Notes for ECE 20001 - EE Fundementals I

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Course Introduction

This course covers fundamental concepts and applications for electrical and computer engineers as well as for engineers who need to gain a broad understanding of these disciplines. The course starts by the basic concepts of charge, current, and voltage as well as their expressions with regards to resistors and resistive circuits. Essential concepts, devices, theorems, and applications of direct-current (DC), 1st order, and alternating-current (AC) circuits are subsequently discussed.

Besides electrical devices and circuits, basic electronic components including diodes and transistors as well as their primary applications are also discussed. For more information, see the syllabus.

Equations

1. $P = \frac{dW}{dt} = IV$
2. $I = \frac{dq}{dt}$
3. $V = \frac{W}{q}$
4. $R = \frac{\rho L}{A}$
5. Ohm's Law: $V = IR$
6. Coulomb's Law: $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$
7. Kirchhoff's Voltage Law: $\sum V_i = 0$ (around a closed loop)
8. Kirchhoff's Current Law: $\sum I_i = 0$ (going into a node)
9. Conductance: $G = \frac{1}{R}$
10. Equivalent resistance: $R_{eq} = \frac{V_{test}}{I_{test}}$
11. Series capacitance: $\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$
12. Parallel capacitance: $C_{total} = C_1 + C_2 + \ldots$
13. Series inductor: $L_{total} = L_1 + L_2 + \ldots$
14. Parallel inductor: $\frac{1}{L_{total}} = \frac{1}{L_1} + \frac{1}{L_1} + \ldots$
15. $I_{cap} = C\frac{dV}{dt}$
16. $V_{ind} = L\frac{dI}{dt}$
17. Energy stored in capacitor: $\frac{1}{2}CV^2$
18. Energy stored in inductor: $\frac{1}{2}LI^2$
19. Voltage in RC circuit: $v_c(\infty) + (v_c(t_0) - v_c(\infty))\, e^{(\frac{-1}{RC})(t-t_0)}$
20. Current in RL circuit: $I_L(\infty) + (I_L(t_0) - I_L(\infty))\, e^{(\frac{-R}{L})(t-t_0)}$
21. Impedance of a capacitor: $\frac{-j}{\omega C}$
22. Impedance of an inductor: $j\omega L$
23. Equivalent impedance for impedances in series: $Z_{eq} = \sum_i^n Z_i$
24. Equivalent impedance for impedances in parallel: $\frac{1}{Z_{eq}} = \sum_i^n \frac{1}{Z_i}$
25. RMS value of signal: $S_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} s^2(t)dt}$
26. Maximum power extracted by a load in DC circuits: $\frac{Vth^2}{4R_{th}}$

27. Maximum power extracted by a load in AC circuits: $\frac{|\tilde{V}_{th}|^2}{8R_{th}}$

28. Average power: $P_{avg} = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$

29. Reactive power: $V_{ar} = Im(\frac{1}{2}\tilde{V}\tilde{I}^*)$

30. Apparent power: $P_{app} = \frac{|V_{rms}|^2}{|z|}$

31. Voltage in magnetically coupled coils:

$$v_1(t) = L_1 \frac{di_1(t)}{dt} + L_{12} \frac{di_2(t)}{dt}$$
$$v_2(t) = L_1 \frac{di_2(t)}{dt} + L_{12} \frac{di_1(t)}{dt}$$

32. Voltage in ideal transformer:

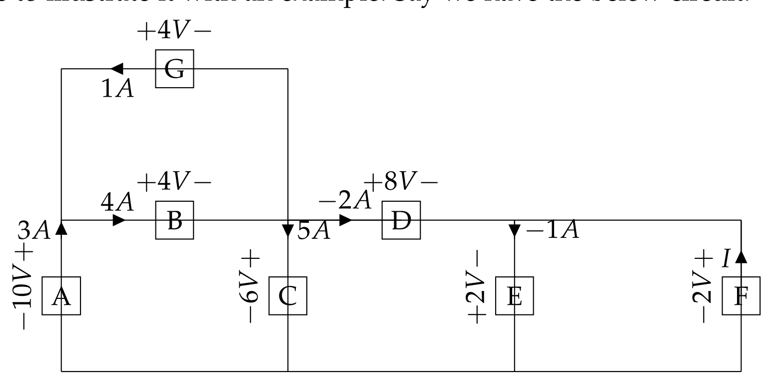
$$\frac{v_2(t)}{v_1(t)} = \frac{N_2}{N_1}$$
$$\frac{i_2(t)}{i_1(t)} = -\frac{N_1}{N_2}$$

33. Terminated transformer circuit equations:

$$\tilde{V}_1 = \frac{\tilde{V}_2}{n}$$
$$\tilde{I}_{out} = \frac{\tilde{I}_{in}}{n}$$
$$\frac{\tilde{V}_{in}}{\tilde{I}_{in}} = Z_{in} + \frac{Z_{out}}{n^2}$$

Charge, current, voltage, and power

Before we begin a discussion of electrical engineering, we will want an understanding of some important concepts.
Charge: A fundemental property of matter. Charge is measured in units of Coulombs (C) and arises from aggregates of charged fundemental particles like electrons.
Current: The rate of flow of charge. Typically, we think of current as occuring in a circuit, a loop through which charge can flow. Current can be mathematically defined as $I = \frac{dq(t)}{dt}$, where $q(t)$ is the charge at time t. Current therefore has units of Coulombs per second (C/s).
Voltage: The difference in electric potential energy between two points, per unit charge. Voltage thus has units of $\frac{J}{C}$. or Volts (V). Intuitively, a stronger voltage source (such as a battery) will result in a higher current on identical wires. Imagine a voltage source as a concentration of negative charges on one side and positive charges on the other. If we hook both ends up to a conducting wire and place an electron in the wire, it will be repelled from the negative side and attracted to the positive side. The electron has a high potential energy near the concentration of negative charge, and a low potential energy near the positive charges. The difference between these potentials is the voltage. This should make it clear that voltage must always be defined as across two points, each with its electric potential. A good note is that voltage can be negative (if the electric potential at the second point measured is higher) and it may not be constant with time.
Power: The rate of doing work, or changing energy. Mathematically, $P = \frac{dW}{dt} = IV$ and has units of Watts (W). In a closed system, power is always balanced: whatever is put out by sources is consumed by loads. Thus, in a closed circuit, $\sum P = 0$ This is an important point, so allow me to illustrate it with an example. Say we have the below circuit:



We know $\sum P_i = \sum V_i I_i = 0$. Let's keep track of each P_i in a table, not forgetting passive sign convention: Summing the rows of this table, we have $\sum P_i = 2I - 2 = 0$, implying $I = 1$.

Symbol	Watts
P_A	-30
P_B	16
P_C	30
P_D	-16
P_E	2
P_F	-2I
P_G	-4

Table 1: Power absorbed

In the previous example, we had current flowing both into and out of the positive terminals of elements. We also had negative currents. Let's define how to handle signage in circuits:

Passive sign convention: Defines current as going into positive voltage node of a component. The component is labeled a *load* or a *passive device*. We call it this because the component consumes power. If a current is negative, it is flowing in the opposite direction shown by the arrow.

It's useful to have an idea of the components of circuit schematics (visual representations of a circuit). Below is a list of the terms that will be used in this course:

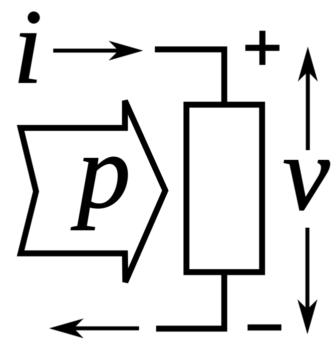


Figure 1: Passive sign convention

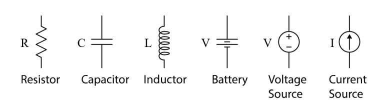
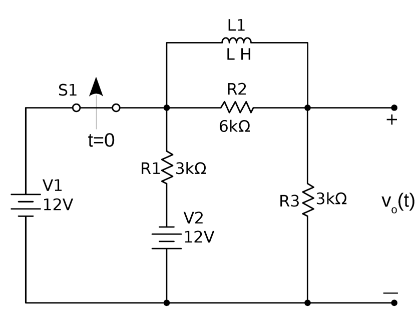


Figure 2: Common circuit symbols

- Elements: The term elements means "components and sources."
- Symbols: Elements are represented in schematics by symbols. Symbols for common 2-terminal elements are displayed to the right.
- Lines: Connections between elements are drawn as lines, which we often think of as "wires". On a schematic, these lines represent perfect conductors with zero resistance. Every component or source terminal touched by a line is at the same voltage.
- Dots: Connections between lines can be indicated by dots. Dots are an unambiguous indication that lines are connected. If the connection is obvious, you don't have to use a dot.

Check out the circuit schematic below and see how many components you can identify!



(In)dependent sources, connections, resistance and Ohm's Law

Now, on to what circuits are doing. For interesting things to happen we need electrons flowing through those wires, and for that we need sources. There are two types: independent and dependent. Independent sources are voltage sources or current sources that maintain a constant value regardless of the rest of the circuit. They are not influenced by the circuit's current or voltage conditions. There are two main types of independent sources:

- **Independent voltage source**: Maintains a constant voltage across its terminals, regardless of the current flowing through it. It is typically represented by a symbol with a plus sign and a minus sign, indicating the polarity of the voltage. A battery maintaining a constant voltage of 9 V is an example of an independent voltage source.
- **Independent current source**: Maintains a constant current through its terminals, regardless of the voltage across it. It is usually represented by a symbol with an arrow indicating the direction of current flow. Consider a current source that provides a constant current of 2 amperes. This source will deliver a current of 2A through any component connected to it, regardless of the voltage across the component.

