

Numerical Computation - Assignment 4

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Q1.

Initially, we can convert it to a matrix:

$$\begin{array}{rcl} 3.03x_1 - 12.1x_2 + 14x_3 = -119 & & [\begin{array}{ccc|c} 3.03 & -12.1 & 14 & -119 \\ -3.03 & 12.1 & -7 & 120 \\ 6.11 & -14.2 & 21 & -139 \end{array}] \\ -3.03x_1 + 12.1x_2 - 7x_3 = 120 & \Rightarrow & \\ 6.11x_1 - 14.2x_2 + 21x_3 = -139 & & \end{array}$$

$$A = [\begin{array}{ccc} 3.03 & -12.1 & 14 \\ -3.03 & 12.1 & -7 \\ 6.11 & -14.2 & 21 \end{array}]$$

$$\text{and } b = \begin{bmatrix} -119 \\ 120 \\ -139 \end{bmatrix}$$

$$(a). [A|B] = \left[\begin{array}{ccc|c} 3.03 & -12.1 & 14 & -119 \\ 0 & 0 & 7.00 & 1.00 \\ 0 & 10.1 & -7.10 & 100 \end{array} \right] \text{ through } R_2 + R_1 \text{ and } R_3 - \frac{a_{31}}{a_{11}}R_1$$

$$\text{Exchange } R_3 \text{ and } R_2 \text{ and we can get } \left[\begin{array}{ccc|c} 3.03 & -12.1 & 14 & -119 \\ 0 & 10.1 & -7.10 & 100 \\ 0 & 0 & 7.00 & 1.00 \end{array} \right]$$

Using the back substitution and each calculation is cropping 3 digits left. We can get:

$$x_3 = \frac{1}{7} = 0.142$$

$$x_2 = \frac{100 + 0.142 \cdot 7.10}{10.1} = 10.0$$

$$x_1 = \frac{-119 + 10.0 \cdot 12.1 - 14 \cdot 0.142}{3.03} = 0.00660$$

(b).

In the first column, $6.11 > 3.03 = 3.03$. So, Exchange R_3 to R_1 .

$$[A|b] = \left[\begin{array}{ccc|c} 6.11 & -14.2 & 21 & -139 \\ -3.03 & 12.1 & -7 & 120 \\ 3.03 & -12.1 & 14 & -119 \end{array} \right] \text{ through } R_2 - \frac{a_{21}}{a_{11}}R_1 \text{ and } R_3 - \frac{a_{31}}{a_{11}}R_1.$$

$$\text{And we can get } \left[\begin{array}{ccc|c} 6.11 & -14.2 & 21 & -139 \\ 0 & 5.08 & 3.30 & 51.2 \\ 0 & -5.08 & 3.70 & -50.2 \end{array} \right]$$

$|5.05| = |-5.05|$, there no need to exchange. Then through $R_3 + R_2$, we can get:

$$\left[\begin{array}{ccc|c} 6.11 & -14.2 & 21 & -139 \\ 0 & 5.08 & 3.30 & 51.2 \\ 0 & 0 & 7.00 & 1.00 \end{array} \right].$$

Using the back substitution and each calculation is cropping 3 digits left. We can get:

$$x_3 = \frac{1}{7.00} = 0.142$$

$$x_2 = \frac{51.2 - 3.30 \cdot 0.142}{5.05} = 9.98$$

$$x_1 = \frac{-139 - 0.142 \cdot 21 + 14.2 \cdot 9.98}{6.11} = -0.160$$

(c).

$$\text{scalar} = \max\left(\left|-\frac{6.11}{14.2}\right|, \left|-\frac{3.03}{12.1}\right|, \left|-\frac{3.03}{-12.1}\right|\right) = \left|-\frac{6.11}{14.2}\right| = 0.291 \rightarrow R_3 \text{ exchange to } R_1.$$

$$[A|b] = \begin{bmatrix} 6.11 & -14.2 & 21 & -139 \\ -3.03 & 12.1 & -7 & 120 \\ 3.03 & -12.1 & 14 & -119 \end{bmatrix}. \text{ Though } R_2 - \frac{a_{21}}{a_{11}}R_1 \text{ and } R_3 - \frac{a_{31}}{a_{11}}R_1, \text{ we}$$

$$\text{can get } \begin{bmatrix} 6.11 & -14.2 & 21 & -139 \\ 0 & 5.06 & 3.40 & 51.1 \\ 0 & -5.06 & 3.60 & -50.1 \end{bmatrix}. \text{ scalar} = \max\left(\left|\frac{5.06}{5.06}\right|, \left|-\frac{5.06}{5.06}\right|\right) = \left|\frac{5.06}{5.06}\right|. \text{ So, we}$$

do not need to exchange two rows. Though $R_3 + R_2$, we can get the final matrix:

$$\begin{bmatrix} 6.11 & -14.2 & 21 & -139 \\ 0 & 5.06 & 3.40 & 51.1 \\ 0 & 0 & 7.00 & 1.00 \end{bmatrix}.$$

