Optimization
Loops & Prefetching

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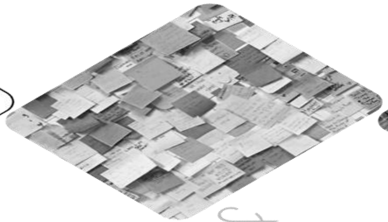
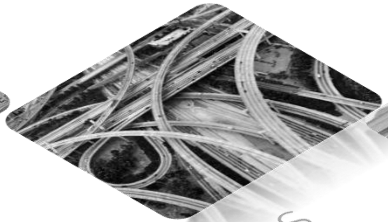
Outline

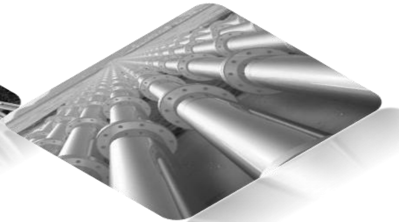
First things first



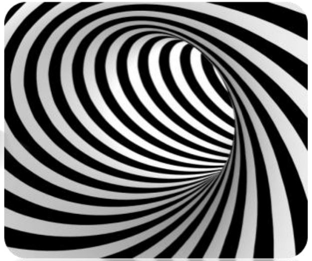
Cache & Memory



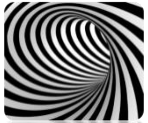
Branches



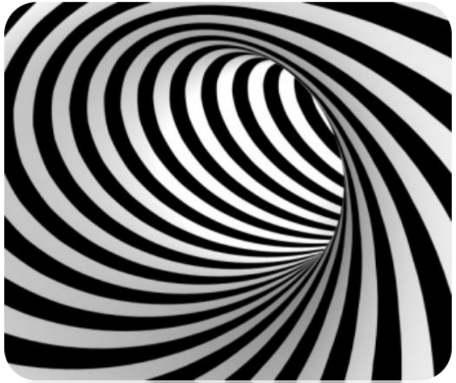
Pipelines



Loops



Outline



Loops techniques

Prefetching

Optimizing cache access in loops

Loop classification

$$A_I = \frac{f(n)}{n}$$

Arithmetic Intensity: the ratio between the number of performed operations and the amount of the data.

1. O(N) / O(N)
   scanning arrays, 1D vector ops, ..
   optimization potential limited

2. $O(N^2)$ / $O(N^2)$
   matrix × vect, matr. transp, matr. add, ...
   some more opportunities for opt.

3. $O(N^3)$ / $O(N^2)$
   matrix × matr,...
   significant optimization potential

Cache access in loops: *O(N)/O(N)*

Example

1-level loops: Scalar products, vector additions, sparse matrix-vector multiplication

Inevitably memory-bound for very large *N*; in general, improvements come from *avoiding unnecessary* operations and/or *repeated memory accesses*, and increasing **data reuse**

O(N) / O(N)

```
for(int j=0; j<2; j++)
  A[i] = B[i] × C[i]

for(int j=0; j<2; j++)
  Q[i] = B[i] + D[i]
```

➔

```
for(int j=0; j<2; j++)
  {
    A[i] = B[i] × C[i]
    Q[i] = B[i] + D[i]
  }
```

Loop fusion: *in the version on the right, B is recalled from memory only once.*

[check the possibility for loops fusion]

Cache access in loops: $O(N^2)/O(N^2)$

Example

2-levels loops: dense matrix-vector mul, matrix transpos., matrix add, …

Improvements comes again from increasing *data reuse*, exploiting *locality* and *avoiding unnecessary* operations and memory accesses.

```
for(int i=0; i < N; i++)
   for(int j=0; j<N; j++)
      C[i] += A[i][j] * B[j];
```

N×N N×N N×N

→ $3 \times N^2$ memory accesses

Cache access in loops: *O(N²)/O(N²)*

Step 1:
Avoid unecessary loads /stores

```
for(int i=0; i < N; i++)
   for(int j=0; j<N; j++)
      C[i] += A[i][j] * B[j];
```

→

```
for(int i=0; i < N; i++) {
   c_temp = C[i];
   for(int j=0; j < N; j++)
      c_temp += A[i][j] * B[j];
   C[i] = c_temp; }
```

Now it is clear for the compiler that `C[i]` needs to be loaded and stored only 1 time

→ $2 \times N^2 + N$ memory accesses

Step 2:
Unroll outern loop and *fuse* in the inner loop; there is potential for *vectorisation*.

```
for(int i=0; i < N; i++)
   for(int j=0; j<N; j++)
      C[i] += A[i][j] * B[j];
```

→

```
for(int i=0; i < N; i += m)
   for(j = 0; j < N; j++){
      b_temp  = B[j];
      C[i]   += A[i][j] * b_temp;
      C[i+1] += A[i+1][j] * b_temp;
      …
      C[i+m] += A[i+m][j] * b_temp; }
```

$N \times N/m$

$N \times N$

N

$\boxed{\rightarrow N^2 \times (1+1/m) + N}$

Note: unrolling and register spill

Using a too large m in the previous example while the target CPU does not have enough registers to keep all the needed operands results in a "code bloating".
In this case, the CPU has to spill registers' content to cache and viceversa, slowing down the computation.

→ **learn to inspect the compiler's *log***

A too much involute and obscure loop body may hamper the compiler to effectively perform *unroll & jam* optimizations targeted to the CPU it runs on.

→ **hand code effort to clarify the code**
→ **hints / directives to the compiler**
(directives are generally not portable across different compilers)

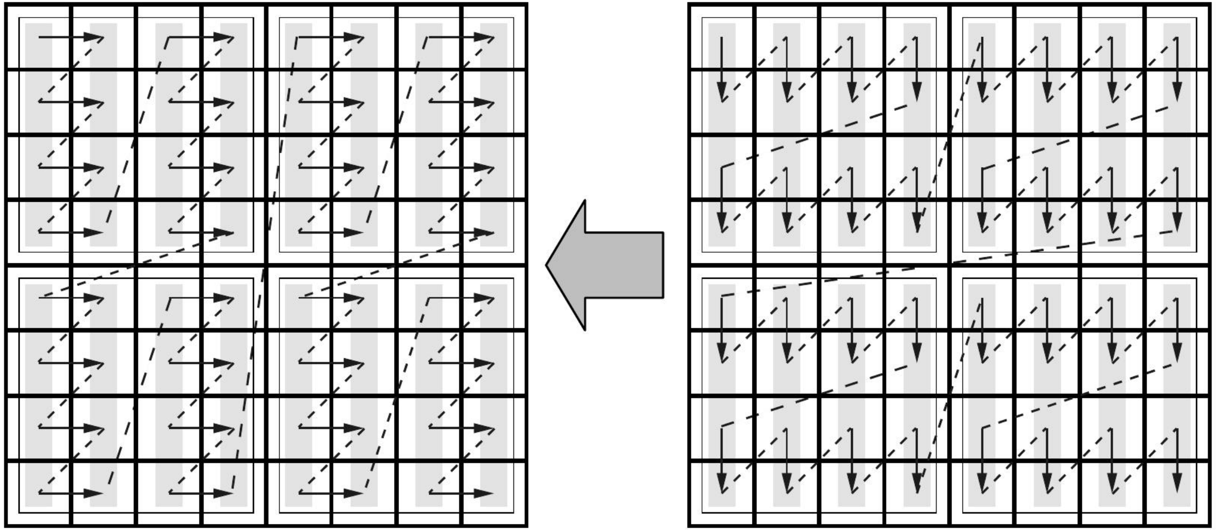
Cache access in loops: *$O(N^2)/O(N^2)$*

Sometimes no magic wand can cure the fact that you have to access N^2 memory locations.
Example: in matrix transpose you have to access all the source matrix and all the destination matrix once.

Unroll & Jam strategy can bring benefits as long as the cache can hold N lines.
An L_C-way unrolling is too much aggressive and may easily result in register pressure.

Loop tailing (or blocking) is a good strategy that does not save memory loads but increase dramatically the cache hit ratio.
We have mentioned it in the past, we'll se it in more detail at the end of this lecture.

Cache access in loops: $O(N^2)/O(N^2)$



Step 3:
Fully exploit locality of referenced data; cut TLB misses by accessing 2D arrays by blocks

Image taken from: introduction to HPC for scientists and engineers

Loop unrolling

Loop unrolling is a fundamental code transformation which usually helps significantly in improving your code performance:

- It reduces the loop overhead (counter update, branching)
- It exposes *critical data path* and dependencies
- It helps in exploiting ILP, especially in case of memory aliasing

We have already seen this technique in the examples from past lecture, although we did not really focus on it.
Now, let's understand it with some more detail.

Loop unrolling

Let's examine this simple *reduction*:

```
for ( int i=0; j<N; i++ )
  S += A[i];
```

-O0

```
.L3:
# reduction.c:41:      acc = acc OP array[ii];
        mov     rax, QWORD PTR -24[rbp] # ii
        lea     rdx, 0[0+rax*8]
        mov     rax, QWORD PTR -48[rbp] # array
        add     rax, rdx
        movsd   xmm0, QWORD PTR [rax]
        movsd   QWORD PTR -56[rbp], xmm0
        fld     QWORD PTR -56[rbp]
# reduction.c:41:      acc = acc OP array[ii];
        fld     TBYTE PTR -16[rbp]      # acc
        faddp   st(1), st
        fstp    TBYTE PTR -16[rbp]      # acc
# reduction.c:40:    for ( uLint ii = 1; ii < N; ii++ )
        add     QWORD PTR -24[rbp], 1   # ii,
.L2:
# reduction.c:40:    for ( uLint ii = 1; ii < N; ii++ )
        mov     rax, QWORD PTR -24[rbp] # ii
        cmp     rax, QWORD PTR -40[rbp] # N
        jb      .L3
```

-O3 -march=native

```
.L3:
# reduction.c:41:      acc = acc OP array[ii];
        fadd    QWORD PTR [rax] # array
        add     rax, 8          #
# reduction.c:40:    for ( uLint ii = 1; ii < N; ii++ )
        cmp     rdx, rax        # N
        jne     .L3

        ret
```

With optimization turned on the compiler manages much better the loop overhead, but does not optimize the FP ops in any way.

Loop unrolling

If we compile *exactly the same code* but using **`int`** as data type instead of **`double`**, we obtain a quite different result:

-O0

```
.L3:
# reduction.c:43:      acc = acc OP array[ii];
        movq    -8(%rbp), %rax  # ii
        leaq    0(,%rax,4), %rdx
        movq    -32(%rbp), %rax # array
        addq    %rdx, %rax
        movl    (%rax), %eax
        movl    %eax, %eax
# reduction.c:43:      acc = acc OP array[ii];
        addq    %rax, -16(%rbp)
# reduction.c:42:    for ( uLint ii = 1; ii < N; ii++ )
        addq    $1, -8(%rbp)    #, ii
.L2:
# reduction.c:42:    for ( uLint ii = 1; ii < N; ii++ )
        movq    -8(%rbp), %rax  # ii
        cmpq    -24(%rbp), %rax # N
        jb      .L3
```

-O3 -march=native

```
.L4:
# reduction.c:43:      acc = acc OP array[ii];
        vmovdqu 4(%rax), %ymm0
        addq    $32, %rax
        vpmovzxdq       %xmm0, %ymm1
        vextracti128    $0x1, %ymm0, %xmm0
        vpmovzxdq       %xmm0, %ymm0
# reduction.c:43:      acc = acc OP array[ii];
        vpaddq  %ymm0, %ymm1, %ymm0
        vpaddq  %ymm0, %ymm2, %ymm2
        cmpq    %rdx, %rax
        jne     .L4
```

Now the compiler opts for the complete *vectorization* of the loop, with a very strong impact on performances !!

