

Notes for ECE 30500 - Semiconductor Devices

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Properties of Silicon

The core of semiconductors lies in the silicon transistor. But, why silicon (Si)?

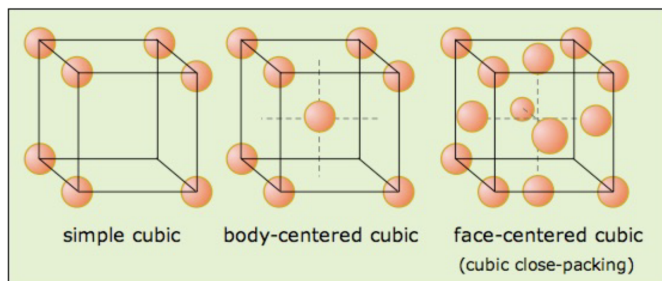
- Si is the second most common element on Earth.
- It is easily purified, and grown defect free, with less than 1 impurity in 10^9 atoms.
- Reasonably good electronic properties
- Resilient to harsh environments
- Excellent mechanical properties
- There are three forms of Si:
 - In a Si crystal, atoms are arranged in an orderly array, allowing arrangements to be easily reproduced.
 - In poly-crystalline Si, many crystalline subsections exist.
 - In amorphous Si, there are no long range patterns or arrangements.

The unit cell is a portion of any crystal that could be used to reproduce the crystal.

The primitive cell is the smallest possible unit cell.

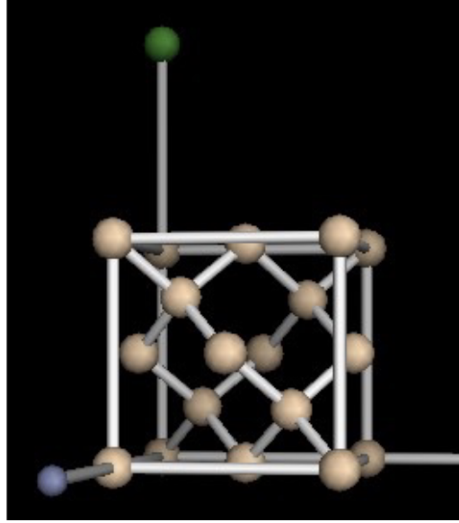
In essence, a unit cell is a subset of a lattice that can be moved in the x, y, z axis, and cover the entire lattice. (*Note the absence of rotation movements.*)

Some examples of cells are the following:



Note: the image is missing a corner atom.

Another important cubic unit cell is the diamond cubic unit cell with 8 silicon atoms in the cell:



Density (diamond cube cell)

Lattice constant: $a = 5.3407 \text{ \AA}$

Atomic mass: 28.055 amu

Density: $\rho = \frac{8 \times 28.055 \times 1.6605 \times 10^{-23}}{(5.3407 \times 10^{-10})^3} \text{ kg/m}^3 = 2.3296 \text{ g/cm}^3$

Miller Indices

Let us consider a plane that intercepts the axes at $x_{int}, y_{int}, z_{int}$. The equation of the plane is:

$$\frac{x}{x_{int}} + \frac{y}{y_{int}} + \frac{z}{z_{int}} = 1$$

The vector that is perpendicular to this plane will have the same components as the Miller indices.

The Miller indices are defined as $LCM * (\frac{1}{x_{int}}, \frac{1}{y_{int}}, \frac{1}{z_{int}})$.

Energy Bands

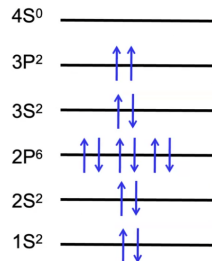
Quantization of Energy Levels

Bohr hypothesized, in his atomic model, that there was a quantization of electron angular momentum and energy levels.

In the context of this class, these levels will be described by $n \in \mathbb{N}$.

Additionally, the energy of these levels is $E_H = -\frac{13.6}{n^2} \text{ eV}$.

