Numerical computation – Assignment 10

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Q1.

$m = 1, 2\ and\ 4\ subintervals$ which is nonequivalence. Apply the composite Trapezoid Rule for $\int_0^{\frac{\pi}{2}} \cos x\, dx$:

When $m = 1, h = \frac{\frac{\pi}{2}-0}{1} = \frac{\pi}{2}$

$\int_0^{\frac{\pi}{2}} \cos x\, dx \approx \frac{h}{2}\left[f(0) + f\left(\frac{\pi}{2}\right)\right] = \frac{\pi}{4}(1+0) = \frac{\pi}{4} \approx 0.7854$

And the true value is: $\int_0^{\frac{\pi}{2}} \cos x\, dx = sinx|_0^{\frac{\pi}{2}} = 1.$

$E = I - T = 1 - 0.7854 = 0.2146$

When $m = 2, h = \frac{\frac{\pi}{2}-0}{2} = \frac{\pi}{4}$

$\int_0^{\frac{\pi}{2}} \cos x\, dx \approx \frac{h}{2}\left[f(0) + f\left(\frac{\pi}{4}\right) + f\left(\frac{\pi}{4}\right) + f\left(\frac{\pi}{2}\right)\right] = \frac{\pi}{8}\left(1 + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + 0\right) \approx 0.9481$

And the true value is: $\int_0^{\frac{\pi}{2}} \cos x\, dx = sinx|_0^{\frac{\pi}{2}} = 1.$

$E = I - T = 1 - 0.9481 = 0.0519$

When $m = 4, h = \frac{\frac{\pi}{2}-0}{4} = \frac{\pi}{8}$

$\int_0^{\frac{\pi}{2}} \cos x\, dx \approx \frac{h}{2}\left[f(0) + f\left(\frac{\pi}{8}\right) + f\left(\frac{\pi}{8}\right) + f\left(\frac{\pi}{4}\right) + f\left(\frac{\pi}{4}\right) + f\left(\frac{3\pi}{8}\right) + f\left(\frac{3\pi}{8}\right) + f\left(\frac{\pi}{2}\right)\right] = \frac{\pi}{16}(1 + \frac{\sqrt{2+\sqrt{2}}}{2} + \frac{\sqrt{2+\sqrt{2}}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2-\sqrt{2}}}{2} + \frac{\sqrt{2-\sqrt{2}}}{2} + 0) \approx 0.9871$

And the true value is: $\int_0^{\frac{\pi}{2}} \cos x\, dx = sinx|_0^{\frac{\pi}{2}} = 1.$

$E = I - T = 1 - 0.9871 = 0.0129$

Q2.

$\int_2^3 e^x dx = e^x \quad |_2^3 \approx 12.6965$

Apply the composite Trapezoid Rule for $\int_2^3 e^x dx$ with equal subintervals:

When $h = 1$, $\frac{1}{2} * 1 * [f(2) + f(3)] \approx \frac{1}{2} * 27.4746 = 13.7373$

When $h = \frac{1}{2}$, $\frac{1}{2} * \frac{1}{2} * \left[f(2) + f\left(\frac{5}{2}\right) + f\left(\frac{5}{2}\right) + f(3)\right] \approx \frac{1}{4} * (51.8396) = 12.9599.$

$|E| \leq |\frac{-(b-a)h^2}{12} * \max_{2\leq x\leq 3} f''(x)|$ and $\max_{2\leq x\leq 3} f''(x) = \max_{2\leq x\leq 3} e^x = e^3 \approx 20.0855$

$|E| \leq \frac{3-2}{12n^2}e^3 \leq 10^{-3}, n^2 \geq 1{,}673.7916 \rightarrow n \geq 40.9120 \rightarrow n \geq 41$

Q3.

When $m = 2, h = \frac{\pi-0}{2}$

$\int_0^\pi x\, cosxdx \approx \frac{h}{3}\left[f(x_0) + 4\sum_{i=1,3,5\ldots}^{n-1} f(x_i) + 2\sum_{i=2,4,6\ldots}^{n-2} f(x_j) + f(n)\right] = \frac{\pi}{6}[f(0) +$

$4f\left(\frac{\pi}{2}\right) + f(\pi)\Big] = \frac{\pi}{6} * (0 + 0 - \pi) =- 1.6449$

$\int_0^\pi x \cos x\, dx = xsinx|_0^\pi - \int_0^\pi sinx\, dx = cos\,\pi - cos\,0 =- 2$

$E = I - T =- 2 + \frac{\pi^2}{6} =- 0.3551$

When $m = 4, h = \frac{\pi-0}{4}$

$\int_0^\pi x\, cosxdx \approx \frac{h}{3}\left[f(x_0) + 4\sum_{i=1,3,5\ldots}^{n-1} f(x_i) + 2\sum_{i=2,4,6\ldots}^{n-2} f(x_j) + f(n)\right] = \frac{\pi}{6}[f(0) +$