Loop unrolling

Why does the compiler choose to optimize the loop when the data are of type `int` and it does not when the data are of type `double` ?

It is due to the fact that, as we have seen the compiler is NOT free to restructure the code that deals with floating-point numbers.
Since the math with floating point is not associative, changing the exact order of the operations in your code may – from what the compiler may judge at compile time – change the correctness of the calculations at run time.

For instance, in the example that we have considered, changing the order of the operations obviously impacts on the result. From the point of view of the compiler, you may have chosen a given workflow exactly because *you know* that it is the most correct with respect to the data it will apply to!

Loop unrolling

Then, we are left with the responsibility of optimizing this simple code. Since we are traversing it continuously in natural memory order, the cache is not an issue.

Our aim is to re-structure the code so that the compiler could exploit the CPU's ILP *(Instruction-Level Parallelism)*.

```
for ( int i=0; j<N; i++ )
  S += A[i];
```

SCO/examples_on_pipelines
reduction

Step 1: unrolling 2×1

Our first attempt is to reduce the loop overhead and expose some parallelism among the data by explicitly processing 2 elements per iteration.

```
for ( int i=0; i<N; i++ )
  S = S OP A[i];
```

↓

```
for ( int i=0; i<N-2; i+=2 )
  S = (S OP A[i]) OP A[i+1] ;
```

Step 1: unrolling 2×1

Our first attempt is to reduce the loop overhead and expose some parallelism among the data by explicitly processing 2 elements per iteration.

```
for ( int i=0; i<N; i++ )
  S = S OP A[i];
```

```
for ( int i=0; i < N-2; i+=2 )
  S = (S OP A[i]) OP A[i+1] ;
```

Note: when unrolling, you always have to care about the final iterations that would be left behind. A common way to do it for un unrolling factor U (usually U ranges in [2..16] of a loop with N iterations is:

```
int N_ = (N/U)*U;      // by contruction this is the largest multiple of U
                       // smaller than N
for ( int i = 0; i < N_; i += U )
   iteration_ops;

for ( int i = N_; i < N; i++ )
   iteration_ops;
```

Step 1: unrolling 2×1

The compiler generates the following assembly code:

```
.L17:
        vmovupd ymm1, YMMWORD PTR [rax]
        add         rax, 32
        vaddsd      xmm0, xmm0, xmm1
        vunpckhpd xmm2, xmm1, xmm1
        vextractf128     xmm1, ymm1, 0x1
        vaddsd      xmm0, xmm0, xmm2
        vaddsd      xmm0, xmm0, xmm1
        vunpckhpd xmm1, xmm1, xmm1
        vaddsd      xmm0, xmm0, xmm1
.LVL18:
        cmp      rax, rcx
        jne      .L17
```

load 32B (4 double) starting from *i*th element.
`ymm1` has 256bits.

`rax` contains the address to the ith element of the array, `[rax]` means "the address pointed by `rax`"

registers' reshuffle to move each double at the begin, in order to use **`vaddsd`**

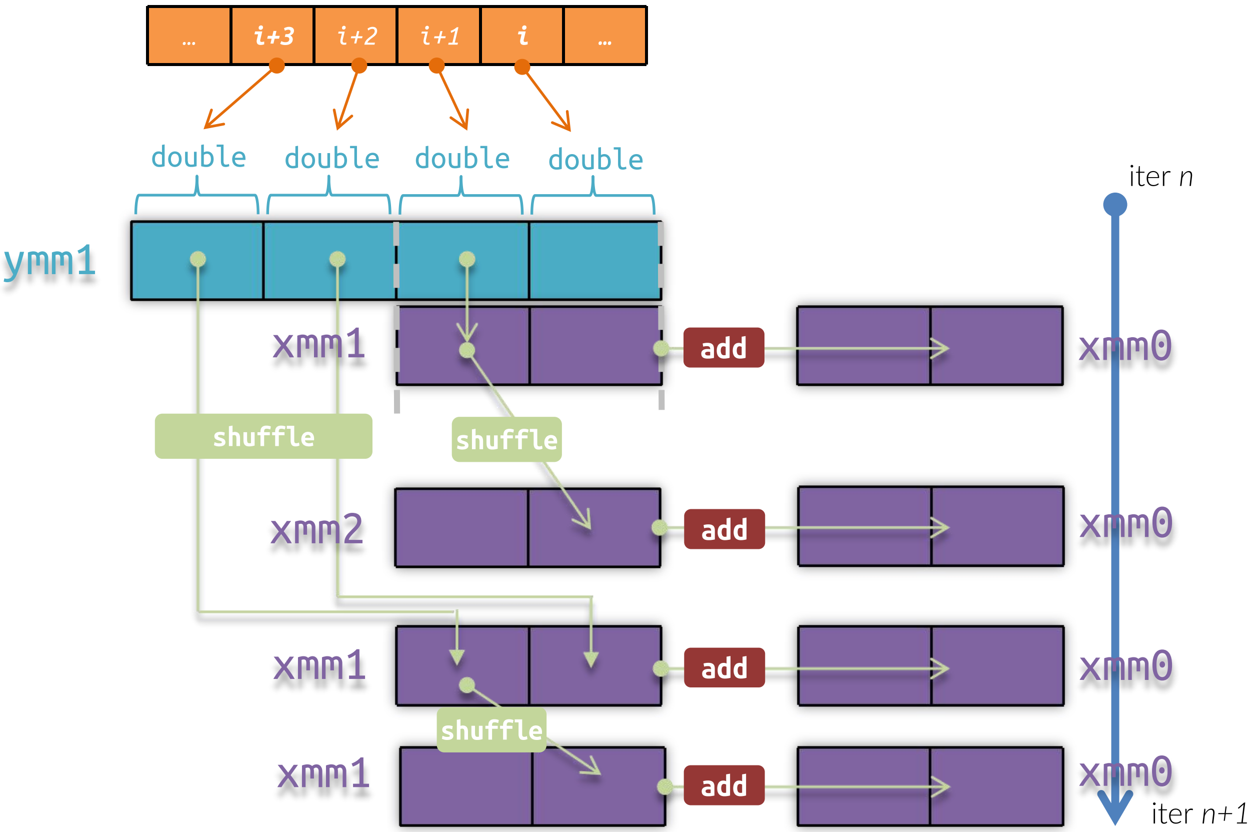
all these instructions sum with, and store in, **`xmm0`**

in the following we will always comment the code generated with `-O3 -march=native`

Step 1: unrolling 2×1

In a (hopefully) simpler view, the scheme of what happens is the following

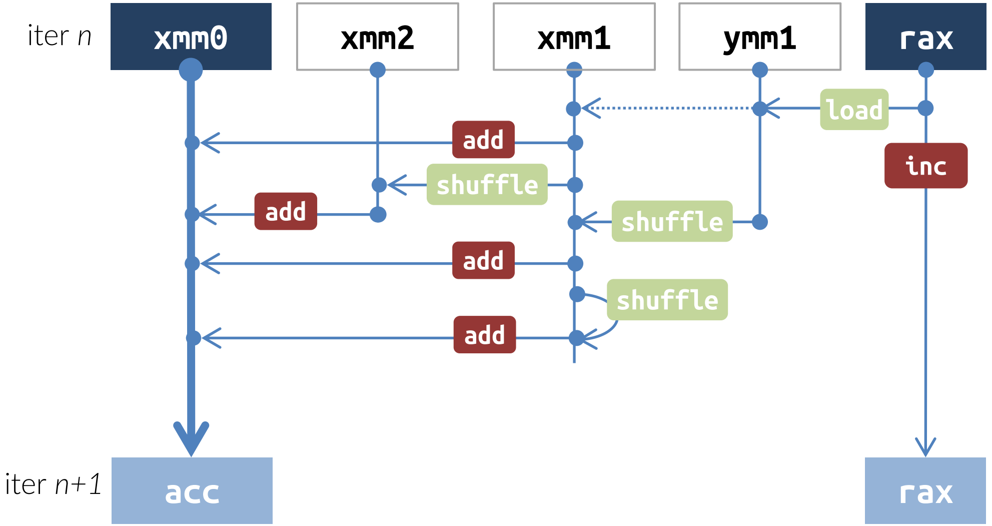
```
.L17:
        vmovupd ymm1, YMMWORD PTR [rax]
        add         rax, 32
        vaddsd      xmm0, xmm0, xmm1
        vunpckhpd xmm2, xmm1, xmm1
        vextractf128    xmm1, ymm1, 0x1
        vaddsd      xmm0, xmm0, xmm2
        vaddsd      xmm0, xmm0, xmm1
        vunpckhpd xmm1, xmm1, xmm1
        vaddsd      xmm0, xmm0, xmm1
.LVL18:
        cmp       rax, rcx
        jne       .L17
```



Step 1: unrolling 2×1

Then, we have the following abstraction:
(arrows indicate a dependency)

```
.L17:
        vmovupd ymm1, YMMWORD PTR [rax]
        add        rax, 32
        vaddsd     xmm0, xmm0, xmm1
        vunpckhpd xmm2, xmm1, xmm1
        vextractf128    xmm1, ymm1, 0x1
        vaddsd     xmm0, xmm0, xmm2
        vaddsd     xmm0, xmm0, xmm1
        vunpckhpd xmm1, xmm1, xmm1
        vaddsd     xmm0, xmm0, xmm1
.LVL18:
        cmp      rax, rcx
        jne      .L17
```



`xmm0` carries a loop dependency because its value is to be used in the next iteration (that is true for `rax` too, but its latency is smaller than that of FP operations)
It forms a ***critical path*** that limits the efficiency.

Step 2: unrolling 2×1+ reshuffle

Let's explore what happens if we apport a semantic change to our code, just using the fact that our `OP` is associative.

