Compiler Design

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Control-Flow Analysis

Sample Exercises and Solutions

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Problem1: Control-Flow Graph, Dominators and Natural Loops

For the program below:

1. Determine the corresponding control flow graph.
2. Determine the dominators of each node in the CFG
3. Show the immediate dominator tree
4. Identify the set of nodes in each natural loop. Are the loops nested? Why

```
00:   i = 0;
01:   while (i < N) do
02:     for(i=0; j < i; j++)
03:        if(a[i] < a[j]) then
04:           tmp = a[j];
05:           a[j] = a[i]
06:           a[i] = tmp;
07:         else
08:             if(a[i] == a[j]) then
09:               continue;
10:             else
11:               break;
12:      end for
13:      i = i + 1;
14:   end while
```

Note: Assume there are *entry* and *exit* nodes corresponding to the entry point and exit points of the code segment, which would correspond to the prologue and epilogue sections of the procedure's generated code.

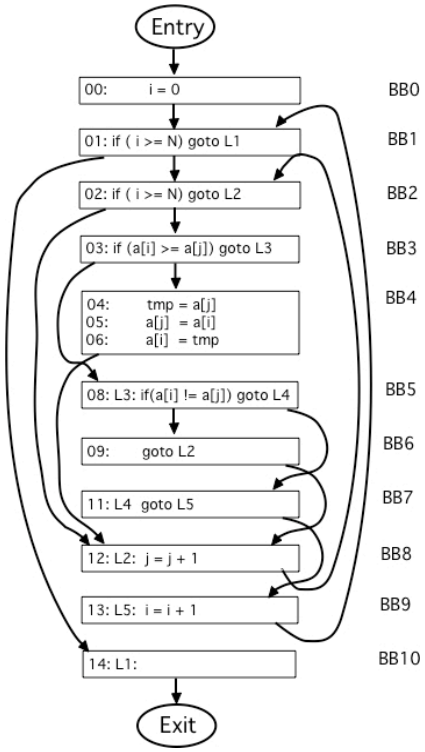
Solution:

1. A possible control flow graph is as show on the right.
2. Node Dominators

0	{0}
1	{0, 1}
2	{0, 1, 2}
3	{0, 1, 2, 3}
4	{0, 1, 2, 3, 4}
5	{0, 1, 2, 3, 5}
6	{0, 1, 2, 3, 5, 6}
7	{0, 1, 2, 3, 5, 7}
8	{0, 1, 2, 8}
9	{0, 1, 2, 3, 5, 7, 9}
10	{0, 1, 10}

For instance node 4 does not dominate node 8 as there is a path from the entry node that goes to node 8 and does not pass through node 4.

While there is an algorithm to determine the dominator relationship one can derive the dominator tree by inspection of control-flow paths.



3. Immediate dominator tree edges: 0→1, 1→2, 1→10, 2→3, 3→4, 3→5, 2→8, 5→6, 5→7, 7→9

4. Natural loops: The natural loops are determined by observing the back-edges. A back-edge is an edge in the original control-flow graph such that the node pointed to by its head dominates the node pointed to by its tail. To find a natural loop we traverse the back-edge against the control flow and all the edges in the CFG until e reach the node pointed to by the head in a breath-first traversal.

Back edge	Nodes in natural loop
9→1	{1, 2, 3, 4, 5, 6, 7, 8, 9}
8→2	{2, 3, 4, 5, 6, 8}