$4\left(f\left(\frac{\pi}{4}\right) + f\left(\frac{3\pi}{4}\right)\right) + 2f\left(\frac{\pi}{2}\right) + f(\pi)\Big] = \frac{\pi}{12} * (-1 - \sqrt{2})\pi =- 1.9856$

$\int_0^\pi x \cos x\, dx = xsinx|_0^\pi - \int_0^\pi sinx\, dx = cos\,\pi - cos\,0 =- 2$

$E = I - T =- 2 + \frac{\pi^2}{6} =- 0.0144$

$m = 8, h = \frac{\pi}{8}, x_0 = 0$

$\int_0^\pi x\, cosxdx \approx \frac{h}{3}\left[f(x_0) + 4\sum_{i=1,3,5\ldots}^{n-1} f(x_i) + 2\sum_{i=2,4,6\ldots}^{n-2} f(x_j) + f(n)\right] = \frac{\pi}{24}[f(0) +$

$4\left(f\left(\frac{\pi}{8}\right) + f\left(\frac{3\pi}{8}\right) + f\left(\frac{5\pi}{8}\right) + f\left(\frac{7\pi}{8}\right)\right) + 2\left(f\left(\frac{\pi}{4}\right) + f\left(\frac{\pi}{2}\right) + f\left(\frac{3\pi}{4}\right)\right) + f(\pi)\Big] = \frac{\pi}{24} * (-\pi -$

$4\left(\frac{\pi}{8}\cos\left(\frac{\pi}{8}\right) + \frac{3\pi}{8}\cos\left(\frac{3\pi}{8}\right) + \frac{5\pi}{8}\cos\left(\frac{5\pi}{8}\right) + \frac{7\pi}{8}\right) + 2\left(\frac{\pi}{4}\cos\left(\frac{\pi}{4}\right) + \frac{\pi}{2}\cos\left(\frac{\pi}{2}\right) + \frac{3\pi}{4}\cos\left(\frac{3\pi}{4}\right)\right)\Big) \approx$

$- 1.9992$

$\int_0^\pi x \cos x\, dx = xsinx|_0^\pi - \int_0^\pi sinx\, dx = cos\,\pi - cos\,0 =- 2$

$E = I - T =- 2 - (-1.9992) =- 0.0008$

Q4.

Using the Trapezoidal Rule$\rightarrow$ $m = 0$, when $n = 1 \rightarrow h = 4$

$R(0,0) = \frac{h}{2}[f(1) + f(5)] = 2(2.4142 + 3.2804) = 11.3892$

When $n = 2 \rightarrow h = 2$:

$R(1,0) = \frac{h}{2}[f(1) + f(3) + f(3) + f(5)] = (2.4142 + 2.8974 * 2 + 3.2804) = 11.4894$

When $n = 4 \rightarrow h = 1$:

$R(2,0) = \frac{h}{2}[f(1) + f(2) + f(2) + f(3) + f(3) + f(4) + f(4) + f(5)] = (2.4142 + 2 * 2.6734 + 2.8974 * 2 + 2 * 3.0976 + 3.2804) = 11.5157$

Using Simpson's Rule→ $m = 1$, when $h = 2$, we have x_0, x_2, x_4

$R(1,1) = \frac{1}{3}\left[f(1) + f(5) + 4\big(f(2) + f(4)\big) + 2\big(f(3)\big)\right] = 11.5245$

$R(2,1) = \frac{2}{3}\left[f(1) + f(5) + 4\big(f(3)\big)\right] = 11.5228$

$R(2,2) = \frac{4^2 R(2,1) - R(1,1)}{4^2 - 1} = 11.5246$

The most accurate value of $\int_1^5 f(x)\, dx$ is 11.5246.

Q5.

The function code is:

$f = x\, e^{2x}, a = 0\ and\ b = 4, n\ is\ the\ shape$

```
function r=romberg(f, a, b, n)
    r=zeros(n,n);
    interval=b-a;
    row=0;
    % Assign tintervale R(0,0)
    r(1,1)=interval*((f(a))+f(b))/2;
    step = 1
    while (row < n)
        interval=interval/2;
        sum=0;
        row=row+1
        for i=1:step
            x=interval*(2*i-1)+a;
            sum=sum+f(x);
        end
        r(row+1,1)=r(row,1)/2 + interval*sum;
        step = step*2;
        for j=1:row
            r(row+1,j+1)=r(row+1,j)+(r(row+1, j)-r(row, j))/(4^j-1);
        end
    end
end
```

Then we can get the result like this:

Function f

```
function y=fun(x)
    y=x*exp(1)^(2*x);
```

Result table

```
>> f = @fun

f =

  包含以下值的 function_handle:

    @fun

>> romberg(f, 0,4,5)

ans =

   1.0e+04 *

    2.3848         0         0         0         0         0
    1.2142    0.8240         0         0         0         0
    0.7289    0.5671    0.5500         0         0         0
    0.5765    0.5257    0.5229    0.5225         0         0
    0.5356    0.5220    0.5217    0.5217    0.5217         0
    0.5252    0.5217    0.5217    0.5217    0.5217    0.5217
```