Theoretical Computer Science
- You will learn the basic concepts of Linear Algebra and how they apply to various aspects of computer science.

Why are we here?



Blue: Data Science
Orange: Machine Learning

Reach Us!

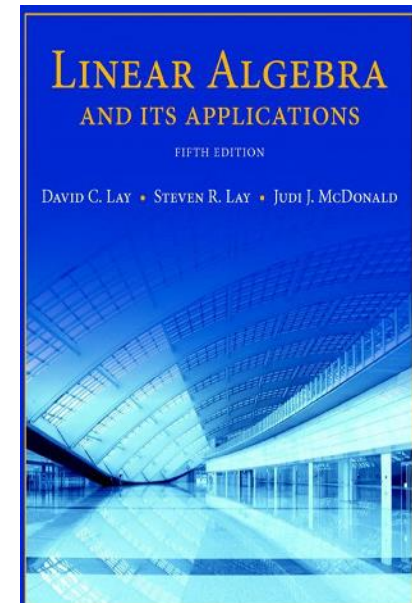
- Office Room: 803-804
- Email:
 - Hamid R. Rabiee: (rabiee@sharif.edu)
 - Maryam Ramezani: (maryam.ramezani@sharif.edu)
- Course notes, homework and solutions, handouts, and other useful resources are available on the Quera page:
 - <https://quera.ir/course/11826/>
- Room: (Sunday & Tuesday: 10:30-12)
 - Ebnesina Alef 12
 - <https://vc.sharif.edu/ch/rabiee>
- Lead TA:
 - Aryan Ahadinia: (aryan.ahadinia@sharif.edu)
- Feedback
 - <https://forms.gle/ktngLcnxRSeKtV1z8>

Resources

■ Textbooks:

- David C. Lay. Linear Algebra and Its Applications. University of Maryland, 2016.
- Stephen Boyd and Lieven Vandenberghe. Introduction to Applied Linear Algebra – Vectors, Matrices, and Least Squares. Cambridge University Press, 2018.
- Gilbert Strang. Introduction to Linear Algebra. Wellesley-Cambridge Press, 2016.
- Gilbert Strang. Linear Algebra and Learning From Data. Wellesley-Cambridge Press, 2019.

+Other textbooks and course materials.



Structure of the Course

■ Lectures

- **Goal:** To introduce concepts in linear algebra, and motivate their use and importance.
- **Note:** We try to cover useful materials in class, but we recommend you reading more!

■ Assignments

- **Purpose:** To give you a chance to exercise your mind, and to solidify the concepts introduced to you in class.
- **Structure:** Six theoretical problems, and one linear algebra practical problems.
- **Importance:** Not important unless you want to learn the material and get a good grade.

■ Project (Application)

■ Exams: Midterm + Final

Lecture Notes

- Lecture slide will be uploaded.
- Many times we will write on board, in real-time, during lecture to prove a theory or answer a question or add some additional explanations. It will be your responsibility to take notes.
- Slides links will be provided on Quera.

Homework

- Six series
 - 6 theory questions
 - 1 practical question
 - Basic Python for programming
 - Basic NumPy for array manipulation
 - Basic programming with Pytorch
 - Learning linear algebra and application of machine learning.
- Assignments will be released on Tuesday every two weeks. Students will have 12 days for the theoretical and 14 days for the practical questions to submit answers in Quera
- Homework Upload:
 - <https://quera.ir/course/11826/>

TA

- TA class: (Sundays: 12:30 to 13:20. - Start: 1401/07/10)
 - CE??
 - <https://vc.sharif.edu/ch/rabiee-ta>

Date	Topic
1401/07/10	Python Workshop: Introduction to Python, installation and elementaries
1401/07/17	Python Workshop: Python elementaries and numerical computation using NumPy
1401/07/24	Python Workshop: Pandas and Visualization
1401/08/01	HW1
1401/08/08	Extra
1401/08/15	HW2
1401/08/22	Extra
1401/08/29	HW3
1401/09/06	Extra
1401/09/13	Midterm Exclusive
1401/09/20	HW4
1401/09/23	Python Workshop: PyTorch, will be held on Wednesday
1401/09/27	Extra
1401/10/04	HW5
1401/10/11	Extra
1401/10/18	HW6
1401/11/01	Final Exclusive

Grading Scheme

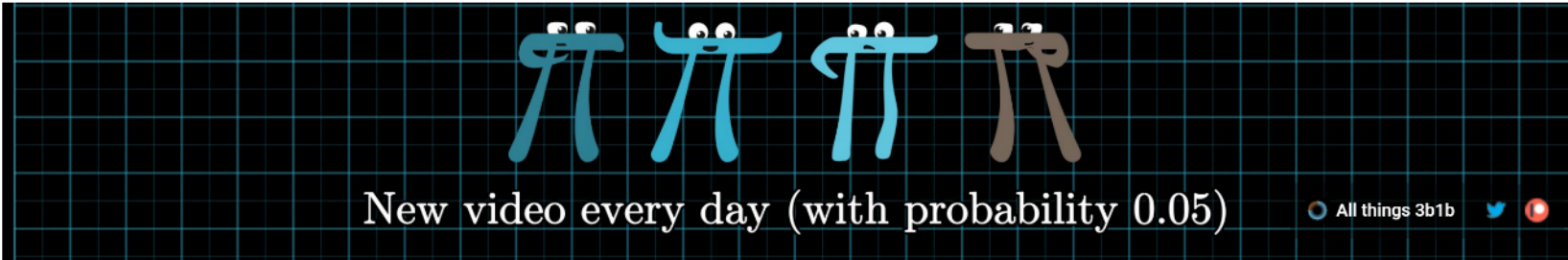
Item	Grade
Homework	50% (10 points)
Final Project	5% (1 point)
Midterm Exam	25% (5 points)
Final Exam	30% (6 points) (Comprehensive)
Class Homework	?
Total	22+?