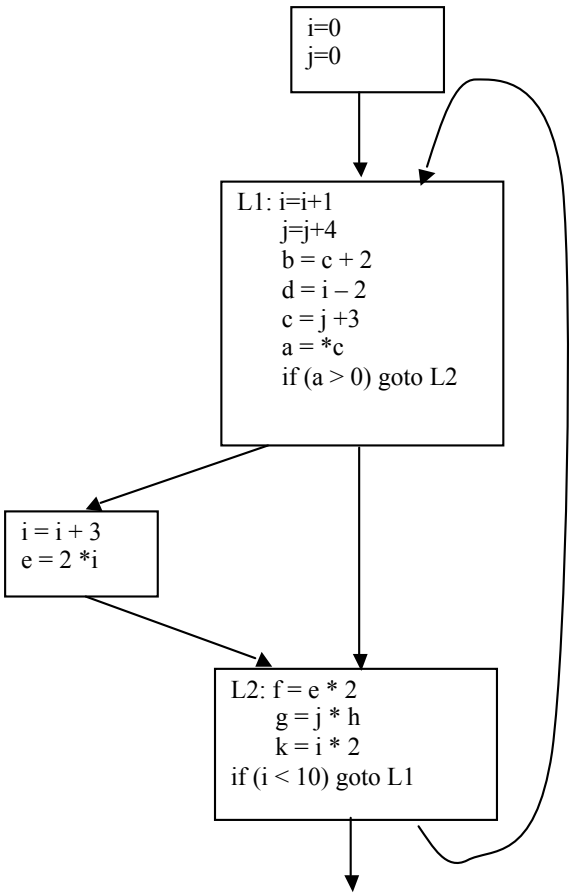
They are nested because they have different headers, and natural loops with different headers can either be nested or disjoint. They are not disjoint, so they must be nested.

Problem 2: Loop Induction Variables and Loop Unrolling

For the program depicted below do the following:

1. Find the induction variables in this program. Show the results after applying strength reduction and loop test replacement.
2. Show the resulting code from applying loop unrolling with an unrolling factor of 2.

Note: The "a = *c" assigns the contents of the memory location pointed to by c to a



Solution:

Assuming that there is no alias among the variables, so h is not changed within the loop. Basic induction variables: {i, j} Induction variable families: {{i, d, e, k}, {j, g, c}} (We can consider h as a constant within the loop, and as a result, g will be an induction variable in the same family with j) Introduce s1 for d, s2 for e and k, s3 for g, s4 for c, and eliminate basic induction variables i and j because they are only used for calculation of other induction variables and loop test. After strength reduction and loop test replacement:

```
        s1 = -2
        s2 = 0
        s3 = 0
        t3 = 4 * h
        s4 = 3

L1:     s1 = s1 + 1
        s2 = s2 + 2
        s3 = s3 + t3
        s4 = s4 + 4
        b = c + 2
        d = s1
        c = s4
        a = *c

        if a > 0 goto L2

        s1 = s1 + 3
        s2 = s2 + 6
        e = s2

L2:     f = e * 2
        g = s3
        k = s2
        if s1 < 8 goto L1

        i = d + 2
        j = c – 3
```

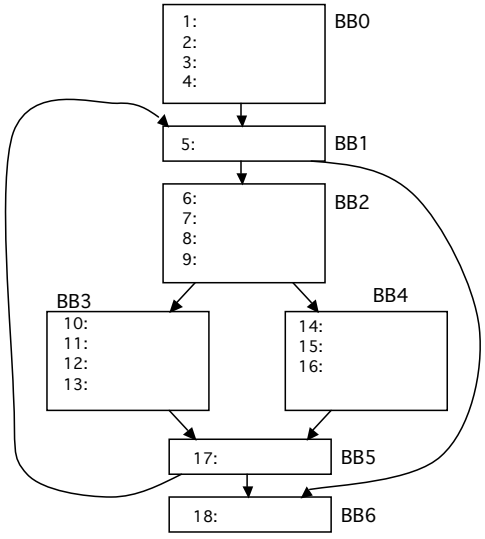
1) After loop unrolling:

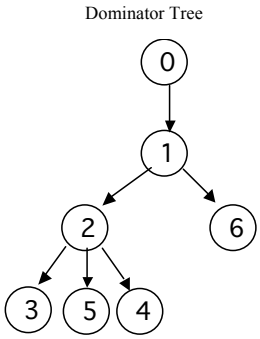
```
        s1 = -2
        s2 = 0
        s3 = 0
        t3 = 8 * h
        s4 = 3

        if s1 >= 4 goto L4
L1:     s2 = s2 + 2
        s4 = s4 + 4
        c = s4
        a = *c
        if a > 0 goto L2

        s1 = s1 + 3
        s2 = s2 + 6
        e = s2

L2:     s1 = s1 + 2
        s2 = s2 + 2
        s3 = s3 + t3
        b = c + 2
        d = s1

        s4 = s4 + 4
        c = s4
        a = *c
        if a > 0 goto L3

        s1 = s1 + 3
        s2 = s2 + 6
        e = s2

L3:     f = e * 2
        g = s3
        k = s2

        if s1 < 4 goto L1

L4:     s1 = s1 + 1
        s2 = s2 + 2
        s3 = s3 + t3
        s4 = s4 + 4
        b = c + 2
        d = s1
        c = s4
        a = *c
        if a > 0 goto L5

        s1 = s1 + 3
        s2 = s2 + 6
        e = s2

L5:     f = e * 2
        g = s3
        k = s2
        if s1 < 8 goto L4

        i = d + 2
        j = c –
```

Problem 3: Loops and Loop Optimizations

Consider the code depicted below for a function with integer local variables i, a, b, c, d and an integer input parameters p and n.

```
int i, a, b, c, d;

 1:     i = 0;
 2:     a = 1;
 3:     b = i * 4;
 4:     c = a + b;
 5: L1: if(i > n) goto L2
 6:     c = a + 1;
 7:     i = i + 1;
 8:     b = i * 4;
 9:     if (c <= p)goto L3
10:     e = 1;
11:     c = e - b;
12:     a = e;
13:     goto L4
14: L3: d = 2;
15:     c = d + b;
16:     a = d;
17: L4: goto L1
18: L2: return;
```





For this code fragment determine:

1. The Control Flow Graph (CFG).
2. The dominator tree and the loops (identify the back edges).
3. Use constant propagation to the statements in the body of the loop identified in (2).
4. Are the statements in line 8 and 16 loop invariants? Explain and describe where can they be moved to if they can in fact be moved outside any of the loops.

Solution:

1. See the CFG on the left above. In the CFG each node is a basic block. Recall that a basic block is a maximal sequence of instructions such that if the first instruction of the sequence is executed so are all the instructions in the basic block. Conversely if the first instruction is not executed none of the instructions in the basic block are executed.
2. See the dominator tree on the right above. In this tree an edge indicates the notion of immediate dominance. Recall that a node p dominates another node q if and only if every path from the entry node of the code to node q has to go through node p.
3. Back edge is (5,1) since the node pointed to by its head dominates the node pointed to by its tail. The natural loop is (1,2,3,4,5). The basic blocks in this natural loop are found tracing back the back-edge against the control flow until the node pointed to by the head. In this case and regarding the edge (5,1) we would find nodes 3 and 4, then node 2 and finally node 1. The basic blocks of the natural loop induced by the back edge (5,1) would then be {1,2,3,4,5}.
4. Could remove the statement `a = p1` to the head of the loop. This is only valid because a is not live at the end of the loop so even if the original loop were never to execute and since the head of the loop would execute once this transformation would still be correct.

Problem 4: Algebraic Transformations

For the code below apply the following code transformations: constant propagation, constant folding, copy-propagation, dead-code elimination and strength reduction.

```
          t1 = t1 + 1
L0:       t2 = 0
          t3 = t1 * 8 + 1
          t4 = t3 + t2
          t5 = t4 * 4
          t6 = *t5
          t7 = FP + t3
          *t7 = t2
          t8 = t1
          if (t8 > 0) goto L1
L1:       goto L0
L2:       t1 = 1
          t10 = 16
          t11 = t1 * 2
          goto L1
```

Solution:

The code in red is unreachable and can thus be eliminated. The goto L1 in the end is a goto to another goto instruction and thus can be converted into a goto L0 instruction.

```
          t1 = t1 + 1
L0:       t2 = 0
          t3 = t1 * 8 + 1
          t4 = t3 + t2
          t5 = t4 * 4
          t6 = *t5
          t7 = FP + t3
          *t7 = t2
          t8 = t1
          if (t8 > 0) goto L0
L1:       goto L0
L2:       t1 = 1
          t10 = 16
          t11 = t1 * 2
          goto L1
```

after dead-code elimination

```
          t1 = t1 + 1
L0:       t2 = 0
          t3 = (t1 << 3) + 1
          t4 = t3
          t5 = (t4 << 2)
          t6 = *t5
          t7 = FP + t3
          *t7 = t2
          t8 = t1
          if (t8 > 0) goto L0
```

after symbolic propagation for (t2 = 0) and strength reduction of the various multiplications by 4 and 8.

```
          t1 = t1 + 1
L0:       t2 = 0
          t3 = (t1 << 3) + 1
          t4 = t3
          t5 = (t3 << 2)
          t6 = *t5
          t7 = FP + t3
          *t7 = t2
          t8 = t1
          if (t1 > 0) goto L0
```

after copy propagation; now we can eliminate the variables t8 and t4 and t2 as they are never used in this code.

In reality you can also see that t1 is now loop invariant and so is the sub-expression `(t1 <<3) + 1` which can now be hoisted outside the loop. Along the same reasoning t5 is also loop invariant and so is everything else. This is a very uninteresting example.

Problem 5: Control-Flow Graph Analysis and Data-Flow Analysis

```
01      a = 1
02      b = 2
03 L0:  c = a + b
04      d = c - a
05      if c < d goto L2
06 L1:  d = b + d
07      if d < 1 goto L3
08 L2:  b = a + b
09      e = c – a
10      if e == 0 goto L0
11      a = b + d
12      b = a – d
13      goto L4
14 L3:  d = a + b
15      e = e + 1
16      goto L1
17 L4:  return
```

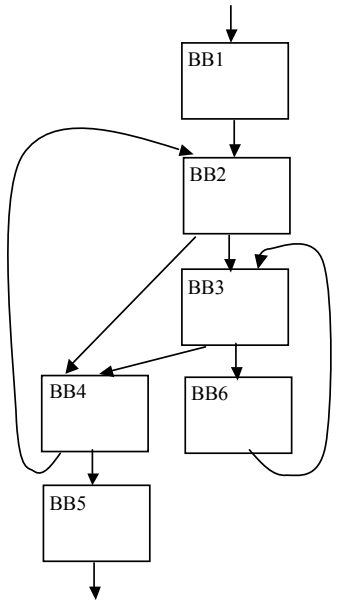
For the code shown above, determine the following:

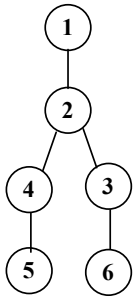
a) The basic blocks of instructions and the control-flow graph (CFG).
b) The live variables at the end of each basic block. You do not need to determine the live variables before and after each basic block and justify your answer for the value presented for the basic block containing instructions at line 6 and 7.
c) Is the live variable analysis a forward or backward data-flow analysis problem? Why and what does guarantee its termination when formulated as a data-flow analysis iterative problem?

***Solution**:*

a) We indicate the instructions in each basic block and the CFG and dominator tree on the RHS.

```
BB1: {01, 02}
BB2: {03, 04, 05}
BB3: {06, 07}
BB4: {08, 09, 10}
BB5: {11, 12, 13}
BB6: {14, 15, 16}
```





b) The *def-use* chains can be obtained by inspection of which definitions are not killed along all path from its definition point to the use points. For example for variable "a" definition at the instruction 1, denoted by a1 reaches the use points at the instructions 3, 4, 8, 9 and 15, hence the notation {d1, u3, u4, u8, u9, 15}. Definition a1 does not reach use u12 because there is another definition at 11 that masks it, hence another du chain for "a" denoted as {d11, u12}. The complete list is shown below.

```
a: {d1, u3, u4, u8, u9, u15}
a: {d11, u12}
b: {d2, u3, u4, u6, u14, u8}
b: {d8, u11, u8, u14}
b: {d12}
c: {d3, u4, u5, u9}
d: {d4, u5, u6}
d: {d6, u7}
d: {d14, u6}
e: {d9, u10, u15}
e: {d14, u15}
e: {d14, u6}
```

