COMP3023: Design and Analysis of Algorithms
Fall 2021
Assignment 3
Submission: Dec. 15

Question 1. (20pt) There are 6 gifts with weights 5, 3, 2, 1, 6 and 4; and values 8, 2, 5, 13, 16, and 1 respectively. Use dynamic programming to find the most valuable subset of gifts subject to the constraint the total weight cannot exceed 10. Show the entire table for bottom-up computation, together with the keep array, i.e., $V[i,w]$ and $keep[i,w]$ for $0 \le i \le 6$ and $0 \le w \le 10$. Give the optimal value and optimal itemset. (20')

Question2. (20pt) Let A_1, A_2, A_3 and A_4 be matrices of dimensions 8×10, 10×3, 3×5, and 5×4 respectively. Use dynamic programming to find the minimum number of multiplications required to compute $A_1\times A_2\times A_3\times A_4$. Show each step by a table. And give the optimal solution.

Question3. (20pt) (Longest Common Substring) Given two strings, use dynamic programming to find the length of longest common substring. A common substring is a sequence that appears in the same order and necessarily contiguous in both the strings.

- Input: X= 'abcdxyz', Y='xyzabcd'
  - Optimal value: 4
  - Optimal solution: 'abcd'
- Input: X='zxabcdezy', Y='yzabcdezx'
  - Optimal value: 6
  - Optimal solution: 'abcdez'

Question4. (20pt) (Longest Common Subsequence) Given two strings, use dynamic programming to find the length of longest common subsequence. A common subsequence of two strings is a subsequence that is common to both strings. If there is no common subsequence, return 0.

- A subsequence is a sequence that appears in the same relative order, but not necessarily contiguous. For example: “acd” is a subsequence of “abcd”.

- Input: X= ‘ABCDGH, Y=‘AEDFHR
  - Optimal value: 3
  - Optimal solution: ‘ADH’
- Input: X=‘AGGTAB, Y=‘GXTXAYB’
  - Optimal value: 4
  - Optimal solution: ‘GTAB’

Question5. (20pt) Consider encoding “dbacadcccbcbcadabacc” as a binary string. Give a prefix-free code that minimizes the encoding length. Show the main steps that you take to reach the solution.