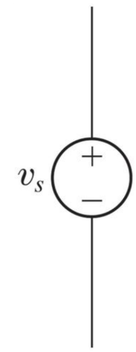


Figure 3: inependent voltage source

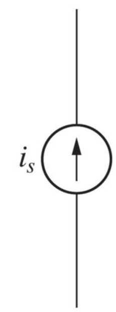


Figure 4: Independent current source

Contrasted with independent sources are dependent sources. Dependent sources are sources whose values are dependent on other variables within the circuit. These sources are used to model components whose behavior changes according to the conditions in the circuit. There are four types of dependent sources:

- **Voltage-Controlled Voltage Source (VCVS)**: This type of dependent source generates a voltage that is proportional to the voltage across a separate part of the circuit.
- **Current-Controlled Current Source (CCCS)**: This type of dependent source generates a current that is proportional to the current flowing through a different part of the circuit.
- **Voltage-Controlled Current Source (VCCS)**: This type of dependent source generates a current that is proportional to the voltage across a different part of the circuit.
- **Current-Controlled Voltage Source (CCVS)**: This type of dependent source generates a voltage that is proportional to the current flowing through a separate part of the circuit.

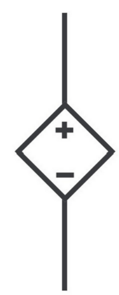


Figure 5: Dependent voltage source

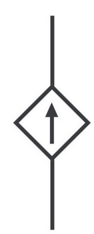


Figure 6: Dependent current source

In both the independent and dependent case, we assume the sources are ideal. There are two critical attributes of ideal sources. First, their value remains unchanged indefi- nitely. Second, they can deliver any amount of power needed by their loads. Turning off a voltage source is equivalent to replacing it with a short circuit (line). Turning off a current source is equivalent to replacing it with an open circuit (broken line). Also equivalent to an open circuit is a resistor with infinite resistance. Resistance is a measure of how hard it is to shove current through a resistor. The harder it is, the higher the resistance. It's given by $R = \frac{\rho L}{A} = \frac{V}{I}$, where ρ is the resistivity, L is the length of the resistor, and A is the cross-sectional area of the resistor. The reciprocal of resistance is conductance ($G = \frac{1}{R}$). We can relate voltage, current and resistance with *Ohm's Law*.
Ohm's Law: $V = IR$. This law is fundemental to EE and will occur repeatedly throughout the course.

Kirchhoff's Laws, resistor combinations, and voltage/current division

Kirchhoff's Current Law: The sum of all currents going into a node is 0. Mathematically, $\sum_k i_k = 0$. We may intuit this by imagining electricity as water flowing into a pipe. Anything that flows in must flow out. Say you have a circuit such as fig. 7. By Kirchhoff's current

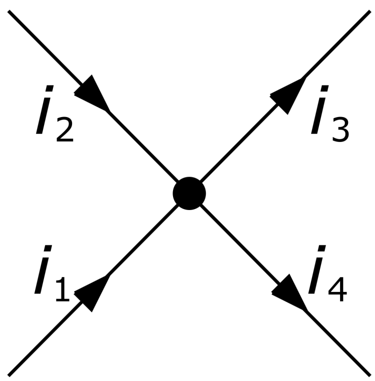


Figure 7: Kirchhoff's Current Law

law, we see that $i_1 + i_2 - i_3 - i_4 = 0 \rightarrow i_1 + i_2 = i_3 + i_4$. The neat thing is that this must be true for any node in a circuit. If you have a node with two known currents going in and one unkown comming out, Kirchhoff's current law tells us the unknown current is simply the sum of the known currents.

Contrasted with Kirchhoff's current law, we have Kirchhoff's voltage law.

Kirchhoff's Voltage Law: In a closed loop, the sum of all voltage drops is zero. Mathematically, $\sum_k v_k = 0$. A useful way to visualize this is to think of voltage as potential energy. No matter where you go and how the voltage drops and rises, when you get back to where you began, the voltage must be the same there. You end up with the same gravitational potential if you return to the same spot, even if you run up and down a mountain. Imagine a closed loop such as fig. 8. Say the voltage across the battery is $15V$ and each resistor is identical. Although we don't know for sure yet, we can guess that the current will be the same through each, and ergo by Ohm's Law ($V = IR$) so will each voltage. Therefore,

$$15 + 3V = 0 \rightarrow V = -5$$

Now that we have a good idea of circuit components and how voltage/current behave, we can look at important configurations. These are the most useful:

Series Combination: In a series combination, the elements are connected with end to end in contact. We can use Kirchhoff's current law to show that the current at each point in the circuit is the same. Since current flows into a resistor it has no option but to flow out the same resistor. Say the resistances are R_1, R_2, and R_3. By Kirchhoff's voltage

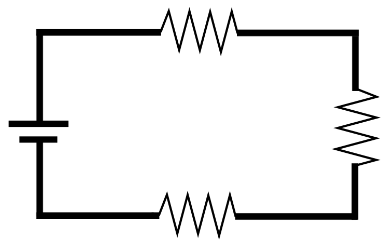


Figure 8: Series Combination

law, we have

$$\begin{aligned} V &= IR_1 + IR_2 + IR_3 \\ &= I(R_1 + R_2 + R_3) \\ &= IR_{total} \\ &\rightarrow R_{total} = R_1 + R_2 + R_3 \end{aligned}$$

Therefore for resistors in series, the total resistance is equal to the sum of each individual resistance.

Parallel Combination: When two or more resistances are connected between the same two points, they are said to be connected in parallel. Here, voltage is equal across all elements. In fig. 9, the total current

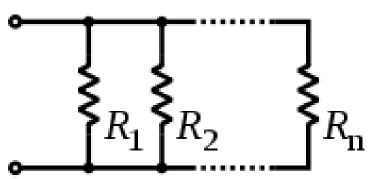


Figure 9: Parallel Combination

is split among n different resistors. That is, $I = \sum_k^n i_k$. With a bit of manipulation and Ohm's Law, we have that

$$
\begin{aligned}
I &= \frac{V}{R_{total}} \\
&= \sum_{k}^{n} i_k \\
&= \sum_{k}^{n} V/R_k \\
&\rightarrow \frac{1}{R_{total}} \\
&= \sum_{k}^{n} \frac{1}{R_k}
\end{aligned}
$$

So for resistors in parallel, the reciprocal of the total resistance is the sum of the reciprocals of the individual resistances.

Now would be an excellent point to introduce the idea of

Equivalent resistance

Equivalent resistance: a simplification of a circuit with multiple resistors, where the resistance of the equivalent resistor maintains the same voltage and current relationship. We have already seen formulas for when the resistors are in parallel or series, and can use them to determine the equivalent resistance across two nodes. For instance, consider fig. 10. We can see that the 1Ω and the 10Ω resistor

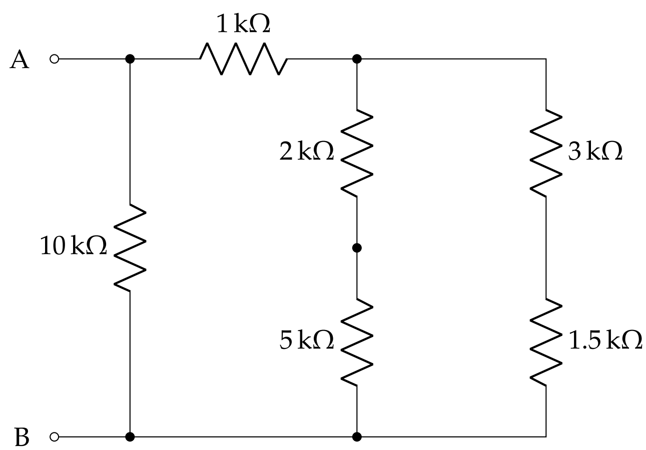


Figure 10: Finding equivalent resistance in circuit with resistors in series and parallel, step 1

are in series, so their equivalent resistance is simply $1 + 10\Omega = 11\Omega$. We can condense the circuit to fig 11. We next combine the 2Ω and

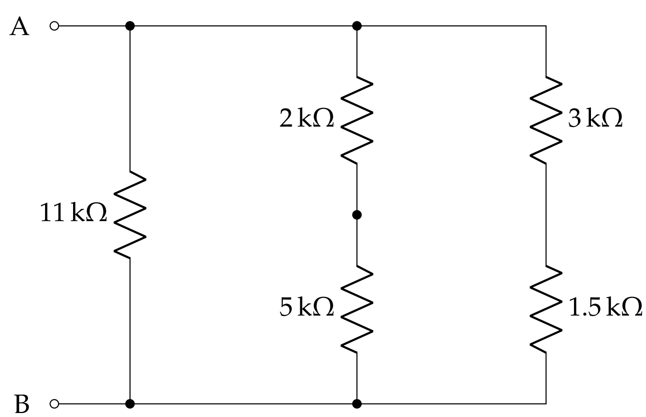


Figure 11: Finding equivalent resistance in circuit with resistors in series and parallel, step 2

the 5Ω resistor to obtain an equivalent resistance of 7Ω, and the 3Ω and the 1.5Ω to obtain 4.5Ω (fig. 12) Now that we are left with only resistors in parallel, we may use the formula $\frac{1}{R_{eq}} = \sum_k \frac{1}{R_k}$ to obtain $\frac{1}{R_eq} = \frac{1}{11} + \frac{1}{2} + \frac{1}{1.5} = \frac{83}{66}$, yielding $R_{eq} = \frac{66}{83}\Omega$. Note that this method is only possible when the circuit has resistors in series or parallel. For more complex situations, such as when souces are present or resistors are combined in neither series nor parallel, we must use Kirchhoff's

