

# *Notes for ECE 30411 - Electromagnetics I*

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*January 24, 2025*

These are lecture notes for Fall 2025 ECE 30500 by professor Elliott at Purdue. Modify, use, and distribute as you please.

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## Gradient, Divergence, and Curl

### Gradient

The gradient describes the spatial slope of a 3-dimensional function. It can only be applied to a scalar field.

Rectangular:

$$\nabla f = a_x \frac{\delta f}{\delta x} + a_y \frac{\delta f}{\delta y} + a_z \frac{\delta f}{\delta z}$$

Cylindrical:

$$\nabla f = a_\rho \frac{\delta f}{\delta \rho} + a_\phi \frac{1}{\rho} \frac{\delta f}{\delta \phi} + a_z \frac{\delta f}{\delta z}$$

Spherical:

$$\nabla f = a_R \frac{\delta f}{\delta R} + a_\theta \frac{1}{R} \frac{\delta f}{\delta \theta} + a_\phi \frac{1}{R \sin(\theta)} \frac{\delta f}{\delta \phi}$$

### Divergence

Describes the rate of change of a vector function.

Rectangular:

$$\nabla \cdot D = \left( a_x \frac{\delta}{\delta x} + a_y \frac{\delta}{\delta y} + a_z \frac{\delta}{\delta z} \right) \cdot (a_x D_x + a_y D_y + a_z D_z) = \frac{\delta D_x}{\delta x} + \frac{\delta D_y}{\delta y} + \frac{\delta D_z}{\delta z}$$

### Curl

Describes the rotation of a vector function.

Rectangular:

$$\nabla \times D = a_x \left( \frac{\delta D_z}{\delta y} - \frac{\delta D_y}{\delta z} \right) + a_y \left( \frac{\delta D_x}{\delta z} - \frac{\delta D_z}{\delta x} \right) + a_z \left( \frac{\delta D_y}{\delta x} - \frac{\delta D_x}{\delta y} \right)$$

Cylindrical:

$$\nabla \times D = a_\rho \left( \frac{\delta D_z}{\delta \phi} - \frac{\delta \rho D_\phi}{\delta z} \right) + \rho a_\phi \left( \frac{\delta D_\rho}{\delta z} - \frac{\delta D_z}{\delta \rho} \right) + a_z \left( \frac{\delta \rho D_\phi}{\delta \rho} - \frac{\delta D_\rho}{\delta \phi} \right)$$

### Identities

1.  $\nabla \times \nabla V = 0$ : the gradient does not rotate.
2.  $\nabla \cdot (\nabla \times A) = 0$ : the curl of a vector function does not diverge (grow).
3. A vector field whose divergence and curl are known is completely determined.

### *Electrostatics and Coulomb's Law*

Coulomb's law was determined using a torsion pendulum.

Using his experimental results, the following properties were derived:

- Direction is always along  $\mathbf{r}_2 - \mathbf{r}_1$
- Decreases in magnitude proportional to  $|\mathbf{r}_2 - \mathbf{r}_1|^{-2}$
- It can be repulsive or attractive depending on the sign of  $q_1 q_2$ .

$$\mathbf{F}_{12} = \frac{q_1 q_2 \mathbf{a}_{12}}{4\pi\epsilon_0 |\mathbf{r}_2 - \mathbf{r}_1|^2}$$

On the other hand the electric field at a point, due to a charge is defined as:

$$\mathbf{E}(\mathbf{r}_2) = \frac{q_1 \mathbf{a}_{12}}{4\pi\epsilon_0 |\mathbf{r}_2 - \mathbf{r}_1|^2} = \frac{q_1 (\mathbf{r}_2 - \mathbf{r}_1)}{4\pi\epsilon_0 |\mathbf{r}_2 - \mathbf{r}_1|^3}$$

Which in turn leads to the superposition property, which states:

$$\mathbf{E}_{total} = \sum_{\text{all charges}, i} \mathbf{E}_i$$

### *Mathematical properties of E*

The electric field has the following properties:

- $\nabla \times \mathbf{E} = 0$ : this is later modified and becomes Faraday's Law
- $\nabla \cdot \mathbf{E} = \frac{\rho_v}{\epsilon_0}$ : Also known as Gauss's Law

These two laws allow for the creation of Coulomb's Law, as well as superposition.

**Surface Integral**  $\int_s \mathbf{A} \cdot d\mathbf{s}$ : it is the integral across a surface  $A$ . In it  $\mathbf{A} \cdot d\mathbf{s} = A_n ds$ . That is, it integrates across the normal components of the surface. It is also known as the flux across the surface.