

Notes for PHYS 27200 - Electric And Magnetic Interactions

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Course Introduction

This is a calculus-based physics course using concepts of electric and magnetic fields and an atomic description of matter to describe polarization, fields produced by charge distributions, potential, electrical circuits, magnetic forces, induction, and related topics, leading to Maxwell's equations and electromagnetic radiation and an introduction to waves and interference. 3-D graphical simulations and numerical problem solving by computer are employed throughout. For more information, consult the syllabus.

Equations

1. Coulomb's Law: $\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$
2. Electric field due to a point or charged sphere: $\vec{E}_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r^2} \hat{r}$
3. Force due to electric field: $\vec{F}_2 = E_1 q_2$
4. Dipole moment between charges $-q$ and q separated by \vec{s} : $\vec{p} = q\vec{s}$
5. Electric field on dipole axis: $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{2sq}{r^3} \hat{p}$
6. Electric field on dipole bisecting plane: $\vec{E} = \frac{-1}{4\pi\epsilon_0} \frac{sq}{r^3} \hat{p}$

Electric charge

Electric Charge: Electric charge is an intrinsic characteristic of the fundamental particles that make up objects.

Conservation of charge: The net charge of a *closed system* never changes

Objects can have negative, zero, or positive charge. Charges are always multiples of the *elementary charge* $e = 1.60217662(63) \times 10^{-19} \text{C}$

Coulomb (C): One coulomb is the amount of charge that is transferred through the cross section of a wire in 1 second when there is a current of 1 ampere in the wire.

The charges of elementary particles are listed below.

Particle	Charge (elementary charge, e)
Electron (e^-)	-1
Positron (e^+)	+1
Proton (p^+)	+1
Anti-proton (p^-)	-1
Neutron	0
Anti-neutron	0
Photon	0

Point Charge: A charged object whose radius is much smaller than the distance between itself and all other objects of interest.

The magnitude of electric force between two point charges is directly proportional to the magnitude of each charge and inversely proportional to the distance squared. Specifically:

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

This is Coulomb's Law. Note the direction of the force changes with the sign of the charges involved. Like repels like and opposites attract.

Electric field

Consider a charged particle. We can represent its effect by drawing vectors that show the path a positively charged particle would follow if placed within its influence. These lines represent the electric field of the charged particle. The greater the density of the lines, the greater the strength of the electric field. Note that at the origin, the force is undefined (infinite), since $|r| = 0$.

There are many types of fields, which can be either scalar or vector. For example, a temperature map is a scalar field, since each point

It was mentioned in class that mass is likewise intrinsic. However, when discussing the Higgs field's role in giving particles mass, the distinction between mass as an intrinsic or emergent property becomes more nuanced. The mass of elementary particles like electrons and quarks is emergent in the sense that it arises from their interaction with the Higgs field, which itself is a fundamental aspect of the universe.

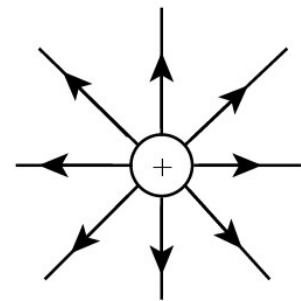


Figure 1: An electric field coming from point charge. Notice how the densities of the lines vary with distance from the source.

Technically, fields can be the more general tensor, or even the fascinating and exotic spinor!

is associated with a scalar (the temperature at that point). A map of fluid velocity is a vector field, since each point is associated with a vector (the velocity of the fluid at that point). For a particle, the electric field vector at any point is given by

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r^2} \hat{r}$$

The direction in which the field lines point depends on the sign of the charge. If the charge is negative, the field lines point in. If it is positive, the field lines point out. A useful mnemonic is to think of the charge as someone's STD test results. If it's negative, others will go for them and the lines point in. If it's positive, everyone will try to get away and the lines point out.