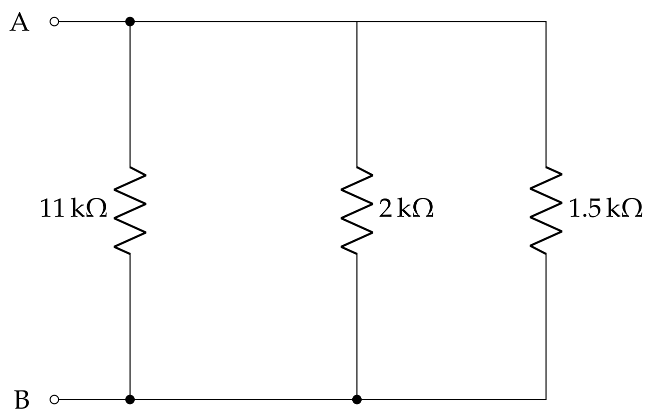


Figure 12: Finding equivalent resistance in circuit with resistors in series and parallel, step 3

laws and good judgement to find the equivalent resistance. Consider fig. 13, where we have a circuit with both dependent and independent sources. The process for finding the equivalent resistance of a circuit

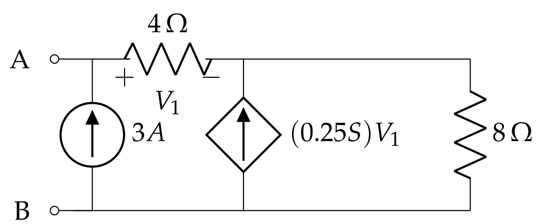


Figure 13: Finding equivalent resistance in cirucit with sources

with sources is as follows:

1. "Turn off" all independent sources. That is, replace independent current sources with opens and independent voltage sources with shorts.

2. Apply a test current or voltage across a and b.

3. Use $R_{eq} = \frac{V_test}{I_test}$, circuit laws, and algebra to solve.

Let's apply a test current of $1A$ across $a, b,$ as shown in fig. 14 By

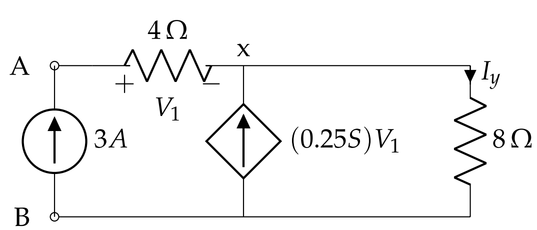


Figure 14: Finding equivalent resistance in cirucit with sources

using Kirchhoff's voltage law on the outermost loop, we have

$$V_{test} = V_1 + 8I_y$$

We also know that the current flowing into node x must be equal to the current flowing out. That is,

$$1 + (0.25S)V_1 = I_y$$

But since we know R_1 and I_1, we can find V_i. We simply have

$$V_1 = 1A \times 4\Omega = 4V$$

Meaning

$$I_y = 4 * 0.25 + 1 = 2$$

We can plug V_1 and I_y into $V_{test} = V_1 + 8I_y$ to get $V_{test} = 4 + 16 = 20$, and via $R_{eq} = \frac{V_{test}}{I_{test}}$ we have $R_{eq} = \frac{20V}{1A} = 20\Omega$.

Analysis

Let's now examine a couple of extremely powerful techniques we can use to analyze circuits, *nodal analysis* and *mesh analysis*.
Nodal analysis: a systematic method used to determine the voltage at every (essential) node in a circuit. After picking our reference (ground) node, we systematically apply KCL at every (essential) node in the circuit except, of course, for the ground node. By finding every nodal voltage in the circuit, we can find every branch current, which enables us to determine the power absorbed or delivered by every element. Because of its ability to be applied to any circuit, nodal analysis is the method most often used in circuit analysis computer programs. Here are the steps to perform nodal analysis:

1. Number all (essential) nodes of the given circuit.
2. Write KCL for every (essential) node by keeping in mind that only nodal voltages should be used (no currents). If other unknowns are involved (e.g. dependent source equations) express them as a function of the unknown nodal voltages (e.g. by using Ohm's law).
3. Group the resulting equations together in a matrix form.
4. Solve for the unknown nodal voltages by inverting the resulting linear equation.
5. Calculate any quantity of interest (e.g power consumption) from the known nodal voltages.

Consider an example cirucit, fig. 15. Let's apply our steps to find the

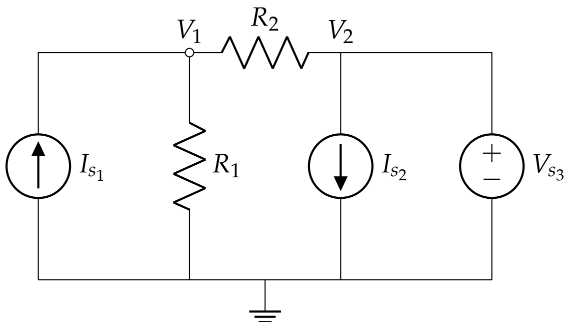


Figure 15: Using nodal analysis

nodal voltages V_1 and V_2.

1. Number the essential nodes (fig. 16).
2. Apply Kirchhoff's current law to each node

$$-I_{s_1} - \frac{0 - V_1}{R_1} - \frac{V_2 - V_1}{R_2} = 0$$

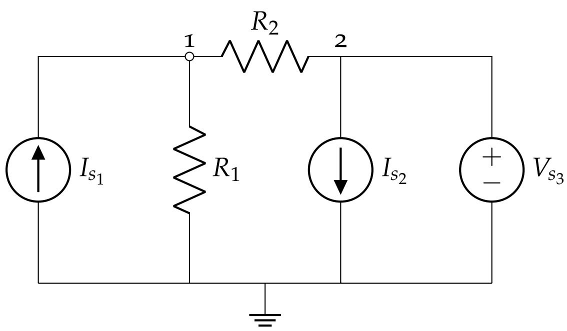


Figure 16: Numbering nodes

Here we can actually take a shortcut. Since the negative terminal of V_{s_3} is grounded, then the voltage at the positive terminal must be V_{s_3}. Since this terminal is connected directly to V_2, we know that $V_2 = V_{s_3}$. That makes our equations

$$\begin{aligned} -I_{s_1} - \frac{0 - V_1}{R_1} - \frac{V_2 - V_1}{R_2} &= 0 \\ V_2 &= V_{s_3} \end{aligned}$$

3. Let's group these by the variables we wish to find now.

$$\begin{aligned} V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_2\frac{1}{R_2} &= I_{s_1} \\ 0V_1 + V_2 &= V_{s_3} \end{aligned}$$

   Which in matrix form becomes

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_{s_1} \\ V_{s_3} \end{bmatrix}$$

4. From here, we could use a computer program to find the inverse matrix and multiply both sides by it to get the vector $[V_1, V_2]$ by itself. I'm not going to do this because I'm lazy, but if you're interested try using Mathematica or MATLAB.

If our circuit lacks a ground, we may simply choose some node as ground, since voltages are relative and having a grounded node makes analysis easier.

Mesh analysis: This method is used to determine every loop current in a circuit. We use KVL around meshes (loops) to find the mesh (loop) currents. We can then calculate any voltage or any branch current from the resulting mesh currents. (Basic) mesh analysis has a limitation in that it can only be applied to planar circuits.

Here are the steps to perform mesh analysis:

1. Choose your loops and draw mesh currents in them.
2. Use Kirchhoff's voltage law and the voltages you encounter to create a system of equations. If two mesh currents go against each other, the net current is their difference.
3. Group the resulting equations together in a matrix form.
4. Solve for any quantity of interest.

Note that you don't want two mesh currents flowing through the same current source. If you find a circuit where this occurs, make a larger loop where no current sources are shared.

Source transformations

Source transformation: Source transformation is a wonderful tool that can be used to change the originally given circuit to an equivalent and simpler circuit. Two important theorems are Thevenin's theorem and Norton's theorem.
Thevenin's theorem: states that it is possible to simplify any linear circuit, irrespective of how complex it is, to an equivalent circuit with a single voltage source and a series resistance. Here are the steps to

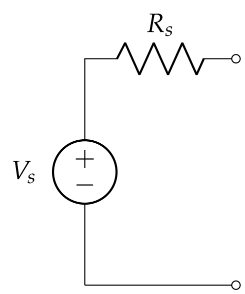


Figure 17: Thevenin equivalent

find the Thevenin equivalent circuit:

1. Remove the load resistor.

2. Find R_{th} by shorting all voltage sources and by open circuiting all the current sources and then see what the resistance looks like from the point of view of the nodes where the load resistor was located.

3. Find V_{Th} by finding the voltage across the nodes the load was originally hooked to, using standard circuit analysis methods.

4. Replace the load and find the current flowing through the load with these new values.

Norton's theorem: states that any linear circuit can be simplified to an equivalent circuit consisting of a single current source and parallel resistance that is connected to a load. Here are the steps to find the

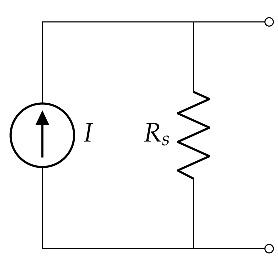


Figure 18: Norton equivalent

Norton equivalent circuit:

1. Remove the load resistor and replace it with a short circuit.

2. Find I by calculating the current through the short circuit where the load was.

3. Find R by creating an open circuit where the load resistor is, shorting all voltage sources and by open circuiting all the current sources. Once this is done, calculate the resistance seen by the open circuit.