The energy levels of silicon are as shown below:

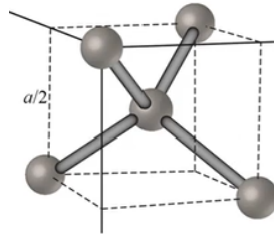


The further from the core, the more energy is within, so, in the image above, $4S^0$ is the layer with the most energy.

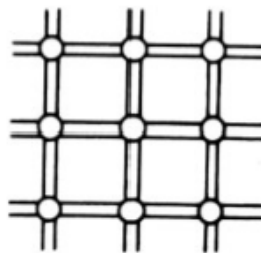
The four electrons in the valence region ($3P^2, 3S^2$) are the easiest to break away from the atom.

Bonding Model

Only the four valence electrons are of interest. Each of these, is shared with one of the four nearest neighbors, forming a structure like this:



This is the base of the silicon lattice, from which more complex lattices can be formed. The full model can be abstracted as follows:



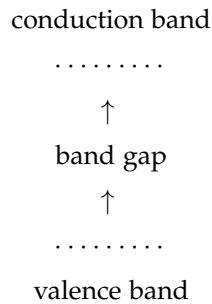
where a line is a shared valence electron and a circle is the core of the semiconductor.

Energy Levels

When going from a single Si atom to a Si crystal with many atoms, the atoms are closely packed enough that we cannot treat them as individual atoms. The structure that is formed is very stable as all the atoms and their energy levels are completely filled.

Note: Current cannot flow from full energy levels.

The energy levels of these atoms are in the following order:



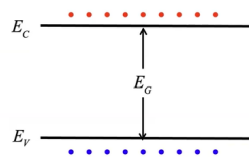
The conduction band and the valence band are energy levels where electrons can reside.

Electrons can be freed from their bonds by heating the lattice to temperatures close to 300K. The thermal energy provided by this is defined by:

$$E = \frac{3}{2}kT$$

The freed electrons are analyzed in terms of their energy states. When freed, they are moving from the valence band (leaving behind a "hole" in the valence band), jumping over the band gap and arriving at the conduction band.

The band representation can be further abstracted to look like this:



Where the red dots are conduction band electrons (n) and the blue dots are valence band holes (p).

Note: an intrinsic semiconductor is that in which $n = p = n_i$.

$$n_i = BT^{\frac{3}{2}} e^{-\frac{E_g}{2k_B T}}$$

Constants to Remember

$$E_G(\text{Si}) = 1.12\text{eV}$$

$$E_G(\text{GaAs}) = 1.4\text{eV}$$

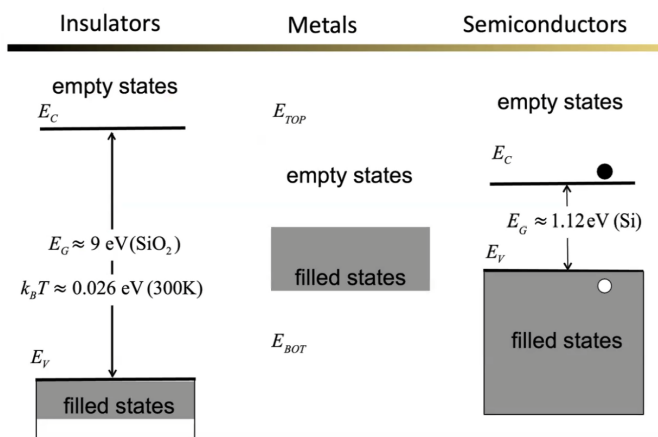
$$k_B T = 0.026\text{eV at } (T = 300\text{K})$$

$$P \approx e^{-E_G/k_B T}$$

$$n_i(\text{Si}) = 1 \times 10^{10}\text{cm}^{-3} \text{ at } (T = 300\text{K})$$

$$n_i(\text{GaAs}) = 2 \times 10^6\text{cm}^{-3} \text{ at } (T = 300\text{K})$$

Material Representations



Note that the main difference between an insulator and a semiconductor is the size of the band gap. On the other hand, metals have no band gap at all, and all the electrons are free by default.

