Notes for PHYS 27200 - Electric And Magnetic Interactions

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These are lecture notes for fall 2023 PHYS 27200 at Purdue. Modify, use, and distribute as you please.

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Course Introduction

This is a calculus-based physics course using concepts of electric and magnetic fields and an atomic description of matter to describe polarization, fields produced by charge distributions, potential, electrical circuits, magnetic forces, induction, and related topics, leading to Maxwell's equations and electromagnetic radiation and an introduction to waves and interference. 3-D graphical simulations and numerical problem solving by computer are employed throughout. For more information, consult the syllabus.

Equations

1. Coulomb's Law: $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$
2. Electric field due to a point or charged sphere: $\vec{E}_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}\hat{r}$
3. Force due to electric field: $\vec{F}_2 = E_1 q_2$
4. Dipole moment between charges $-q$ and q separated by $\vec{s}$: $\vec{p} = q\vec{s}$
5. Electric field on dipole axis: $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{-2sq}{r^3}\hat{p}$
6. Electric field on dipole bisecting plane: $\vec{E} = \frac{-1}{4\pi\epsilon_0}\frac{sq}{r^3}\hat{p}$
7. Electric field from point charge-induced dipole:
8. Electric field from dipole-induced dipole:

Electric charge

Electric Charge: Electric charge is an intrinsic characteristic of the fundamental particles that make up objects.

Conservation of charge: The net charge of a *closed system* never changes

It was mentioned in class that mass is likewise intrinsic. However, when discussing the Higgs field's role in giving particles mass, the distinction between mass as an intrinsic or emergent property becomes more nuanced. The mass of elementary particles like electrons and quarks is emergent in the sense that it arises from their interaction with the Higgs field, which itself is a fundamental aspect of the universe.

Objects can have negative, zero, or postive charge. Charges are always multiples of the *elementary charge* $e = 1.60217662(63) \times 10^{-19}C$

Coulomb (C): One coulomb is the amount of charge that is transferred through the cross section of a wire in 1 second when there is a current of 1 ampere in the wire.

The charges of elementary particles are listed below.

Particle	Charge (elementary charge, e)
Electron (e^-)	-1
Positron (e^+)	$+1$
Proton (p^+)	$+1$
Anti-proton (p^-)	-1
Neutron	0
Anti-neutron	0
Photon	0

Point Charge: A charged object whose radius is much smaller than the distance between itself and all other objects of interest.

The magnitude of electric force between two point charges is d irectly proportional to the magnitude of each charge and inversely proportional to the distance squared. Specifically:

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

This is Coulomb's Law. Note the direction of the force changes with the sign of the charges involved. Like repels like and opposites attract.

Electric field

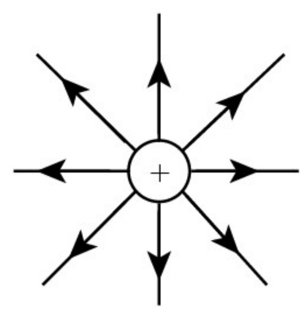

Figure 1: An eletric field coming from point charge. Notice how the densities of the lines vary with distance from the source.

Consider a charged particle. We can represent its effect by drawing vectors that show the path a positively charged particle would follow if placed within its influence. These lines represent the electric field of the charged particle. The greater the density of the lines, the greater the strength of the electric field. Note that at the origin, the force is undefined (infinite), since $|r| = 0$.

Technically, fields can be the more general tensor, or even the fascinating and exotic spinor!

There are many types of fields, which can be either scalar or vector. For example, a temperature map is a scalar field, since each point is associated with a scalar (the temperature at that point). A map of fluid velocity is a vector field, since each point is associated with a vector (the velocity of the fluid at that point). For a particle, the eletric field vector at any point is given by

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}\hat{r}$$

The direction in which the field lines point depends on the sign of the charge. If the charge is negative, the field lines point in. If it is positive, the field lines point out. A useful mnemonic is to think of the charge as someone's STD test results. If it's negative, others will go for them and the lines point in. If it's positive, everyone will try to get away and the lines point out.

