Notes for ECE 20001 - EE Fundementals I

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These are lecture notes for fall 2023 ECE 20001 at Purdue. Modify, use, and distribute as you please.

Contents

Course Introduction

This course covers fundamental concepts and applications for electrical and computer engineers as well as for engineers who need to gain a broad understanding of these disciplines. The course starts by the basic concepts of charge, current, and voltage as well as their expressions with regards to resistors and resistive circuits. Essential concepts, devices, theorems, and applications of direct-current (DC), 1st order, and alternating-current (AC) circuits are subsequently discussed. Besides electrical devices and circuits, basic electronic components including diodes and transistors as well as their primary applications are also discussed. For more information, see the syllabus.

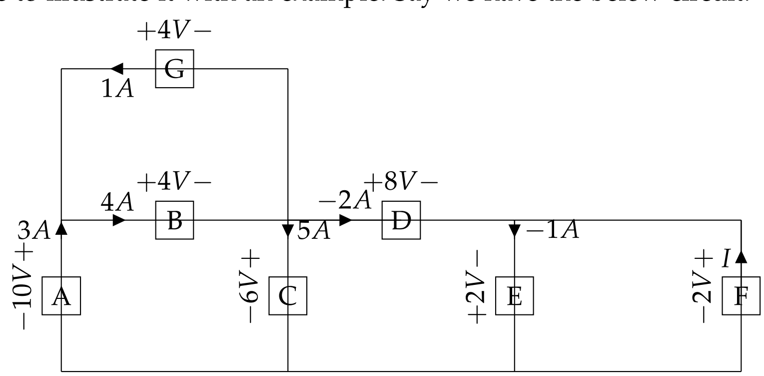
Equations

1. $P = \frac{dW}{dt} = IV$
2. $I = \frac{dq}{dt}$
3. $V = \frac{W}{q}$
4. $R = \frac{\rho L}{A}$
5. Ohm's Law: $V = IR$

6. Coulomb's Law: $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$
7. Kirchhoff's Voltage Law:
8. Conductance: $G = \frac{1}{R}$
9. Equivalent resistance: $R_{eq} = \frac{V_{test}}{I_{test}}$

Charge, current, voltage, and power

Before we begin a discussion of electrical engineering, we will want an understanding of some important concepts.
Charge: A fundemental property of matter. Charge is measured in units of Coulombs (C) and arises from aggregates of charged fundemental particles like electrons.
Current: The rate of flow of charge. Typically, we think of current as occuring in a circuit, a loop through which charge can flow. Current can be mathematically defined as $I = \frac{dq(t)}{dt}$, where $q(t)$ is the charge at time t. Current therefore has units of Coulombs per second (C/s).
Voltage: The difference in electric potential energy between two points, per unit charge. Voltage thus has units of $\frac{J}{C}$. or Volts (V). Intuitively, a stronger voltage source (such as a battery) will result in a higher current on identical wires. Imagine a voltage source as a concentration of negative charges on one side and positive charges on the other. If we hook both ends up to a conducting wire and place an electron in the wire, it will be repelled from the negative side and attracted to the positive side. The electron has a high potential energy near the concentration of negative charge, and a low potential energy near the positive charges. The difference between these potentials is the voltage. This should make it clear that voltage must always be defined as across two points, each with its electric potential. A good note is that voltage can be negative (if the electric potential at the second point measured is higher) and it may not be constant with time.
Power: The rate of doing work, or changing energy. Mathematically, $P = \frac{dW}{dt} = IV$ and has units of Watts (W). In a closed system, power is always balanced: whatever is put out by sources is consumed by loads. Thus, in a closed circuit, $\sum P = 0$ This is an important point, so allow me to illustrate it with an example. Say we have the below circuit:



We know $\sum P_i = \sum V_i I_i = 0$. Let's keep track of each P_i in a table, not forgetting passive sign convention: Summing the rows of this table, we have $\sum P_i = 2I - 2 = 0$, implying $I = 1$.

