به نام خداوند بخشنده مهربان

جبر خطی

دانشکده مهندسی کامپیوتر

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پاییز ۱۴۰۲

قضایای اسلاید اول

قضیه ۱

Orthogonality of the Row Space and the (Right) Null Space
Orthogonality of the Column Space and the Left Null Space (Null space of transpose)

According to definition

$$\mathcal{N}(A) = z \in R^{n\times 1} : Az = 0 \tag{1}$$

assume

$$A = \left[\begin{array}{c} a_1^T a_2^T \ldots \ldots a_m^T \end{array}\right] \tag{2}$$

then the row space of A would be

$$\mathcal{R}(A) = y \in R^{n\times 1} : y = \sum_{i=1}^{m} a_i x_i \text{،} \ x_i \in R \text{،} \ a_i \in R^{n\times 1} \tag{3}$$

since z is in null space of A we know $a_i^T z = 0$ So if we take a $y \in \mathcal{R}(A)$ ،then

$$y = \sum_{k=1}^{m} a_i x_i \text{ where} \text{،} \ x_i \in R \tag{4}$$

$$y^T z = (\sum_{k=1}^{m} a_i x_i)^T z = (\sum_{k=1}^{m} x_i a_i^T) z = \sum_{k=1}^{m} x_i (a_i^T z) = 0 \tag{5}$$

قضیه ۲

If A has linearly independent columns(full rank) AA^T then is invertible.

since columns of A are linearly independent the equation Ax = ۰ has ”only” the trivial answer and we want to show

$$A^TAx = ۰ \tag{۶}$$

has only trivial answer first multiply (left) both side by x^T and that brings us to this equation

$$x^TA^TAx = (x^TA^T)(Ax) = (Ax).(Ax) = ۰ \tag{۷}$$

because of inner product property ($u.u = ۰$ means $u = ۰$) we know Ax = ۰ and this equation has only trivial answer thus $A^TAx = ۰$ has only trivial answer $A^TA = ۰$ is invertible.

مثال ۱

در اینجا مثال اول مطرح می‌شود.

در اینجا حل مثال اول قرار داده می‌شود.