- Mid-Term Exam: 25% (1401/09/17 9:00 AM)
- Final Exam: 30% (Comprehensive) (1401/11/02 9:00 AM)

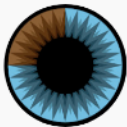
Homework Rules

- Homework and Project: 50%
 - Homework Assignments: 10 points, first 5 series with 1.4 points for theoretical questions and 0.3 point for practical questions, last series with 1.5 for theoretical questions
 - Final Project: 1 point
 - You have a total **15 days of allowed late** submission (for both theoretical and code part). **0.5%** of the assignment grade will be subtracted for each **hour of delay**
- For each homework, you can use your late submission up to **5 days. No submissions are accepted after TA class.**
- Discussing the problems with each other is encouraged. Copying each others assignments or submitting solutions/answers on the web is strictly prohibited. First time, you **will receive -100% of grade for the task at hand**. If you are caught for a second time, **you will fail the course.**

Some Useful Links



New video every day (with probability 0.05) All things 3b1b

 **3Blue1Brown** ✓
3.94M subscribers

HOME VIDEOS PLAYLISTS COMMUNITY CHANNELS **ABOUT**

Description

3Blue1Brown, by Grant Sanderson, is some combination of math and entertainment, depending on your disposition. The goal is for explanations to be driven by animations and for difficult problems to be made simple with changes in perspective.

For more information, other projects, FAQs, and inquiries see the website:
<https://www.3blue1brown.com>

Stats

Joined Mar 4, 2015

217,063,334 views

Details

Location: United States

What we'll cover

Date	Session	Instructor	Topic	HW	Midterm				
1401/06/27	1	MR	Introduction	HW1	Midterm Materials				
1401/06/29	2	MR	Vector Space						
1401/07/03	Holiday		Nationwide Holiday						
1401/07/05	Holiday		Nationwide Holiday						
1401/07/10	3	MR	Vector Space						
1401/07/12	4	MR	Vector Space						
1401/07/17	5	HRR	Linear Equations	HW2		Midterm Materials			
1401/07/19	6	HRR	Scaler-valued Functions						
1401/07/24	7	HRR	Independence						
1401/07/26	8	HRR	Independence						
1401/08/01	9	MR	Combinations	HW3			Midterm Materials		
1401/08/03	10	MR	Euclidean Norm						
1401/08/08	11	MR	Norm Space						
1401/08/10	12	MR	Inner Product and Orthogonality						
1401/08/15	13	MR	Matrix Properties	HW4				Midterm Materials	
1401/08/17	14	MR	Transformations						
1401/08/22	15	MR	Transformations						
1401/08/24	16	MR	Dimention and Rank						
1401/08/29	17	MR	Dimention and Rank (and Inverse)						
1401/09/01	18	MR	Inverse						
1401/09/06	19	MR	Eigenvalue, Eigenvector, Diagonalization	HW5					Midterm Materials
1401/09/08	20	MR	Matrix Inner Product and Norm Vector, Matrix, and Tensor Derivatives						
1401/09/13	21	HRR	Determinant						
1401/09/15	22	HRR	Decomposition and Matrix Factorization						
1401/09/17	Midterm		Midterm		Midterm Materials				
1401/09/20	23	HRR	Symmetric Matrices and Quaratic Form	HW6					
1401/09/22	24	HRR	Least Squares						
1401/09/26	Withdraw/Survey		Educational Deadline						
1401/09/27	25	HRR	Least Squares						
1401/09/29	26	MR	Singular Value and Singular Vector						
1401/10/04	27	MR	Principal Components and the Best Low Rank Matrix						
1401/10/06	Holiday		Nationwide Holiday			Midterm Materials			
1401/10/07	28 (Optional)	HRR/MR	Graph and Network						
1401/11/02	Final Exam		Final Exam						
1401/11/13	Grading Deadline		Educational Deadline						

Some (Interesting) and Important Applications

- [Coding theory](#)
- [Cryptography](#)
- [Tomography](#)
- [Vector graphics](#)
- [Games Theory](#)
- [Quantum computing](#)
- [Economics](#)
- [Chemistry](#)
- [Genetics](#)
- [Image processing](#)
- [Markov chains](#)
- [Linear Regression](#)

What is linear algebra?

- Linear algebra is a branch of mathematics that studies vector spaces, also called linear spaces, along with linear functions that input one vector and output another.
- It has extensive applications in engineering, physics, natural sciences, computer science, and the social sciences (particularly in economics).
- Nonlinear mathematical models can often be approximated by linear ones.
- Linear algebra is a branch of mathematics, but the truth of it is that linear algebra is the mathematics of data.
- Matrices and vectors are the language of data

What is linear algebra?