Symbol	Watts
P_A	-30
P_B	16
P_C	30
P_D	-16
P_E	2
P_F	$-2I$
P_G	-4

Table 1: Power absorbed

In the previous example, we had current flowing both into and out of the positive terminals of elements. We also had negative currents. Let's define how to handle signage in circuits:
Passive sign convention: Defines current as going into positive voltage node of a component. The component is labeled a *load* or a *passive device*. We call it this because the component consumes power. If a current is negative, it is flowing in the opposite direction shown by the arrow.

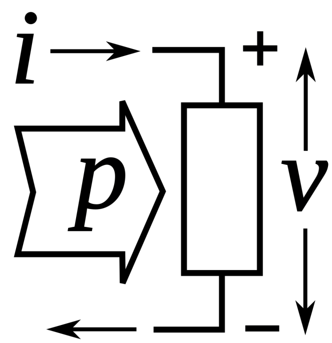


Figure 1: Passive sign convention

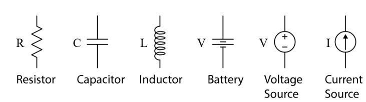
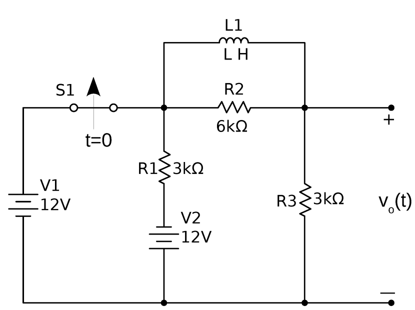


Figure 2: Common circuit symbols

It's useful to have an idea of the components of circuit schematics (visual representations of a circuit). Below is a list of the terms that will be used in this course:

- Elements: The term elements means "components and sources."
- Symbols: Elements are represented in schematics by symbols. Symbols for common 2-terminal elements are displayed to the right.
- Lines: Connections between elements are drawn as lines, which we often think of as "wires". On a schematic, these lines represent perfect conductors with zero resistance. Every component or source terminal touched by a line is at the same voltage.
- Dots: Connections between lines can be indicated by dots. Dots are an unambiguous indication that lines are connected. If the connection is obvious, you don't have to use a dot.

Check out the circuit schematic below and see how many components you can identify!



(In)dependent sources, connections, resistance and Ohm's Law

Now, on to what circuits are doing. For interesting things to happen we need electrons flowing through those wires, and for that we need sources. There are two types: independent and dependent. Independent sources are voltage sources or current sources that maintain a constant value regardless of the rest of the circuit. They are not influenced by the circuit's current or voltage conditions. There are two main types of independent sources:

- **Independent voltage source**: Maintains a constant voltage across its terminals, regardless of the current flowing through it. It is typically represented by a symbol with a plus sign and a minus sign, indicating the polarity of the voltage. A battery maintaining a constant voltage of 9 V is an example of an independent voltage source.

- **Independent current source**: Maintains a constant current through its terminals, regardless of the voltage across it. It is usually represented by a symbol with an arrow indicating the direction of current flow. Consider a current source that provides a constant current of 2 amperes. This source will deliver a current of 2A through any component connected to it, regardless of the voltage across the component.

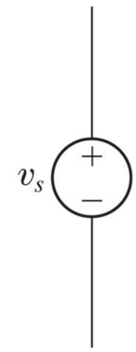


Figure 3: inependent voltage source

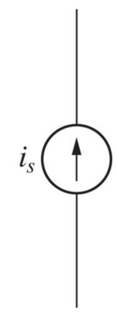


Figure 4: Independent current source

Contrasted with independent sources are dependent sources. Dependent sources are sources whose values are dependent on other variables within the circuit. These sources are used to model components whose behavior changes according to the conditions in the circuit. There are four types of dependent sources:

- **Voltage-Controlled Voltage Source (VCVS)**: This type of dependent source generates a voltage that is proportional to the voltage across a separate part of the circuit.