4. Replace load and find the current flowing through the load or voltage across the load with these new values.

Essentially, there are three quantities we may be interested in finding.

(R_{eq}) Turn off all independent sources (dependent sources remain unchanged) and calculate the resulting resistance at the desired port. Notice that you may have to apply the i-v test if resistors cannot be combined through series and parallel connections, or if the circuit includes dependent sources.

(V_{th}) Leave the desired port open-circuited (i.e. no load connected) and find the voltage across it.

(I_N) Short-circuit the desired port (i.e. connect a short circuit across the port) and find the current through it.

Let's see how to transform between Norton and Thevenin circuits. Fig. 19 shows a circuit with one voltage source and a resistor in series.

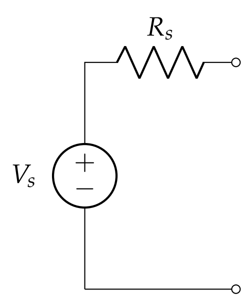


Figure 19: Thevenin to Norton

This series resistance normally represents the internal resistance of a practical voltage source. Let us short circuit the output terminals of the voltage source circuit as shown in fig. 20 We know that $V_s = IR_s$, where I is the current delivered by the voltage source when it is short circuited. Now, let's take a current source of the same current I which produces same open-circuit voltage at its open terminals as shown in fig. 21 Now we have $I = \frac{V_s}{R_t}$, meaning $R_s = R_t$. The open circuit voltage of both the sources is V_s and short circuit current of both sources is I. The same resistance connected in series in voltage source is connected in parallel in its equivalent current source. So, the voltage

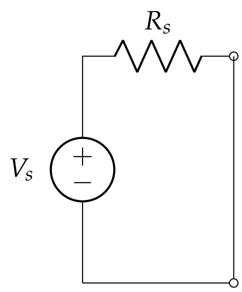


Figure 20: Thevenin to Norton

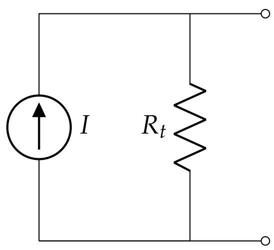


Figure 21: Thevenin to Norton

source and current source are equivalent to each other. Changing between forms is called a source transformation and can be used to simplify an electric circuit, since any place a voltage source and resistor are in series we can transform to a current source and resistor in parallel (and vice versa). If the voltage of the voltage source is V_{th} and the resistance is R, then the current of the equivalent current source will be $\frac{V_{th}}{R}$.

Let's do an example. Say we have the circuit in fig. 22 and we wish to find the Thevenin voltage and Norton current with respect to terminals a and b. Let's simplify this circuit a bit. First, notice that we

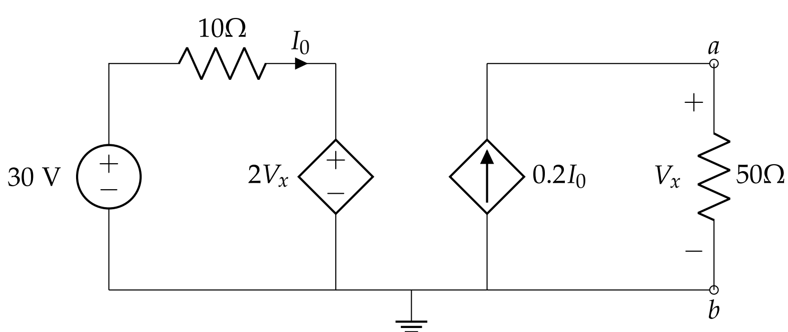


Figure 22: Source transformation example

have $30 - 2V_x$ V across the 10Ω resistor. By Ohm's law, $I_0 = \frac{30-2V_x}{10}$. Looking at the right loop, we have that the current flowing across the resistor is $0.2\frac{30-2V_x}{10}$. That means $V_x = 50 * 0.2\frac{30-2V_x}{10} = 10\frac{30-2V_x}{10}$. Let's organize our equations.

$$I_0 = \frac{30 - 2V_x}{10}$$
$$V_x = 10\frac{30 - 2V_x}{10}$$

In this case, we don't even need linear algebra to solve. The second equation yields $V_x = 10$ V. The Norton current is the current through a and b when their load resistance is shorted, so let's replace that 50Ω resistor with a short. That makes $V_x = 0$, and using KVL on the leftmost loop we find that $-30 + 2 \times 0 + 10I_0 = 0 \rightarrow I_0 = 3$. The Norton current is the current through that short between terminals a and b, which is simply $0.2 \times 3 = 0.6$ A. Notice that $V_x = V_{th}$ and we are done.

Well and good, but what if we want to try source transformations? Let's do just that on circuit 23. We could pretty easily solve this with

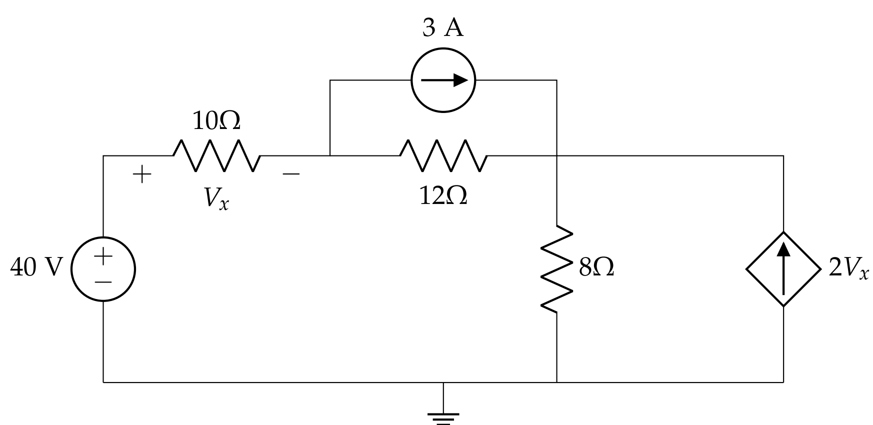


Figure 23: Source transformation example

nodal analysis, but we know that a current source I and resistor R in parallel can be simplified to a resistor and an RI voltage source in series. Performing this transformation, we have fig. 24. Now what? Looking at the 10Ω resistor, we see that the voltage across it is $40 - 36 = 4$ V. Since this is what we wished to find, we are done.

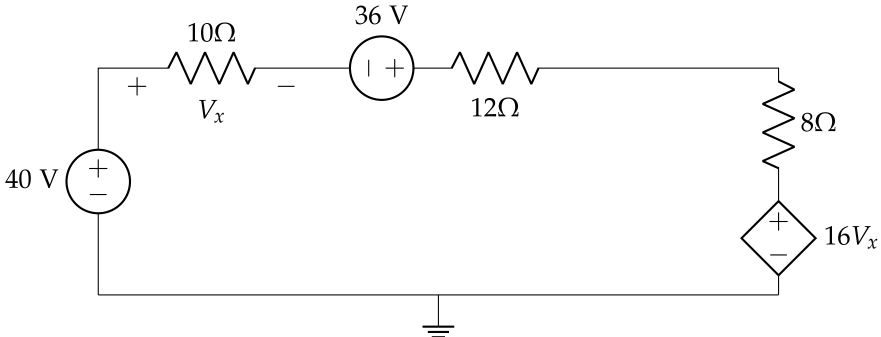


Figure 24: Source transformation example, with transformed sources

Linearity, superposition, and max power transfer

Linear system: a linear circuit is one in which the electronic components' values (such as resistance, capacitance, inductance, gain, etc.) do not change with the level of voltage or current in the circuit. Linear circuits are so called because they can be expressed as a linear combination of sources.
Superposition: for any linear and bilateral network or circuit with multiple independent sources, the response of an element will be equal to the sum of the responses of that element by considering one source at a time. For instance, say we have a circuit such as fig. 25 If

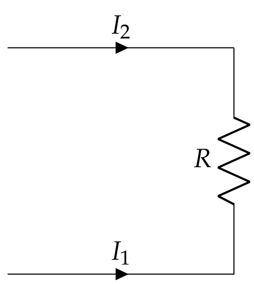


Figure 25: Superposition

we want to calculate the voltage across R, then we can simply sum the currents to get $V = (I_1 + I_2)R$ by the principle of superposition.

Say we have a linear system with two terminals, like fig. 26. We

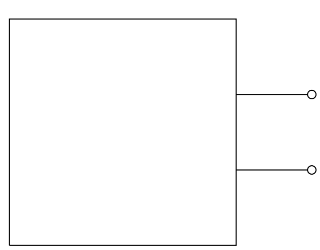


Figure 26: Linear system with two terminals

may be interested in finding the value of R_L that maximizes the power extracted across the terminals by a load resistor placed there. To see how to do this, let's consider the example fig. 27. We know that the

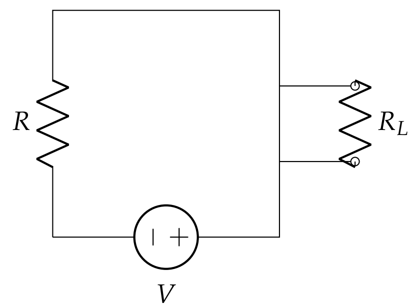


Figure 27: Max power extracted

amount of power dissapated is $P_L = I^2 R_L$. Substitute $I = \frac{VTh}{RTh+RL}$ in the above equation, and obtain

$$P_L = \left(\frac{V_{Th}}{(R_{Th} + R_L)}\right)^2 R_L.$$

For maximum or minimum, the first derivative will be zero. So, differentiate P_L with respect to R_L and make it equal to zero.

$$\begin{aligned}
\frac{dP_L}{dR_L} &= {V_{Th}}^2 \{ \frac{(R_{Th} + R_L)^2 \times 1 - R_L \times 2(R_{Th} + R_L)}{(R_{Th} + R_L)^4} \} = 0 \\
&\rightarrow (R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L) = 0 \\
&\rightarrow (R_{Th} + R_L)(R_{Th} + R_L - 2R_L) = 0 \\
&\rightarrow (R_{Th} - R_L) = 0 \\
&\rightarrow R_{Th} = R_L
\end{aligned}$$

Note that this implies $P_{max} = \frac{V_{th}^2}{4R_{th}}$.