- The study of linear algebra and matrices has a long history. It first emerged from determinants, which were used to solve systems of linear equations.
- Linear algebra is the study of lines and planes, vector spaces and mappings that are required for linear transforms.

$$\begin{array}{l} y_1 = 0.1 \times x_1 + 0.4 \times x_2 \\ y_2 = 0.3 \times x_1 + 0.9 \times x_2 \\ y_3 = 0.2 \times x_1 + 0.3 \times x_2 \\ \vdots \end{array} \longrightarrow y = A \cdot b$$

Numerical Linear Algebra

- The application of linear algebra in computers is often called **numerical linear algebra**
- Computers are good at performing linear algebra calculations, and much of the dependence on Graphical Processing Units (GPUs) by modern machine learning methods such as deep learning is because of their ability to compute linear algebra operations fast.

Numerical Linear Algebra and Statistics

- The application of linear algebra in computers is often called **numerical linear algebra**: Graphical Processing Units (GPUs)
- Some clear fingerprints of linear algebra on **statistics** and statistical methods include:
 - Use of vector and matrix notation, especially with multivariate statistics.
 - Solutions to least squares and weighted least squares, such as for linear regression.
 - Estimates of mean and variance of data matrices.
 - The covariance matrix that plays a key role in multinomial Gaussian distributions.
 - Principal component analysis for data reduction that draws many of these elements together.

Applications of Linear Algebra

- Matrices in Engineering
- Graphs and Networks, such as analyzing networks
- Markov Matrices, Population, and Economics, such as population growth
- Linear Programming, the simplex optimization method
- Fourier Series: Linear Algebra for functions, used widely in signal processing
- Linear Algebra for statistics and probability, such as least squares for regression
- Computer Graphics, such as the various translation, rescaling and rotation of images

Linear Algebra in Machine Learning

- Images and Photographs
 - Each image that you work with is itself a table structure with a width and height and one pixel value in each cell for black and white images or 3 pixel values in each cell for a color image.
 - A photo is yet another example of a matrix from linear algebra.
 - Operations on the image, such as cropping, scaling, shearing, and so on are all described using the notation and operations of linear algebra.

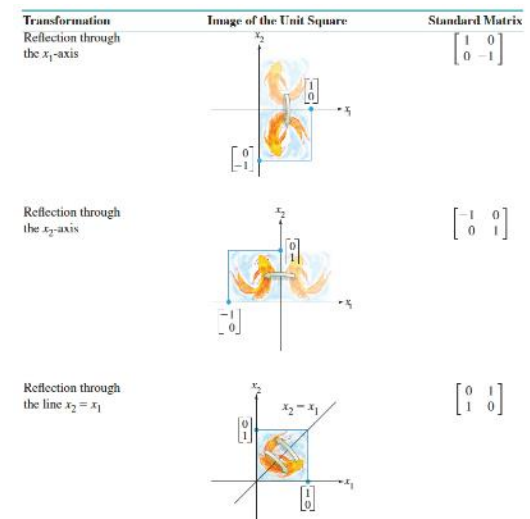
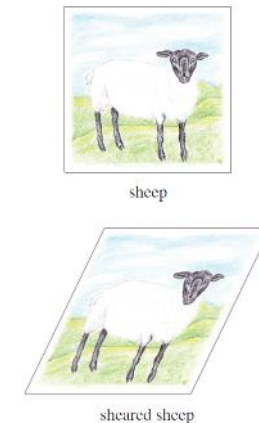


Image Processing

- Image Rotation



(a)



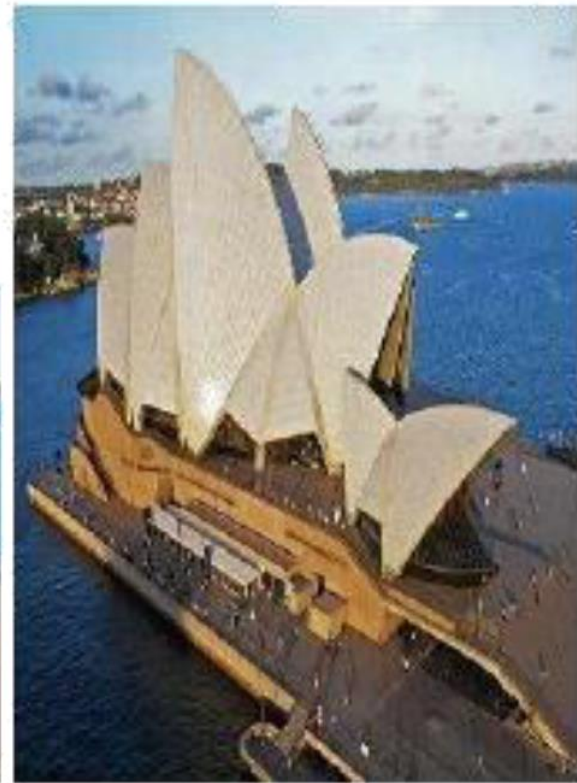
(b)

Image Processing

- Image Scaling



(a)



(b)

Linear Algebra in Machine Learning

■ Dataset and Data Files



Attribute Information:

1. sepal length in cm
2. sepal width in cm
3. petal length in cm
4. petal width in cm
5. class:
 - Iris Setosa
 - Iris Versicolour
 - Iris Virginica

Linear Algebra in Machine Learning

■ Dataset and Data Files

- In machine learning, you fit a model on a dataset.
- This is the table-like set of numbers where each row represents an observation and each column represents a feature of the observation.