Consider the relative strengths of the electric and gravitational fields. The gravitational force is given by $F_g = G \frac{m_1 m_2}{r^2} \hat{r}$, with $m_{electron} = 9 \times 10^{-31} \text{kg}$ and $m_{proton} = 1.7 \times 10^{-27} \text{kg}$. If we consider a hydrogen atom, then $r = 5.3 \times 10^{-11} \text{m}$. With $G = 6.7 \times 10^{-11}$, we have

$$F_g = \frac{(1.7 \times 10^{-27})(9 \times 10^{-31})(6.7 \times 10^{-11})}{(5.3 \times 10^{-11})^2} \approx O(10^{-46}) \text{N}$$

Now, the electric force is given by $\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$. The charge of a proton and electron are $q_1 \approx q_2 \approx 1.6 \times 10^{-19} \text{C}$. Ergo, since $\frac{1}{4\pi\epsilon_0} \approx 8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}$,

$$F_e = \frac{(8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2})(1.60 \times 10^{-19} \text{C})^2}{(5.3 \times 10^{-11} \text{m})^2} \approx O(10^{-17}) \text{N}$$

This means that $\frac{F_e}{F_g} \approx 2.27 \times 10^{39}$, meaning the electric force is much stronger for these masses and charges than gravity. On scales as large as humans and planets, gravity is the dominant force because gravity is strictly additive.

For sufficient distances, the electric field of a uniformly charged spherical shell resembles the electric field of a point charge.

That means for $r \gg R$, $E_{sphere} = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r^2} \hat{r}$. This holds only for outside the sphere. Inside, it can be shown that the electric field is zero.

Superposition Principle: The net electric field at a location in space is a vector sum of the individual electric fields contributed by all charged particles located elsewhere.

To introduce systems with multiple sources of electric field lines, consider the particle pair known as a dipole. Dipoles consist of one negatively charged and one positively charged particle, like so:

Dipole Moment: The dipole moment is a way of expressing asymmetrical charge distribution. It is a vector quantity, i.e. it has magnitude as



Figure 2: Notice how a circle resembles a point from a great distance.

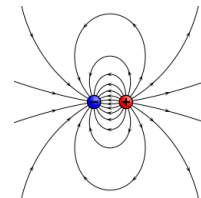


Figure 3: Two oppositely charged particles distanced from one another

well as definite directions. The dipole moment is given by the expression $\vec{p} = q\vec{d}$, where q is the charge on one end of the dipole and \vec{d} is the distance between dipoles.

On the axis of the dipole (i.e. the lines formed by the two particles), the electric field is

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{2sq}{r^3} \hat{p}$$

On the bisecting plane (i.e., the plane exactly halfway from each point) the field is given by

$$\vec{E} = \frac{-1}{4\pi\epsilon_0} \frac{sq}{r^3} \hat{p}$$

Where r is the distance from the point to either dipole. The force on a positive point charge q_1 a distance of d away from the dipole, aligned with the dipole, and on the side of the negative charge is given by

$$\vec{F} = q_1 \vec{E}_{dipole} = q_1 \left(\frac{-1}{4\pi\epsilon_0} \frac{2qs}{d^3}, 0, 0 \right)$$

Note that the field will be parallel to the axis of the dipole: this can simplify vector calculations. Now consider a dipole in a uniform electric field, like so: The positive end will be pulled to the right and the

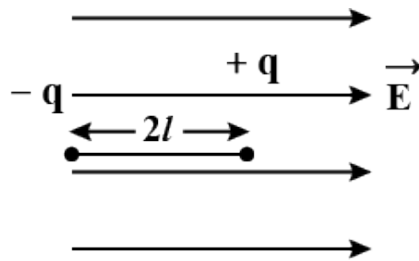


Figure 4: Dipole in uniform electric field

negative end to the left, exerting a torque given by $\vec{\tau} = \vec{p} \times \vec{E}_{uniform}$. Note that by the definition of \times , $\tau = pE_{uniform} \sin \theta$, where θ is the dipole's angle from horizontal. It can be shown that the potential energy of a dipole in a uniform electric field is $U = -\vec{p} \cdot \vec{E}_{uniform}$ and similarly as before (but now with \cdot) $U = -pE_{uniform} \cos \theta$.