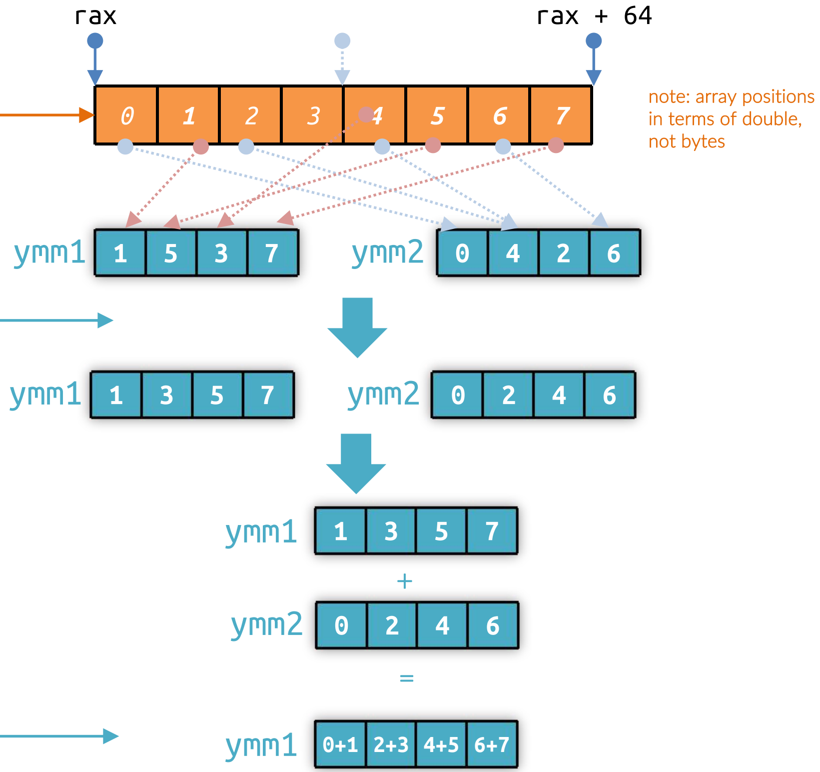
```
for ( int i=0; i<N-2; i+=2 )
  S = (S OP A[i]) OP A[i+1] ;
```

```
for ( int i=0; i<N-2; i+=2 )
  S = S OP (A[i] OP A[i+1]) ;
```

Step 2: unrolling 2×1+ reshuffle

```
for ( int i=0; i<N-2; i+=2 )
   S = S OP (A[i] OP A[i+1]) ;
```

```
.L33:
        vmovupd ymm4, YMMWORD PTR [rax]
        add     rax, 64
        vunpcklpd       ymm1, ymm4, YMMWORD PTR -32[rax]
        vunpckhpd       ymm2, ymm4, YMMWORD PTR -32[rax]
        vpermpd ymm1, ymm1, 216
        vpermpd ymm2, ymm2, 216
        vaddpd  ymm1, ymm1, ymm2
        vaddsd  xmm0, xmm0, xmm1
        vunpckhpd       xmm3, xmm1, xmm1
        vextractf128    xmm1, ymm1, 0x1
        vaddsd  xmm0, xmm0, xmm3
        vaddsd  xmm0, xmm0, xmm1
        vunpckhpd       xmm1, xmm1, xmm1
        vaddsd  xmm0, xmm0, xmm1
```



8 doubles are processed per iteration; thanks to the re-association of operations, the compiler can reshuffle operations in a more efficient way

4 summation of subsequent elements in the array!

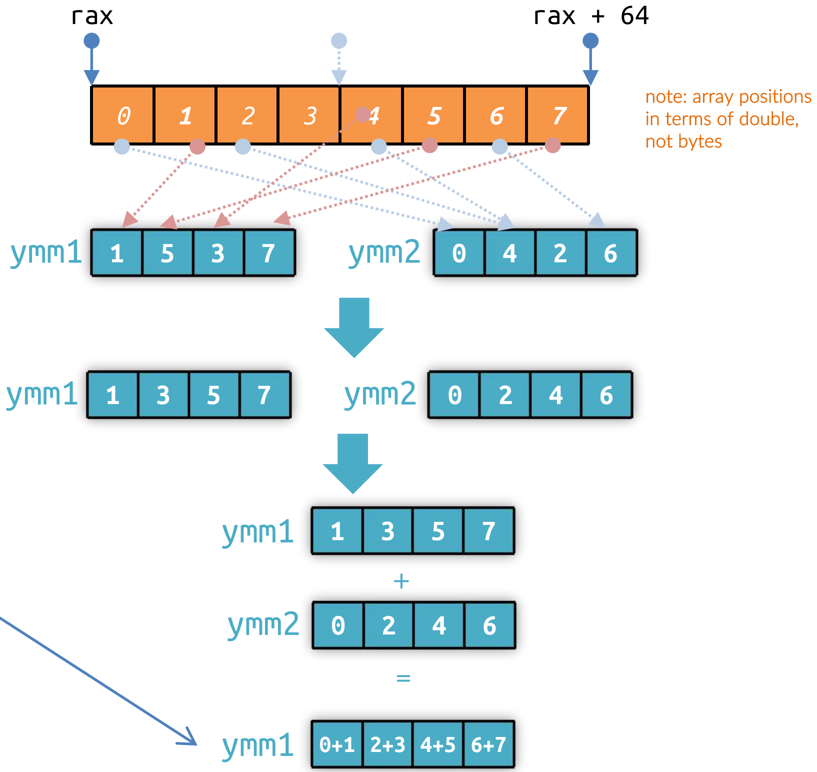
Step 2: unrolling 2×1+ reshuffle

To be clearer: just because we re-associated the math expression in the loop,
from `S = (S OP A[i]) OP A[i+1] ;`
to `S = S OP (A[i] OP A[i+1]) ;`

the compiler is entitled to exploit the fact that *in the semantics that **now** we are giving*, the operation order will be

`Sum= ((([0]+[1]) + ([2]+[3])) + ([4]+[5])) + ([6]+[7])…`

And the result is what we have just discussed, more efficient than what we obtained with the previous code



4 summation of subsequent elements in the array!

Let's inquire more (still accounting for the fact that in floating point the OP is *not* associative) the difference among

```
for ( int i = 0; i < N; i++ )       [A]
   S = S OP a[i];

for ( int i = 0; i < N; i+=2 )      [B]
   S = S OP (a[i] OP a[i+1]);
```

If we define OP to be '$+$' and S_i to be the i_{th} partial result, i.e. the value of S after the first $i-1$ iterations, the cases $[A]$ and $[B]$ expand to (square brackets cluster what happens in each single iteration)

$$
\begin{aligned}
&[A] \rightarrow [\,[\,[\,[0 + a_0] + a_1] + a_2] + a_3] + \cdots \$\$ \\
&S_0 = a_0 \\
&S_1 = S_0 + a_1 \\
&S_2 = S_1 + a_2 \\
&\cdots \\
&S_i = S_{i-1} + A_i
\end{aligned}
$$

$$
\begin{aligned}
&[B] \rightarrow [\,[\,[0 + (a_1 + a_2)] + (a_3 + a_4)] + (a_5 + a_6)] + \cdots \\
&S_0 = 0 + (a_0 + a_1) \\
&S_1 = S_0 + (a_2 + a_3) \\
&S_3 = S_2 + (a_4 + a_5) \\
&\cdots \\
&S_i = S_{i-1} + (a_i + a_{i+1})
\end{aligned}
$$

It is evident that for $[A]$ the basic "*element*" of each iteration is the single array entry a_i, whereas in $[B]$ the basic element is the sum $a_i + a_{i+1}$. Hence, while in the first case there is nothing we can do but subsequently calculate the S_i, in the second case we can *separately* calculate as many $[a_i + a_{i+1}]$ elements as possible and *then* sum them up subsequently. Actually, in the second case (because of how we specified the operations!) each a_i, a_{i+1} pair *must* be summed up *before* being summed to S_i.

Q : *And how many* $[a_i + a_{i+1}]$ *elements can we separately calculate in a cycle?*

A : In this case, since we are using only 1 accumulator, most probably the choice of the compiler will depend on how wide a vector register is. In fact, since

1. there is the computation of only 1 S_i per iteration, with a subsequent summation
2. it is most effective to exploit a single vector load

the most effective choice is the one made by the compiler, i.e. to use a 2 vector registers to load 8 a's entries and to reshuffle them in order to obtain 4 $[a_i + a_{i+1}]$ elements in just one $+$ operation, and to to sum them up subsequently. If the vector register was 512bits instead of 256bits, it would have loaded 16 entries in order to obtain 8 $[a_i + a_{i+1}]$ elements and so on.

Step 3: unrolling 2×2

As we have seen, what is the blocking element is the critical path of the accumulator, because we are using a unique place to store the summation.
A logical step is to *separate* partial results in multiple accumulators.

```
for ( int i=0; i<N; i++ )
  S = S OP A[i];
```

```
for ( int i=0; i<N-2; i+=2 )
  {
     s0 = s0 OP A[i];
     s1 = s1 OP A[i+1];
  }
return s0 = s0 OP s1;
```

Step 3: unrolling 2×2

As we have seen, what is the blocking element is the critical path of the

```
for ( int i=0; i<N; i++ )
  S = S OP A[i];
```

NOTE:

The unrolling is expressed in general as $n\times m$ (in this slide $n=2$, $m=2$; in the previous slides, $n=2$, $m=1$).

n refers to the number of iterations that are unrolled

m refers to the number of accumulators that are being used

So, both the case presented in this slide and the one discussed in the previous slide unroll 2 iterations (in other words: the iteration counter is increased by 2!). However, this case uses 2 accumulators, and so it is 2×2, while the previous one uses only one and then it is 2×1.