Capacitors and inductors

Summary:

1. The *voltage* of a capacitor is always continous.
2. The *current* of an inductor is always continous.
3. $I_{cap} = C\frac{dV}{dt}$
4. $V_{ind} = L\frac{dI}{dt}$
5. Series capacitance: $\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$
6. Parallel capacitance: $C_{total} = C_1 + C_2 + \dots$
7. Series inductor: $L_{total} = L_1 + L_2 + \dots$
8. Parallel inductor: $\frac{1}{L_{total}} = \frac{1}{L_1} + \frac{1}{L_1} + \dots$

Capacitor: two conducting plates with a gap in between, connected to a voltage source. One plate will have positive charge Q and the other plate will have charge $-Q$. The units of capacitance are Faradays. The

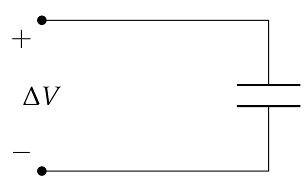


Figure 28: Capacitor

formula for capacitance is $i = C\frac{dV}{dt}$. This formula is true *only if* V is continuous.

Take the example of a current source hooked up to a capacitor, as in fig. 29. Say $C = 0.5F$. We know that $i = C\frac{dV}{dt}$, so

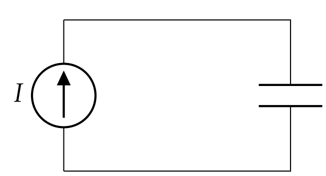


Figure 29: Capacitor with current source

$$V(t) = \int_0^t \frac{I}{C} dt.$$

If $I = 2t - 2$, for example, then we have that

$$\begin{aligned} V(t) &= 4\int_0^t (2t-2)dt \\ &= 4t^2 - 8t \end{aligned}$$

and we are done.

Say we have two capacitors in series. It can be shown that $\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2}$. Similarly, if the capacitors are in parallel instead, $C_{total} = C_1 + C_2$.

If the reader is familiar with the practical application of capacitors, they will know that capacitors are often used to store energy. To see how much energy can be stored, recall that $P = IV$ and $I = C\frac{dV}{dt}$. Therefore,

$$\begin{aligned} E(t_2) - E(t_1) &= \int_{t_1}^{t_2} P(t)dt \\ &= \int_{t_1}^{t_2} VC\frac{dV}{dt} \\ &= \frac{CV^2(t_2)}{2} - \frac{CV^2(t_1)}{2} \end{aligned}$$

Inductor: a passive two-terminal electrical component that stores energy in a magnetic field when electric current flows through it, according to the formula $V = L\frac{dI}{dt}$. The units of inductance are Henrys (H). Note that, comparable to capacitors, the given formula is true only if *current* is continous. We could show that if the current is continuous, then energy stored is equal to $\frac{1}{2}LI^2(t_2) - \frac{1}{2}LI^2(t_1)$ We could also show that for series inductance $L_{total} = L_1 + L_2 + \ldots$ and for parallel inductance $\frac{1}{L_{total}} = \frac{1}{L_1} + \frac{1}{L_1}$. To find the equivalent inductance between two terminals, simply turn off all independent sources and apply these rules.

First order circuit analysis

First order circuit: contains only one energy storage element (capacitor or inductor), and that can, therefore, be described using only a first order differential equation

As we have seen, the three major circuit components are resistors, capacitors, and inductors. We can combine these components into RC (resistor and capacitor) circuits, LR (inductor and resistor), and so on (though if we have more than one energy-storing component then we will need a high-order differential equation in order to solve the circuit). Recall

$$V = L\frac{di}{dt}$$
$$i = C\frac{dV}{dt}$$

In the steady state, an inductor acts as a short circuit, while a capacitor acts as an open circuit. In the first case we require that i be continuous and in the second that V be continuous. With this information in mind, we can start on first-order circuit analysis. Let's see an example with fig. 30.

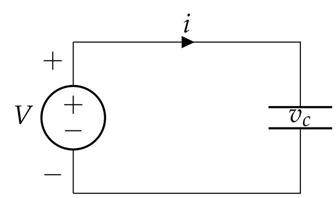


Figure 30: Capacitor with voltage source

It can be shown (I apologize for not showing it) that this circuit can be modeled by

$$\begin{aligned} v_c(t) &= v_c(\infty) + (v_c(t_0) - v_c(\infty))\, e^{(\frac{-1}{RC})(t-t_0)} \\ &= x(t_0)e^{\lambda(t-t_0)} \end{aligned}$$

Where $\lambda = \frac{-1}{RC}$ for RC and $-\frac{R}{L}$ for RL (we also have that $\lambda = -\frac{1}{\tau}$, where τ is called the time constant), R_{th} is the Thevenin resistance, and $v_c(t)$ is the voltage across the capacitor. RC and RL circuits are the only first-order circuits. LC, RLC, and so on all require higher-order differential equations to solve. In general, the equation for a circuit with a source is

$$\frac{dx}{dx} = \lambda x + f(t)$$

with solution

$$x(t_0)e^{\lambda(t-t_0)} + \int_{t_0}^{t} e^{\lambda(t-t_0)} f(\tau)d\tau$$

A useful function to know for diff. eq.s is the *unit step function*, $u(t)$.

$$u(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases}$$

To change when the function turns off, you can add an offset of c, like $u(t-c)$.

If the source is constant, the equation becomes

$$\frac{F}{-\lambda} + (x_0 - \frac{F}{-\lambda})e^{\lambda(t-t_0)}$$

where $F = f(\tau)$ is constant. This is where the equation

$$v_c(t) = v_c(\infty) + (v_c(t_0) - v_c(\infty))\, e^{(\frac{-1}{RC})(t-t_0)}$$

comes from.

Let's see an example. In fig. 31 the switch has been open for a long time. Thus, at t_0, the voltage $v_c(t_0)$ will be equal to the source

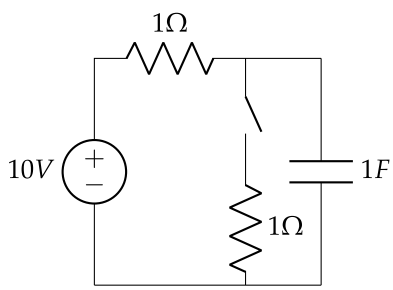


Figure 31: First order circuit example

voltage, 10. After the switch is closed and sufficient time has passed, the voltage will be across the two 1 Ω resistances, which are in series. Therefore $v_c(\infty) = 10V \times \frac{1\Omega}{1\Omega+1\Omega} = 5V$. Plugging these values into our equation for $v_c(t)$, we get

$$v_c(t) = 5 + (10-5)\, e^{(\frac{-1}{RC})(t-t_0)}$$

and all that remains is to find the resistance R. R is always calculated with respect to the capacitor, so in this case the capacitor will see two resistors in parallel and the total resistance will be $(\frac{1}{\frac{1}{1}+\frac{1}{1}}) = 1/2$, so our final equation is

$$v_c(t) = 5 + 5e^{-2t}$$

Response classification

We may write the general equation for voltage across a capacitor or current across an inductor as

$$x(\infty) + (x(t_0) - x(\infty))e^{\lambda(t-t_0)}.$$

This can itself be rewritten, as

$$x(t_0)e^{\lambda(t-t_0)} + x(\infty)(1 - e^{\lambda(t-t_0)})$$

The first term is called the *zero input response*, while the second is called the *zero state response*.
Zero input response: the natural response of a system to its initial conditions, that is, how the system behaves without any external inputs. It is the response that arises from the internal dynamics of the system, such as the energy stored in its capacitors or inductors.
Zero state response: the output of a system when the input to the system is zero.

This splitting can be immensely useful when your circuit has sources that turn on at different times. The total response will simply be the zero input response plus each zero state response. Allow me to show you what I mean with the circuit in fig. 32. Say the initial

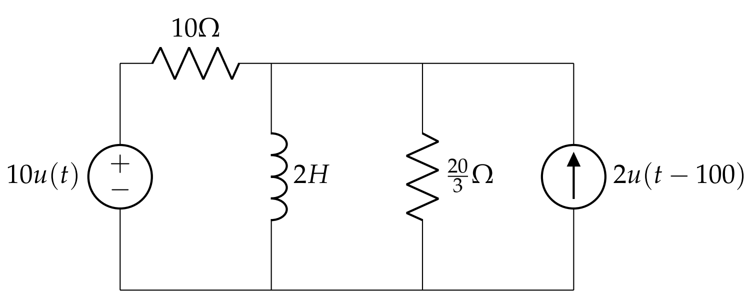


Figure 32: Response splitting example

current through the inductor is 1 amp upwards. Let's build our entire equation by finding the zero input response and then each of the zero state responses.

Zero input: Fig. 33 displays the zero input circuit. From the perspective of the inductor, there is a 10Ω and $\frac{20}{3}\Omega$ resistor in parallel, so the total resistance will be 4Ω. Thus we have

$$\begin{aligned} x(t_0)e^{\lambda(t-t_0)} &= I_L(t_0)e^{\frac{-R}{L}(t-t_0)} \\ &= 1 \times e^{\frac{-4}{2}(t-0)} \\ &= e^{-2t} \end{aligned}$$

as the zero input response.

