Task 2.1 Analysis of Co-occurrence Patterns of Points of Interest (POI)

Below are the frequent POIs found (occurring at least 35% of the time).

	Frequent Itemsets (35% support)
City A	['Transit Station'] ['Real Estate'] ['Home Appliances'] ['Hair Salon'] ['Church'] ['Building Material'] ['Heavy Industry']
City B	['Transit Station'] ['Church']
City C	['Park'] ['Transit Station'] ['Elderly Care Home'] ['Real Estate'] ['Home Appliances'] ['Accountant Office'] ['Building Material'] ['Heavy Industry'] ['Park', 'Transit Station'] ['Transit Station', 'Building Material']
City D	['Church']

We can see that ['Church'] occurs as a frequent itemset in cities A, B, and D but not C. This might suggest that City C has fewer churchgoers. We also observe that ['Transit Station'] is a frequent itemset of cities A, B, and C but not D, suggesting that city D is not as well connected.

Uniquely, City C has ['Elderly Care Home'] as a frequent itemset, suggesting that it has a larger elderly population. ['Accountant Office'] is also a frequent itemset in City C, suggesting that it might be a business hub where there is a lot of accounting to do. City A is also the only one with ['Hair Salon'] as a frequent itemset, suggesting that its population might be more conscious of their hair condition.

['Real Estate'], ['Home Appliances'], ['Building Material'] and ['Heavy Industry'] are also frequent itemsets in cities A and C, suggesting that cities A and C are undergoing some construction projects.

['Park', 'Transit Station'] occurs together frequently in City C, which suggests that many transit stations in City C are near parks. ['Transit Station', 'Building Material'] also occur together frequently in City C, which makes sense as it would be convenient to transport building material from an area near a transit station. Generating rules and picking confidence above 0.75, we get 'Park -> Transit Station' and 'Building Material -> Transit station', with both having interest above 0.45, which suggests that if there are parks or building materials around, there is likely a transit station in the same area for City C.

In code, we implemented the apriori algorithm and checked its results against the mlxtend library's implementation for correctness.

Task 2.2 Mining Sequential Patterns

Data Preprocessing

We converted the (x,y) coordinates from strings to Shapely Point objects and converted d and t into pandas datetime (nanoseconds) format to resolve formatting differences. We edited the code to add the L2 distance calculation as the trackintel source code only provides haversine distance.

Several parameters then had to be set for the trackintel functions. For the distance threshold, we settled on L2 >= 50, roughly representing an x shift of 5 and a y shift of 5. We set the time threshold at 31 to accommodate the temporal resolution of 30 minutes, and the gap threshold at 59 to create a new tripleg if an hour has passed since the last positionfix.

We observed that consecutive positionfixes that do not meet L2 >= 50 will be grouped together as a staypoint and the resultant staypoint coordinate is the average of their coordinates. This resulted in many tripleg coordinates containing non-integer values. We rounded all values to the nearest integer to avoid removing these points and losing data.

	A	B	C	D
All triplegs	14,687,943	4,227,889	3,207,578	1,112,211
Month 1 triplegs	5,798,250	1,717,501	1,318,140	495,921

The table above shows the total number of generated triplegs and the number of triplegs in month 1, which was used for analysis.

