Numerical Computation
Assignment 6

1. Let A be an $n \times n$ matrix. Assume that your computer can solve 100 problems $Ax = b_1, ..., Ax = b_{100}$ by the LU method in the same amount of time it takes to solve the first problem $Ax = b_0$. Estimate n.

2. Let $A = (a_{ij}) \in R^{n \times n}$, Prove that

$$\|A\|_\infty = \max_{1 \le i \le n} \left[\sum_{j=1}^{n} |a_{ij}| \right]$$

3. What is the norm of

$$A = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1/2 & 0 & \cdots & 0 \\ 0 & 0 & 1/3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1/n \end{bmatrix}$$

4. The linear system is $x_1 - 2x_2 = 3, 3x_1 - 4x_2 = 7$.
(a) Find the relative forward and backward errors and error magnification factor for the approximate solutions $[-1, -1]$. (infinity norm)
(b) What is the condition number of the coefficient matrix.(infinity norm)

5. (a) Find the (infinity norm) condition number of the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 4.001 \end{bmatrix}$.
(b) Let $b = \begin{bmatrix} 3 \\ 6.001 \end{bmatrix}$ and let $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ denote the exact solution of $Ax = b$. Find the relative forward error, relative backward error, and error magnification factor of the approximate solution $x_a = \begin{bmatrix} -6000 \\ 3001 \end{bmatrix}$.
(c) Show that for any $\delta > 0$, the error magnification factor of the approximation solution $x_a = \begin{bmatrix} 1 - 6001\delta \\ 1 + 3000\delta \end{bmatrix}$ is equal to the condition number of A..

6. Let A is a n-by-n upper triangular matrix with elements

$$a_{ij} = \begin{cases} -1 & i < j \\ 1 & i = j \\ 0 & i > j \end{cases}$$

Show how to generate this matrix in MATLAB with **eye**, **ones**, and **triu**.
Show that $\kappa_1(A) = n2^{n-1}$ (L^1 norm).