Consider the relative strengths of the electric and gravitational fields. The gravitational force is given by $F_g = G\frac{m_1 m_2}{r^2}\hat{r}$, with $m_{electron} = 9 \times 10^{-31}kg$ and $m_{proton} = 1.7 \times 10^{-27}kg$. If we consider a hydrogen atom, then $r = 5.3 \times 10^{-11}m$. With $G = 6.7 \times 10^{-11}$, we have

$$F_g = \frac{(1.7 \times 10^{-27})(9 \times 10^{-31})(6.7 \times 10^{-11})}{(5.3 \times 10^{-11})^2} \approx O(10^{-46})N$$

Now, the eletric force is given by $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$. The charge of a proton and eletric are $q_1 \approx q_2 \approx 1.6 \times 10^{-19}C$. Ergo, since $\frac{1}{4\pi\epsilon_0} \approx 8.99 \times 10^9 \frac{Nm^2}{C^2}$,

$$F_e = \frac{(8.99 \times 10^9 \frac{Nm^2}{C^2})(1.60 \times 10^{-19}C)^2}{(5.3 \times 10^{-11}m)^2} \approx O(10^{-}17)N$$

This means that $\frac{F_e}{F_g} \approx 2.27 \times 10^{39}$, meaning the eletric force is much stronger for these masses and charges than gravity. On scales as large as humans and planets, gravity is the dominant force because gravity is strictly additive.

For sufficient distances, the eletric field of a uniformly charged spherical shell resembles the eletric field of a point charge.

That means for $r >> R$, $\vec{E_{sphere}} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}\hat{r}$ This holds only for outside the sphere. Inside, it can be shown that the electric field is zero.

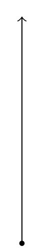

Figure 2: Notice how a circle resembles a point from a great distance.

Superposition Principle: The net electric field at a location in space is a vector sum of the individual electric fields contributed by all charged particles located elsewhere.

To introduce systems with multiple sources of electric field lines, consider the particle pair known as a dipole. Dipoles consist of one negatively charged and one positively charged particle, like so:

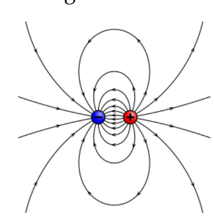

Figure 3: Two oppositely charged particles distanced from one another

Dipole Moment: The dipole moment is a way of expressing asymmetrical charge distribution. It is a vector quantity, i.e. it has magnitude as well as definite directions. The dipole moment is given by the expression $\vec{p} = q\vec{d}$, where q is the charge on one end of the dipole and $\vec{d}$ is the distance between dipoles.

On the axis of the dipole (i.e. the lines formed by the two particles), the electric field is

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2sq}{r^3}\hat{p}$$

where r is the distance from the point in consideration to the center of the dipole. On the bisecting plane (i.e., the plane exactly halfway from each point) the field is given by

$$\vec{E} = \frac{-1}{4\pi\epsilon_0}\frac{sq}{r^3}\hat{p}$$

Where r is the distance from the point to either dipole. The force on a positive point charge q_1 a distance of d away from the dipole, aligned with the dipole, and on the side of the negative charge is given by

$$\vec{F} = q_1\vec{E_{dipole}} = q_1(\frac{-1}{4\pi\epsilon_0}\frac{2qs}{d^3}, 0, 0)$$

Note that the field will be parallel to the axis of the dipole: this can simplfy vector calculations. Now consider a dipole in a uniform electric field, like so: The positive end will be pulled to the right

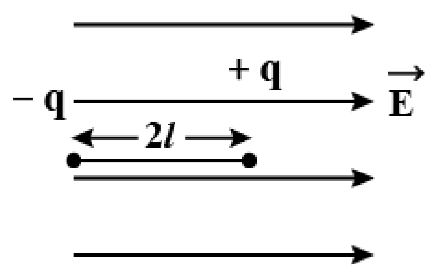

Figure 4: Dipole in uniform electric field

and the negative end to the left, exerting a torque given by $\vec{\tau} = \vec{p} \times \vec{E_{uniform}}$. Note that by the definition of $\times$, $\tau = pE_{uniform}\sin\theta$, where θ is the dipole's angle from horizontal. It can be shown that the potential energy of a dipole in a uniform eletric field is $U = -\vec{p} \cdot \vec{E_{uniform}}$ and similarly as before (but now with $\cdot$) $U = -pE_{uniform}\cos\theta$. Usefully, this means dipoles can be used to measure the direction of an electric field.