At the end of each basic block we can inspect the code and ask if a given variable is still used after the control flow leaves that basic block. The results of this inspection is depicted below:

```
BB1: {a, b}
BB2: {a, b, c, d, e}
BB3: {a, b, c, d, e}
BB4: {a, b, d, e}
BB5: { }
BB6: {a, b, c, d, e}
```

For BB6 the live variable solution at the exit of this basic block has {a, b, c, d, e} as there exists a path after BB6 where all the variables are being used. For example variable "e" is used in the basic block following the L1 label and taking the jump to L3 to BB6 again where is used before being redefined.

c) The live variable analysis is a backward data-flow problem as we propagate the information about a future use of a variable backward to specific points of the program. If there is a definition at a specific point backward the solution kills all other uses and resets the information associated with that variable. As with many other iterative formulations of data-flow analysis problems termination is guaranteed by the fact that the lattice, in this case the set of variables, has finite cardinality or length. The flow-function, in this case set-union is monotonic.

Problem 6: Control-Flow Graph Analysis and Data-Flow Analysis

```
01 func:  a = 1
02        b = 2
03 L0:    c = a + b
04        d = c - a
05        if c < d goto L2
06        d = b + d
07        if d < 1 goto L3
08 L2:    b = a + b
09        e = c - a
10        if e == 0 goto L0
11        a = b + d
12        b = a - d
13        goto L1
14 L3:    d = a + b
15        e = e + 1
16        goto L2
17 L1:    return
```

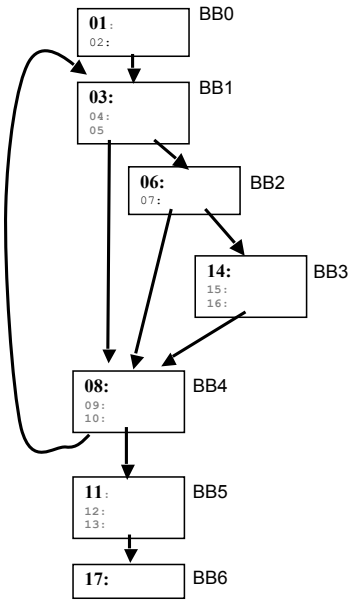
For the code shown on the left, determine the following:

a) The basic blocks of instructions identifying the instructions that constitutes each basic block. Clearly identify the leader instruction of each basic block.
b) The control-flow graph (CFG) and the corresponding dominator tree.
c) For each variable, its *use-def* chains.
d) The set of available expressions at the beginning of each basic block. You do not need to compute the DFA solution at each step of the iterative formulation but rather argue that your solution is correct by explaining the specific Available Expressions DFA problem.
e) Is the Available Expressions DFA a forward or backward data-flow analysis problem? Why and what does guarantee its termination when formulated as a data-flow analysis iterative problem?

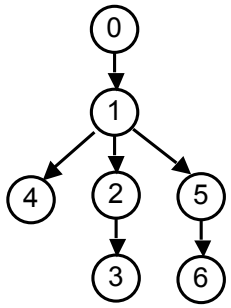
Solution:

a) The basic blocks of instructions identifying the instructions that constitutes each basic block. Clearly identify the leader instruction of each basic block.
b) The control-flow graph (CFG) and the corresponding dominator tree.

See the answer to both these questions below where the leader of each basic block is in bold font.



Control-Flow Graph



Dominator Tree

c) For each variable, its *use-def* chain.
For each variable we indicate for a given definition "d" its uses "u" as the line numbers in which they occur.

```
a: {d1, u3, u4, u8, u9} {d11, u12} {d1, u3, u4, u14}
b: {d2, u3, u6, u8} {d8, u11} {d8, u3, u6, u8}
c: {d3,  u4, u5, u9}
d: {d4, u5, u11, u12} {d6, u7, u11, u12}
e: {d9, u10, u15} {d15}
```

d) The set of available expressions at the beginning of each basic block. You do not need to compute the DFA solution at each step of the iterative formulation but rather argue that your solution is correct by explaining the specific Available Expressions DFA problem. Justify your answer by showing the GEN and KILL sets of each of the basic blocks.