Attribute Information:

1	5.1,3.5,1.4,0.2,Iris-setosa
2	4.9,3.0,1.4,0.2,Iris-setosa
3	4.7,3.2,1.3,0.2,Iris-setosa
4	4.6,3.1,1.5,0.2,Iris-setosa
5	5.0,3.6,1.4,0.2,Iris-setosa

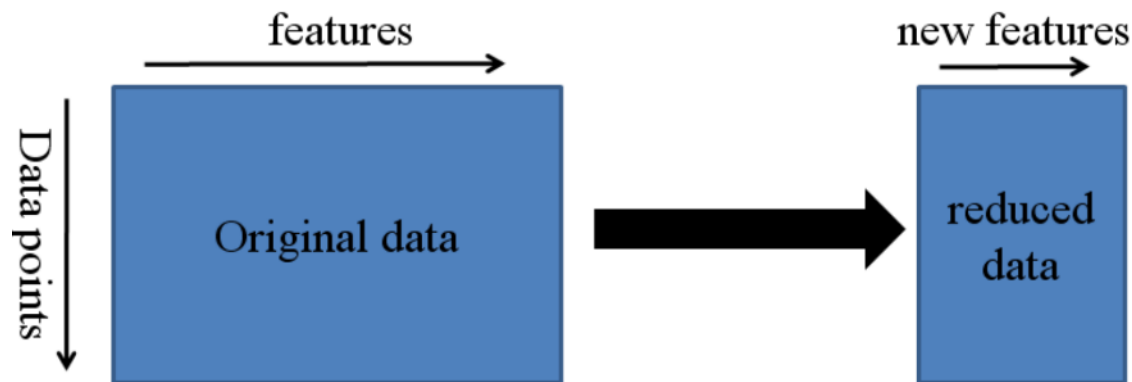
1. sepal length in cm
2. sepal width in cm
3. petal length in cm
4. petal width in cm
5. class:
-- Iris Setosa
-- Iris Versicolour
-- Iris Virginica

- This data is in fact a matrix: a key data structure in linear algebra.