Using the back substitution and each calculation is rounding 3 digits left. We can get:

$$x_3 = \frac{1}{7} = 0.143$$

$$x_2 = \frac{51.1 - 3.40 \cdot 0.143}{5.06} = 10.0$$

$$x_1 = \frac{-139 - 0.143 \cdot 21 + 14.2 \cdot 10}{6.11} = 0.00$$

Q2.

```
function x = gauss_no_pivot(A, b)
    AA=[A b];
    [n,m] = size(A);
    x=zeros(n,1);
    if n ~= m
        error(1);
    elseif n ~= size(b)
        error(1);
    else
        for k=1:n-1
            if A(k,k) == 0
                error(1);
            end

            for i=k+1:n
                m = AA(i, k)/AA(k,k);
                a=cell(1);
                for j=k:n+1
                    AA(i, j)=AA(i, j)-m*AA(k, j);
                end
                % displace the result
                a{1}=AA;
                disp(a{1});
            end
        end
        x(n)=AA(n, n+1)/AA(n, n);
        for i = n-1:-1:1
            s=0;
            for j=i+1:n
                s=s+AA(i, j)*x(j);
            end
            x(i)=(AA(i, n+1)-s)/AA(i, i);
        end
    end
end
```

Q3.

```
guass_no_pivot.m  guass_partial_pivot.m  guass_scaled_partial_pivot.m  +
1  function x = guass_partial_pivot(A, b)
2  AA=[A b];
3  [n,m] = size(A);
4  x=zeros(n,1);
5  if n ~= m
6      error(1);
7  elseif n ~= size(b)
8      error(1);
9  else
10     for k=1:n-1
11         % Initialize the max information
12         max_value=abs(AA(k,k));
13         max_row=k;
14         % Select the row which contains the maximum number in the rows
15         for a=k:n
16             if abs(AA(a,k))>max_value
17                 max_value=abs(AA(a,k));
18                 max_row=a;
19             end
20         end
21         AA([max_row k], :) = AA([k max_row], :);
22         disp(max_row);
23         % Exchange the two rows to pivot
24         %for b=1:n+1
25         %     temp = AA(k,b);
26         %     AA(k, b) = AA(max_row, b);
27         %     AA(max_row, b) = temp;
28         %end
29         if A(k,k) == 0
30             error(1);
31         end
32
33         for i=k+1:n
34             m = AA(i, k)/AA(k,k);
35             a=cell(1);
36             for j=k+1:n
37                 AA(i, j)=AA(i, j)-m*AA(k, j);
38             end
39             % displace the result
40             a{1}=AA;
41             disp(a{1});
42         end
43     end
44     x(n)=AA(n,n+1)/AA(n,n);
45     for i = n-1:-1:1
46         s=0;
47         for j=i+1:n
48             s=s+AA(i, j)*x(j);
49         end
50         x(i)=(AA(i, n+1)-s)/AA(i,i);
51     end
52 end
53
```

Q4.

```
1 function x = gauss_scaled_partial_pivot(A, b)
2     AA=[A b];
3     [n,m] = size(A);
4     x=zeros(n,1);
5     if n ~= m
6         error(1);
7     elseif n ~= size(b)
8         error(1);
9     else
10        for k=1:n-1
11            % Initialize the max information
12            scalar = abs(AA(k,k)/max(AA(k,:)));
13            max_row=k;
14            % Select the row which contains the max scalar in each rows
15            for a=k:n
16                if AA(a,k)/max(AA(k,:))>scalar
17                    scalar=abs(AA(a,k)/max(AA(a,:)));
18                    max_row=a;
19            end
20            end
21            AA([max_row k], :) = AA([k max_row], :);
22            % Exchange the scaled to the kth row
23            %for b=1:n+1
24            %     temp = AA(k,b);
25            %     AA(k, b) = AA(max_row, b);
26            %     AA(max_row, b) = temp;
27            % end
28            if A(k,k) == 0
29                error(1);
30            end
```

```
31
32        for i=k+1:n
33            m = AA(i, k)/AA(k, k);
34            a=cell(1);
35            for j=k:n+1
36                AA(i, j)=AA(i, j)-m*AA(k, j);
37            end
38            % displace the result
39            a{1}=AA;
40            disp(a{1});
41        end
42    end
43    x(n)=AA(n, n+1)/AA(n, n);
44    for i = n-1:-1:1
45        s=0;
46        for j=i+1:n
47            s=s+AA(i, j)*x(j);
48        end
49        x(i)=(AA(i, n+1)-s)/AA(i, i);
50    end
51 end
```