Step 2: unrolling 2×2

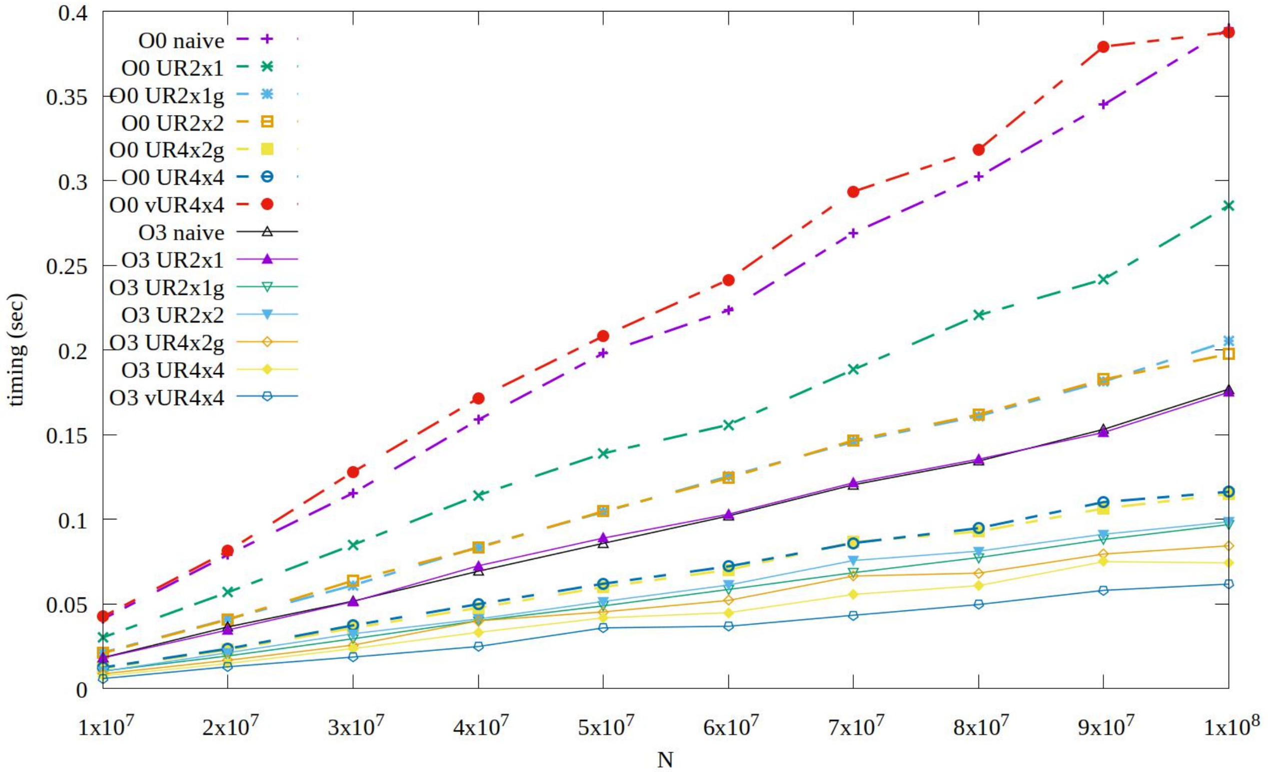
```
for ( int i=0; i<N-2; i+=2 ) {
  s0 = s0 OP A[i];
  s1 = s1 OP A[i+1]; }
```

```
.L49:
        vmovupd         ymm4, YMMWORD PTR [rax]
        vmovupd         ymm3, YMMWORD PTR 32[rax]
        vmovapd         xmm2, xmm4
        vaddsd          xmm1, xmm1, xmm4
        vunpckhpd       xmm2, xmm2, xmm2
        vaddsd          xmm0, xmm0, xmm2
        vextractf128    xmm4, ymm4, 0x1
        vaddsd          xmm1, xmm1, xmm4
        vunpckhpd       xmm4, xmm4, xmm4
        vaddsd          xmm0, xmm0, xmm4
        vmovapd         xmm6, xmm3
        vaddsd          xmm5, xmm1, xmm3
        vunpckhpd       xmm6, xmm6, xmm6
        vaddsd          xmm0, xmm0, xmm6
        vextractf128    xmm3, ymm3, 0x1
        vaddsd          xmm1, xmm5, xmm3
        add             rax, 64
        vunpckhpd       xmm3, xmm3, xmm3
        vaddsd          xmm0, xmm0, xmm3

        cmp             rax, rcx
        jne      .L49
```

As before (2x1g) the compiler feels free to load 8 doubles per iteration, and then reshuffling them appropriately in order to respect the semantics of our coding, it sums them up using `xmm0` and `xmm1` as separate accumulators.

Reduction: results (timing)



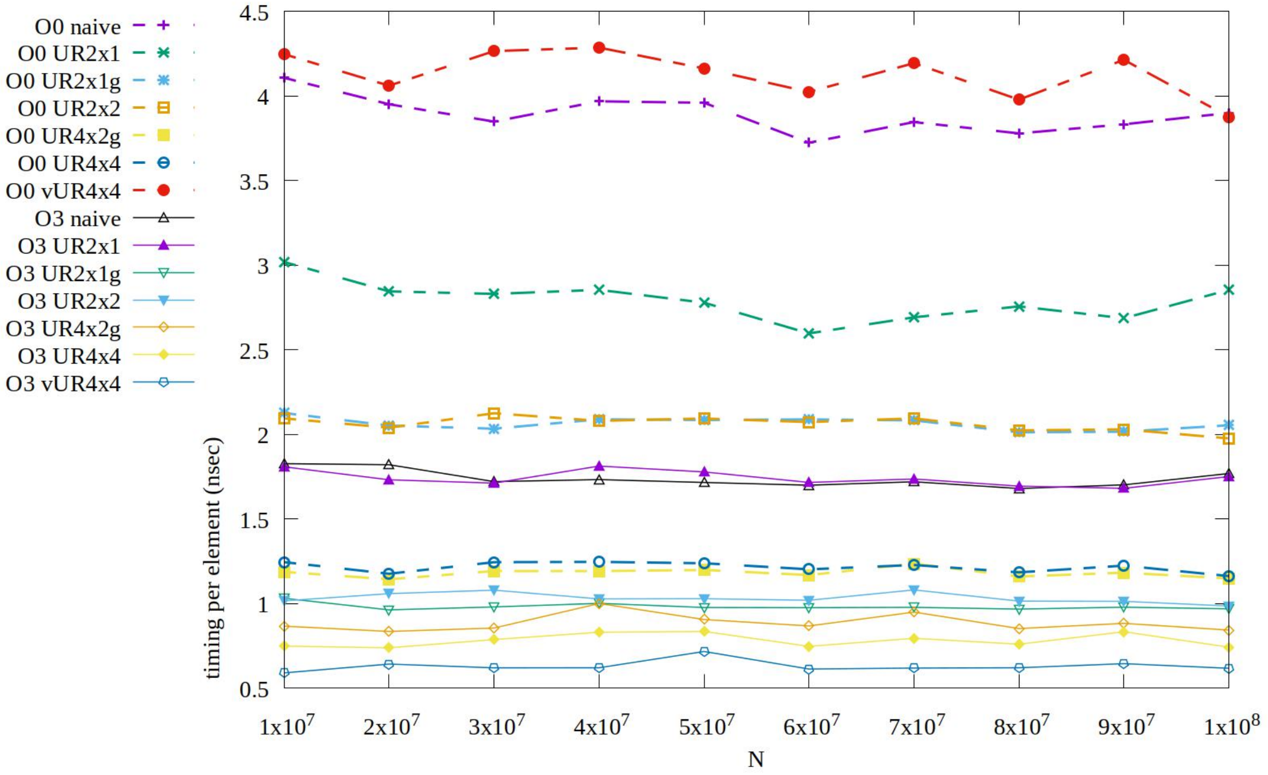
Run time of different implementation with and without compiler's optimization

UR NxM: unrolled *N* times using *M* accumulators.

vUR4x4: UR4x4 with explicit vectorization.

In this plot and in the following ones dashed lines are for `-O0` and solid line for `-O3 -march=native` (only `gcc` has been used)

Reduction: results (timing-per-element)



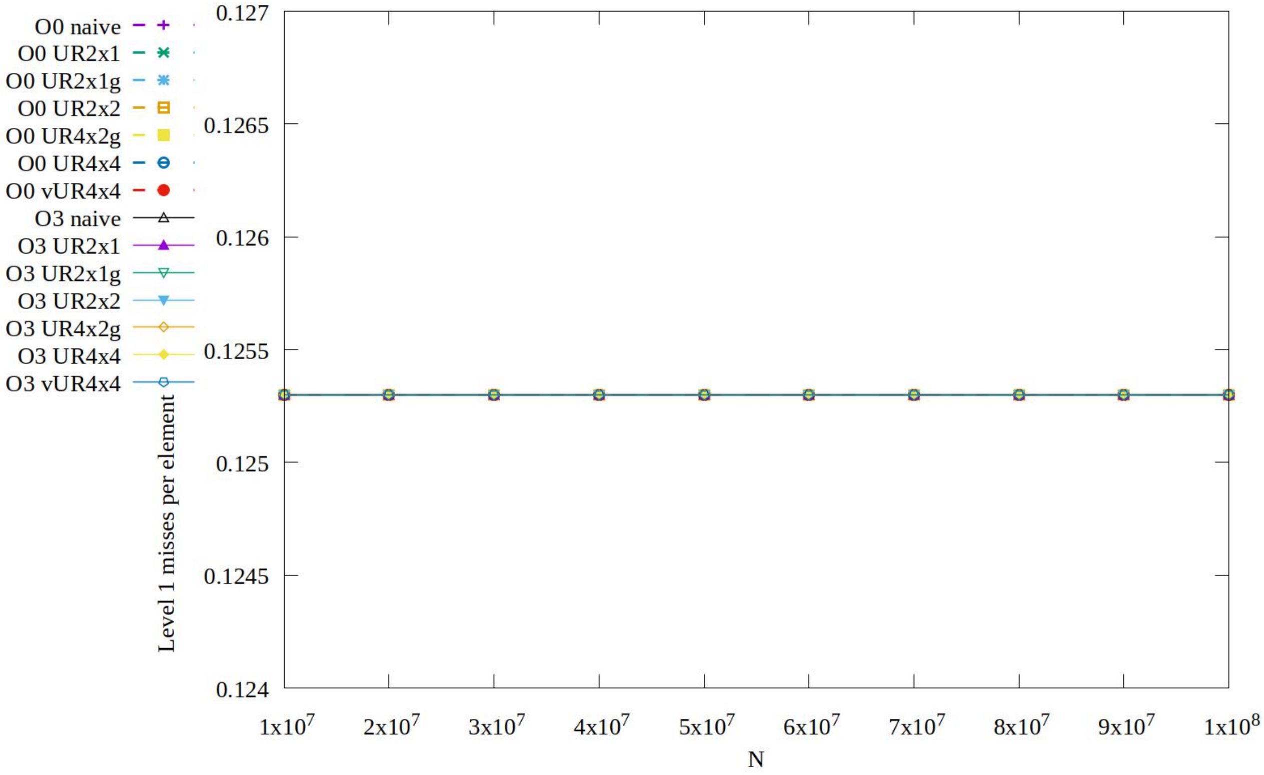
Run time of different implementation with and without compiler's optimization

UR NxM: unrolled *N* times using *M* accumulators.

vUR4x4: UR4x4 with explicit vectorization.

