Calculo dos pesos para $\xi = (-1, 1, 1)$

$\xi_1 = -1$; $\xi_2 = 1$; $\xi_3 = 1$ e $N = 3$

$w_{ij} = \frac{1}{N} \xi_i \xi_j$

$w_{12} = \frac{1}{3} \cdot -1 \cdot 1 = \frac{-1}{3}$

$w_{13} = \frac{1}{3} \cdot -1 \cdot 1 = \frac{-1}{3}$

$w_{21} = \frac{1}{3} \cdot 1 \cdot -1 = \frac{-1}{3}$

$w_{23} = \frac{1}{3} \cdot 1 \cdot 1 = \frac{1}{3}$

$w_{31} = \frac{1}{3} \cdot 1 \cdot -1 = -1/3$

$w_{32} = \frac{1}{3} \cdot 1 \cdot 1 = \frac{1}{3}$

Resumo:

$w_{12} = \frac{-1}{3}$ $w_{13} = \frac{-1}{3}$ $w_{21} = \frac{-1}{3}$ $w_{23} = \frac{1}{3}$ $w_{31} = \frac{-1}{3}$ $w_{32} = \frac{1}{3}$

Agora iremos proceder com a dinâmica do sistema usando os pesos obtidos, aplicadas à equação de movimento

$$S_i = \mathrm{sgn}\left(\sum_j^n w_{ij} S_j\right)$$

Para todos os possíveis estados em uma rede Hopfield com 3 neurônios utilizando atualizações síncronas e assíncronas.

0)

Atualização sincrona. $\xi = (-1, -1, -1)$

$w_{12} = \frac{-1}{3}$ $w_{13} = \frac{-1}{3}$ $w_{21} = \frac{-1}{3}$ $w_{23} = \frac{1}{3}$ $w_{31} = \frac{-1}{3}$ $w_{32} = \frac{1}{3}$

$\xi = (-1, -1, -1) = S(0)$; $S_1(0) = -1$; $S_2(0) = -1$; $S_3(0) = -1$

$S_1(1) = sgn(w_{12} S_2(0) + w_{13} S_3(0)) = sgn\left(\frac{-1}{3} \cdot -1 + \frac{-1}{3} \cdot -1\right) = sgn\left(\frac{1}{3} + \frac{1}{3}\right) = sgn\left(\frac{2}{3}\right) = 1$

$S_2(1) = sgn(w_{21} S_1(0) + w_{23} S_3(0)) = sgn\left(\frac{-1}{3} \cdot -1 + \frac{1}{3} \cdot -1\right) = sgn\left(\frac{1}{3} + \frac{-1}{3}\right) = sgn(0) = 1$

$S_3(1) = sgn(w_{31} S_1(0) + w_{32} S_2(0)) = sgn\left(\frac{-1}{3} \cdot -1 + \frac{1}{3} \cdot -1\right) = sgn\left(\frac{1}{3} + \frac{-1}{3}\right) = sgn(0) = 1$

$S(1) = (1, 1, 1)$ (0.1)

$$w = \begin{pmatrix} 0 & \frac{-1}{3} & \frac{-1}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

$S_1(2) = sgn\left(-\frac{1}{3} \cdot 1 + \frac{-1}{3} \cdot 1\right) = sgn\left(\frac{-2}{3}\right) = -1$

$S_2(2) = sgn\left(\frac{-1}{3} \cdot 1 + \frac{1}{3} \cdot 1\right) = sgn(0) = 1$

$S_3(2) = sgn\left(-\frac{1}{3} \cdot 1 + \frac{1}{3} \cdot 1\right) = sgn(0) = 1$

$S(2) = (-1, 1, 1) = \xi$ // (0.2)

1)

Sincrona. $\xi = (-1, -1, 1)$

$$\omega = \begin{pmatrix} 0 & \frac{-1}{3} & \frac{-1}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

$S_1(1) = sgn\left(\frac{-1}{3} \cdot -1 + \frac{-1}{3} \cdot 1\right) = sgn\left(\frac{+1}{3} + \frac{-1}{3}\right) = sgn(0) = 1$