- **Current-Controlled Current Source (CCCS)**: This type of dependent source generates a current that is proportional to the current flowing through a different part of the circuit.

- **Voltage-Controlled Current Source (VCCS)**: This type of dependent source generates a current that is proportional to the voltage across a different part of the circuit.

- **Current-Controlled Voltage Source (CCVS)**: This type of dependent source generates a voltage that is proportional to the current flowing through a separate part of the circuit.

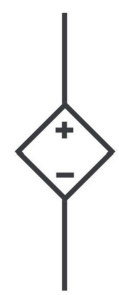


Figure 5: Dependent voltage source

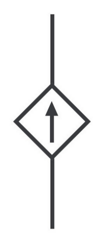


Figure 6: Dependent current source

In both the independent and dependent case, we assume the sources are ideal. There are two critical attributes of ideal sources. First, their value remains unchanged indefi- nitely. Second, they can deliver any amount of power needed by their loads. Turning off a voltage source is equivalent to replacing it with a short circuit (line). Turning off a current source is equivalent to replacing it with an open circuit (broken line). Also equivalent to an open circuit is a resistor with infinite resistance. Resistance is a measure of how hard it is to shove current through a resistor. The harder it is, the higher the resistance. It's given by $R = \frac{\rho L}{A} = \frac{V}{I}$, where ρ is the resistivity, L is the length of the resistor, and A is the cross-sectional area of the resistor. The reciprocal of resistance is conductance ($G = \frac{1}{R}$). We can relate voltage, current and resistance with *Ohm's Law*.
Ohm's Law: $V = IR$. This law is fundemental to EE and will occur repeatedly throughout the course.

Kirchhoff's Laws, resistor combinations, and voltage/current division

Kirchhoff's Current Law: The sum of all currents going into a node is 0. Mathematically, $\sum_k i_k = 0$. We may intuit this by imagining electricity as water flowing into a pipe. Anything that flows in must flow out. Say you have a circuit such as fig. 7. By Kirchhoff's current

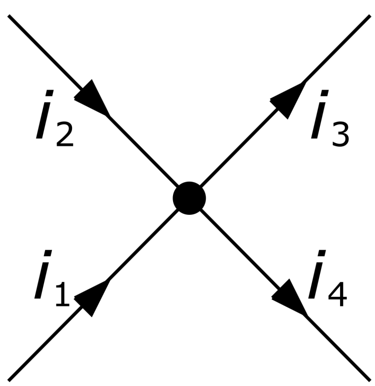


Figure 7: Kirchhoff's Current Law

law, we see that $i_1 + i_2 - i_3 - i_4 = 0 \rightarrow i_1 + i_2 = i_3 + i_4$. The neat thing is that this must be true for any node in a circuit. If you have a node with two known currents going in and one unkown comming out, Kirchhoff's current law tells us the unknown current is simply the sum of the known currents.

Contrasted with Kirchhoff's current law, we have Kirchhoff's voltage law.
Kirchhoff's Voltage Law: In a closed loop, the sum of all voltage

drops is zero. Mathematically, $\sum_k v_k = 0$. A useful way to visualize this is to think of voltage as potential energy. No matter where you go and how the voltage drops and rises, when you get back to where you began, the voltage must be the same there. You end up with the same gravitational potential if you return to the same spot, even if you run up and down a mountain. Imagine a closed loop such as fig. 8. Say the voltage across the battery is $15V$ and each resistor is identical. Although we don't know for sure yet, we can guess that the current will be the same through each, and ergo by Ohm's Law ($V = IR$) so will each voltage. Therefore,

$$15 + 3V = 0 \rightarrow V = -5$$

Now that we have a good idea of circuit components and how voltage/current behave, we can look at important configurations. These are the most useful:

Series Combination: In a series combination, the elements are connected with end to end in contact. We can use Kirchhoff's current law to show that the current at each point in the circuit is the same. Since current flows into a resistor it has no option but to flow out the same resistor. Say the resistances are R_1, R_2, and R_3. By Kirchhoff's voltage

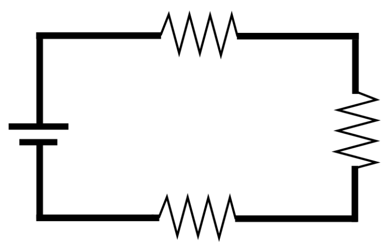


Figure 8: Series Combination

law, we have

$$\begin{aligned} V &= IR_1 + IR_2 + IR_3 \\ &= I(R_1 + R_2 + R_3) \\ &= IR_{total} \\ &\rightarrow R_{total} = R_1 + R_2 + R_3 \end{aligned}$$

Therefore for resistors in series, the total resistance is equal to the sum of each individual resistance.

Parallel Combination: When two or more resistances are connected between the same two points, they are said to be connected in parallel. Here, voltage is equal across all elements. In fig. 9, the total current is split among n different resistors. That is, $I = \sum_k^n i_k$. With a bit of

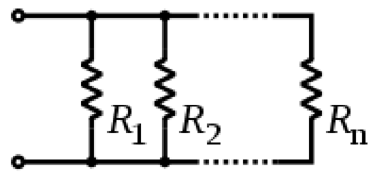


Figure 9: Parallel Combination

manipulation and Ohm's Law, we have that

$$\begin{aligned} I &= \frac{V}{R_{total}} \\ &= \sum_k^n i_k \\ &= \sum_k^n V/R_k \\ \rightarrow \frac{1}{R_{total}} \\ &= \sum_k^n \frac{1}{R_k} \end{aligned}$$

So for resistors in parallel, the reciprocal of the total resistance is the sum of the reciprocals of the individual resistances.

Now would be an excellent point to introduce the idea of

Equivalent resistance

Equivalent resistance: a simplification of a circuit with multiple resistors, where the resistance of the equivalent resistor maintains the same voltage and current relationship. We have already seen formulas for when the resistors are in parallel or series, and can use them to determine the equivalent resistance across two nodes. For instance, consider fig. 10. We can see that the 1Ω and the 10Ω resistor are in series, so their equivalent resistance is simply $1 + 10\Omega = 11\Omega$. We can condense the circuit to fig 11. We next combine the 2Ω and the 5Ω resistor to obtain an equivalent resistance of 7Ω, and the 3Ω and the 1.5Ω to obtain 4.5Ω (fig. 12) Now that we are left with only resistors in parallel, we may use the formula $\frac{1}{R_{eq}} = \sum_k \frac{1}{R_k}$ to obtain $\frac{1}{R_{eq}} = \frac{1}{11} + \frac{1}{2} + \frac{1}{1.5} = \frac{83}{66}$, yielding $R_{eq} = \frac{66}{83}\Omega$. Note that this method is only possible when the circuit has resistors in series or parallel. For more complex situations, such as when souces are present or resistors are combined in neither series nor parallel, we must use Kirchhoff's laws and good judgement to find the equivalent resistance. Consider

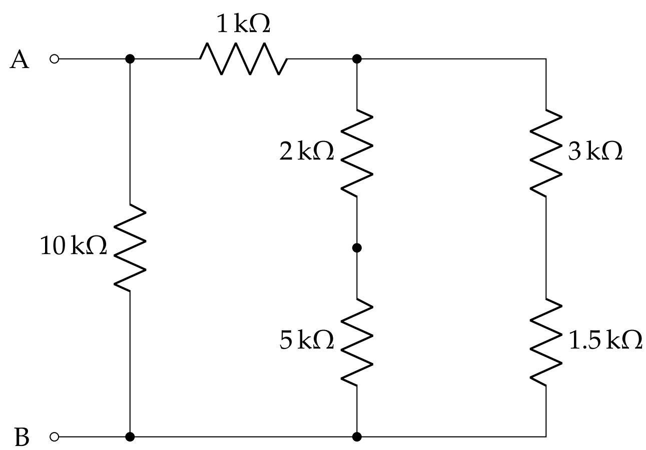


Figure 10: Finding equivalent resistance in circuit with resistors in series and parallel, step 1