Data Analysis

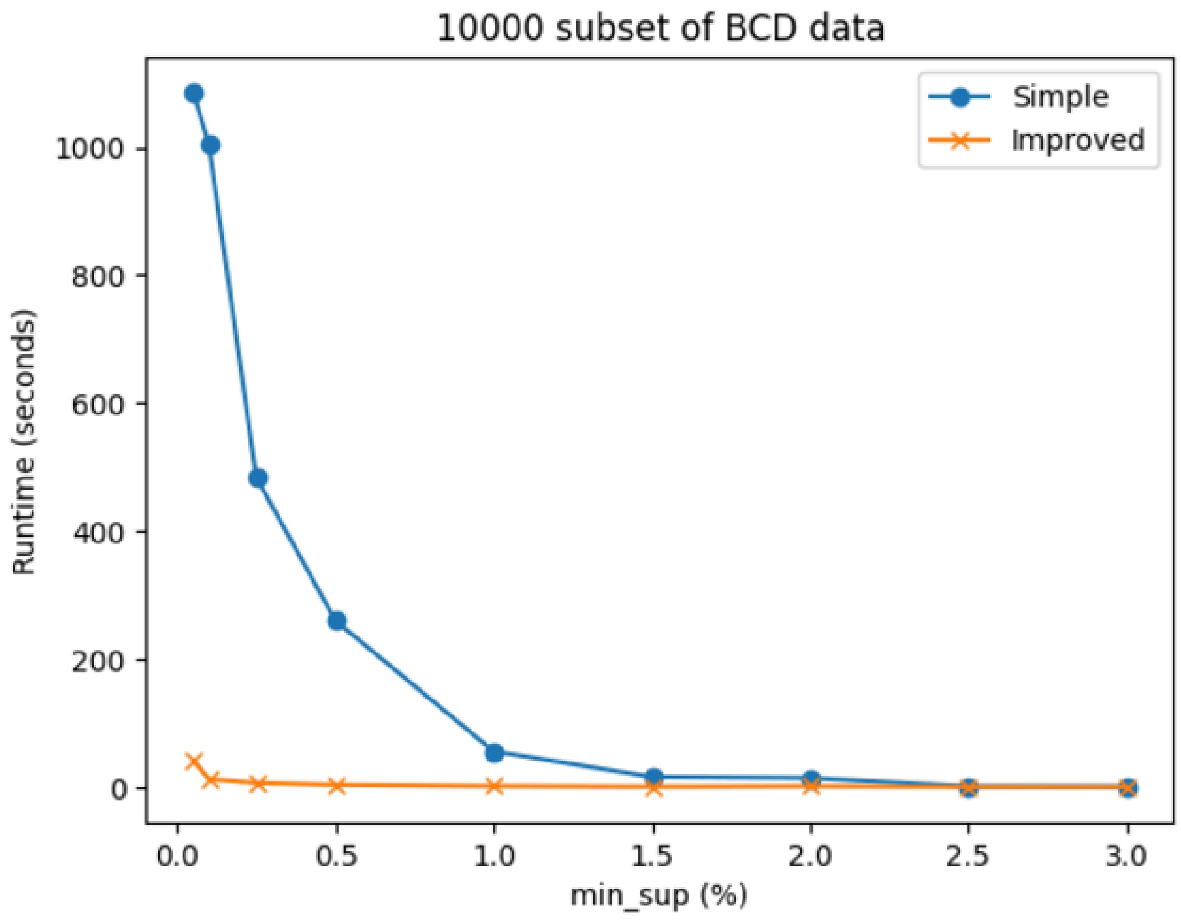
The potential candidate space is relatively large (40,000 for C_1, 1,600,000,000 for C_2). Thus we propose 2 hypotheses:

1. The time complexity can be reduced by recording the indexes of all support sequences for F_{k-1} and only scanning these sequences to compute C_k and F_k
2. The minimum support level should be low

GSP Algorithm

We manually implemented GSP to test hypothesis 1, using arrays to record support in the format [[elements], support, [support_indexes]] and only iterating over the support_indexes arrays of F_1 parents when generating their F_2 children and counting support. Since the data is sparse, the increase in space complexity is manageable. We also recorded all support counts for F_1 so that multiple support levels could be tested efficiently without recomputing it.

For each itemset, simple GSP makes n (number of sequences) passes whereas the improved GSP only makes s_{avg} (average support in F_1) number of passes. This saves a significant amount of time, especially when the minimum support level is low.



This graph shows the advantage of improved GSP over simple GSP at various supports.

Minimum Support

min_sup	F_A (F_{A2})	F_B (F_{B2})	F_C (F_{C2})	F_D (F_{D2})
5%	0	0	0	0
1%	0	6	6	0
0.1%	0	709 (11)	976 (24)	587 (1)

For hypothesis 2, we experimented with a few support levels to find one that yielded potentially useful results. The table shows the frequent subsequence counts at each support (with F_2 counts in brackets). 0.1% was the first support level that returned some F_2.

Results

Common mobility patterns we found were 2-sequences with both items at the same point. Some of these points were: (79, 93) and (80, 93) for B, from (24, 150) to (26, 153) for C, and (101, 102) for D. We deduce that these are densely populated residential areas.

Task 3 Open Advanced Tasks

Problem Statement

Following definitions by Hong et al. (in Appendix), we aim to design an algorithm that, given an input of a user_id and his fixed-length (99 in our model) sequence of consecutive staypoints, together with corresponding sequences of location_id, arrival_time, duration stayed, location area (area of convex hull), location centre and Place Of Interest (POI) context, will output a prediction of the next location to be visited. For a sample of inputs, our output will be a sample_size x 22179 probability matrix, where the $(i,j)^{th}$ element gives the probability that the next location of data i is location j. next_location_id is used as the ground truth class labels.

