Notes for ECE 30411 - Electromagnetics I

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These are lecture notes for Fall 2025 ECE 30500 by professor Elliott at Purdue. Modify, use, and distribute as you please.

Contents

Gradient, Divergence, and Curl

Gradient

The gradient describes the spatial slope of a 3-dimensional function.
It can only be applied to a scalar field.

Rectangular:

$$\nabla f = a_x \frac{\delta d}{\delta x} + a_y \frac{\delta d}{\delta y} + a_z \frac{\delta d}{\delta z}$$

Cylindrical:

$$\nabla f = a_\rho \frac{\delta d}{\delta \rho} + a_\phi \frac{1}{\rho} \frac{\delta d}{\delta \phi} + a_z \frac{\delta d}{\delta z}$$

Spherical:

$$\nabla f = a_R \frac{\delta d}{\delta R} + a_\theta \frac{1}{R} \frac{\delta d}{\delta \theta} + a_\phi \frac{1}{R \sin(\theta)} \frac{\delta d}{\delta \phi}$$

Divergence

Describes the rate of change of a vector function.

Rectangular:

$$\nabla \cdot D = \left(a_x \frac{\delta}{\delta x} + a_y \frac{\delta}{\delta y} + a_z \frac{\delta}{\delta z} \right) \cdot \left(a_x D_x + a_y D_y + a_z D_z \right) = \frac{\delta D_x}{\delta x} + \frac{\delta D_y}{\delta y} + \frac{\delta D_z}{\delta z}$$

Curl

Describes the rotation of a vector function.
Rectangular:

$$\nabla \times D = a_x \left(\frac{\delta D_z}{\delta y} - \frac{\delta D_y}{\delta z} \right) + a_y \left(\frac{\delta D_x}{\delta z} - \frac{\delta D_z}{\delta x} \right) + a_z \left(\frac{\delta D_y}{\delta x} - \frac{\delta D_x}{\delta y} \right)$$

Cylindrical:

$$\nabla \times D = a_\rho \left(\frac{\delta D_z}{\delta \phi} - \frac{\delta \rho D_\phi}{\delta z} \right) + \rho a_\phi \left(\frac{\delta D_\rho}{\delta z} - \frac{\delta D_z}{\delta \rho} \right) + a_z \left(\frac{\delta \rho D_\phi}{\delta \rho} - \frac{\delta D_\rho}{\delta \phi} \right)$$

Identities

1. $\nabla \times \nabla V = 0$: the gradient does not rotate.
2. $\nabla \cdot (\nabla \times A) = 0$: the curl of a vector function does not diverge (grow).
3. A vector field whose divergence and curl are known is completely determined.

Electrostatics and Coulomb's Law

Coulomb's law was determined using a torsion pendulum.

Using his experimental results, the following properties were derived:

- Direction is always along $\mathbf{r_2} - \mathbf{r_1}$
- Decreases in magnitude proportional to $|\mathbf{r_2} - \mathbf{r_1}|^{-2}$
- It can be repulsive or attractive depending on the sign of $q_1 q_2$.

$$\mathbf{F_{12}} = \frac{q_1 q_2 \mathbf{a_{12}}}{4\pi\epsilon_0 |\mathbf{r_2} - \mathbf{r_1}|^2}$$

On the other hand the electric field at a point, due to a charge is defined as:

$$\mathbf{E}(r_2) = \frac{q_1 \mathbf{a_{12}}}{4\pi\epsilon_0 |\mathbf{r_2} - \mathbf{r_1}|^2} = \frac{q_1(\mathbf{r_2} - \mathbf{r_1})}{4\pi\epsilon_0 |\mathbf{r_2} - \mathbf{r_1}|^3}$$

Which in turn leads to the superposition property, which states:

$$\mathbf{E}_{total} = \sum_{\text{all charges},i} \mathbf{E}_i$$

Mathematical properties of E

The electric field has the following properties:

- $\nabla \times E = 0$: this is later modified and becomes Faraday's Law
- $\nabla \cdot E = \frac{\rho_v}{\epsilon_0}$: Also known as Gauss's Law

These two laws allow for the creation of Coulomb's Law, as well as superposition.

> **Surface Integral** $\int_s A \cdot ds$: it is the integral across a surface A. In it $A \cdot ds = A_n ds$. That is, it integrates across the normal components of the surface. It is also known as the flux across the surface.

Using Gauss Law

1. Determine whether the enclosed charge has the required symmetry
2. Construct an equally symmetric Gaussian Surface
3. Determine the relevant variables, that is, the variables that still have an effect.
4. Evaluate using Gauss's Law.

Electric Potential and Electric Dipole

$V(r) = \frac{q}{4\pi r\epsilon_0}$ for a point charge. Superposition applies to electric potential as well.

$$V = \sum_i \frac{q_i}{4\pi r_i \epsilon_0}$$

We cannot find the electric field using just the potential at one point. However, if we know the complete function of V:

$$E = -\nabla V$$

For a dipole with an inter-charge distance d:

$$V(r) = \frac{p \cdot a_r}{4\pi\epsilon_0 R^2}$$

where $p = qd$.