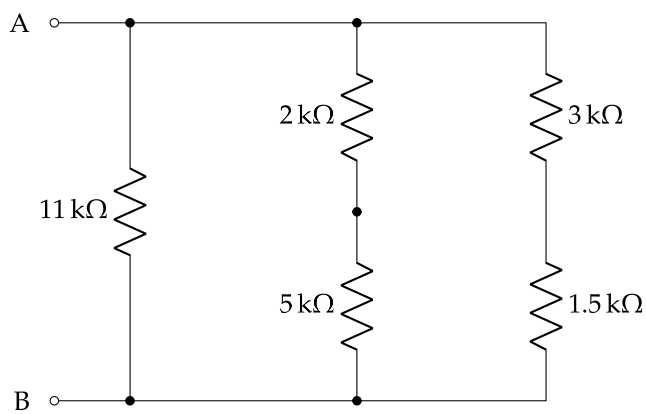


Figure 11: Finding equivalent resistance in circuit with resistors in series and parallel, step 2

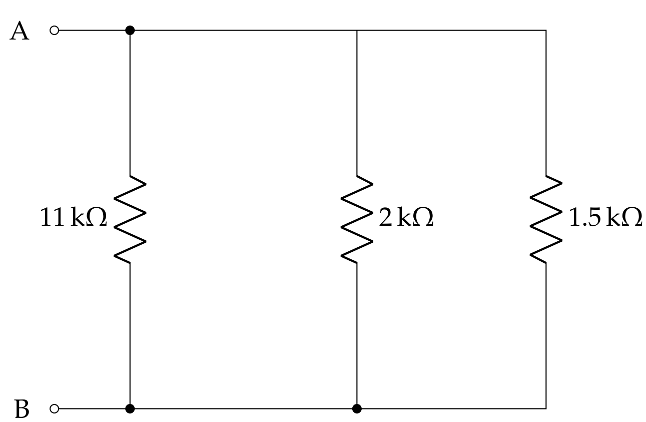


Figure 12: Finding equivalent resistance in circuit with resistors in series and parallel, step 3

fig. 13, where we have a circuit with both dependent and independent sources. The process for finding the equivalent resistance of a circuit

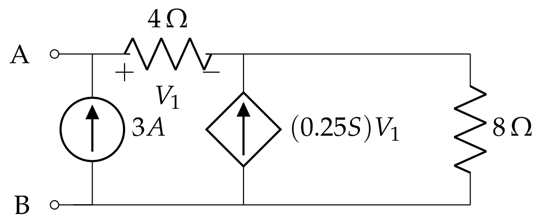


Figure 13: Finding equivalent resistance in cirucit with sources

with sources is as follows:

1. "Turn off" all independent sources. That is, replace independent current sources with opens and independent voltage sources with shorts.
2. Apply a test current or voltage across a and b.
3. Use $R_{eq} = \frac{V_test}{I_test}$, circuit laws, and algebra to solve.

Let's apply a test current of $1A$ across a, b, as shown in fig. 14 By

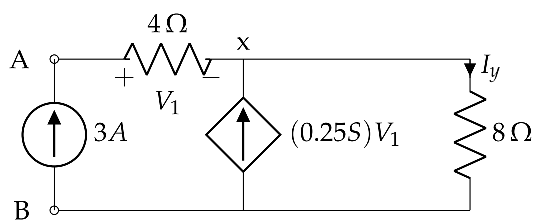


Figure 14: Finding equivalent resistance in cirucit with sources

using Kirchhoff's voltage law on the outermost loop, we have

$$V_{test} = V_1 + 8I_y$$

We also know that the current flowing into node x must be equal to the current flowing out. That is,