Data Preprocessing

Cleaning

We removed rows where x,y = ±999 as this means missing data. Then we multiplied x and y by 500 as 1 grid unit = 500m, then multiplied t by 30 to convert it to minutes before combining with d to form a proper DateTime (assuming UTC timezone & that d = 0 starts from 1st Jan 2024). We then filtered out the last half of the time period and users as the dataset was too large. We finally obtained 2175462 positionfixes.

Parameter Settings

Staypoints Generation

In Trackintel, we need to set gap_threshold ≥ temporal_resolution = minimum duration between any 2 positionfixes = 30 min, otherwise no staypoints will be generated. Based on Trackintel, a set with only 1 positionfix is allowed to be a staypoint. To avoid this, we used time_threshold = temporal_resolution + 1 > temporal_resolution & gap_threshold = 2*temporal_resolution - 1 < 2*temporal_resolution (i.e. do not allow tracking gaps). We set distance_threshold = spatial_resolution = minimum distance between any 2 positionfixes = 500m so that for each staypoint, its positionfixes have identical geometries. For our data with poor spatial resolution, this is ideal as 1 positionfix covers $500^2 = 250000m^2$ of area which is already too big so we cannot allow for any more discrepancy in positionfixes' geometries in a staypoint. Trackintel only supports haversine distance which is for latitude-longitude coordinates but we are using flat cartesian coordinates for geometry, so we did monkey patching to use the euclidean distance function. We finally obtained 171105 staypoints.

Locations Generation

We used DBSCAN clustering as it accommodates non-spherical clusters of staypoints. We used a euclidean distance function as explained. We set num_samples (minPts for DBSCAN) to 1 so that every staypoint will be a core point and thus belong to exactly 1 location and there will be no outliers, which makes sense since every staypoint, even if isolated, is at least a location in itself. If epsilon < spatial_resolution then every individual staypoint is a unique location, which is not very meaningful, so we need to avoid this by ensuring that epsilon ≥ spatial_resolution. Also min_intercluster_distance = (⌊epsilon / spatial_resolution⌋+ 1)*spatial_resolution, so we set this minimum separation distance between locations to 1000m, then set epsilon = min_intercluster_distance - spatial_resolution. We finally obtained 22179 locations

Preparation of Input Data

POI Context Vector Generation (Hong et al., 2023)

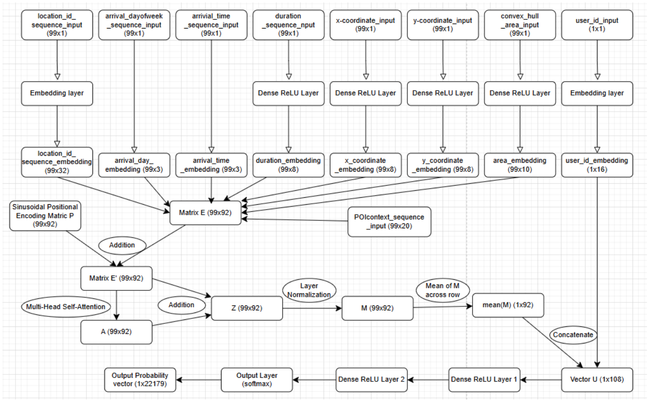
We first set the vicinity_radius to 1000m. Then for each location, compute the POI vector, where element j of the vector gives the count of POIs of category j within the vicinity_radius of the location's centre. We then treat each location as a document and each POI category as a word, so each POI vector can then be viewed as a Bag-Of-Word (BOW) vector. We stack them as row vectors of BOW matrix. We then fit and transform this BOW matrix using Latent Dirichlet Allocation (LDA) algorithm to obtain topic distribution matrix, whose $(i,j)^{th}$ element gives probability of topic j occuring in location i.

Obtaining Historical Sequences

Afterwards, we also calculated the convex hull area of each location. Then we filter out positionfixes which do not belong to any staypoint, then merge additional information from staypoints using staypoint_id, and from locations using location_id. We then sort this complete dataframe by user_id and tracked_at, then group by user_id, and for each user, obtain his full ordered sequence of staypoints and relevant information. Arrival timestamp was split into day & hour to enable the model to identify weekly and daily patterns respectively. We then filter out users whose sequence length < 100 as we want our input sequence length = 99 (excluding ground truth). If on average each user has about 10 staypoints/day, then duration span of the input sequence is about 10 days, which is reasonable as up to the previous 7 days of historical data is relevant in next location prediction (Hong et al., 2023). For each user, we then obtain all possible 100-length subsequences of consecutive elements from his full sequences, then remove the last element from each sequence as it is used as ground truth (next_location_id)