In this plot and in the following ones dashed lines are for `-O0` and solid line for `-O3 -march=native` (only `gcc` has been used)

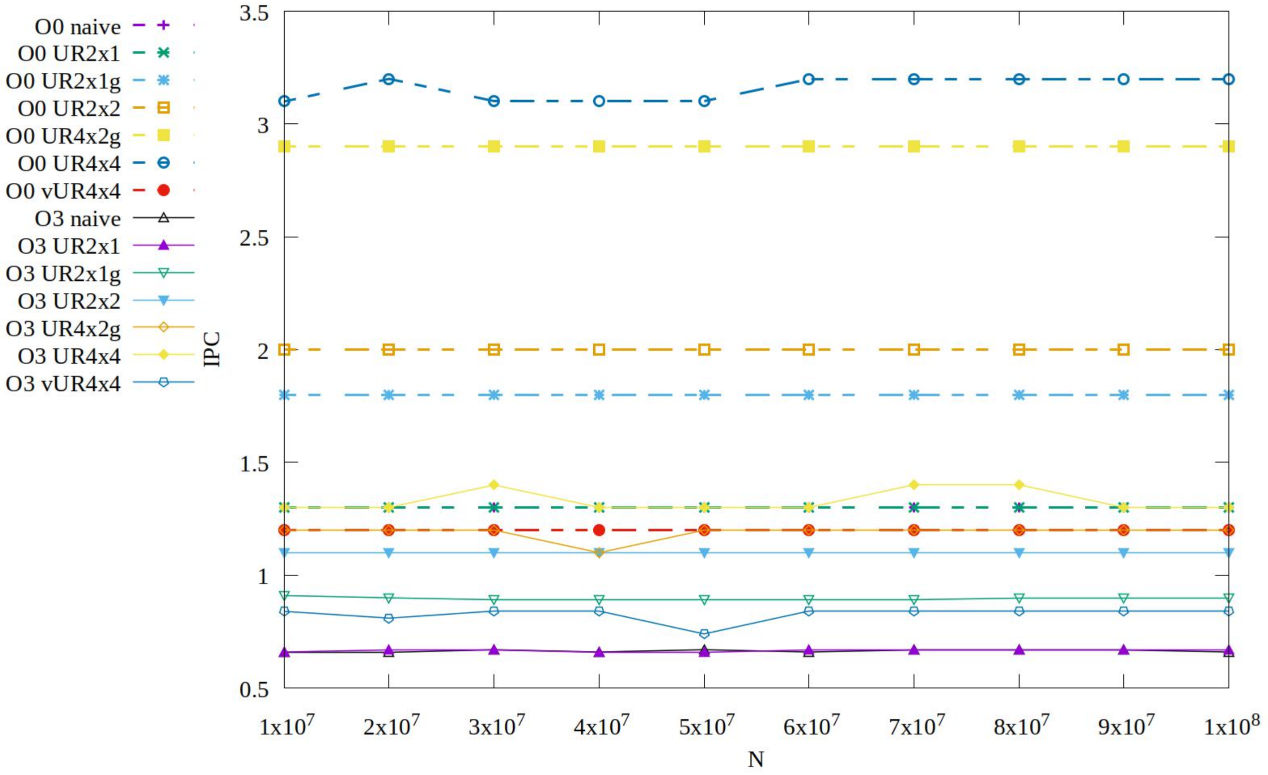
Reduction: results



Level 1 Data cache misses

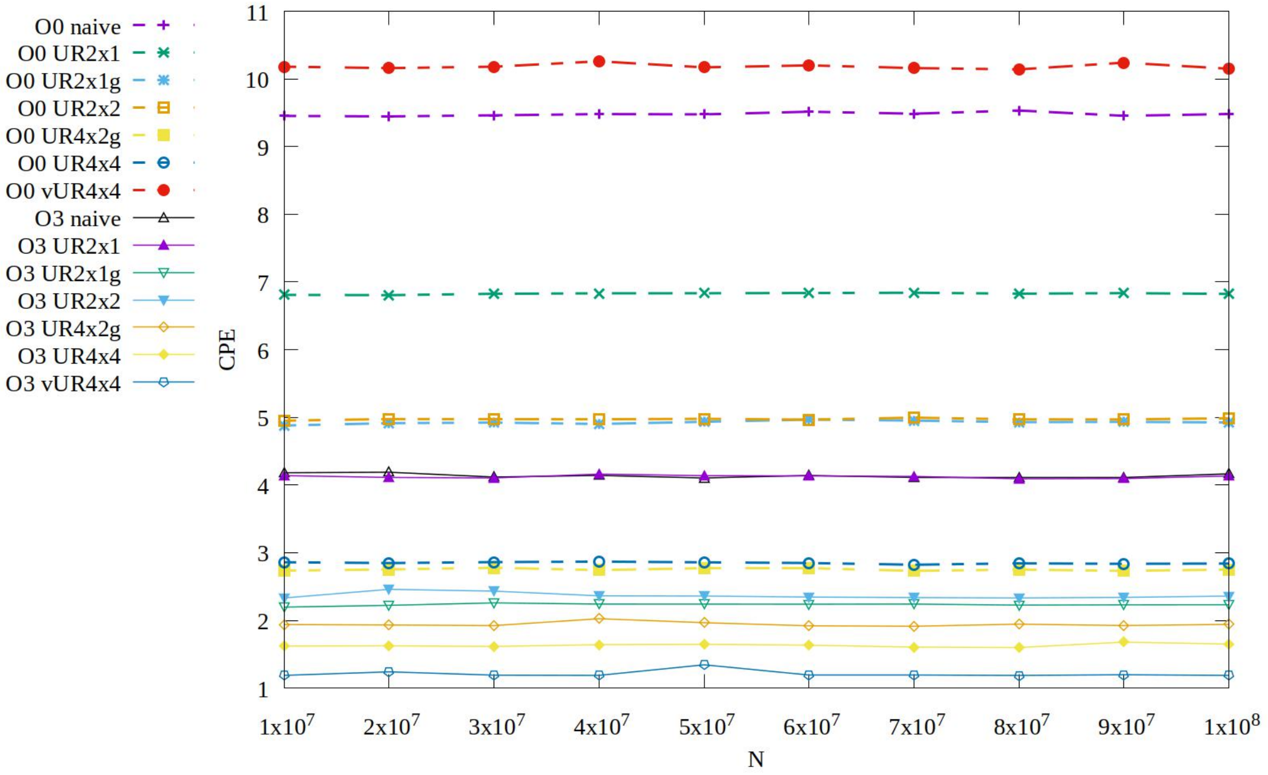
of course, we get the expected 1/8 since we are processing the array continously.

Reduction: results



Instructions per cycle

Reduction: results



Cycles per element

Cache access in loops: $O(N^3)/O(N^2)$

These algorithms (ex: matrix-matrix multiplication or dense matrix diagonalization) are very good candidates for optimizations that lead flop/s performance very close to the theoretical peak (in fact, MMM is at the core of `linpack`).

Tailing, unroll&jam + vectorization of operations, reorganization of ops to exploit CPU’s pipelines and out-of-order capability, are all used by extremely **specialized libraries**.

→ It is a brilliant idea to link those library instead of developing your own algorithm, unless some very special needs must be met.

$O(N^3)/O(N^2)$ example

matrix-matrix multiplication is a very common task in HPC.
Although there are highly optimized library that performs the job, it is a very classical and useful case study to understand the loop tiling and the cache-oblivious algorithms.

Let's start from the definition of the problem.
Given 2 matrices, A and B, having respectively (m, n) and (n, p) rows and columns respectively, their product is defined as the matrix $C(m, p)$

$$C_{i,j} = \sum_{k=0}^{n} A_{i,k} \times B_{k,j}$$

$O(N^3)/O(N^2)$ example

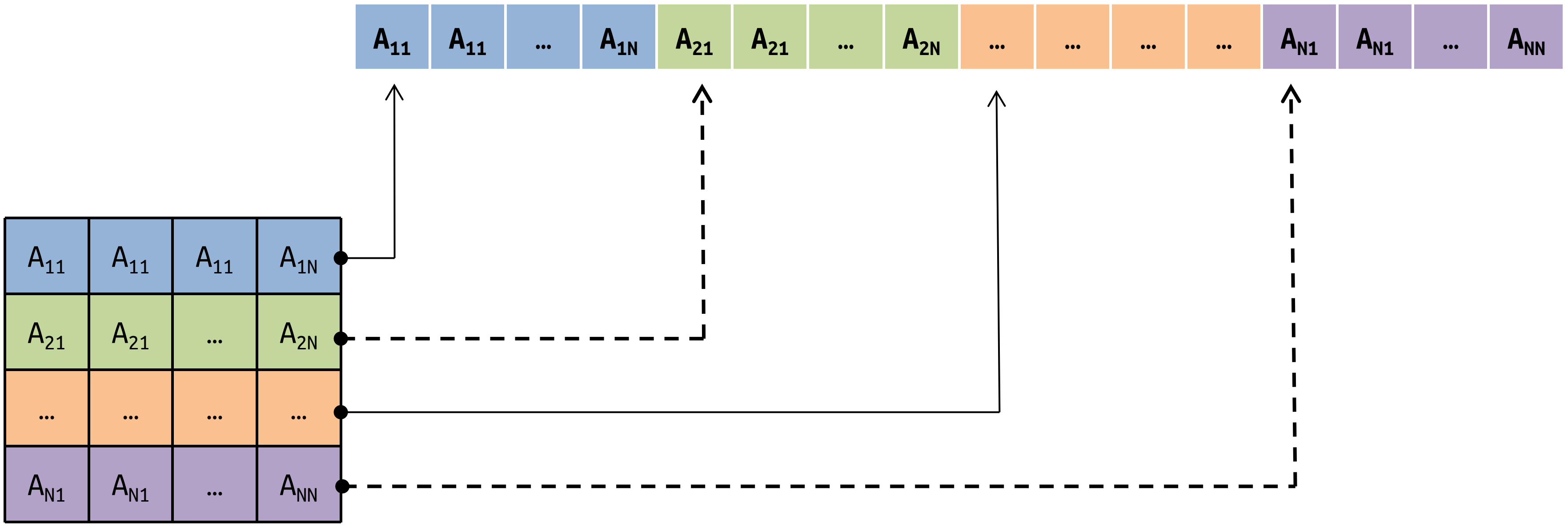
$$C_{i,j} = \sum_{k=0}^{n} A_{i,k} \times B_{k,j}$$

A possible obvious straightforward implementation of this algorithm is as follows:

```
for ( int i = 0; i < m; i++ )              // traverse the A's (and C’s) rows
    for ( int k = 0; k < p; k++ )          // traverse the B's rows ( Ac = Br )
        for ( int j = 0; j < n; j++ )
            C[i][k] += A[i][j] * B[j][k];
```

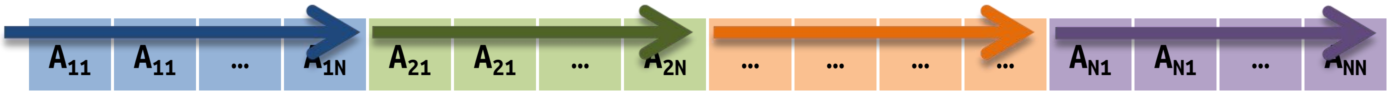
Note: how a matrix is stored in memory

Remember the obvious fact that your memory is a continuous 1-dimensional stream of bytes.

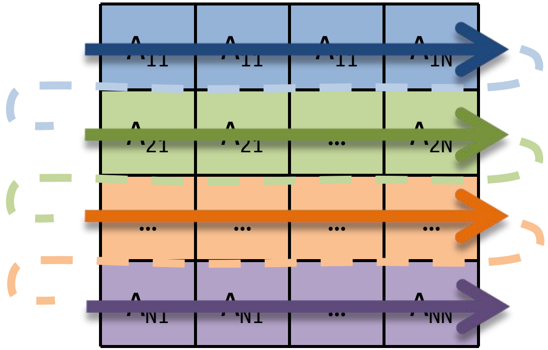


This convention is the C/C++ convention, which is labelled as *row-major order*. Note that the Fortran convention is opposite, with columns being contigous in memory (*column-major*).