The set of expressions in the program are outlined below on the left. On the right we have for each Basic Block BB0 through BB6 we have the following GEN and KILL sets.

(1): (a+b) line 3
(2): (c-a) line 4
(3): (b+d) line 6
(4): (e+1) line 15
(5): (a-d) line 12
(6): (a+b) line 8
(7): (c-a) line 9
(8): (a+b) line 14
(9): (b+d) line 11

BB0: gen = { }, kill = {1,2,3,5,6,7,8,9}
BB1: gen = {1,2}, kill = {2,3,5,7,9}
BB2: gen = {3}, kill = {3,5,9}
BB3: gen = {8}, kill = {3,4,5,9}
BB4: gen = {6,7}, kill = {1,3,4,6,8,9}
BB5: gen = {5}, kill = {1,2,3,5,6,7,8,9}
BB6: gen = { }, kill = { }

Applying the iterative fixed-point computation with set intersection corresponding to this problem's DFA formulation as described in class we would arrive at the following final solution for the set of available expression at the input of each basic block:

BB0: { }by definition
BB1: {4}
BB2: {1,2,4}
BB3: {1,2,3,4}
BB4: {2,4}
BB5: {2,4}
BB6: {4,5,9}

Recall that for each basic block the input is the intersection of the outputs of all its predecessors and the output of each basic block is computed by the data-flow equation out = gen + (in – kill).

e) Is the Available Expressions DFA a forward or backward data-flow analysis problem? Why and what does guarantee its termination when formulated as a data-flow analysis iterative problem?

This DFA problem is a forward data-flow problem as the expression become un-available as soon as one of its operands is redefined. At the beginning no expressions are available as they are generated as soon as the expression that defined them are encountered. A given expression is only available at a specific execution point is all path leading to that specific point have that expression available. As such the meet function is the set intersection. With an initial solution of the set of all available expressions (except for the initial node whose initial solution is empty) and applying the set intersection operation, given that the number of expressions is finite the iterative application of the set intersection will eventually converge. Hence the termination of the fixed-point computation is guaranteed.

Problem 7: Control-Flow Analysis and Basic Blocks

Consider the code depicted below in 3-address instructions already with specific register allocation and assignments.

```
01:      r0 = 0
02:      r1 = $fp - 4
03:      r1 = *r1
04:      r2 = 0
05:      r4 = 1
06:      r5 = r1
07: L1:  if r0 = r1 goto L5
08:      if r2 = r5  goto L3
09:      goto L2
10: L2:  r3 = 1
11:      r0 = r0 + 1
12:      goto L6
13: L3:  r4 = 0
14: L4:  goto L7
15: L5:  r3 = 1
16:      r0 = r0 + 1
17:      r4 = 0
18: L6:  r4 = r0 * 2
19:      goto L8
20: L7:  r1 = FP + 4
21:      *r1 = r4
22:      r1 = $gp + 20
23:      *r1 = r5
24:      return
25: L8:  r5 = r5 - r3
26:      if r5 != 0 goto L1
27:      r4 = 1
28:      goto L7
```

For this code fragment determine:

a. [10 points] The corresponding Control Flow Graph (CFG) identifying the basic blocks with their instructions.

b. [10 points] The dominator tree and the natural loops (identify the back edges). The dominator tree is a simple relationship between nodes. A node in this tree is directly connected to the nodes it immediately dominates.

c. [10 points] Check and transform the code taking advantage of loop-invariant code motion arguing about the correctness of your transformations.