$$1 + (0.25S)V_1 = I_y$$

But since we know R_1 and I_1, we can find V_i. We simply have

$$V_1 = 1A \times 4\Omega = 4V$$

Meaning

$$I_y = 4 * 0.25 + 1 = 2$$

We can plug V_1 and I_y into $V_{test} = V_1 + 8I_y$ to get $V_{test} = 4 + 16 = 20$, and via $R_{eq} = \frac{V_{test}}{I_{test}}$ we have $R_{eq} = \frac{20V}{1A} = 20\Omega$.

Analysis

Let's now examine a couple of extremely powerful techniques we can use to analyze circuits, *nodal analysis* and *mesh analysis*.

Nodal analysis: a systematic method used to determine the voltage at every (essential) node in a circuit. After picking our reference (ground) node, we systematically apply KCL at every (essential) node in the circuit except, of course, for the ground node. By finding every nodal voltage in the circuit, we can find every branch current, which enables us to determine the power absorbed or delivered by every element. Because of its ability to be applied to any circuit, nodal analysis is the method most often used in circuit analysis computer programs. Here are the steps to perform nodal analysis:

1. Number all (essential) nodes of the given circuit.
2. Write KCL for every (essential) node by keeping in mind that only nodal voltages should be used (no currents). If other unknowns are involved (e.g. dependent source equations) express them as a function of the unknown nodal voltages (e.g. by using Ohm's law).
3. Group the resulting equations together in a matrix form.
4. Solve for the unknown nodal voltages by inverting the resulting linear equation.
5. Calculate any quantity of interest (e.g power consumption) from the known nodal voltages.

Consider an example cirucit, fig. 15. Let's apply our steps to find the

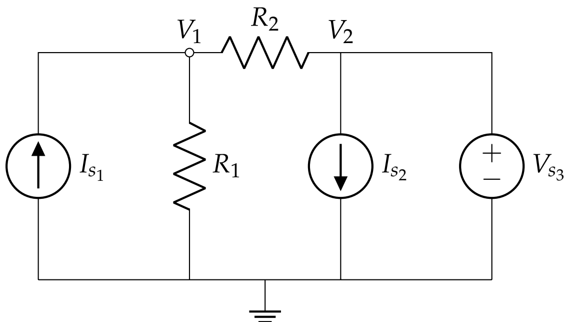


Figure 15: Using nodal analysis

nodal voltages V_1 and V_2.

1. Number the essential nodes (fig. 16).
2. Apply Kirchhoff's current law to each node

$$-I_{s_1} - \frac{0 - V_1}{R_1} - \frac{V_2 - V_1}{R_2} = 0$$

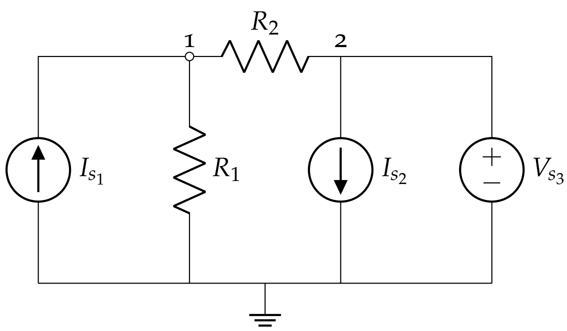


Figure 16: Numbering nodes

Here we can actually take a shortcut. Since the negative terminal of V_{s_3} is grounded, then the voltage at the positive terminal must be V_{s_3}. Since this terminal is connected directly to V_2, we know that $V_2 = V_{s_3}$. That makes our equations

$$
\begin{aligned}
-I_{s_1} - \frac{0 - V_1}{R_1} - \frac{V_2 - V_1}{R_2} &= 0 \\
V_2 &= V_{s_3}
\end{aligned}
$$

3. Let's group these by the variables we wish to find now.

$$
\begin{aligned}
V_1(\frac{1}{R_1} + \frac{1}{R_2}) - V_2\frac{1}{R_2} &= I_{s_1} \\
0V_1 + V_2 &= V_{s_3}
\end{aligned}
$$

Which in matrix form becomes

$$
\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_{s_1} \\ V_{s_3} \end{bmatrix}
$$

4. From here, we could use a computer program to find the inverse matrix and multiply both sides by it to get the vector $[V_1, V_2]$ by itself. I'm not going to do this because I'm lazy, but if you're interested try using Mathematica or MATLAB.