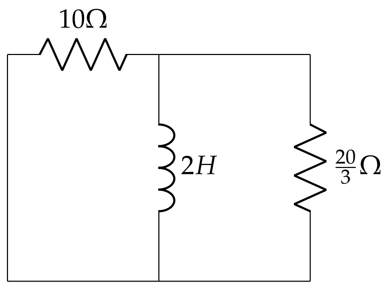


Figure 33: Zero input response

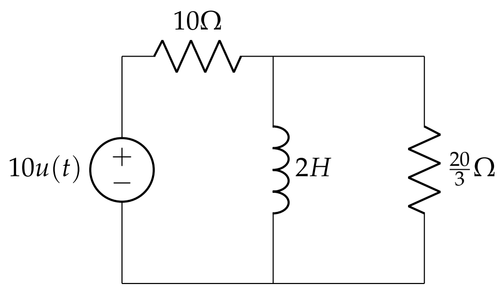


Figure 34: Zero state response for voltage source

Zero state response for voltage source: Fig. 34 displays the circuit for the zero state response in the case of the voltage source.

Let us now calculate the zero state response. At infinity, the inductor will behave as a short circuit and so $I_L(\infty) = \frac{10V}{10\Omega} = 1A$. From the perspective of the inductor, the resistance is again 4Ω.

In fact, the resistance in every response will be the same, since the find resistance we turn off every source anyway.

$$\begin{aligned} x(\infty)(1 - e^{\lambda(t-t_0)}) &= I_L(\infty)(1 - e^{\frac{-R}{L}(t-t_0)}) \\ &= 1 \times (1 - e^{\frac{-R}{2}(t-0)}) \\ &= 1 - e^{-2t} \end{aligned}$$

But we're not quite done! Because this source turns on at $t = 0$, we must multiply the zero state response by $u(t)$, to obtain

$$(1 - e^{-2t})u(t)$$

Zero state response for current source: Fig. 35 displays the circuit for the current source. By now we're experts. $R = 4$, $L = 2$, and $t_0 = 100$, so we already have

$$\begin{aligned} x(\infty)(1 - e^{\lambda(t-t_0)}) &= I_L(\infty)(1 - e^{\frac{-R}{L}(t-t_0)}) \\ &= I_L(\infty) \times (1 - e^{\frac{-4}{2}(t-100)}) \end{aligned}$$

$I(\infty)$ will simply be the current from the source, $2A$. Ergo,

$$\begin{aligned} I_L(\infty)(1 - e^{\frac{-R}{L}(t-t_0)}) &= I_L(\infty) \times (1 - e^{\frac{-4}{2}(t-100)}) \\ &= 2 \times (1 - e^{-2(t-100)}) \end{aligned}$$

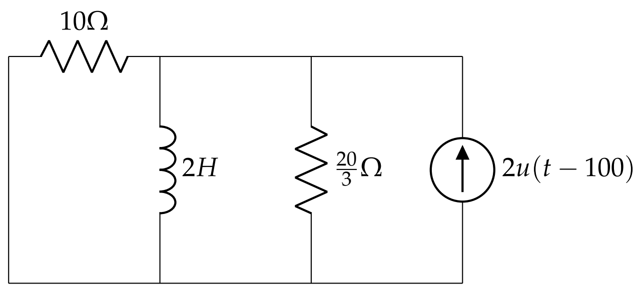


Figure 35: Zero state response for current source

Again we must multiply by $u(t-100)$, yielding a final answer of

$$2 \times (1 - e^{-2(t-100)})u(t-100)$$

Putting this all together, we obtain

$$I_L(t) = e^{-2t} + (1 - e^{-2t})u(t) + 2 \times (1 - e^{-2(t-100)})u(t-100)$$

and we are done.

Waveform generation

Again, take the equation

$$x(\infty) + (x(t_0) - x(\infty))e^{\lambda(t-t_0)}.$$

Select two value on this graph at times t_1 and t_2, x_1 and x_2. It can be shown that

$$\frac{x_1 - x_\infty}{x_2 - x_\infty} = e^{\lambda(t_t - t_2)}$$
$$t_1 - t_2 = \frac{1}{\lambda} \ln\left(\frac{x_1 - x_\infty}{x_2 - x_\infty}\right)$$
$$t_2 - t_1 = \tau \ln\left(\frac{x_1 - x_\infty}{x_2 - x_\infty}\right)$$

Phasors

Until this point we have been studying direct current (DC) circuits. DC circuits are easier to analyze and were the first to be studied historically, but most circuits you find in the real world are alternating current (AC) circuits. An AC source produce sinusoidal signals (current or voltage) of the form

$$v(t) = V_m \cos(\omega t + \theta)$$

for voltage, and

$$i(t) = I_m \cos(\omega t + \theta)$$

for current. Here is the name for each term in the sinusoidal equation:

V_m Magnitude of voltage: amplitude of voltage graph.

I_m Magnitude of current: amplitude of current graph.

ω Radial frequency: equal to $\frac{2\pi}{T}$, where T is the period.

ϕ Phase shift: horizontal displacement of the graph from the origin.

Someone with sufficient background in math may see a sinusoidal equation and immediately think of complex numbers. Complex numbers can vastly simplify circuit analysis, as we will shortly see. Complex numbers may be expressed in rectangular or polar form interchangeably, via Euler's identity. That is,

In circuit analysis, i is usually reserved for current. Thus the symbol used for $\sqrt{-1}$ is j: $j = \sqrt{-1}$.

$$e^{j\theta} = \cos(\theta) + j\sin(\theta).$$

The utility of complex numbers comes from a couple interesting properties. First, if the real part of one complex number is equal to the real part of another, then the two complex numbers must be equal. That is,

$$\Re[Ae^{j\omega t}] = \Re[Be^{j\omega t}]$$
$$\Longleftrightarrow$$
$$Ae^{j\omega t} = Be^{j\omega t}$$

The second property relates to the sum of derivatives and integrals of a complex number z.

$$Az + B\frac{dz}{dt} + c\frac{d^2z}{dt^2} + \cdots + \int zdt + \int\int zdt + \cdots = \alpha z$$

We will also find out that any circuit element will transform into what we will call an impedance Z which will help us treat all elements as resistors.

Now, on to the meat of this section: phasors.

Phasor: a complex number representing a sinusoidal function whose amplitude (A), angular frequency (ω), and initial phase (θ) are time-invariant. A phasor representation is a method or a way to treat sinusoids as complex numbers or vectors. We represent a sinusoidal signal as a complex number which will make operations of sinusoids much easier. The only requirement of this method is that all sinusoids involved have to oscillate at the same frequency. If you have a signal

If we had multiple frequencies in our circuit, we would have to use our powerful method of superposition instead.

$$x(t) = Acos(\omega t + \theta^\circ)$$

the phasor is given by

$$\tilde{X} = Ae^{j\theta}$$

Let's examine a few different signals and how they compare.

Note that if the signal is in terms of sine, then it must be transformed to cosine before the phasor can be calculated (the relationship $\sin(\theta) = \cos(90 - \theta)$ is useful here).

$$y(t) = 2Acos(\omega t + \phi^\circ)$$
$$z(t) = \frac{1}{2}Acos(\omega t)$$

In the lingo of electrical engineering, we say $y(t)$ *leads* $z(t)$ by ϕ°, while $z(t)$ *lags* $y(t)$ by ϕ°.

Impedance: a complex number (not a phasor) that is equal to the ratio of the phasor voltage over the phasor current of an element ($\frac{V(\omega)}{I(\omega)}$). It is defined only for AC signals and is measured in Ω. In general it is a function of frequency and is a measure of the difficulty that AC current faces when flowing through a device.

- Impedance of resistor R: $Z = R$
- Impedance of capacitor C: $Z = \frac{-j}{\omega C}$

- Impedance of inductor L: $Z = j\omega L$

Impedances are basically resistances for AC current. Just as with resistances, series impedances are summed while impedances in parallel follow the same reciprocal pattern as resistances.

Series $Z_{eq} = \sum_{i=0}^{n} Z_i$

Parallel $\frac{1}{Z_{eq}} = \sum_{i=0}^{n} \frac{1}{Z_i}$

Again, just as with resistances, the reciprocal of impedance has units of mhos or siemens, although in this case it is called *admittance*. Since impedance is defined as

$$\begin{aligned} Z &= \frac{\tilde{V}}{\tilde{I}} \\ &= \frac{V_0 e^{j\theta_v}}{I_0 e^{j\theta_i}} \\ &= \frac{V_0}{I_0} e^{j(\theta_v - \theta_i)} \end{aligned}$$

we have for free that

$$\angle Z = \theta_v - \theta_i$$

So the phase of the impedance tells us about the phase difference between the current and the voltage.

Every technique that we used to analyze DC circuits can be applied to AC circuits, with some small changes.

- Series and parallel combinations of impedance elements
- Mesh analysis
- Nodal analysis
- Superposition
- Source transformations
- Thevenin equivalents

Superposition allows us to solve circuits with sources of different frequencies, since normally we can only add phasors if the frequencies are identical.

