Numerical Computation
Assignment 5

1. Find the LU factorization of the matrix

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 0 \\ 1 & 3 & 4 & 4 \\ 0 & 2 & 1 & -1 \end{bmatrix}$$

2. Solve the system by finding the LU factorization and then carrying out the two-step back substitution.

$$\begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

3. Please modify the Matlab function **lu_gauss** shown in lecture so we can compute the solution to $Ax = b$ using LU factorization (Show the m-file and highlight the modification(addition) parts.)

4. Implement Gaussian Jordan method in a Matlab M-file named **inverse**. And then modify the M-file **inverse** you found and named **inverse_solve** so we can compute the solution to $Ax = b$ using $x = A^{-1}b$.(Only show the **inverse_solve** m-file).

5. Solving the linear system below use the function you found in Q3 and Q4. Show the LU factors, the solution x, and z where $Lz = b$ when using **lu_gauss**. And show A^{-1} and the solution x when using **inverse_solve**.

$$\begin{bmatrix} 2 & 1 & -5 & 1 & 4 \\ 1 & -3 & 0 & -6 & 2 \\ 0 & 2 & -1 & 2 & 17 \\ 1 & 4 & -7 & 6 & 0 \\ 6 & 2 & 0 & 10 & 55 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 8 \\ 9 \\ -5 \\ 0 \\ 12 \end{bmatrix}$$

6. Find the inverse of the following matrix using the function **inverse** you found in Q4. In Matlab we can use $A \wedge -1$ to find the inverse directly. Compare the result **inverse**(A) and $A \wedge -1$, are they the same? Could you explain why.

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{bmatrix}$$