$S_2(1) = sgn\left(\frac{-1}{3} \cdot -1 + \frac{1}{3} \cdot 1\right) = sgn\left(\frac{1}{3} + \frac{1}{3}\right) = sgn\left(\frac{2}{3}\right) = 1$

$S_3(1) = sgn\left(\frac{-1}{3} \cdot -1 + \frac{1}{3} \cdot 1\right) = sgn\left(\frac{1}{3} + \frac{1}{3}\right) = sgn(2/3) = 1$

$S(1) = (1, 1, 1)$ $\qquad (1.1)$

$S_1(2) = sgn\left(-\frac{1}{3} \cdot 1 + \frac{-1}{3} \cdot 1\right) = sgn\left(\frac{-2}{3}\right) = -1$

$S_2(2) = sgn\left(\frac{-1}{3} \cdot 1 + \frac{1}{3} \cdot 1\right) = sgn(0) = 1$

$S_3(2) = sgn\left(-1/3 \cdot 1 + 1/3 \cdot 1\right) = sgn(0) = 1$

$S(2) = (-1, 1, 1) = \xi$ // $\qquad (1.2)$

2)

Simetroma. $\zeta = (-1, 1, -1)$

$$\omega = \begin{pmatrix} 0 & \frac{-1}{3} & \frac{-1}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

$S_1(1) = sgn\left(\frac{-1}{3} \cdot 1 + \frac{-1}{3} \cdot -1\right) = sgn\left(\frac{-1}{3} + \frac{1}{3}\right) = sgn(0) = 1$

$S_2(1) = sgn\left(-\frac{1}{3} \cdot -1 + \frac{1}{3} \cdot -1\right) = sgn\left(\frac{1}{3} + \frac{-1}{3}\right) = sgn(0) = 1$

$S_3(1) = sgn\left(\frac{-1}{3} \cdot -1 + \frac{1}{3} \cdot 1\right) = sgn\left(\frac{1}{3} + \frac{1}{3}\right) = sgn\left(\frac{2}{3}\right) = 1$

$S(1) = (1, 1, 1)$ (2.1)

$S_1(2) = sgn\left(-\frac{1}{3} \cdot 1 + \frac{-1}{3} \cdot 1\right) = sgn\left(\frac{-2}{3}\right) = -1$

$S_2(2) = sgn\left(\frac{-1}{3} \cdot 1 + \frac{1}{3} \cdot 1\right) = sgn(0) = 1$

$S_3(2) = sgn\left(-\frac{1}{3} \cdot 1 + \frac{1}{3} \cdot 1\right) = sgn(0) = 1$

$S(2) = (-1, 1, 1) = \xi$ // (2.2)

3)

Sincrona. $\zeta = (1, -1, 1)$ (3.0)

$$\omega = \begin{pmatrix} 0 & \frac{-1}{3} & \frac{-1}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

$S_1(1) = sgn\left(\frac{-1}{3} \cdot -1 + \frac{-1}{3} \cdot 1\right) = sgn\left(\frac{+1}{3} + \frac{-1}{3}\right) = sgn(0) = 1$

$S_2(1) = sgn\left(\frac{-1}{3} \cdot 1 + \frac{1}{3} \cdot 1\right) = sgn\left(\frac{-1}{3} + \frac{1}{3}\right) = sgn(0) = 1$

$S_3(1) = sgn\left(-\frac{1}{3} \cdot 1 + \frac{1}{3} \cdot -1\right) = sgn\left(-\frac{1}{3} + -\frac{1}{3}\right) = sgn\left(-\frac{2}{3}\right) = -1$

$S(1) = (1, 1, -1)$ (3.1)

Por (3.1) instável.

4

Simetria. $\zeta = (1, 1, -1)$ $\quad (4.0)$

$$\omega = \begin{pmatrix} 0 & \frac{-1}{3} & \frac{-1}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

$S_1(1) = sgn(-1/3 \cdot 1 + -1/3 \cdot -1) = sgn(-1/3 + 1/3) = sgn(0) = 1$

$S_2(1) = sgn(-1/3 \cdot 1 + 1/3 \cdot -1) = sgn(-1/3 + -1/3) = sgn(-2/3) = -1$

$S_3(1) = sgn(-1/3 \cdot 1 + 1/3 \cdot 1) = sgn(-1/3 + 1/3) = sgn(0) = 1$

$S(2) = (1, -1, 1)$ $\quad (4.1)$

Por (3.0) e (3.1)

$S(2) = (1, 1, -1)$

Por (4.0) e (4.1)

$S(3) = (1, -1, 1)$ ....

