Notes for ECE 30100 - Signals and Systems

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Contents

Introduction

A signal can be continuous time (CT) signal, which has an independent continuous variable indexed $t \in \mathbb{R}$, or discrete time (DT) which has a discrete independent variable indexed $n \in \mathbb{N}(\mathbb{Z})$.
$(.) \rightarrow CT$
$[.] \rightarrow DT$

A system, on the other hand, is something that transforms inputs into outputs.

$$input \rightarrow [\textbf{SYSTEM}] \rightarrow output$$

Another way this could be represented is:

$$system(input, t) = output$$

These can also be divided into CT and DT.
A CT system is of the form:

$$x(t) \rightarrow [CT] \rightarrow y(t)$$

On the other hand, a DT is of the form:

$$x[n] \rightarrow [DT] \rightarrow y[n]$$

Note : For most of the course, continuous and discrete will be analyzed separately. That is, only a couple topics will have CT inputs with DT outputs, or DT inputs with CT outputs.
Note : Most of the analyzed systems we will be linear and time invariant.

Linearity

A system is linear if superposition holds. That is, if it can be analyzed by analyzing the individual components of the system and combining it.
A more formal definition would be "given an input, which can be represented as the weighted sum of several inputs, the output can be represented as the sum of several weighted outputs".
The necessary and sufficient conditions for linearity are:

- CT: $\alpha_1 x_1(t) + \alpha_2 x_2(t) \cdots \xrightarrow{S} \alpha_1 y_1(t) + \alpha_2 y_2(t) \cdots$
- DT: $\alpha_1 x_1[n] + \alpha_2 x_2[n] \cdots \xrightarrow{S} \alpha_1 y_1[n] + \alpha_2 y_2[n] \cdots$

for all values of α.
Linearity gives us an alternative way to represent and analyze a system. That is, if we know the responses of all the subcomponents of the input, we can calculate the response of the input without having to calculate it for the input directly.

Signal Classification

1. DT vs. CT:

   - DT: $x[n]$ is a sequence of either real or complex valued numbers. $x[n]$ can be written as $x_{Re}[n] + jx_{Im}[n]$ or as $A[n]e^{j\phi[n]}$.
     We can transform between the two notations using Euler's formula:

     $$x[n] = A[n]e^{j\phi[n]} = A[n]\cos(\phi[n]) + jA[n]\sin(\phi[n])$$

   - CT: $x(t)$ behaves similarly, with the only difference being that it is in terms of t. Euler's formula still applies.

2. Energy and Power:

   - Energy is the area under the squared magnitude of the signal, and it represents how costly it is to store and/or transmit the signal.
     For CT signals, energy over times $t \in (t_1, t_2)$ is

     $$E = \int_{t_1}^{t_2} |x(t)|^2 dt = \int_{t_1}^{t_2} (x_{Re}^2(t) + jx_{Im}^2(t))dt$$

     For DT signals, energy over $n \in [n_1, n_2]$ is:

     $$E = \sum_{n=n_1}^{n_2} |x[n]|^2 = \sum_{n=n_1}^{n_2} (x_{Re}^2[n] + jx_{Im}^2[n])$$

     The total energy can be written as:

     $$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 dt = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

   - Power:
     For CT signals, average power over (t_1, t_2) is

     $$P = \frac{1}{t_2 - t_1} E(t_1, t_2)$$

     For DT signals:

     $$P = \frac{1}{n_2 - n_1 + 1} E[n_1, n_2]$$

     And the overall average power will be:

     $$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

     And finally, instantaneous power is $|x(t)|^2$ or $|x[n]|^2$

   There are three realistic types of signals:

(a) E_∞ finite: Must have $P_\infty = 0$

(b) P_∞ finite: Must have $e_\infty = \infty$, since we are integrating over time.

(c) Neither E_∞ nor P_∞ are finite: not practical, but mathematically possible.

Note: We cannot have finite energy, ∞ power signals.