If our circuit lacks a ground, we may simply choose some node as ground, since voltages are relative and having a grounded node makes analysis easier.

Mesh analysis: This method is used to determine every loop current in a circuit. We use KVL around meshes (loops) to find the mesh (loop) currents. We can then calculate any voltage or any branch current from the resulting mesh currents. (Basic) mesh analysis has a limitation in that it can only be applied to planar circuits.

Here are the steps to perform mesh analysis:

1. Choose your loops and draw mesh currents in them.
2. Use Kirchhoff's voltage law and the voltages you encounter to create a system of equations. If two mesh currents go against each other, the net current is their difference.
3. Group the resulting equations together in a matrix form.
4. Solve for any quantity of interest.

Note that you don't want two mesh currents flowing through the same current source. If you find a circuit where this occurs, make a larger loop where no current sources are shared.

Source transformations

Source transformation: Source transformation is a wonderful tool that can be used to change the originally given circuit to an equivalent and simpler circuit. Two important theorems are Thevenin's theorem and Norton's theorem.
Thevenin's theorem: states that it is possible to simplify any linear circuit, irrespective of how complex it is, to an equivalent circuit with a single voltage source and a series resistance.

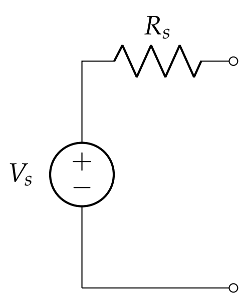


Figure 17: Thevenin equivalent

Norton's theorem: states that any linear circuit can be simplified to an equivalent circuit consisting of a single current source and parallel resistance that is connected to a load. Let's see how to transform

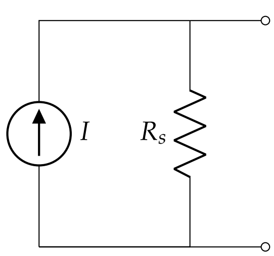


Figure 18: Norton equivalent

between these two. Fig. 19 shows a circuit with one voltage source and a resistor in series. This series resistance normally represents

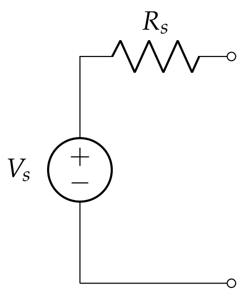


Figure 19: Thevenin to Norton

the internal resistance of a practical voltage source. Let us short circuit the output terminals of the voltage source circuit as shown in fig. 20 We know that $V_s = IR_s$, where I is the current delivered

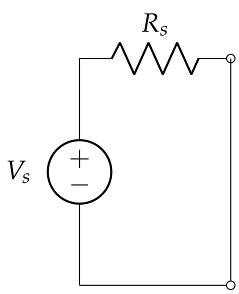


Figure 20: Thevenin to Norton

by the voltage source when it is short circuited. Now, let's take a current source of the same current I which produces same open-circuit voltage at its open terminals as shown in fig. 21 Now we have

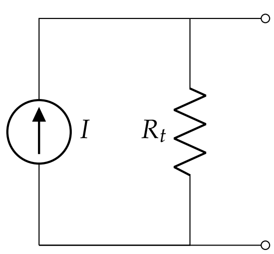


Figure 21: Thevenin to Norton

$I = \frac{V_s}{R_t}$, meaning $R_s = R_t$. The open circuit voltage of both the sources is V_s and short circuit current of both sources is I. The same resistance connected in series in voltage source is connected in parallel in its equivalent current source. So, the voltage source and current source are equivalent to each other. Changing between forms is called a source transformation and can be used to simplify an electric circuit, since any place a voltage source and resistor are in series we can transform to a current source and resistor in parallel (and vice versa). If the voltage of the voltage source is V_{th} and the resistance is R, then the current of the equivalent current source will be $\frac{V_{th}}{R}$.

