Numerical Computation – Assignment 2

薛劭杰 1930026143

1. $f \in O\left(h^k\right)$ and $gO(h^m)$, m < k.
$|f(h)| \leq C * h^k,\ |g(h)| \leq C * h^m.$
$|f(h)| + |g(h)| \leq\ * (h^k + h^m)$
$|(f+g)(h)| = |f(h) + g(h)| \leq |f(h)| + |g(h)| \leq C * (h^k + h^m)$
Because k>m and for all h sufficiently small which means that h is smaller than 1, so $h^m > h^k,\ f + g \in O(h^m)$

2.
Method1:
$f(h) = h^3$
The Taylor's polynomial of the f(h) is:

$$f(x) = f(a) + f(a)^{'}(x-a) + \frac{f^{''}(a)(x-a)^2}{2!} + \dots + \frac{f^{n-1}(a)(x-a)^{n-1}}{(n-1)!} + \frac{f^{n}(a)(\xi-a)^{n}}{(n)!}$$

$$= a^3 + 3a^2(x-a) + 3a(x-a)^2 + 6(x-a)^3 + 0 \qquad (a \leq\ \xi \leq x)$$

Taylor expansion of the function, the series after the third order is 0. The $f(h) = h^3$ is not in$O(h^4)$.

Method2:
$|f(h)| \leq c|h|^k$ for h sufficiently small, so h is close to the zero.
If the upper bound of the $f(h) = h^3$ is $O(h^4)$, then we have the:
$C \geq \frac{|h^3|}{|h^4|} = |\frac{1}{h}|$, and we know that the h is sufficiently small. So, $\left|\frac{1}{h}\right| is\ very\ large$ which means that we cannot find a constant to make $C * |h|^k$ always be larger than or equal to the $f(h)$.
To conclude, the $f(h) = h^3\ is\ not\ in\ O(h^4)$.

3. $\sqrt{x+1} - \sqrt{x} = \frac{(\sqrt{x+1}-\sqrt{x})*(\sqrt{x+1}+\sqrt{x})}{\sqrt{x+1}+\sqrt{x}} = \frac{1}{\sqrt{x+1}+\sqrt{x}}$

When x is very large, and even approaching positive infinity, $\frac{1}{\sqrt{x+1}+\sqrt{x}}$ is approaching 0.

4. let $f(x) = 1 - \cos^2 x = \sin^2 x$
$f^{'}(x) = \sin(2x), f^{''}(x) = 2\cos(2x),\ f^3(x) =- 4\sin(2x), f^4(x) =- 8\cos(2x)$
$f^5(x) = 16\sin(2x), f^6(x) = 32\cos(2x).$
The Taylor's series of the f(x) at order 6 is:

$$f(x) = f(a) + f(a)^{'}(x-a) + \frac{f^{''}(a)(x-a)^2}{2!} + \frac{f^3(a)(x-a)^3}{3!} + \dots + \frac{f^6(a)(\xi-a)^6}{6!}$$

$$= (\sin^2 a) + \sin(2x)(x-a) + \frac{2\cos(2a)(x-a)^2}{2!} - \frac{4\sin(2a)(x-a)^3}{3!} - \frac{8\cos(2a)(x-a)^4}{4!} +$$

$\frac{16\sin(2a)(x-a)^5}{5!}+\frac{32\cos(2a)\,(\xi-a)^6}{6!}$. **Let a=0 and we can get:**

$$f(x)=0+0+x^2-0-\frac{x^4}{3}+0+2*\frac{\xi^6}{45}\ =\ x^2-\frac{x^4}{3}+O(x^6)$$

(5) $x^2+3x-8^{-14}=0$

According to the root finding formula, the root of the function is $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$.

We can find that $b\gg c>0$**, one of roots is:**

$$\frac{-b+\sqrt{b^2-4ac}}{2a}=\frac{\left(-b-\sqrt{b^2-4ac}\right)*(-b+\sqrt{b^2-4ac})}{2a*(-b-\sqrt{b^2-4ac})}=\frac{b^2-b^2+4ac}{2a*(-b-\sqrt{b^2-4ac})}=\frac{4c}{-4*b}=7.579*10^{-14}.$$

Another root is:

$$\frac{-b-\sqrt{b^2-4ac}}{2a}=\frac{-2b}{2a}=-3.000.$$