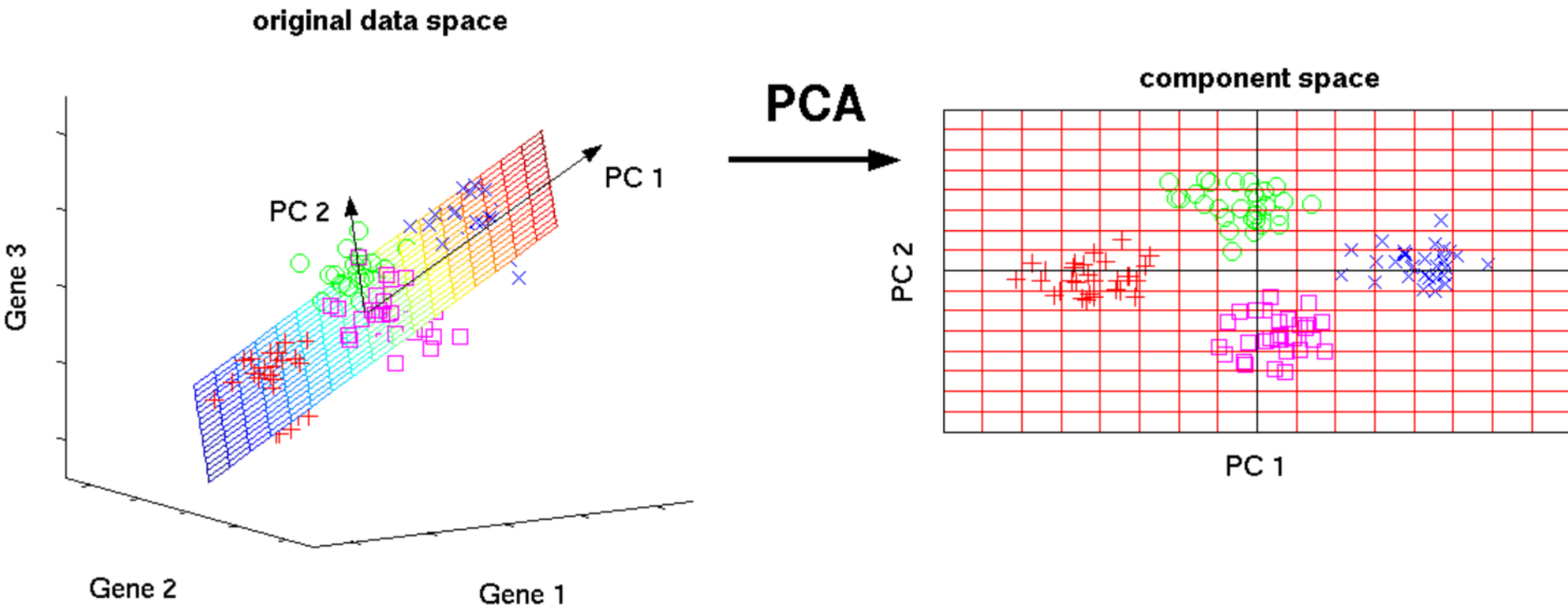
Linear Algebra in Machine Learning

■ Principal Component Analysis

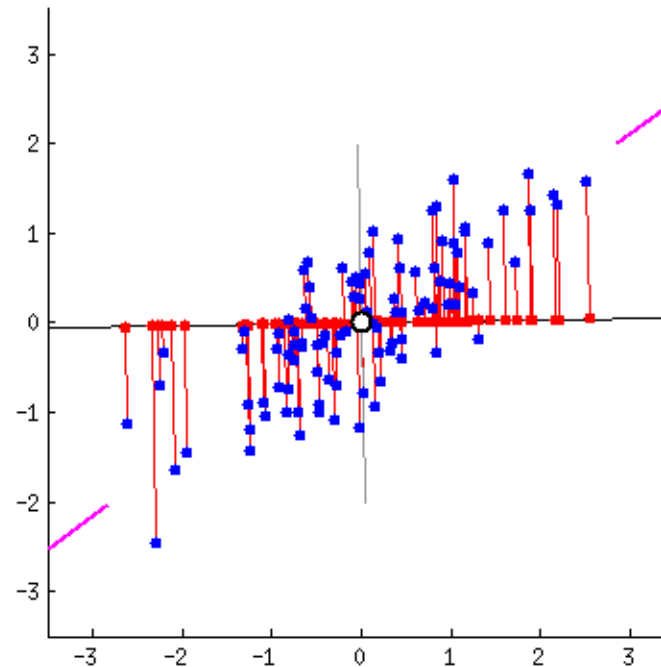
- Often, a dataset has many columns, perhaps tens, hundreds, thousands, or more.
- Methods for automatically reducing the number of columns of a dataset are called dimensionality reduction, and perhaps the most popular method is called the principal component analysis, or PCA for short
- The core of the PCA method is a matrix factorization method from linear algebra.



Principal Component Analysis (PCA)



Principal Component Analysis (PCA)



Linear Algebra in Machine Learning

■ Singular-Value Decomposition

- It is a matrix factorization method from the field of linear algebra.
- It has wide use in linear algebra and can be used directly in applications such as feature selection, visualization, noise reduction, and more.

Linear Algebra in Machine Learning

■ One Hot Encoding

- Sometimes you work with categorical data in machine learning.
- It is common to encode categorical variables to make them easier to work with and learn by some techniques. A popular encoding for categorical variables is the one hot encoding.
- A one hot encoding is:

For example, the color variable with the 3 rows:

1	red
2	green
3	blue
4	...

Might be encoded as:

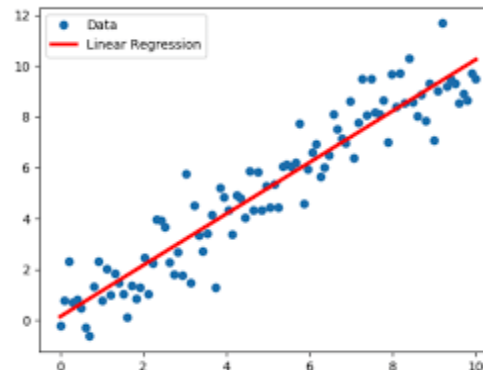
1	red, green, blue
2	1, 0, 0
3	0, 1, 0
4	0, 0, 1
5	...

Linear Algebra in Machine Learning

■ Linear Regression

- Describing the relationships between variables.
- Predicting numerical values in simpler regression problems.
- The most common way of solving linear regression is via a least squares optimization that is solved using matrix factorization methods from linear regression, such as an LU decomposition or a singular-value decomposition, or SVD.

$$y = A \cdot b$$



Linear Algebra in Machine Learning

■ Latent Semantic Analysis

- In the sub-field of machine learning for working with text data called natural language processing, it is common to represent documents as large matrices of word occurrences
- Matrix factorization methods, such as the singular-value decomposition can be applied to this sparse matrix. Documents processed in this way are much easier to compare, query, and use as the basis for a supervised machine learning model.