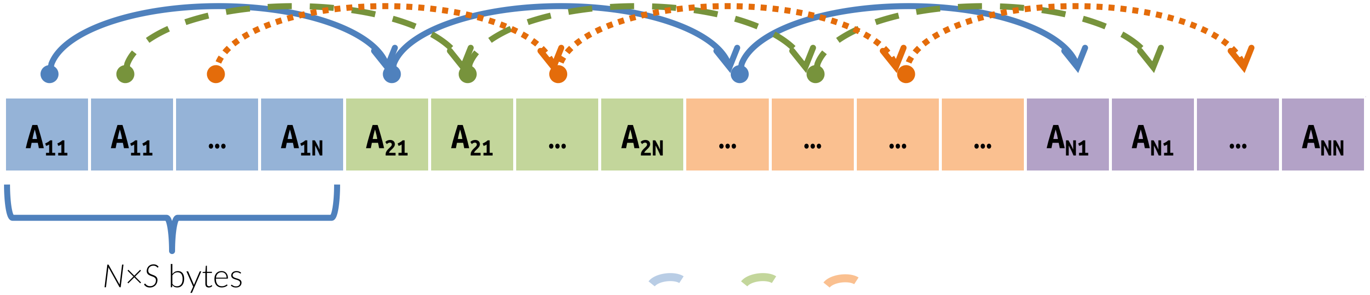
Note: how a matrix is stored in memory



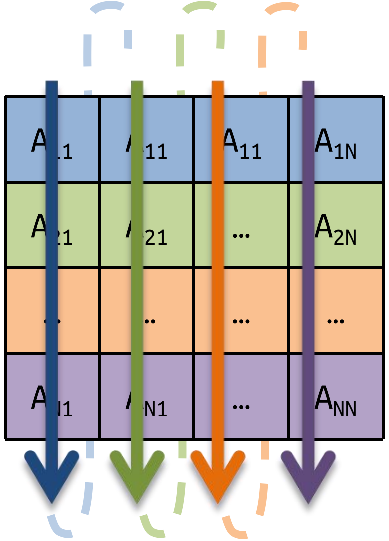
Then, traversing the matrix in the same row-major order amounts to traverse the memory in contiguous order.



Note: how a matrix is stored in memory



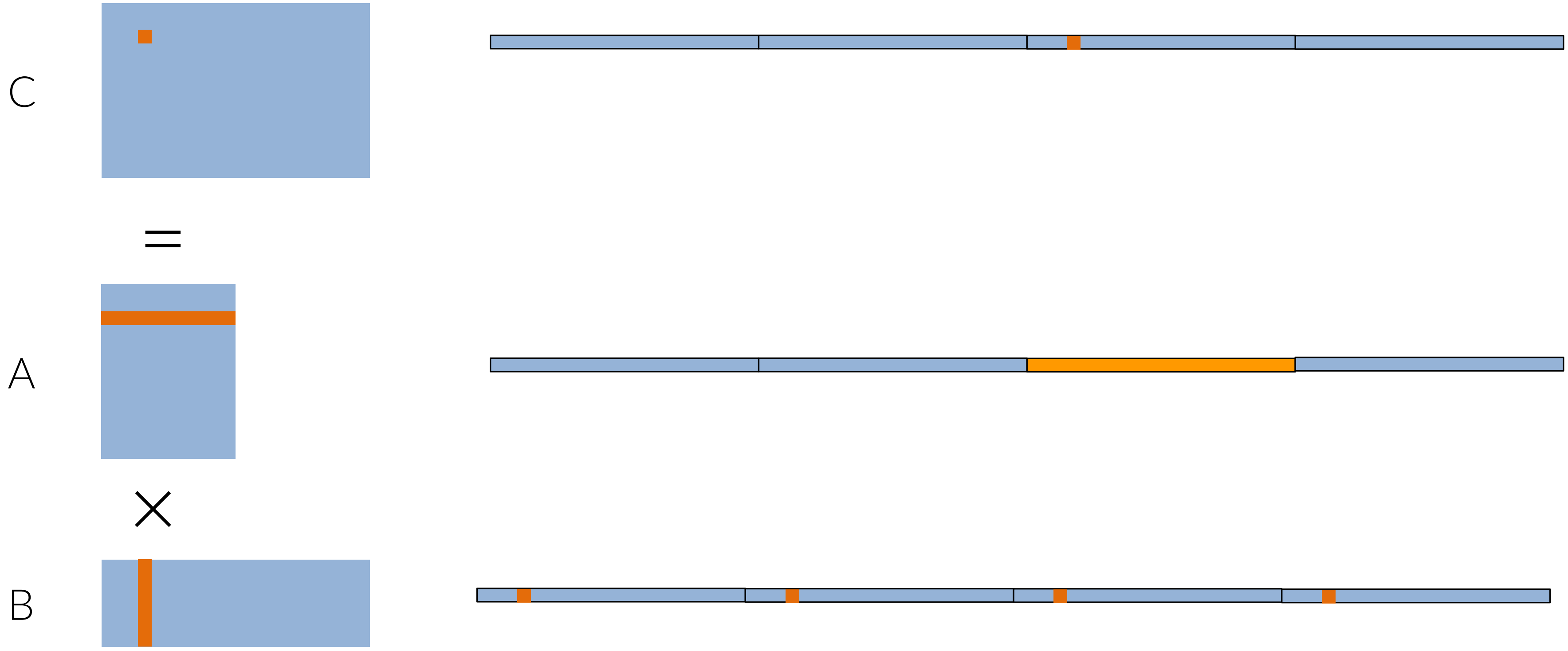
Whereas, traversing the matrix in the opposite column-major order amounts to jump in memory by *N* positions, i.e. *N*×*S* bytes is *S* is the size of each element.



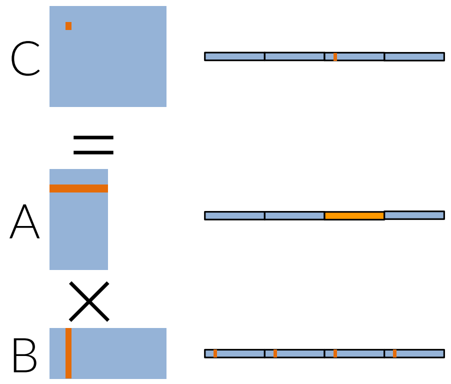
$O(N^3)/O(N^2)$ example

Matrix representation

Memory representation



$O(N^3)/O(N^2)$ example



The naïve implementation has an obvious issue with data locality for large enough matrixes.
For each C's element, possibly all accesses to B result in a cache miss.
Then, we may have *mnp*/**L** cache misses (if **L** is the line capacity of the cache in terms of the data type used) only to traverse B.
The total number of expected misses is:

mp/L +	C is traversed once
mnp/L +	A is scanned entirely *p* times
mnp	B is accessed sparsely *p* times

In fact, the naïve implementation is never used for any large matrix multiplication: since *2mnp* flop are required, it amounts to have nearly a cache miss per each flop.
How can we fix this problem ?

Transposing the matrix B before entering the loop should alleviate the problem; although the transposition requires some additional work, for large enough matrices there is still a performance gain.

$O(N^3)/O(N^2)$ example

A different strategy may consist in swapping the two inner loops:

```
for ( int i = 0; i < m; i++ )          // traverse the A's (and C’s) rows
   for ( int k = 0; k < p; k++ )       // traverse the B's rows ( Ac = Br )
      for ( int j = 0; j < n; j++ )
         C[i][k] += A[i][j] * B[j][k];
```

Now we are still having lots of cache misses due to the fact that we are re-loading C[i][k] many times (Ac times).

```
for ( int i = 0; i < m; i++ )
   for ( int j = 0; j < n; j++ )
      for ( int k = 0; k < p; k++ )
         C[i][k] += A[i][j] * B[j][k];
```

Now we expect to have

mnp/L + running over C

mnp/L + running over A

mnp/L running over B

cache misses. Then, with respect to the previous nesting scheme we expect to have

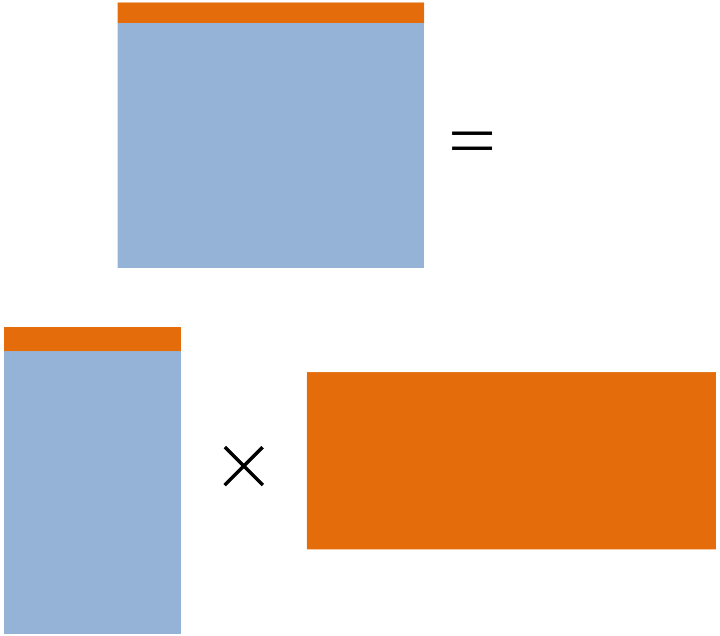
~ mnp

less cache misses.

SCO/examples_on_pipelines/matrix_multiplication

$O(N^3)/O(N^2)$ example

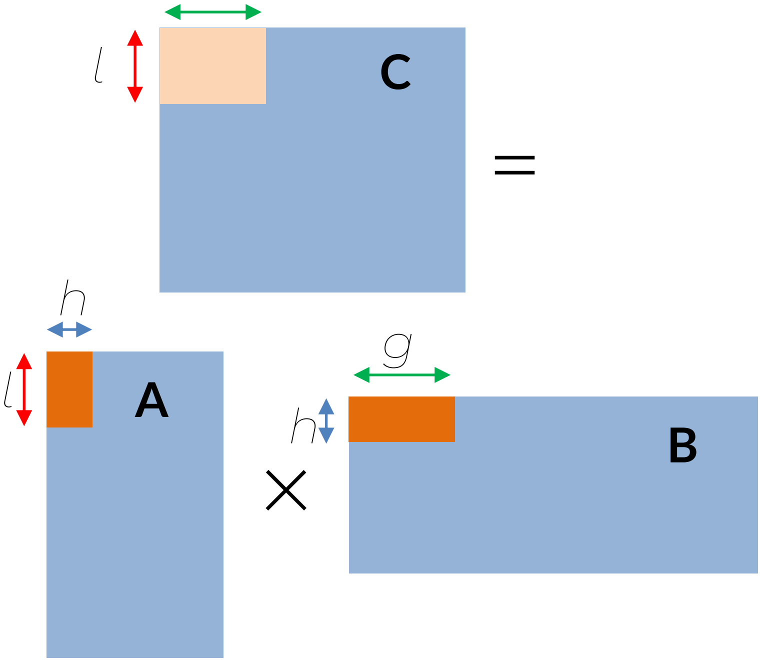
We can do even better by optimizing both the memory accesses and the data contiguity by *tailing* the loops:



To compute a single line in C we need to access the corresponding line in A *p* times. In the hypothesis that A,B and C can not fit in memory, each time the cache will have been flushed and the A's line will not be there anymore.
This amounts to have *n/L* compulsory misses per each column if B, i.e. *np/L* cache misses for each C's line, as we have already calculated.
The same holds for the B' s columns and so on…

$O(N^3)/O(N^2)$ example

We can do even better by optimizing both the memory accesses and the data contiguity by *tailing* the loops:



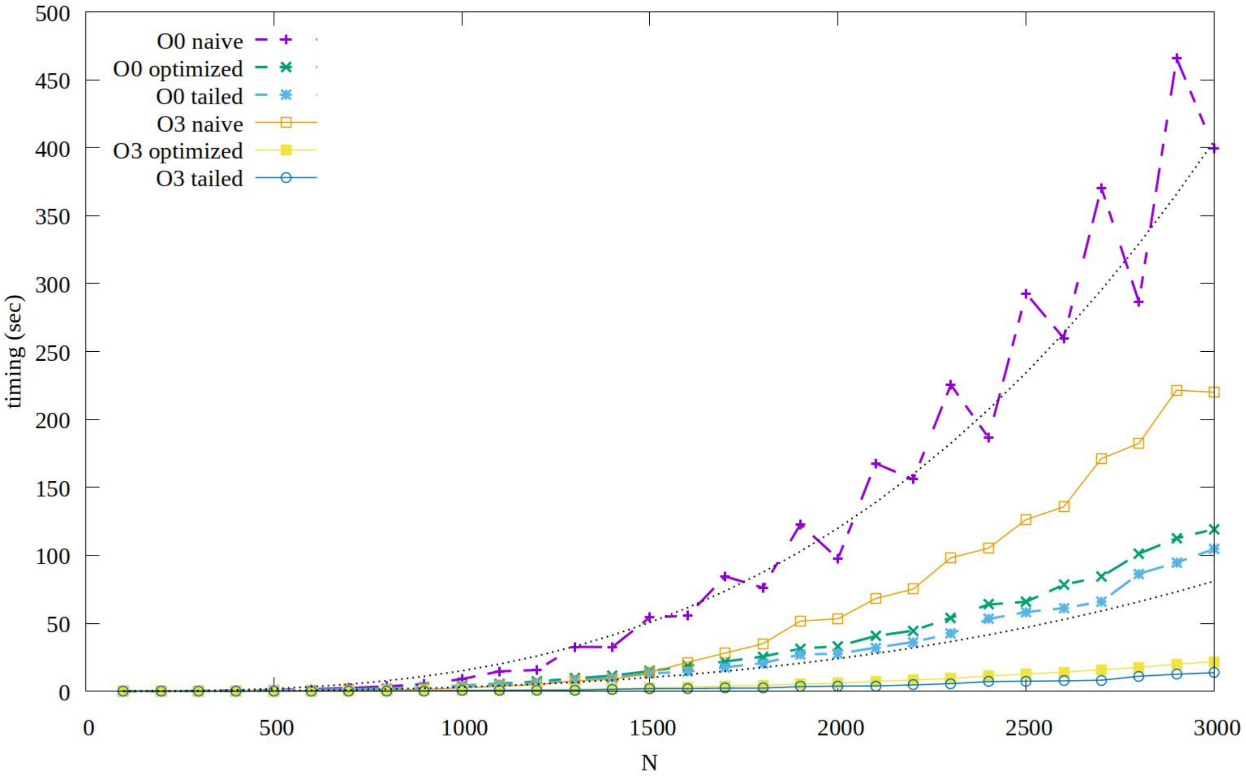
If, instead, we keep in the cache a segment of the A's line, re-using it against the columns of B – or, better, against a columns section tall as the line segment is large (blue arrows in the figure) – we could greatly reduce the amount of cache misses per C's element.

Traversing A and B by *blocks* as in the figure allows to accumulate partial results in the corresponding C's area while decreasing the number of cache misses by a factor

$L \,/\, (\, l \times h \times g \,)$

where L is the cache capacity and are the block factors. With standard value, this figure becomes of the order of 0.001.

Matrix multiplication results



Run time of different implementation with and without compiler's optimization

the results are for the case of 2 square matrix of dimension N

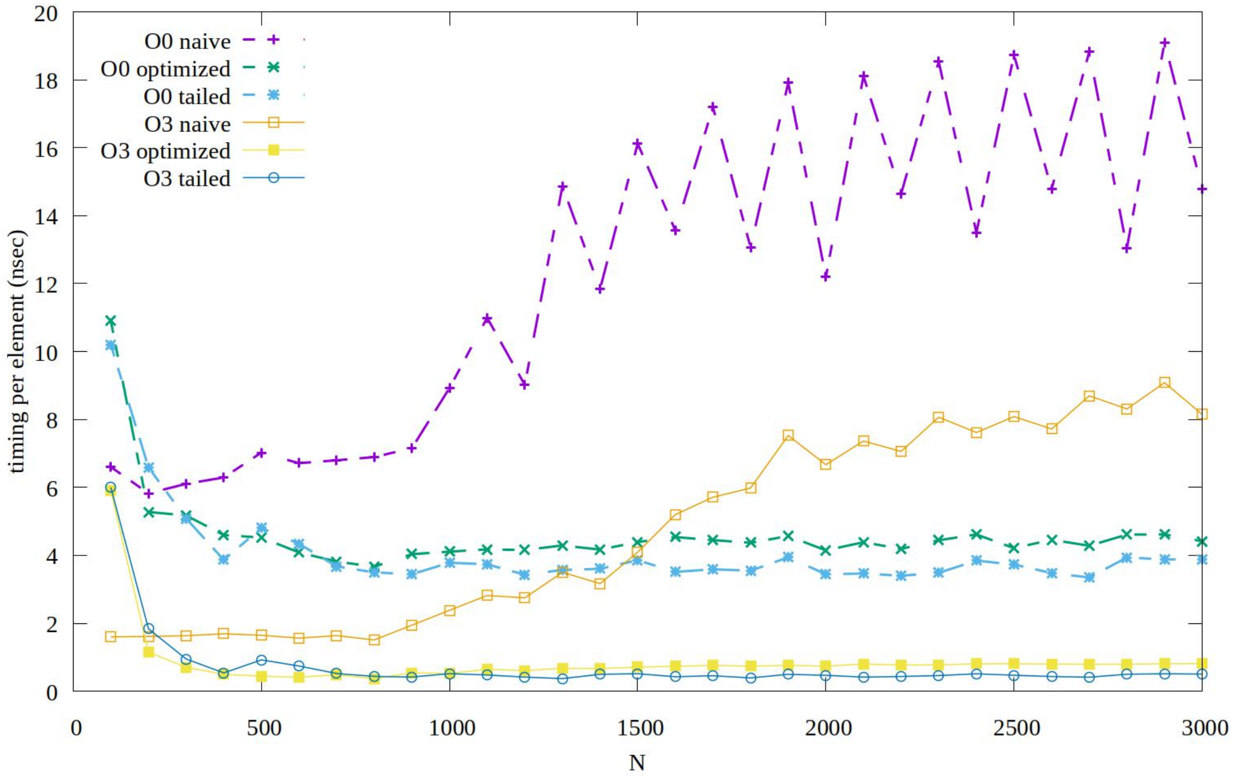
Naïve: the schoolbook's implementation

Optimized: inner loops swapped

Tailed: M-M by blocks

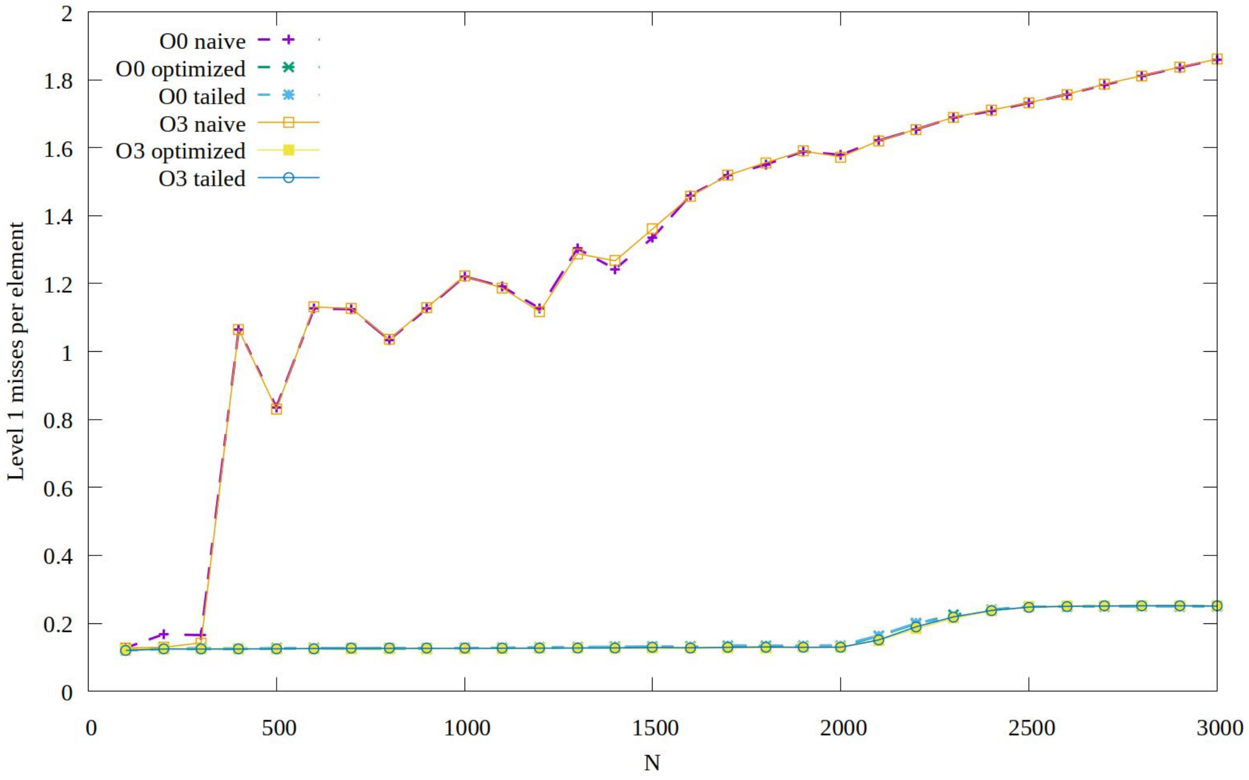
In this plot and in the following ones dashed lines are for `-O0` and solid line for `-O3 -march=native`
(only `gcc` has been used)