Throughout these examples, we have been assuming the associated speeds are much less than the speed of light. If the velocities approach a significant fraction of the speed of light Coulomb's law no longer holds, and we must account for relativity.

Conservation of Charge: Charge cannot be created nor destroyed, with the exception of electron-positron annihilation and other such quantum hijinks. We can use conservation of charge to predict the

Interestingly, in annihilation between positrons and electrons (or any other subatomic particles) the total energy and momentum of the initial pair are conserved in the process and distributed among a set of other particles in the final state (photons in the example of the electron-positron pair). Antiparticles have exactly opposite additive quantum numbers from particles, so the sums of all quantum numbers of such an original pair are zero. Hence any set of particles may be produced whose total quantum numbers are also zero as long as conservation of energy, conservation of momentum, and conservation of spin are obeyed.

behavior of many systems. For example, consider tape pulled from a roll. You may have noticed when dangling strips of freshly-pulled tape they tend to drift towards nearby surfaces to stick and become tangled: this is because peeling a strip of tape off a roll strips electrons from the tape, resulting in a net positive charge because of conservation of charge. When this charge approaches a net neutral object, such as your hand, the electrons in the atoms of your hand are attracted to the positively charged tape and congregate closer to the tape. This results in a negative charge buildup near the tape. The positive tape is attracted to the negative charges, and so the tape moves towards your hand and becomes tangled and useless. The process of one charge inducing a charge on a neutral object occurs often enough for the phenomenon to be named. We call it **Polarization**: The process by which a dipole is formed in a neutral object by an electric field. The dipole moment is given by $\vec{p} = \alpha\vec{E}$, where α is a material-dependent constant called polarizability. Such a dipole is *induced*. Consider a point charge near a neutral atom. The point charge creates an electric field given by

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}\hat{r}$$

Inducing a dipole given by the expression

$$\vec{p} = \alpha\vec{E}_1 = \frac{1}{4\pi\epsilon_0}\frac{\alpha q_1}{r^2}\hat{r}$$

This dipole creates a field at the point charge of

$$\begin{aligned}\vec{E}_2 &= \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3} \\ &= \frac{1}{4\pi\epsilon_0}\frac{2\alpha\vec{E}_1}{r^3} \\ &= \frac{1}{4\pi\epsilon_0}\frac{2\alpha}{r^3}(\frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}\hat{r}) \\ &= \left(\frac{1}{4\pi\epsilon_0}\right)^2\frac{2\alpha q_1}{r^5}\hat{r}\end{aligned}$$

This formula is valid provided the electric field that induced the dipole is from a point charge. If the electric field is instead from, say, another (permanent) dipole then $\vec{p} = \alpha\vec{E}_1$ is still valid. However, in this case the formula for $\vec{E}_1$ will be given by the equation for the electric field along the axis of a dipole instead. Following this logic,

$$\begin{aligned}\vec{p} &= \alpha\vec{E}_1 \\ &= \alpha\frac{1}{4\pi\epsilon_0}\frac{2\vec{p}_1}{r^3}\end{aligned}$$

If we let $'vecr'$ be the location of the permanent dipole from the induced dipole, we have the electric field at the permanent dipole as

$$\begin{aligned} E(\vec{r}') &= \left(\frac{1}{4\pi\epsilon_0}\frac{2}{r^3}\right)\left(\alpha\frac{1}{4\pi\epsilon_0}\frac{2\vec{p}_1}{r'^3}\right) \\ &= \left(\frac{1}{4\pi\epsilon_0}\right)^2\frac{4\alpha\vec{p}_1}{r^3r'^3} \end{aligned}$$