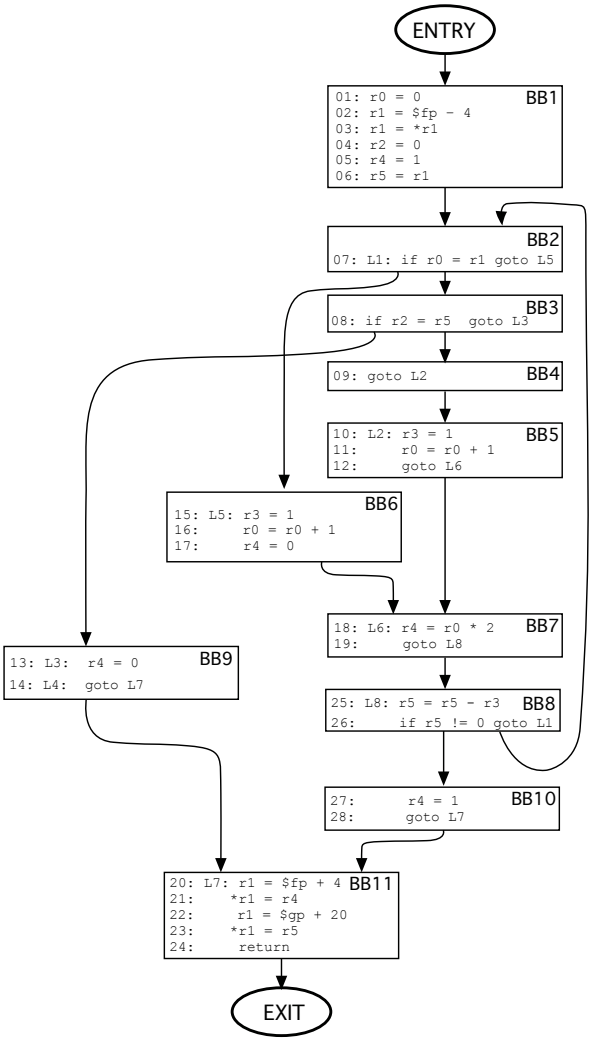
d. [10 points] Check and transform the code taking advantage of induction variables in this loop arguing about the correctness of your transformations.

Solution:

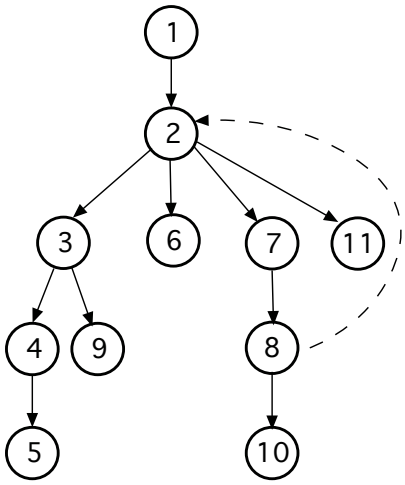
a) The control-flow graph is as shown below on the left-hand-side.

b) The dominator tree is shown on the right-hand side. The natural loop induced by the back-edge (8-2). Tracing backwards this edge from the basic block BB8 towards BB2 we encounters the basic blocks { 2, 3, 4, 5, 6, 7, 8}

c) There are really no loop-invariant instructions in the only loop in this code. The only candidate instruction which one could move to the head of the loop would be the instructions "r3 = 1" in the basic blocks BB5 and BB6. Unfortunately, none of these basic blocks by themselves dominate the basic blocks where there are uses of the variable r3 in BB8. Similarly, we can see that the assignment " r4 = 0" in BB6 would be a candidate for loop invariant code motion. Again this statement does not dominate the exit of the loop (BB2). In fact that are other assignment statement to variable r4.



Control-Flow Graph (CFG)



Dominator Tree, Back-edge and Natural Loop

d) There are some opportunities for induction variables in this loop, which are not trivial to grasp, and for which the help of data-flow analysis (as seen in the next problem) are required. There is an increment on r0 by 1 at every iteration of the loop although the increment occurs in different basic blocks. At each iteration of the loop, however, only one of them is executed. This means that r0 is a *basic induction variable*. As such the variable r4 is a *derived* induction variable with increment by 2. The fact that we have an assignment to r4 in basic block BB6 is irrelevant as that assignment is dead in the sense that is immediately overwritten in BB7.