Matrix multiplication results



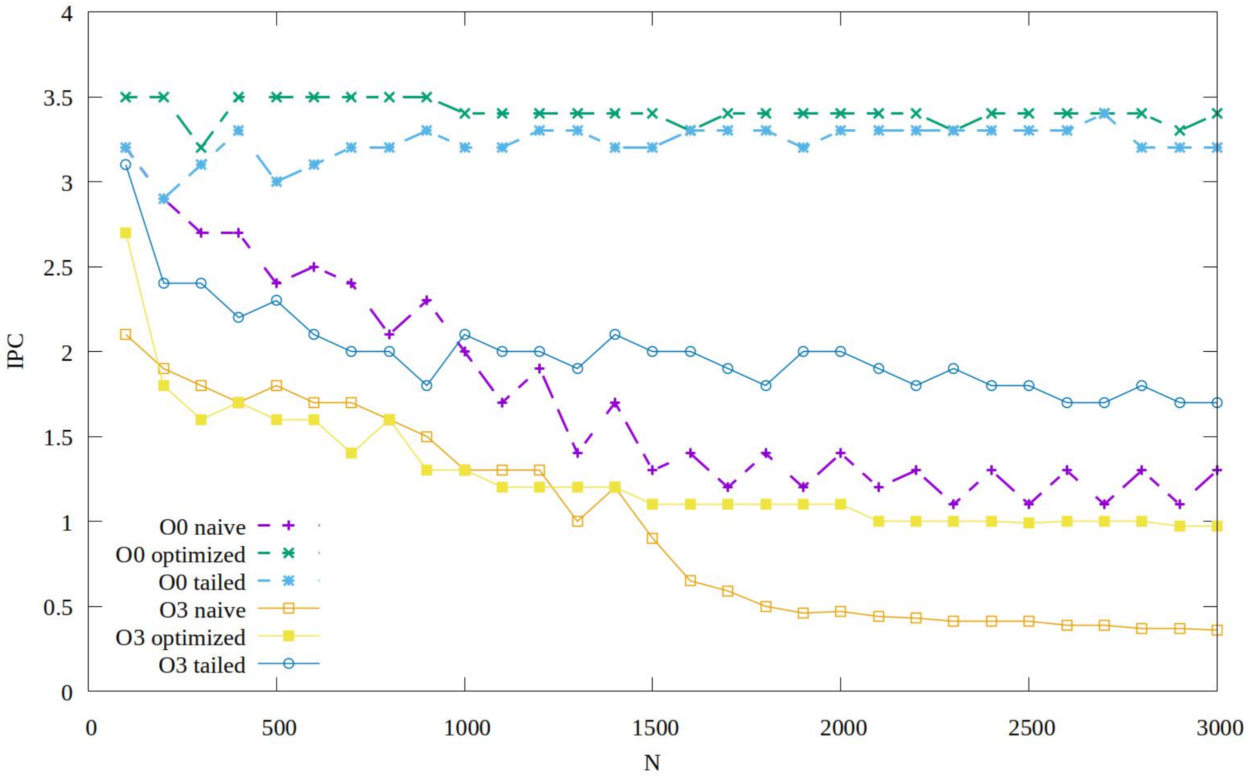
Time per element accessed (N^3)

Matrix multiplication results



Level 1 Data cache misses per element

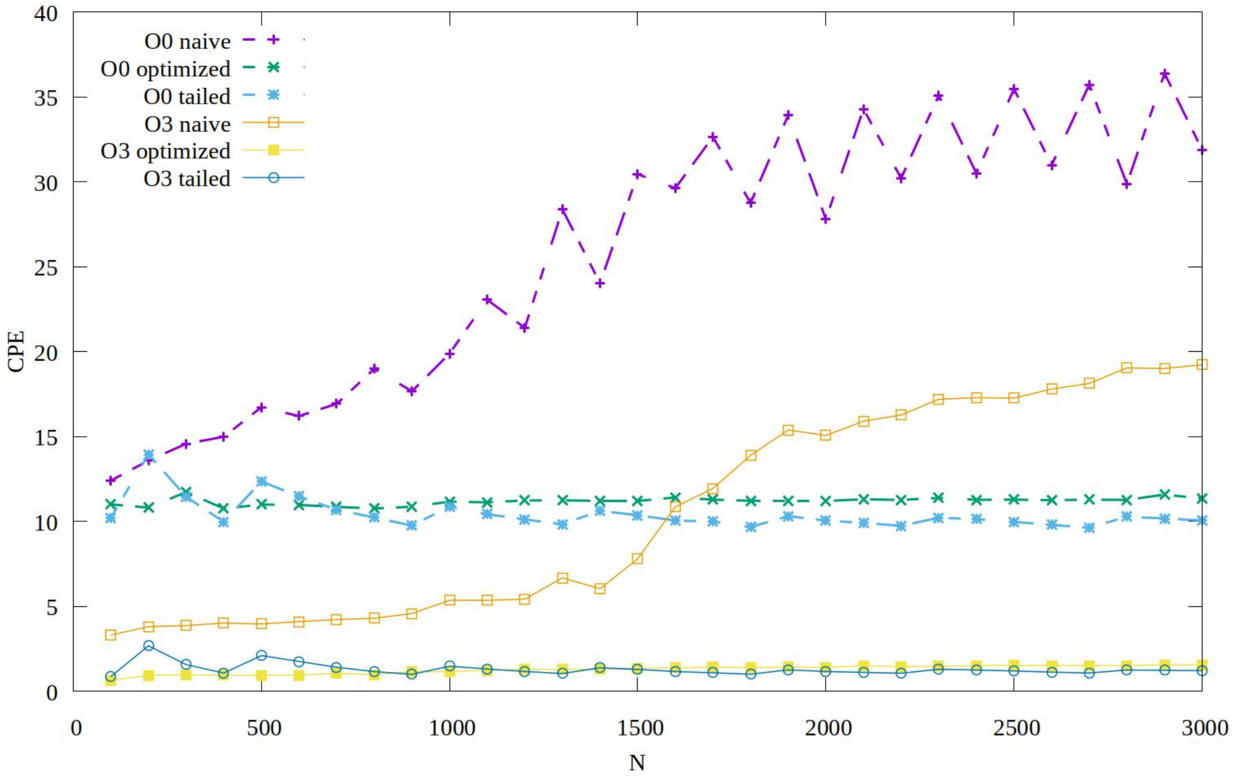
Matrix multiplication results



Instructions per cycle

(the larger the better)

Matrix multiplication results



Cycles per element

(the smaller the better)

Outline

Advanced

Prefetching

At the right moment, at the right place

Advanced

We know that waiting for data and instructions is a major performance killer.

Modern CPUs have the capability of pre-emptively bring from memory into cache levels data that **will be needed shortly afterwards.**
They can do that following some speculative algorithm based on the current execution flow and assuming spatial locality and temporal locality.

Both *data* and *instructions* can be pre-fetched.

Pre-fetching may be both hardware-based and software-based (tipically the compiler insert pre-fetching instructions at compile-time).

At the right moment, at the right place

Advanced

From the point of view of the programmer, there are 2 possible ways to deal with prefetching:

EXPLICIT	you explicitly insert a pre-fetching directive. *Very difficult to be achieved effectively: the directive must be inserted timely but not too early (data eviction) or too late (load latency).*
INDUCED	you consciously arrange data layout and execution flow so that to make it obvious to the compiler what to prefetch.

Explicit prefetching

Advanced

This is a standard binary search implementation.

Find the median element

Define the next search

```
int mybsearch(int *data, int N, int Key)
 {
   int register low = 0;
   int register high = N;
   int register mid;

   while(low <= high) {
     mid = (low + high) / 2;

     if(data[mid] < Key)
       low = mid + 1;
     else if(data[mid] > Key)
       high = mid-1;
     else
       return mid;
   }
   return -1;
 }
```

Explicit prefetching

Advanced

We can make it better by simply making sure that the element to be compared for (the `mid`) is in the cache when requested

```
int mybsearch(int *data, int N, int Key)
 {
   int register low = 0;
   int register high = N;
   int register mid;

   while(low <= high) {
     mid = (low + high) / 2;

     if(data[mid] < Key)
       low = mid + 1;
     else if(data[mid] > Key)
       high = mid-1;
     else
       return mid;
   }
   return -1;
 }
```

Explicit prefetching

Advanced

We can make it better by simply making sure that the element to be compared for (the `mid`) is in the cache when requested

```
int mybsearch(int *data, int N, int Key)
 {
   int register low = 0;
   int register high = N;
   int register mid;

   while(low <= high) {
     mid = (low + high) / 2;
      __builtin_prefetch (&data[(mid + 1 + high)/2], 0, 3);
      __builtin_prefetch (&data[(low + mid - 1)/2], 0, 3);

     if(data[mid] < Key)
       low = mid + 1;
     else if(data[mid] > Key)
       high = mid-1;
     else
       return mid;
   }   return -1; }
```

SCO/examples_on_prefetching/
examples_on_prefetching_1

Explicit prefetching

Advanced

```
luca@GGG:~/code/HPC_LECTURES/prefetching% ./prefetching_off
performing 13421772 lookups on 134217728 data..
set-up data.. set-up lookups..
start cycle.. time elapsed: 20.7534
luca@GGG:~/code/HPC_LECTURES/prefetching% ./prefetching_on
performing 13421772 lookups on 134217728 data with prefetching enabled..
set-up data.. set-up lookups..
start cycle.. time elapsed: 12.6204
```

Explicit prefetching

Advanced

```
Samples: 71K of event 'cpu/mem-loads,ldlat=30/P', Event count (approx.): 1390[illegible]
Overhead          Samples  Memory access
  71,08%            42196  Local RAM hit
  24,14%            17022  LFB hit
   4,11%            10967  L3 hit
   0,63%             1714  L1 hit
   0,02%               75  L2 hit
   0,01%               15  L3 miss
   0,00%                1  Uncached hit
```

```
Samples: 61K of event 'cpu/mem-loads,ldlat=30/P', Event count (approx.): 11720387
Overhead          Samples  Memory access
  68,74%            29450  LFB hit
  27,04%            28208  L1 hit
   2,72%              909  Local RAM hit
   1,29%             2983  L3 hit
   0,20%              346  L2 hit
```

Explicit prefetching

Advanced

Usage of direct prefetching directive is highly uncertain, since it is difficult to spot the exact point – both in the code and in the execution – where to place them (also because your C code is different than the generated assembly code).

Moreover, the “exact point” is very likely dependent on the system you run on, and then it is susceptible to change significantly.

It is normally much safer to re-organize your code so to have **prefetching by pre-loading.**

Prefetching by "moral suasion"

Advanced

```
elem a = elements[0]
for ( i = 0; i < 4*N_4; i+= 4 )
 {
        elem e = elem[i+4]; // non-blocking miss
        elem b = elem[i+1]; // possible cache-hit
        elem c = elem[i+2]; // possible cache-hit
        elem d = elem[i+3]; // possible cache-hit
        Elaborate(a);
        Elaborate(b);
        Elaborate(c);
        Elaborate(d);
        a = e;
 }
```

Let's discuss together this very simple example before putting the hands on the code you find in the `git`

SCO/examples_on_prefetching/
examples_on_prefetching_2

Hands-on

Advanced

You find code snippets with different flavours of prefetching-by-preloading technique on our GitHub, with some comments about compilation.

Compile and run them with different options (and possibly different compilers) and try to understand what happens on your laptop and/or on HPC facility.

```
for ( i = 0; i < N; i++ )
    sum += array[ i ];
```

SCO/examples_on_prefetching/
examples_on_prefetching_2

that's all, have fun