Instável.

5)

Simetria. $\xi = (1, 1, 1)$

$$\omega = \begin{pmatrix} 0 & \frac{-1}{3} & \frac{-1}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

Por (0.1) e (0.2)

$S(1) = (-1, 1, 1) = \xi$ // (5.1)

6)

Simetroma. $\zeta = (-1, 1, 1)$

$$\omega = \begin{pmatrix} 0 & \frac{-1}{3} & \frac{-1}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

$S_1(1) = \operatorname{sgn}(-1/3 \cdot 1 + -1/3 \cdot 1) = \operatorname{sgn}(-2/3) = -1$

$S_2(1) = \operatorname{sgn}(-1/3 \cdot -1 + 1/3 \cdot -1) = \operatorname{sgn}(0) = 1$

$S_3(1) = \operatorname{sgn}(-1/3 \cdot -1 + 1/3 \cdot 1) = \operatorname{sgn}(2/3) = 1$

$S(1) = (-1, 1, 1) = \xi$ //

(6.1)

7)

Simetroma. $\zeta = (1, -1, -1)$

$$\omega = \begin{pmatrix} 0 & \frac{-1}{3} & \frac{-1}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

$S_1(1) = sgn(-1/3 \cdot -1 + -1/3 \cdot -1) = sgn(2/3) = 1$

$S_2(1) = sgn(-1/3 \cdot 1 + 1/3 \cdot -1) = sgn(-2/3) = -1$

$S_3(1) = sgn(-1/3 \cdot 1 + 1/3 \cdot -1) = sgn(-2/3) = -1$

$S(1) = (1, -1, -1) = -\xi$ // (7.1)

8)

Assimetroma. $\zeta = (-1, 1, 1)$

$$\omega = \begin{pmatrix} 0 & \frac{-1}{3} & \frac{-1}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

$s_1(1) = sgn(-1/3 \cdot 1 + -1/3 \cdot 1) = sgn(-2/3) = -1$

$s_2(2) = sgn(-1/3 \cdot -1 + 1/3 \cdot 1) = sgn(\frac{1}{3} + \frac{1}{3}) = sgn(2/3) = 1$

$s_3(3) = sgn(-1/3 \cdot -1 + 1/3 \cdot 1) = sgn(\frac{1}{3} + \frac{1}{3}) = sgn(2/3) = 1$

$s(3) = (-1, 1, 1) = \xi$ //

(8.1)

9)

Assimetroma. $\zeta = (1, -1, -1)$

$$\omega = \begin{pmatrix} 0 & \frac{-1}{3} & \frac{-1}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

$S_1(1) = sgn\left(-\frac{1}{3} \cdot -1 + -\frac{1}{3} \cdot -1\right) = sgn\left(\frac{1}{3} + \frac{1}{3}\right) = sgn\left(\frac{2}{3}\right) = 1$

$S_2(2) = sgn\left(-\frac{1}{3} \cdot 1 + \frac{1}{3} \cdot -1\right) = sgn\left(-\frac{1}{3} + -\frac{1}{3}\right) = sgn\left(-\frac{2}{3}\right) = -1$

$S_3(3) = sgn\left(-\frac{1}{3} \cdot 1 + \frac{1}{3} \cdot -1\right) = sgn\left(-\frac{1}{3} + -\frac{1}{3}\right) = sgn\left(-\frac{2}{3}\right) = -1$

$S(3) = (1, -1, -1) = -\xi$ //

(9.1)

10)

Assimetroma. $\zeta = (-1, -1, -1)$

$$\omega = \begin{pmatrix} 0 & \frac{-1}{3} & \frac{-1}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

$S_1(1) = sgn(-1/3 \cdot -1 + -1/3 \cdot -1) = sgn(-2/3) = -1$

$S_2(2) = sgn(-1/3 \cdot -1 + 1/3 \cdot -1) = sgn(1/3 + -1/3) = sgn(0) = 1$

$S_3(3) = sgn(-1/3 \cdot -1 + 1/3 \cdot 1) = sgn(1/3 + 1/3) = sgn(2/3) = 1$

$S(3) = \xi$ //

(10.1) ✗ ?