Q5.*guass_no_oivot :*

```
>> guass_no_pivot(A, b)
```

```
3.0000    9.0000    4.0000    8.0000   11.0000    0.6000
      0    6.1000   12.6667    5.0333    9.3333   12.4000
10.0000  100.0000    0.3000   -1.0000   17.0000  102.0000
6.0000    5.0000   11.0000    0.1000         0   33.0000
0.0100   -0.6000    1.0000         0   55.0000    1.0000
```

```
3.0000    9.0000    4.0000    8.0000   11.0000    0.6000
      0    6.1000   12.6667    5.0333    9.3333   12.4000
      0   70.0000  -13.0333  -27.6667  -19.6667  100.0000
6.0000    5.0000   11.0000    0.1000         0   33.0000
0.0100   -0.6000    1.0000         0   55.0000    1.0000
```

```
3.0000    9.0000    4.0000    8.0000   11.0000    0.6000
      0    6.1000   12.6667    5.0333    9.3333   12.4000
      0   70.0000  -13.0333  -27.6667  -19.6667  100.0000
      0  -13.0000    3.0000  -15.9000  -22.0000   31.8000
0.0100   -0.6000    1.0000         0   55.0000    1.0000
```

```
3.0000    9.0000    4.0000    8.0000   11.0000    0.6000
      0    6.1000   12.6667    5.0333    9.3333   12.4000
      0   70.0000  -13.0333  -27.6667  -19.6667  100.0000
      0  -13.0000    3.0000  -15.9000  -22.0000   31.8000
      0   -0.6300    0.9867   -0.0267   54.9633    0.9980
```

```
3.0000    9.0000    4.0000    8.0000   11.0000    0.6000
      0    6.1000   12.6667    5.0333    9.3333   12.4000
      0         0 -158.3885  -85.4262 -126.7705  -42.2951
      0  -13.0000    3.0000  -15.9000  -22.0000   31.8000
      0   -0.6300    0.9867   -0.0267   54.9633    0.9980
```

```
3.0000    9.0000    4.0000    8.0000   11.0000    0.6000
      0    6.1000   12.6667    5.0333    9.3333   12.4000
      0         0 -158.3885  -85.4262 -126.7705  -42.2951
      0         0  29.9945   -5.1732   -2.1093   58.2262
      0   -0.6300    0.9867   -0.0267   54.9633    0.9980
```

3.0000	9.0000	4.0000	8.0000	11.0000	0.6000
0	6.1000	12.6667	5.0333	9.3333	12.4000
0	0	-158.3885	-85.4262	-126.7705	-42.2951
0	0	29.9945	-5.1732	-2.1093	58.2262
0	0	2.2949	0.4932	55.9273	2.2787

3.0000	9.0000	4.0000	8.0000	11.0000	0.6000
0	6.1000	12.6667	5.0333	9.3333	12.4000
0	0	-158.3885	-85.4262	-126.7705	-42.2951
0	0	0	-21.3507	-26.1162	50.2167
0	0	2.2949	0.4932	55.9273	2.2787

3.0000	9.0000	4.0000	8.0000	11.0000	0.6000
0	6.1000	12.6667	5.0333	9.3333	12.4000
0	0	-158.3885	-85.4262	-126.7705	-42.2951
0	0	0	-21.3507	-26.1162	50.2167
0	0	0	-0.7446	54.0905	1.6658

3.0000	9.0000	4.0000	8.0000	11.0000	0.6000
0	6.1000	12.6667	5.0333	9.3333	12.4000
0	0	-158.3885	-85.4262	-126.7705	-42.2951
0	0	0	-21.3507	-26.1162	50.2167
0	0	0	0	55.0013	-0.0853

ans =

2.0692
0.7852
1.5358
-2.3501
-0.0016

guass_partition_pivot:

```
>> guass_partial_pivot(A,b)
```

3

10.0000	100.0000	0.3000	-1.0000	17.0000	102.0000
0	20.1000	10.0600	-0.5000	5.4000	32.4000
3.0000	9.0000	4.0000	8.0000	11.0000	0.6000
6.0000	5.0000	11.0000	0.1000	0	33.0000
0.0100	-0.6000	1.0000	0	55.0000	1.0000