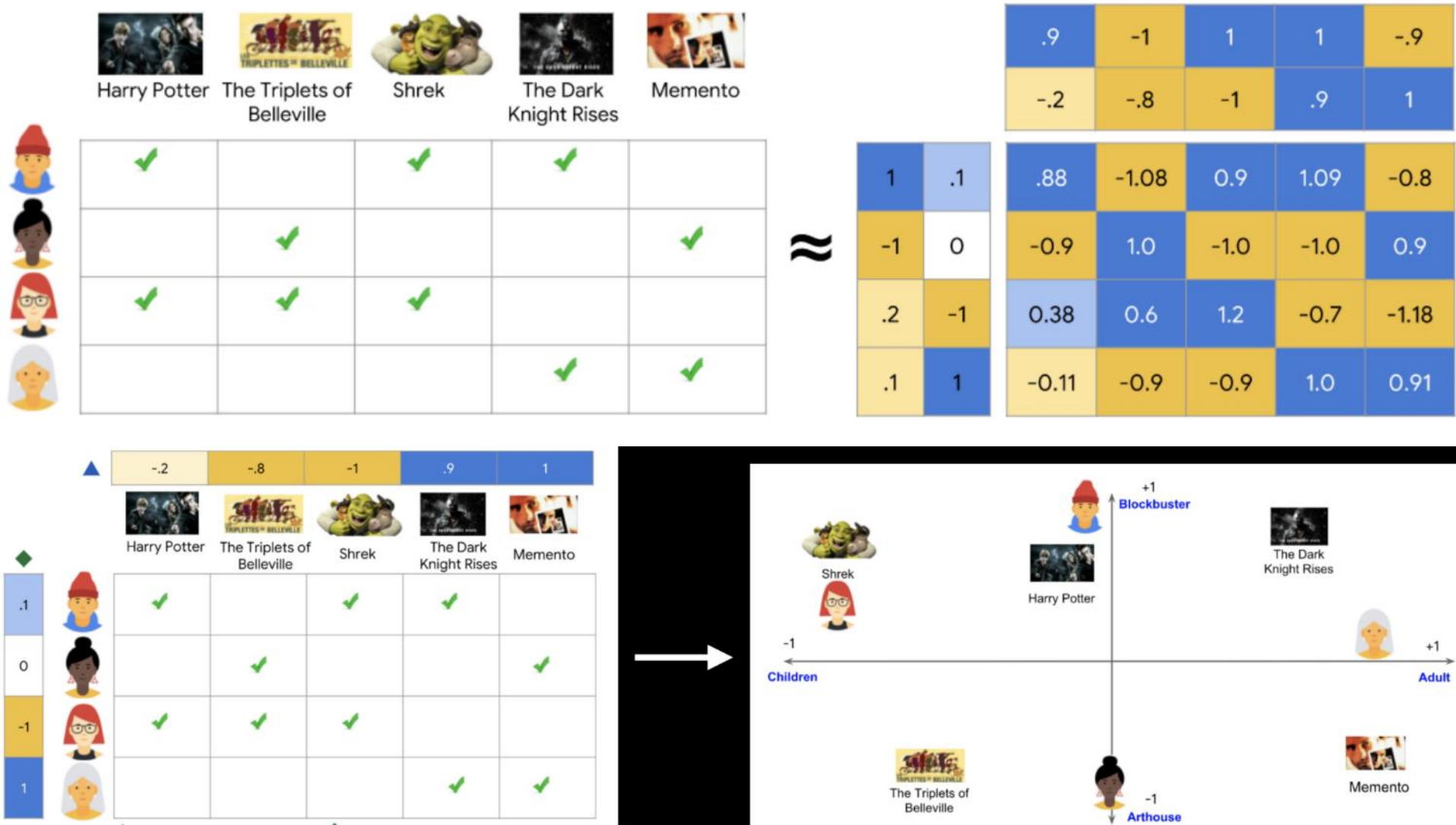
	Quick	Brown	Fox	Jumps	Over	Lazy	Dog
The quick brown fox jumps over the lazy dog	1	1	1	1	1	1	1
If the fox is quick he can jump over the dog.	1	0	1	0	1	0	1
Foxes are quick. Dogs are lazy.	0	1	1	0	0	1	1
Can a fox jump over a dog?	0	0	1	1	1	0	1

Linear Algebra in Machine Learning

■ Recommender Systems

- Predictive modeling problems that involve the recommendation of products are called recommender systems, a sub-field of machine learning.
- The development of recommender systems is primarily concerned with linear algebra methods. A simple example is in the calculation of the similarity between sparse customer behavior vectors using distance measures such as Euclidean distance or dot products.
- Matrix factorization methods like the singular-value decomposition are used widely in recommender systems to distill item and user data to their essence for querying and searching and comparison.

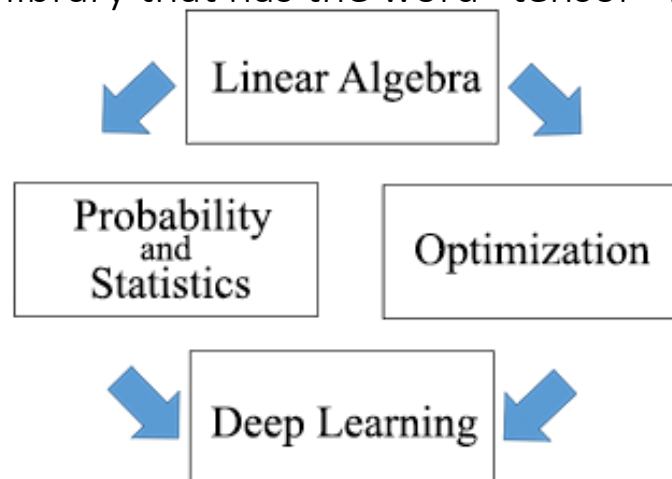
Recommender Systems



Linear Algebra in Machine Learning

■ Deep Learning

- The execution of neural networks involves linear algebra data structures multiplied and added together. Scaled up to multiple dimensions, deep learning methods work with vectors, matrices, and even tensors of inputs and coefficients, where a tensor is a matrix with more than two dimensions.
- Linear algebra is central to the description of deep learning methods via matrix notation to the implementation of deep learning methods such as Google's TensorFlow Python library that has the word "tensor" in its name.