11)

Assimetroma. $\zeta = (-1, -1, 1)$

$$\omega = \begin{pmatrix} 0 & \frac{-1}{3} & \frac{-1}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

$S_1(1) = sgn(-1/3 \cdot -1 + -1/3 \cdot 1) = sgn(1/3 + -1/3) = sgn(0) = 1$

$S_2(2) = sgn(-1/3 \cdot 1 + 1/3 \cdot 1) = sgn(-1/3 + 1/3) = sgn(0) = 1$

$S_3(3) = sgn(-1/3 \cdot 1 + 1/3 \cdot 1) = sgn(-1/3 + 1/3) = sgn(0) = 1$

$S(3) = (1, 1, 1)$ (11.1)

Por (15.1)

$S(6) = (-1, 1, 1) = \xi$ //

12)

Assimetroma. $\zeta = (1, -1, 1)$

$$\omega = \begin{pmatrix} 0 & \frac{-1}{3} & \frac{-1}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

$S_1(1) = sgn(-1/3 \cdot -1 + -1/3 \cdot 1) = sgn(1/3 + -1/3) = sgn(0) = 1$

$S_2(2) = sgn(-1/3 \cdot 1 + 1/3 \cdot 1) = sgn(-1/3 + 1/3) = sgn(0) = 1$

$S_3(3) = sgn(-1/3 \cdot 1 + 1/3 \cdot 1) = sgn(-1/3 + 1/3) = sgn(0) = 1$

$S(3) = (1, 1, 1)$ por (15.1)

$S(6) = (-1, 1, 1) = \xi$ //

X ?

13)

Assimetroma. $\zeta = (-1, 1, -1)$

$$\omega = \begin{pmatrix} 0 & \frac{-1}{3} & \frac{-1}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

$S_1(1) = sgn(-1/3 \cdot 1 + -1/3 \cdot -1) = sgn(-1/3 + 1/3) = sgn(0) = 1$

$S_2(2) = sgn(-1/3 \cdot 1 + 1/3 \cdot -1) = sgn(-1/3 + -1/3) = sgn(-2/3) = -1$

$S_3(3) = sgn(-1/3 \cdot 1 + 1/3 \cdot -1) = sgn(-1/3 + -1/3) = sgn(-2/3) = -1$

$S(3) = (1, -1, -1) = -\xi$ // (13.1)

1(c)

Assimetroma. $\zeta = (1, 1, -1)$

$$\omega = \begin{pmatrix} 0 & \frac{-1}{3} & \frac{-1}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

$S_1(1) = sgn(-1/3 \cdot 1 + -1/3 \cdot -1) = sgn(-1/3 + 1/3) = sgn(0) = 1$

$S_2(2) = sgn(-1/3 \cdot 1 + 1/3 \cdot -1) = sgn(-1/3 + -1/3) = sgn(-2/3) = -1$

$S_3(3) = sgn(-1/3 \cdot 1 + 1/3 \cdot -1) = sgn(-1/3 + -1/3) = sgn(-2/3) = -1$

$S(3) = (1, -1, -1) = -\xi$ //

15)

Assimetroma. $\zeta = (1, 1, 1)$

$$\omega = \begin{pmatrix} 0 & \frac{-1}{3} & \frac{-1}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

$S_1(1) = \operatorname{sgn}(-1/3 \cdot 1 + -1/3 \cdot 1) = \operatorname{sgn}(-2/3) = -1$

$S_2(2) = \operatorname{sgn}(-1/3 \cdot -1 + 1/3 \cdot 1) = \operatorname{sgn}(1/3 + 1/3) = \operatorname{sgn}(2/3) = 1$

$S_3(3) = \operatorname{sgn}(-1/3 \cdot -1 + 1/3 \cdot 1) = \operatorname{sgn}(1/3 + 1/3) = \operatorname{sgn}(2/3) = 1$

$S(3) = (-1, 1, 1) = \xi$ //

(15.1)