10.0000	100.0000	0.3000	-1.0000	17.0000	102.0000
0	20.1000	10.0600	-0.5000	5.4000	32.4000
0	-21.0000	3.9100	8.3000	5.9000	-30.0000
6.0000	5.0000	11.0000	0.1000	0	33.0000
0.0100	-0.6000	1.0000	0	55.0000	1.0000

10.0000	100.0000	0.3000	-1.0000	17.0000	102.0000
0	20.1000	10.0600	-0.5000	5.4000	32.4000
0	-21.0000	3.9100	8.3000	5.9000	-30.0000
0	-55.0000	10.8200	0.7000	-10.2000	-28.2000
0.0100	-0.6000	1.0000	0	55.0000	1.0000

10.0000	100.0000	0.3000	-1.0000	17.0000	102.0000
0	20.1000	10.0600	-0.5000	5.4000	32.4000
0	-21.0000	3.9100	8.3000	5.9000	-30.0000
0	-55.0000	10.8200	0.7000	-10.2000	-28.2000
0	-0.7000	0.9997	0.0010	54.9830	0.8980

4

10.0000	100.0000	0.3000	-1.0000	17.0000	102.0000
0	-55.0000	10.8200	0.7000	-10.2000	-28.2000
0	0	-0.2213	8.0327	9.7945	-19.2327
0	20.1000	10.0600	-0.5000	5.4000	32.4000
0	-0.7000	0.9997	0.0010	54.9830	0.8980

10.0000	100.0000	0.3000	-1.0000	17.0000	102.0000
0	-55.0000	10.8200	0.7000	-10.2000	-28.2000
0	0	-0.2213	8.0327	9.7945	-19.2327
0	20.1000	10.0600	-0.5000	5.4000	32.4000
0	-0.7000	0.9997	0.0010	54.9830	0.8980

10.0000	100.0000	0.3000	-1.0000	17.0000	102.0000
0	-55.0000	10.8200	0.7000	-10.2000	-28.2000
0	0	-0.2213	8.0327	9.7945	-19.2327
0	0	14.0142	-0.2442	1.6724	22.0942
0	-0.7000	0.9997	0.0010	54.9830	0.8980

10.0000	100.0000	0.3000	-1.0000	17.0000	102.0000
0	-55.0000	10.8200	0.7000	-10.2000	-28.2000
0	0	-0.2213	8.0327	9.7945	-19.2327
0	0	14.0142	-0.2442	1.6724	22.0942
0	0	0.8620	-0.0079	55.1128	1.2569

4

10.0000	100.0000	0.3000	-1.0000	17.0000	102.0000
0	-55.0000	10.8200	0.7000	-10.2000	-28.2000
0	0	14.0142	-0.2442	1.6724	22.0942
0	0	0	8.0289	9.8210	-18.8839
0	0	0.8620	-0.0079	55.1128	1.2569

10.0000	100.0000	0.3000	-1.0000	17.0000	102.0000
0	-55.0000	10.8200	0.7000	-10.2000	-28.2000
0	0	14.0142	-0.2442	1.6724	22.0942
0	0	0	8.0289	9.8210	-18.8839
0	0	0	0.0071	55.0100	-0.1021

4

10.0000	100.0000	0.3000	-1.0000	17.0000	102.0000
0	-55.0000	10.8200	0.7000	-10.2000	-28.2000
0	0	14.0142	-0.2442	1.6724	22.0942
0	0	0	8.0289	9.8210	-18.8839
0	0	0	0	55.0013	-0.0853