تعداد واحد: ۳	نوع درس: نظری
پیش‌نیاز: ریاضی عمومی ۲	هم‌نیاز: —

اهداف درس

هدف از این درس، آشنایی دانشجویان با مفاهیم پایه‌ای نظری جبر خطی و همچنین نحوه‌ی استفاده و پیاده‌سازی آن‌ها در بستر نرم‌افزاری مناسب است. آشنایی با مفاهیم این درس امکان تحلیل نگاشت‌ها و سیستم‌های خطی از طریق ماتریس‌ها و اعمال، اپراتورها و مفاهیم تعریف شده مرتبط با آنها را فراهم می‌کند. همین‌طور مساله بهینه‌سازی به عنوان یکی از کاربردهای پراستفاده جبر خطی مورد بررسی قرار می‌گیرد.

ریز مواد

- فضاهای برداری
- نگاشت خطی و ماتریس
 - فضای برداری نگاشت خطی
 - ساختار جبری نگاشت خطی
 - ماتریس و رنک
 - معکوس نگاشت خطی
 - دوگانگی
 - دستگاه‌های خطی
 - حجم و دترمینان
- چندجمله‌ای‌ها
 - صفرهای چندجمله‌ای
 - فاکتورگیری چندجمله‌ای در میدان‌های مختلط و حقیقی
- مقادیر ویژه و بردارهای ویژه
 - زیرفضاهای ثابت
 - بردارهای ویژه و مقادیر ویژه
 - بردارهای ویژه‌ی مستقل خطی
 - فضاهای ویژه و قطری‌سازی ماتریس‌ها
- فضاهای ضرب داخلی
 - ضرب داخلی و تعریف فاصله
 - پایه‌های متعامد
 - عملگرهای فضاهای ضرب داخلی
- عملگرها و تجزیه

- تجزیه‌ی قطبی
- تجزیه‌ی مقدارهای منفرد
- تجزیه‌ی چولسکی
- تجزیه‌ی ال یو
- تجزیه‌ی کیو آر
- عملگرهای الحاقی
- عملگرهای نرمال
- عملگرهای یکانی و ایزومتري
- عملگرهای مثبت

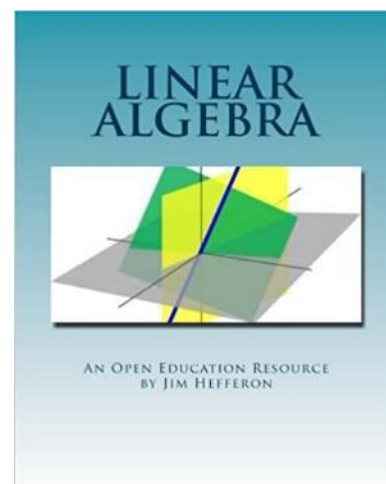
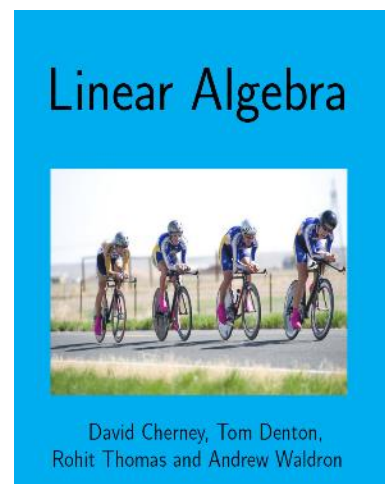
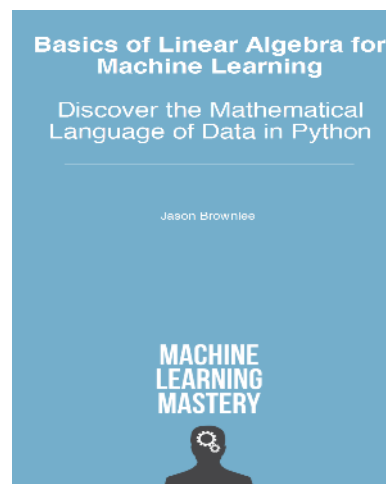
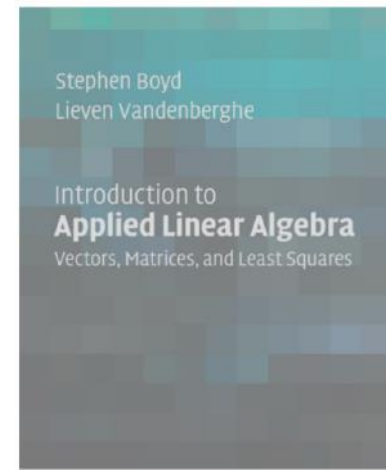
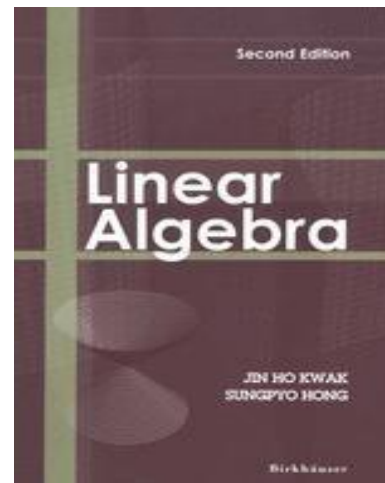
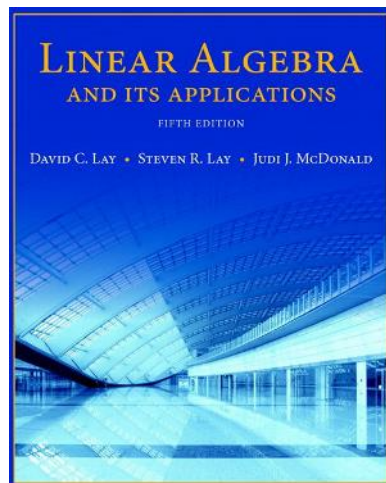
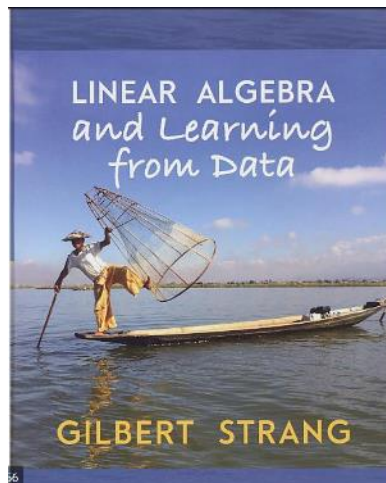
ارزیابی