Let's do an example. Say we have the circuit in fig. 22 and we wish to find the Thevenin voltage and Norton current with respect to terminals a and b. Let's simplify this circuit a bit. First, notice that we

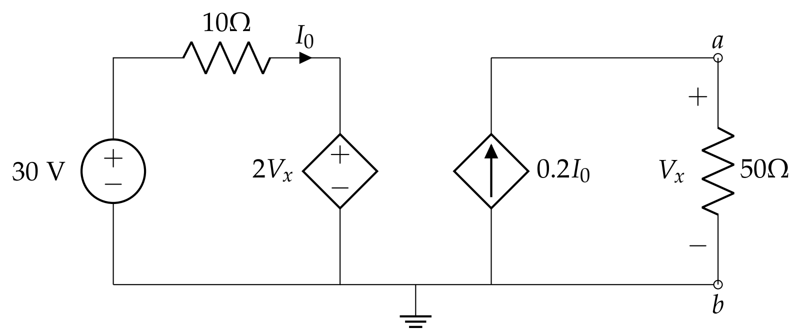


Figure 22: Source transformation example

have $30 - 2V_x$ V across the 10Ω resistor. By Ohm's law, $I_0 = \frac{30-2V_x}{10}$. Looking at the right loop, we have that the current flowing across the resistor is $0.2\frac{30-2V_x}{10}$. That means $V_x = 50 * 0.2\frac{30-2V_x}{10} = 10\frac{30-2V_x}{10}$. Let's organize our equations.

$$I_0 = \frac{30-2V_x}{10}$$
$$V_x = 10\frac{30-2V_x}{10}$$

In this case, we don't even need linear algebra to solve. The second equation yields $V_x = 10$ V, meaning $I_0 = 1$ A. Since the Norton current is the current through a and b when they are shorted, $I_N = 0.2I_0 = 0.2$ A. Likewise, since the Thevenin voltage is the voltage across a and b, the Thevenin voltage is 10 V.

Well and good, but what if we want to try source transformations? Let's do just that on circuit 23. We could pretty easily solve this with nodal analysis, but I think there's an easier way. We know that a current source I and resistor R in parallel can be simplified to a resistor and voltage source in series, with a voltage of RI.

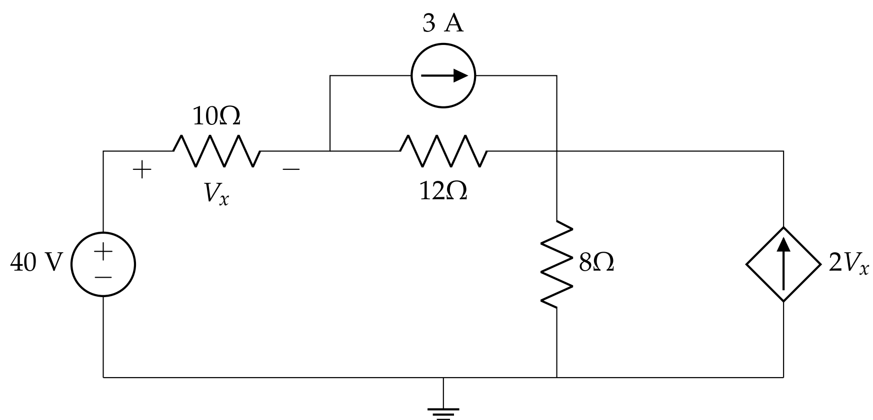


Figure 23: Source transformation example