Calculus I: Quiz 2, 2019 Fall, Time limit: 50 Minutes

Name ______________________ Student # ______________________

1. (10 points) Evaluate

$$\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^9 + \left(\frac{2}{n}\right)^9 + \left(\frac{3}{n}\right)^9 + \cdots + \left(\frac{n}{n}\right)^9\right]$$

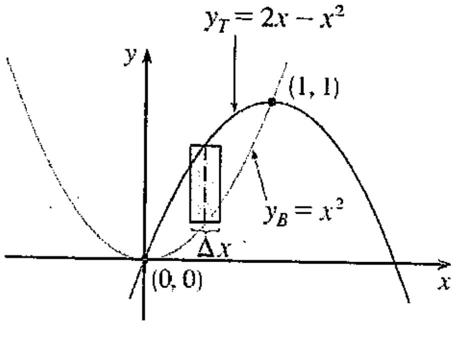
$$= \lim_{h\to 0} h \cdot \sum_{i=1}^{n} (ih)^9$$

Define $f(x) = x^9$, $x \in [0,1]$, then by the definition of integral,

$$\int_0^1 f(x)\,dx = \lim_{h\to 0} \sum_{i=1}^{n} f(x_i)\cdot h, \quad x_i = ih, \quad n = \frac{1}{h}$$

$$\Rightarrow \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^9 = \int_0^1 x^9\,dx = \frac{1}{10}x^{10}\Big|_0^1 = 1/10.$$

2. (10 points) Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.



$$A = \int_0^1 (y_T - y_B)\,dx$$

$$= \int_0^1 \left[(2x - x^2) - (x^2)\right] dx$$

$$= 2\int_0^1 (x - x^2)\,dx$$

$$= x^2 - \frac{2}{3}x^3 \Big|_0^1$$

$$= \frac{1}{3}$$

3. (10 points) The acceleration function (in m/s^2) and the initial velocity are given for a particle moving along a line, $a(t) = t + 4$, $v(0) = 5$. Find the velocity at time t and the distance traveled during the given time interval $0 \le t \le 10$.

$$V(t) = \int_0^t a(t)\,dt + V(0)$$
$$= \int_0^t (t+4)\,dt + 5$$
$$= \frac{1}{2}t^2 + 4t + 5$$

$$S(10) = \int_0^{10} |V(t)|\,dt$$
$$= \frac{1}{6}t^3 + 2t^2 + 5t \Big|_0^{10}$$
$$= 416\frac{2}{3}$$

4. (20 points) First make a substitution and then use integration by parts to evaluate the integral

(a)

$$\int x e^{\sqrt{x}} \mathrm{d}x$$

Let $t = \sqrt{x}$, $2t\,dt = dx$

$$I = \int t^2 e^t\, 2t\,dt = 2t^3 e^t - \int 6t^2\, de^t$$
$$= 2t^3 e^t - 6t^2 e^t + \int 12t\, de^t$$
$$= 2t^3 e^t - 6t^2 e^t + 12t\, e^t - 12 e^t$$
$$= 2(x\sqrt{x} - 3x + 6\sqrt{x} - 6)\, e^{\sqrt{x}}$$

(b)

$$\int_1^e x(\ln x)^2 \mathrm{d}x$$

Let $t = \ln x$, $dt = \frac{1}{x}dx$

$$I = \int_0^1 e^t \cdot t^2 \cdot e^t\,dt$$
$$= \int_0^1 t^2 \frac{1}{2} d e^{2t}$$
$$= \frac{1}{2} t^2 e^{2t} \Big|_0^1 - \int_0^1 e^{2t} \cdot t\,dt$$
$$= \frac{e^2}{2} - \left[\frac{1}{2} t e^{2t}\Big|_0^1 - \int_0^1 \frac{1}{2} e^{2t}\,dt\right] \quad 2$$
$$= \frac{e^2}{4} - \frac{1}{4}$$

(c)

$$\int \frac{x}{\sqrt{2x^2-5}}\mathrm{d}x$$

~~Let $t = x^2$, $dt = 2x\,dx$~~

$\cancel{I = \int \frac{\frac{1}{2}dt}{\sqrt{2t-5}}}$, ~~Let $s = 2t-5$, $ds = 2dt$~~

$\cancel{I = \int \frac{1}{\sqrt{s}} \cdot \frac{1}{4} ds}$

Let $t = 2x^2-5$, $dt = 4x\,dx$, then

$$I = \int \frac{1}{\sqrt{t}} \cdot \frac{1}{4}\,dt = \frac{\sqrt{t}}{2} + c = \frac{\sqrt{2x^2-5}}{2} + C$$

(d)

$$\int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^3 \cos(\theta^2)\mathrm{d}\theta$$

Let $t = \theta^2$, then $dt = 2\theta\, d\theta$

$$I = \int_{\frac{\pi}{2}}^{\pi} t \cdot \cos t \cdot \frac{1}{2}\,dt$$

$$= \frac{1}{2}\int_{\pi/2}^{\pi} t\, d\sin t$$

$$= \frac{1}{2}\left[t\sin t \Big|_{\pi/2}^{\pi} - \int_{\pi/2}^{\pi} \sin t\, dt \right.$$

$$= \frac{1}{2}\left[0 - \frac{\pi}{2} + \cos t \Big|_{\pi/2}^{\pi} \right]$$

$$= -\frac{\pi}{4} - \frac{1}{2}$$

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