- تمرین‌ها: ۶ نمره
- دو آزمون میان‌ترم: ۸ نمره
- آزمون پایانی: ۶ نمره
- آزمونک‌ها: ۱ نمره

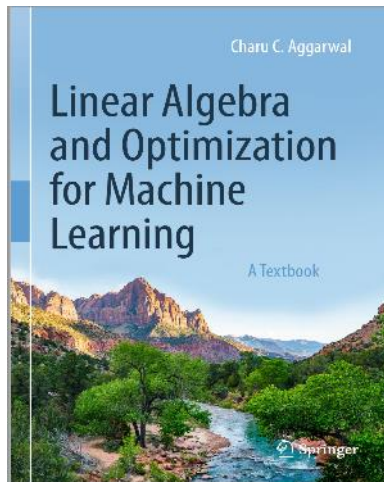
مراجع

- [1] Right Sheldon Axler. *Linear Algebra*. Springer, 2015.
- [2] Gilbert Strang. *Linear Algebra and Its Application*. 4th edition, Cengage Learning, 2006.
- [3] David Clay. *Linear Algebra and Its Application*. 4th edition, Pearson, 2011.

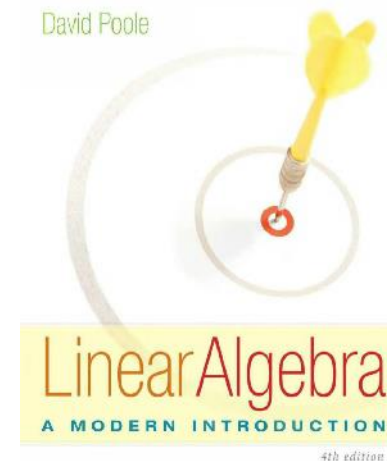
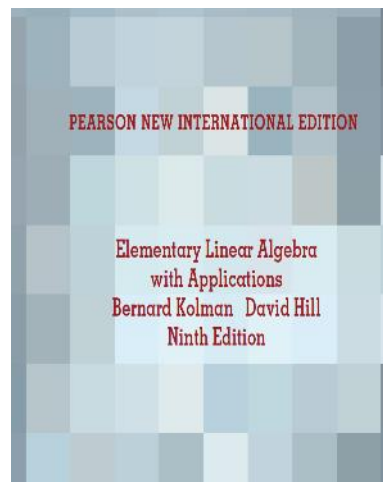
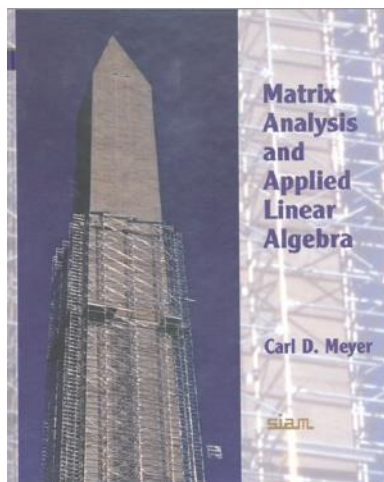
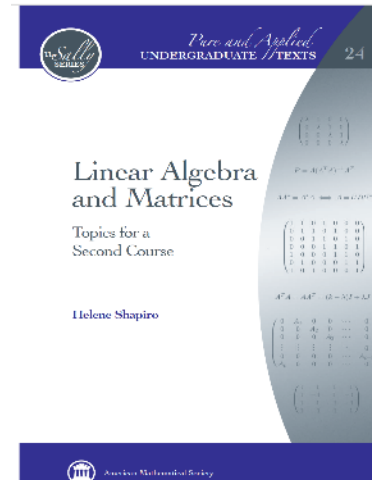
Textbooks



Textbooks



Interactive Linear Algebra



Textbooks

PROBLEMS AND THEOREMS IN LINEAR ALGEBRA

V. PRASOLOV

ABSTRACT. This book contains the basics of linear algebra with an emphasis on non-standard and neat proofs of known theorems. Many of the theorems of linear algebra obtained mainly during the past 30 years are usually ignored in text-books but are quite accessible for students majoring or minoring in mathematics. These theorems are given with complete proofs. There are about 230 problems with solutions.

LINEAR ALGEBRA

Second Edition

KENNETH HOFFMAN

Professor of Mathematics
Massachusetts Institute of Technology

RAY KUNZE

Professor of Mathematics
University of California, Irvine

Next Session

- Vectors

- Background:

who knows what is a vector? How about Vector Space?)