ans =

2.0692

0.7852

1.5358

-2.3501

-0.0016

guass_scaled_partition_pivot:

```
>> guass_scaled_partial_pivot(A, b)
  6.0000   5.0000  11.0000   0.1000         0  33.0000
    0      1.7667  13.6667  -0.2667   2.0000  23.0000
10.0000 100.0000   0.3000  -1.0000  17.0000 102.0000
  3.0000   9.0000   4.0000   8.0000  11.0000   0.6000
  0.0100  -0.6000   1.0000         0  55.0000   1.0000

  6.0000   5.0000  11.0000   0.1000         0  33.0000
    0      1.7667  13.6667  -0.2667   2.0000  23.0000
    0     91.6667 -18.0333  -1.1667  17.0000  47.0000
  3.0000   9.0000   4.0000   8.0000  11.0000   0.6000
  0.0100  -0.6000   1.0000         0  55.0000   1.0000

  6.0000   5.0000  11.0000   0.1000         0  33.0000
    0      1.7667  13.6667  -0.2667   2.0000  23.0000
    0     91.6667 -18.0333  -1.1667  17.0000  47.0000
    0      6.5000  -1.5000   7.9500  11.0000 -15.9000
  0.0100  -0.6000   1.0000         0  55.0000   1.0000

  6.0000   5.0000  11.0000   0.1000         0  33.0000
    0      1.7667  13.6667  -0.2667   2.0000  23.0000
    0     91.6667 -18.0333  -1.1667  17.0000  47.0000
    0      6.5000  -1.5000   7.9500  11.0000 -15.9000
    0     -0.6083   0.9817  -0.0002  55.0000   0.9450

  6.0000   5.0000  11.0000   0.1000         0  33.0000
    0     91.6667 -18.0333  -1.1667  17.0000  47.0000
    0         0    14.0142  -0.2442   1.6724  22.0942
    0      6.5000  -1.5000   7.9500  11.0000 -15.9000
    0     -0.6083   0.9817  -0.0002  55.0000   0.9450

  6.0000   5.0000  11.0000   0.1000         0  33.0000
    0     91.6667 -18.0333  -1.1667  17.0000  47.0000
    0         0    14.0142  -0.2442   1.6724  22.0942
    0         0    -0.2213   8.0327   9.7945 -19.2327
    0     -0.6083   0.9817  -0.0002  55.0000   0.9450

  6.0000   5.0000  11.0000   0.1000         0  33.0000
    0     91.6667 -18.0333  -1.1667  17.0000  47.0000
    0         0    14.0142  -0.2442   1.6724  22.0942
    0         0    -0.2213   8.0327   9.7945 -19.2327
    0         0     0.8620  -0.0079  55.1128   1.2569

  6.0000   5.0000  11.0000   0.1000         0  33.0000
    0     91.6667 -18.0333  -1.1667  17.0000  47.0000
    0         0    14.0142  -0.2442   1.6724  22.0942
    0         0         0     8.0289   9.8210 -18.8839
    0         0     0.8620  -0.0079  55.1128   1.2569

  6.0000   5.0000  11.0000   0.1000         0  33.0000
    0     91.6667 -18.0333  -1.1667  17.0000  47.0000
    0         0    14.0142  -0.2442   1.6724  22.0942
    0         0         0     8.0289   9.8210 -18.8839
    0         0         0     0.0071  55.0100  -0.1021

  6.0000   5.0000  11.0000   0.1000         0  33.0000
    0     91.6667 -18.0333  -1.1667  17.0000  47.0000
    0         0    14.0142  -0.2442   1.6724  22.0942
    0         0         0     8.0289   9.8210 -18.8839
    0         0         0         0  55.0013  -0.0853
```

ans =

```
2.0692
0.7852
1.5358
-2.3501
-0.0016
```

Q6.

guass_no_pivot:

```
>> guass_no_pivot(A, b)
1. 0e+29 *

0. 0000    0. 0000    0. 0000    0. 0000
   0    -0. 1000   -0. 3000   -1. 0000
0. 0000    0. 0000   -0. 0000    0. 0000

1. 0e+29 *

0. 0000    0. 0000    0. 0000    0. 0000
   0    -0. 1000   -0. 3000   -1. 0000
   0    -0. 0030   -0. 0090   -0. 0300

1. 0e+29 *

0. 0000    0. 0000    0. 0000    0. 0000
   0    -0. 1000   -0. 3000   -1. 0000
   0         0     0. 0000         0

ans =

0
10
0
```

guass_partition_pivot:

```
>> guass_partial_pivot(A, b)
2

100    26    11    2
   0     1     3   10
   3    11   -13    4

100. 0000   26. 0000   11. 0000    2. 0000
      0     1. 0000    3. 0000  10. 0000
      0    10. 2200  -13. 3300    3. 9400

3

100. 0000   26. 0000   11. 0000    2. 0000
      0    10. 2200  -13. 3300    3. 9400
      0         0     4. 3043    9. 6145

ans =

-1. 0834
 3. 2989
 2. 2337
```

We can find that the result of two algorithms is different. In the first row, the 10^{-26} is very smaller, close to zero and if it does not exchange the rows with the $A(i, i)$, i is a constant. This will be beyond the accuracy of the computer. Since for each root the loss of the elimination will become larger and larger. If finding the maximum as in question 3 and swap, it will reduce the number of rounds, so they will have the different result.