Carrier Properties

Electrons are in the conduction band, and they can move between states.

Holes are in the valence band, and they can move between states.

Electrons and holes can recombine and regenerate.

When there are no perturbing forces, equilibrium occurs.

When a highly energetic photon hits an electron in the valence band, it pushes it into the conduction band and leaves a hole behind in the valence band.

$$E_{ph} = hf \gg E_G$$

Usually, the force on an object is defined by $F = m_0 a$, but, this cannot be so simply applied to the case of forces on electrons. Instead, the

formula is:

$$F = m_n^* a$$

where m_n^* is the effective mass of the electrons, and it includes

- Cyclotron
- Conductivity
- **Density of States**

Similarly, for a hole, $F = m_p^* a$.

Materials	m_n^*/m_0	m_p^*/m_0
Si	1.18	0.81
Ge	0.55	0.36
GaAs	0.066	0.52

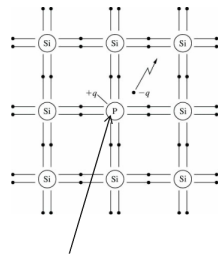
The energy of electrons in vacuum resemble a parabola when compared with momentum:

$$E = \frac{p^2}{2m_0} \rightarrow E_n = E_C + \frac{p^2}{2m_n^*}$$

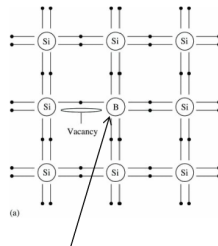
$$p = m_0 v \rightarrow p = \hbar k = \hbar \frac{2\pi}{\lambda}$$

Doping

The following is the process to n-dope a semiconductor sample:

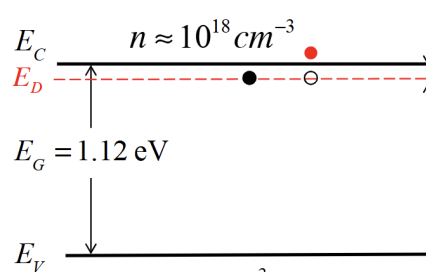


and the process for p-doping:



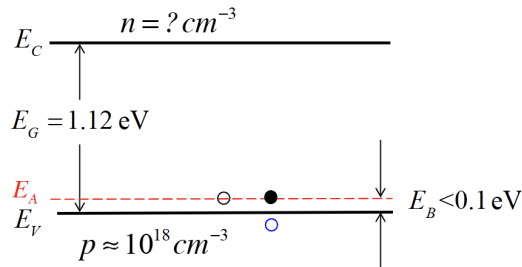
- N-doping: When n-doping, with group V elements, very little energy is necessary to break the fifth electron from the element. The ionized donor is then left with 4 electrons and is bound to the lattice.

The energy band diagram looks like the following:



The released donor electron joins the conduction band. This is easy due to how close E_D and E_C are.

- P-doping: Similar to n-doping, except, that it is with holes instead of electrons. Thus, the band diagram looks like this:



Once again, very little energy is necessary for the transition (accepting, in this case) to occur.

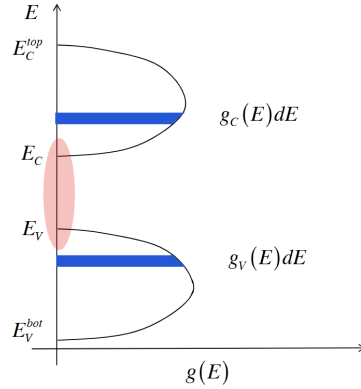
This whole process has a dependance on temperature, with higher temperatures providing more energy to the elements, and thus facilitating the bonding process.

Density of State (DOS)

Density of States ($g(E)$): Number of states per unit of energy per unit volume.

There are $4N_a \frac{\text{states}}{\text{band}}$ where $N_a = 5 \times 10^{22} / \text{cm}^3$.