Average power

Recall from your calculus courses that the average value of a function over an interval $[a, b]$ is

$$\frac{1}{b-a} \int_a^b f(x) dx.$$

The definition of instantaneous power is valid for AC sources.

$$p(t) = v(t) i(t)$$

Thus, we can use the formula for instantaneous power to calculate average power absorbed by an element. Let's first find an expression for power in an AC circuit.

$$\begin{aligned} p(t) &= i(t)v(t) \\ &= (I_m \cos(\omega t + \theta_i))(V_m \cos(\omega t + v\theta_v)) \\ &= \frac{V_m I_m}{(} \cos(\omega t + \theta_i + \omega t + v\theta_v) + \cos((\omega t + v\theta_v) - \omega t + \theta_i)) \\ &= \frac{V_m I_m}{2} cos(2\omega t + \theta_v + \theta_i) + \frac{V_m I_m}{2} cos(\theta_v - \theta_i) \end{aligned}$$

To find the average power, let's use the period as our domain.

$$\begin{aligned} P &= \frac{1}{T} \int_{t_0}^{t_0+T} p(t)dt \\ &= \frac{V_m I_m}{2T} \int_{t_0}^{t_0+T} cos(2\omega t + \theta_v + \theta_i) + cos(\theta_v - \theta_i)dt \end{aligned}$$

Reason that, over a period, the integral of a sinusoidal function is zero. Therefore the only part that contributes to the average power is the $cos(\theta_v - \theta_i)$ term. Therefore the average power is given by

If you'd like, you may compute the integral and confirm this.

$$P = \frac{V_m I_m}{2} cos(\theta_v - \theta_i)$$

We can compute the average power for each element in our toolbox thus far.

R $\bar{P} = \frac{V_m I_m}{2} \cos(0°) = \frac{V_m I_m}{2}$

C $\bar{p} = \frac{V_m I_m}{2} \cos(-90°) = 0$

L $\bar{p} = \frac{V_m I_m}{2} \cos(90°) = 0$

So capacitors and inductors don't ever absorb power, although they do have impedance. The only components able to absorb power in a circuit are resistors. Therefore, the average power we calculate will be the power absorbed by the resistors in the circuit. This fact can be used to simplify calculations when finding average power absorbed.

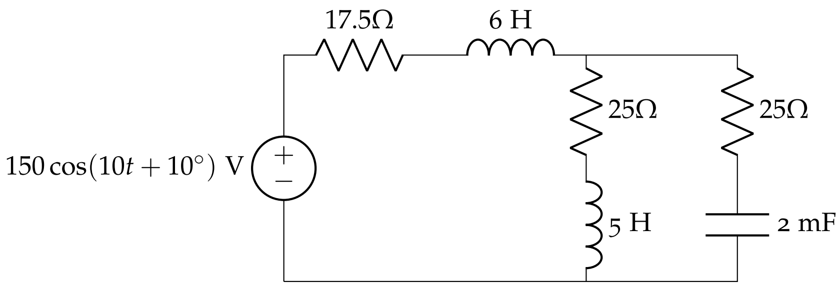
Let's see an example of this in fig. 36. We can calculate the equivalent impedance of the circuit to find the current. It works out to be

$$Z = 80 + j60\Omega,$$

making our current

$$\begin{aligned} I &= \frac{V}{Z} \\ &= \frac{150\angle 10°}{80 + j60} \\ &= 1.5\angle - 26.87° \end{aligned}$$

Figure 36: Average power



We then have that

$$\begin{aligned} P &= \frac{V_m I_m}{cos(\theta_v - \theta_i)} \\ &= \frac{150 \times 1.5}{2} \cos(10^\circ + 26.87^\circ) \\ &= 90W \end{aligned}$$

The effective value (also known as root mean square or RMS) of a signal is given by the formula

$$X_{rms} = X_{emf} = \sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} x^2(t) dt}.$$

If $v(t) = V_m \cos(\omega t + \theta_v)$, then

$$\begin{aligned} V_{RMS} &= \sqrt{\frac{1}{T} \int_{t_0}^{T+t_0} V_m^2 (\cos(\omega t + \theta_v))^2 dt} \\ &= \sqrt{\frac{1}{T} \int_{t_0}^{T+t_0} \frac{1}{2} + \frac{\cos(2\omega t + 2\theta_v)}{2} dt} \\ &= \sqrt{\frac{V_m^2}{T} \frac{T}{2}} \\ &= \frac{V_m}{\sqrt{2}} \end{aligned}$$

Notice that our expression for average power from earlier,

$$P = \frac{V_m I_m}{2} cos(\theta_v - \theta_i),$$

is therefore equal to

$$V_{RMS} I_{RMS} \cos(\theta_v - \theta_i).$$

This is true in general for complex impedance. In fact, it can be shown that

$$\begin{aligned} P &= V_{RMS} I_{RMS} \cos(\theta_v - \theta_i) \\ &= |\tilde{V_{RMS}}|^2 \mathbb{R}(\frac{1}{Z^*}) \end{aligned}$$

The impedance of capacitors and inductors varies based on the frequency of the input signal. If you have a circuit with these elements present, you can plot how the voltage out changes in response to frequency, as in fig. 37.

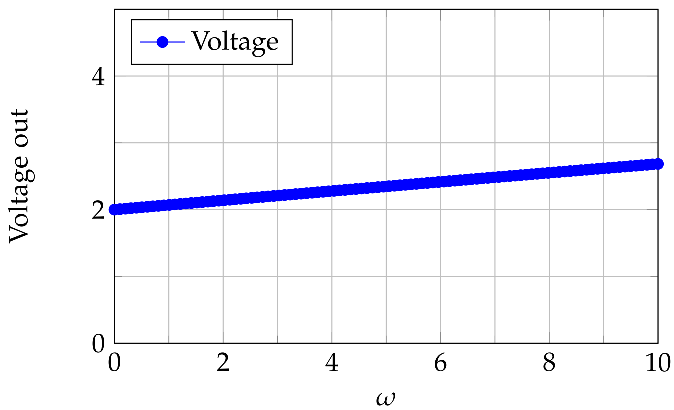


Figure 37: Voltage vs. Frequency

Say we have a system like fig. 38. We are interested in finding the

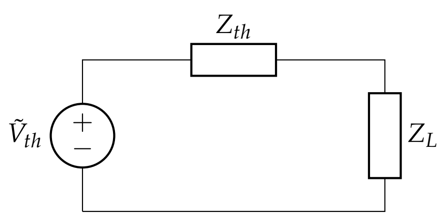


Figure 38: Power

load that extracts the maximum power. Recall that

$$P_{load} = \frac{1}{2} R_{load} |\tilde{I}_{load}|^2.$$

The current through the circuit is

$$\begin{aligned}
\tilde{I} &= \frac{\tilde{V}_{th}}{Z_{th} + Z_L} \\
&= \frac{\tilde{V}_{th}}{R_{th} + R_{load} + j(X_{th} + X_{load})} \\
|\tilde{I}| &= \frac{|\tilde{V}_{th}|}{\sqrt{(R_{th} + R_{load})^2 + (X_{th} + X_{load})^2}}
\end{aligned}$$

Ergo,

$$P_{load} = \frac{R_{load} |\tilde{V}_{th}|^2}{2((R_{th} + R_{load})^2 + (X_{th} + X_{load})^2)}$$

Which has a maximum when $X_{load} = -X_{th}$ and gives us a maximum possible value of

$$P_{load,max} = \frac{|\tilde{V}_{th}|^2}{8R_{th}}$$

Magnetically coupled circuits

Say we have a current-carrying loop, as in fig. 39. The magnetic flux

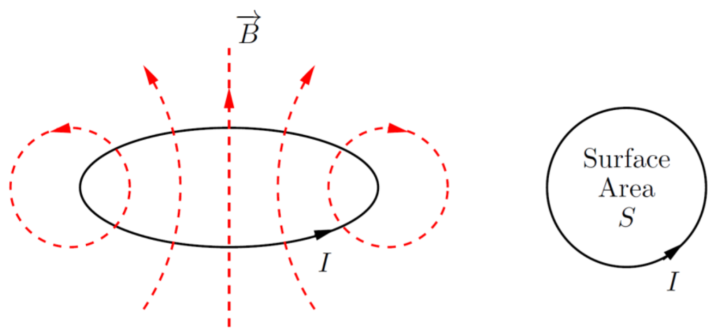


Figure 39: Flux

Φ is mathematically defined as

$$\Phi = \int_S \vec{B} \cdot d\vec{S}.$$

Intuitively, flux is a measure of the total magnetic field which passes through a given area. We can also define the electromotive force (emf) as

$$\epsilon = -\frac{d\Phi}{dt}$$

and the self-inductance as

$$L = \frac{\Phi}{I}.$$

We can also have two loops near each other, as in fig. 40. In this case,

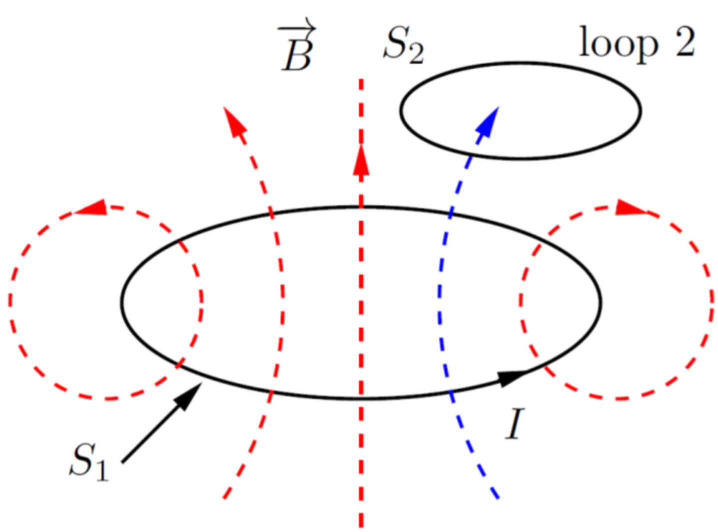


Figure 40: Mutual inductance

the flux through the second loop due to the current in the first is

$$\Phi_{12} = \int_{S_2} \vec{B}_1 \cdot d\vec{S}_2.$$

Similarly as in the case where we had one loop, the *mutual* inductance is defined as

