北京师范大学
香港浸会大学 联合国际学院

# DS4003 Optimization Methods

## Assignment 2 Multivariate Optimization Problems — 2022 Spring

**Student No.:** **Student Name:**

Practical projects: in this project, you are expect to

1. be able to calculate the gradient and Hessian of a multivariate function.
2. be able to use the Newton Method to solve a practical problem.
3. be able to use the logistic regression.
4. please write in Jupyter Note book or Matlab Liverscript, and generate a pdf file.
5. submit your Jupyter notebook or matlab liverscript and the PDF file.

Reference

1. https://en.wikipedia.org/wiki/Logistic_regression

## 1 Question 1

Consider the following function with $\beta_0, \beta_1$ as independent variables:

$$\ell(\beta_0, \beta_1) = y \ln p(x) + (1 - y) \ln(1 - p(x)),$$

where

$$p(x) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}}$$

1. calculate $\nabla \ell = (\frac{\partial \ell}{\partial \beta_0}, \frac{\partial \ell}{\partial \beta_1})^T$.
2. calculate $\nabla^2 \ell$.

## 2 Question 2

Suppose we have some binary observations on whether a student pass the course optimization method, and the the number of hours each student spending studying

| Hours ($x_k$) | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 1.75 | 2.00 | 2.25 | 2.50 | 2.75 | 3.00 | 3.25 | 3.50 | 4.00 | 4.25 | 4.50 | 4.75 | 5.00 | 5.50 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Pass ( $y_k$ ) | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |

Suppose that we want to fit some binary observations with the following function

$$y_k = f(x_k) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_k)}}$$

which is often referred to logistic regression. $p_k$ are the probabilities that corresponding $y_K$ will be (pass) and $1 - p_k$ are the probabilities that will be zero(failed).We wish to find the parameters $\beta_0$ and $\beta_1$ which give the best fit to the data. The measure of goodness fit is given by the likelihood functions

$$L = \prod_{k:y_k=1} p_k \prod_{k:y_k=0} (1 - p_k)$$

and the best fit is obtained when $L$ is maximized. The maximum of $L$ will also be the maximum of the log-likelihood $\ell$,defined as

$$\ell = \sum_{k:y_k=1} \ln(p_k) + \sum_{k:y_k=0} \ln(1 - p_k) = \sum_{k=1}^{K} (y_k \ln(p_k) + (1 - y_k) \ln(1 - p_k)) \tag{1}$$

1. verify (1).
2. calculate $\nabla \ell$
3. calculate $\nabla^2 \ell$
4. use the given data, write a Newton methods to fit the data, make a contour plot to denote how the solutions converge in the $\beta_0 - \beta_1$ plan.
5. specify that how you choose the initial values.
6. compare your results with other fitting tool box, and write your own logistic regression program which can work versatily with as little as human interactions.