$g(E)dE$ is the number of states in an energy range. Energy states are distributed as follows:



And, based on the above equation:

$$\int_{E_c^{top}}^{E_c} g_c(E) dE = 4N_a = \int_{E_v}^{E_v^{bot}} g_v(E) dE$$

which leads to some very complicated expressions which are later approximated using:

$$E = \frac{\hbar^2 k^2}{2m^*}$$

Fermi Level and Function

The following is the Fermi function:

$$f(E) = \frac{1}{1 + e^{(E-E_F)/k_B T}}$$

where $k_B = 8.617 \times 10^{-5} \text{ eV/K}$ is the Boltzmann Constant.

The Fermi function defines the probability of an electron energy state being filled at a given temperature within an atom in equilibrium.

Note: $f(E_F) = \frac{1}{2}$ because it is the very border, so it behaves as a step function, with 1 below and 0 above.

The Fermi Level (E_F) is the top of the energy levels electrons would have if they were at $T = 0\text{K}$. This is because, electrons cannot occupy the same energy level, and thus, at absolute zero, they pack themselves together at the lowest available energy levels, thus filling up all the levels beneath the Fermi Level. It can move up and down based on the composition of the material, being affected by things like doping.

Looking at the Fermi function allows us to notice the temperature dependence of the probability, with probabilities of high energies increasing with higher temperatures.

In an intrinsic semiconductor, the Fermi level is exactly between the conduction and valence bands, making the fermi function probabilities have the following property: $f(E_C) = 1 - f(E_V)$.

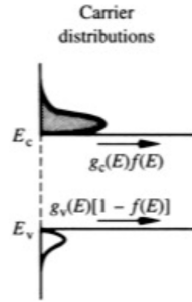
Carrier Distributions

Carriers within a band are distributed as follows:

$$n(E)dE = g_C(E)dE \times f(E)$$

$$p(E)dE = g_V(E)dE \times (1 - f(E))$$

This leads to a distribution that looks like this:



That is, at energies slightly higher than (or lower than) there are the most carriers.

Carrier Concentration

Non-Degenerate Semiconductor

For these, semiconductors, the following approximation can be made:

$$E_F - E_V \geq 3k_B T \implies f_h(E) \approx e^{(E - E_F)/k_B T}$$

$$E_C - E_F \geq 3k_B T \implies f_e(E) \approx e^{(E_F - E)/k_B T}$$

The concentration can be approximated to

$$n = N_C e^{(E_F - E_C)/k_B T}$$

where

$$N_C = 2 \left[\frac{m_n^* k_B T}{2\pi \hbar^2} \right]^{\frac{3}{2}}$$

and the equivalent for holes.

Carrier Concentration in terms of E_F

Intrinsic Density and np product

Calculations and Temperature dependency

Carrier Action: Drift

Devices depend on the motion of free carriers within semiconductors. We know how to calculate the number of free carriers, now we focus on their movement between regions.

Thermal Motion: Thermal Energy

$$E_T = \frac{3}{2}k_B T$$

Motion is completely random in thermal equilibrium, so, there is zero net current.

The average distance between collisions is known as the mean free path l_m . $l_m \approx 10\text{nm} - 1\mu\text{m}$ in Si. By assuming $V_{th} = 10^7\text{cm/s}$, we can get the mean free time, which is $\approx 1\text{ps}$.

Drift: Motion within Electric Field.

On a macroscopic scale, the carriers are moving in a certain direction (e^- towards $+$ and holes in the opposite direction).

On a microscopic scale, the carriers are bouncing around much more. However, they are still moving in the same general direction.

The electrons are moving opposite the electric field. The energy bands are thus modified to account for this by having a slope.

Band Bending

The electric field effectively bends the band diagrams.

E_c and E_v vary in the opposite direction of the voltage, that is, they are higher at lower voltages.

$$E(x) = -\frac{dV}{dx} = \frac{1}{q} \frac{dE_c}{dx} = \frac{1}{q} \frac{dE_v}{dx}$$

Note: In equilibrium, $E(x) = 0 \implies E_F = \text{constant}$

Electrostatic Potential

Electric Field within Semiconductor