$$L_{12} = \frac{\Phi_{12}}{I_1}.$$

We can represent magnetically coupled coils as in fig. 41. and this

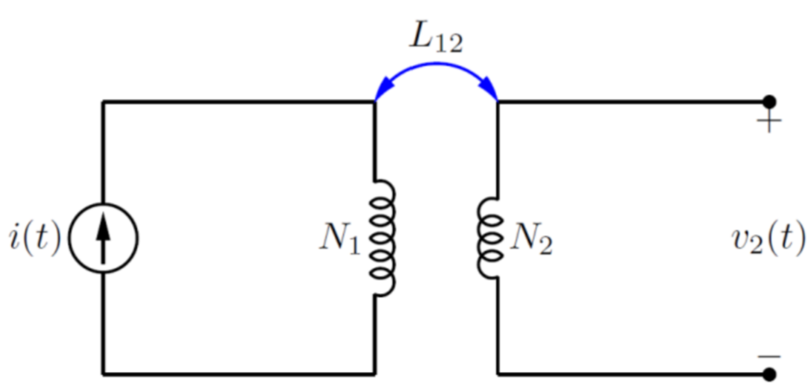


Figure 41: Coupled loops

system has a *coupling coefficient* of $k = \frac{L_{12}}{\sqrt{L_1 L_2}}$. Things can get messy when dealing with inductors, so to keep it straight, let's introduce the dot convention. The dot convention places a dot above an inductor and tells us that the current entering the dotted terminal of one coil (inductor) induces a positive voltage oriented towards the dotted terminal of the other coil. We know that

$$v_1(t) = L_1 \frac{di_1(t)}{dt} + L_{12} \frac{di_2(t)}{dt}$$
$$v_2(t) = L_1 \frac{di_2(t)}{dt} + L_{12} \frac{di_1(t)}{dt}$$

and we can find the direction of the induced voltage using fig. 42

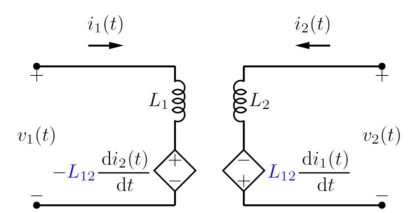


Figure 42: Voltage direction

The transformer is a component that uses magnetic coupling, and is shown in fig. 43. The circuit diagram is shown in fig. 44. Interestingly,

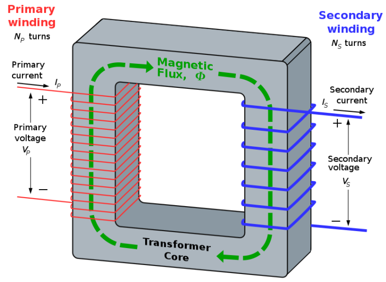


Figure 43: Transformer

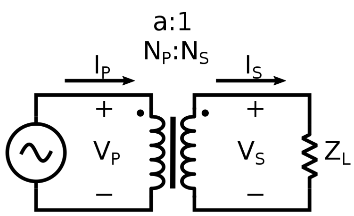


Figure 44: Transformer schematic

for transformers, the ratio of the voltages is equal to the ratio of the number of turns in each coil. That is,

$$\frac{v_2(t)}{v_1(t)} = \frac{N_2}{N_1}.$$

We can also have a terminated transformer circuit, as shown in fig. 45.

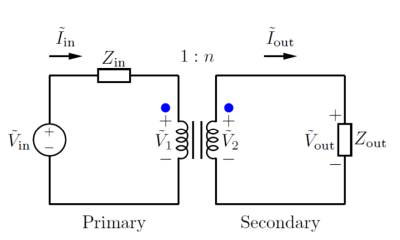


Figure 45: Terminated transformer

In this case, we have that

$$\tilde{V}_{in} = Z_{in}\tilde{I}_{in} + \tilde{V}_1$$
$$\tilde{V}_2 = Z_{out}\frac{\tilde{I}_{in}}{n}$$
$$\frac{\tilde{V}_{in}}{\tilde{I}_{in}} = Z_{in} + \frac{Z_{out}}{n^2}$$

This final expression is the input impedance of a terminated ideal transformer.

This is a lot of new information presented very briefly, so to cement it, let's practice with a few problems. Say we have the circuit in fig. 46. We know that $\tilde{V}_s = 10V_{rms}$ and that the mutual inductance is $j5\Omega$. We

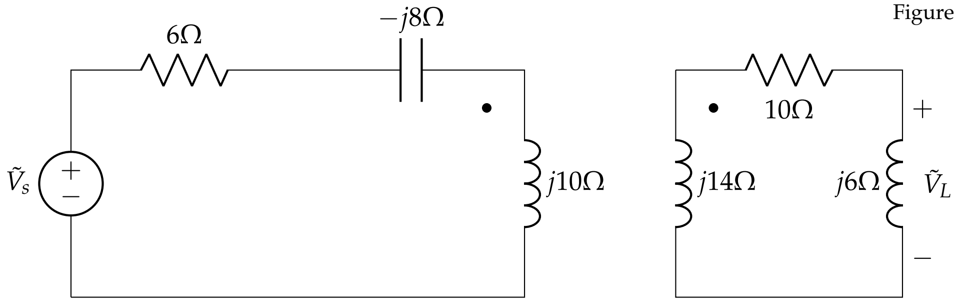


Figure 46: Problem 1

wish to find $\tilde{V}_L$ (since we are not told n_1 and n_2, let's assume they're $1:1$). Recall that

$$v_1(t) = L_1\frac{di_1(t)}{dt} + L_{12}\frac{di_2(t)}{dt}$$
$$v_2(t) = L_1\frac{di_2(t)}{dt} + L_{12}\frac{di_1(t)}{dt}$$

Where $v_1(t)$ is the voltage across the first inductor, and likewise for $v_2(t)$. Ergo,

$$10V_{rms} - \frac{10V_{rms}(6-j8)}{6+j2} =$$

Atomic structure and semiconductors

All materials can be classified as insulators, semiconductors, or conductors. We are familiar with insulators and conductors: the former does not conduct charge at all, while the latter is very good at conducting charge. Semiconductors lie in between in terms of conductivity.
Semiconductors: any of a class of crystalline solids intermediate in electrical conductivity between a conductor and an insulator. Semiconductors can be elemental (silicon, germanium, etc.) or compounds (silicon carbide, gallium nitride, etc.).

If we recall the structure of an atom, we will remember that there are discrete energy levels the electrons occupy. Adding energy to the system will move the electrons into a higher energy level. One of the most important materials in semiconducting is silicon, which luckily makes up around 28.2% of the matter in the earth's crust. Silicon is a crystal with an atomic structure that makes it ideal for semiconducting. By adding energy to silicon, we can excite electrons in the orbitals of the atoms and cause them to flow along the material. When an electron flows away, it leaves behind a postively charged "hole". Electrons that are bound to atoms are in the "valence band", while electrons free to flow are in the "conduction band". We can excite electrons by increasing the temperature of the material. If it is at

0 K, all electrons are in the valence band. If silicon is at room temperature, around 10^10 electrons and locomoting about. For electrons to move between the valence band and the conduction band, they need to cross the "band gap", which is the difference in energy between the top of the valence band and the bottom of the conduction band. Insulators have an extremely wide band gap, while conductors have an overlap between the conduction and valence bands. For silicon

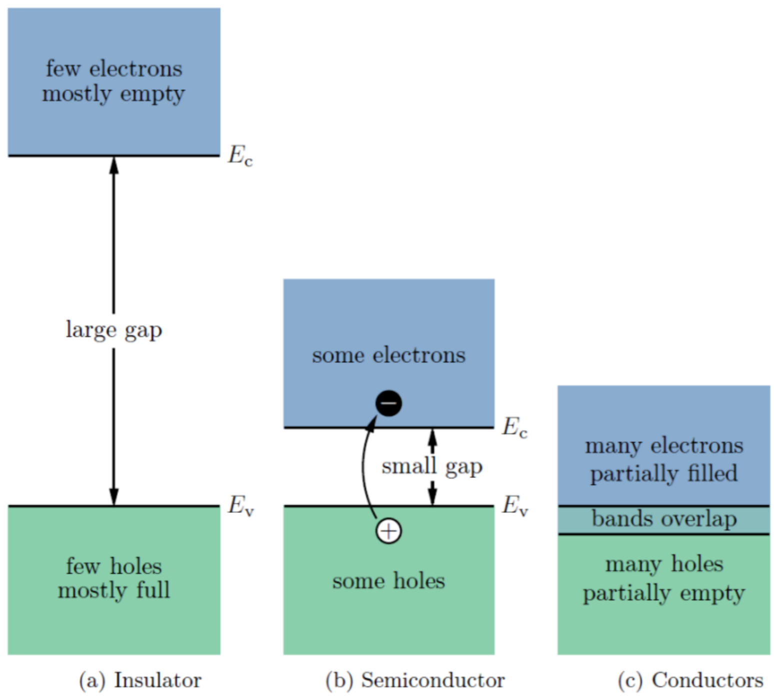


Figure 47: Band gap for different types of materials

dioxide, the energy of the band gap is approximately $8eV$.

To summarize,

- The atomic structure of atoms influences the structure of their crystals.
- The energy bands of materials determine their electrical properties.
- In semiconductors, the valence and conduction band is seperated by the band gap.
- Carrier density in intrinsic semiconductors increases with temperature.