HW 5 Problem 3

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Proof for Lemma 1:

The condition for E' guarantees one edge forms between every node, because every node has at least one minimum weighted edge. If RDMST is a tree, it can not contain any cycle. So if there is a cycle in a digraph, then the underlying graph must also have a cycle. This would be a direct contradiction of the definition for RDMST, because a tree can not have any cycle. On the other hand, if the digraph is an RDMST, then it can not have a cycle because RDMST can't contain any cycle. If an underlying cycle exist, then a node within a cycle must have two in-degree, because a cycle can't exist in RDMST, so the only way to maintain this underlying cycle is for the cycle to travel in two different direction (the node where the direction changes has two in-degree). However, this again goes against the definition of RDMST, because RDMST is a directed tree where every node has one indegree. Therefore, if E' is true, then T is either an RDMST of contains a cycle.

Proof for Lemma 2:

Forward proof: If T is RDMST of g = (V, E, w), then T is also RDMST of g = (V, E, w').

Every node has at least one unique minimum indegree. So when every degree is subtracted by the minimum in-degree to form w', then the minimum in-degree would become zero (w(min) - m(v) = 0 = w'(min), with min being the minimum indegree edge). Therefore, subgraph T will find the same unique RDMST subgraph, because all it has to do is to find the edges that are zero.

Backward proof: If T is RDMST of g = (V, E, w'), then T is RDMST of g = (V, E, w).

Because the minimum of w' is zero, adding it by m(v) will return back to the original minimum weight value. Every node has a unique minimum indegree. So g = (V, E, w) will follow the minimum weight value for each node and result in the exact same RDMST as g = (V, E, w').