Model Architecture

Flow Chart for Single Data Input



location_id_ sequence_input (99x1)
arrival_dayofweek _sequence_input (99x1)
arrivial_time _sequence_input (99x1)
duration _sequence_nput (99x1)
x-coordinate_input (99x1)
y-coordinate_input (99x1)
convex_hull _area_input (99x1)
user_id_input (1x1)
Embedding layer
Dense ReLU Layer
Dense ReLU Layer
Dense ReLU Layer
Dense ReLU Layer
Embedding layer
location_id_ sequence_embedding (99x32)
arrival_day_ embedding (99x3)
arrival_time _embedding (99x3)
duration_embedding (99x8)
x_coordinate _embedding (99x8)
y_coordinate _embedding (99x8)
area_embedding (99x10)
user_id_embedding (1x16)
Sinusoidal Positional Encoding Matric P (99x92)
Matrix E (99x92)
POIcontext_sequence _input (99x20)
Addition
Matrix E' (99x92)
Multi-Head Self-Attention
Addition
A (99x92)
Z (99x92)
Layer Normalization
M (99x92)
Mean of M across row
mean(M) (1x92)
Concatenate
Vector U (1x108)
Dense ReLU Layer 1
Dense ReLU Layer 2
Output Layer (softmax)
Output Probability vector (1x22179)

Referring to our flow chart, in generation of embeddings from inputs, embedding layers were used for categorical features while dense layers with ReLU (Rectified Linear Unit) activation function were used for numeric features (except arrival timestamp). Our embedding row vector of arrival timestamp (formula in Appendix) represents both linear and cyclic (using sinusoidal functions scaled to its period (24 for hour, 7 for day)) nature of time**.** As POI context vector sequence already sufficiently represents land use context information, we will not further embed it and simply take it as it is. All concatenations were done along the column axis to preserve separate feature dimensions. We used sinusoidal positional encoding to get positional_encoding_matrix P (formula in Appendix). For the Multi-Head Self-Attention (MHSA) module (formulas in Appendix), we set key_dim such that A and E' will have the same dimensions. We calculated Z = A+E' so as to retain information from E', then we obtain M by layer-normalisation (formula in Appendix) of Z to prevent values from growing too large as they propagate through layers. The single vector mean(M) is representative of the entire sequence information. Final output layer uses softmax activation as it is producing a probability vector for Multiclass Classification. (model details in Appendix)

Model Explanation

Rationale of LDA

The rationale of using LDA is that we can set the dimension of the POI context vector, which is the number of topics present among the locations and POI categories, to be smaller than the number of POI categories since we can expect them to have some common topics, e.g. "Food". This enables us to not only reduce the dimension of POI vector, but also capture similarities between POIs using common topics which will be beneficial for the model to infer the land use context more effectively.

Rationale of Positional Encoding

Getting E' = E + P is essential in Transformers (Vaswani et al., 2017) because it provides a sense of order of sequence to the input data, which is inherently missing in the Transformer architecture due to its non-sequential design. For the i^{th} staypoint, row i of P gives the embedding row vector for the number i.

Rationale of MHSA

Each row i of E' is now an embedding row vector for the i^{th} staypoint's entire information (except user_id). Each row i of the attention output A is thus an attention (relevance) weighted sum of the value (V) information of all rows of E', with each row contributing proportionally based on its relevance to the row of E'. Therefore, this self-attention mechanism captures both short- and long-range dependencies regardless of sequence length. With multiple heads, MHSA can learn richer, multi-perspective representations of the sequence in parallel as each head has its own set of learnable parameters.

Model Compilation

Metric & Loss Function

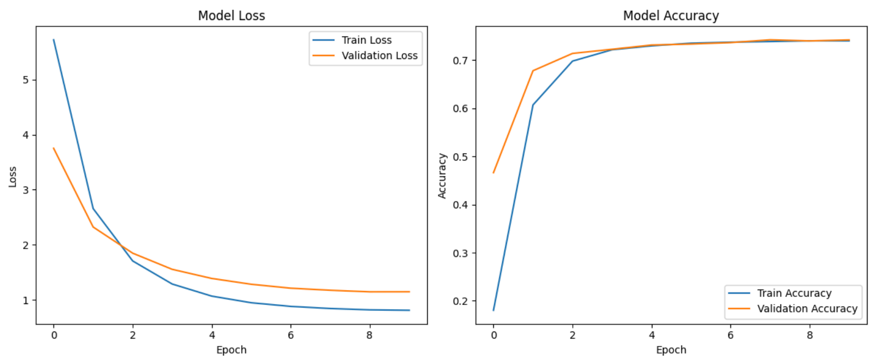
Sparse_categorical_crossentropy (SCCE) was used as loss function (formula in Appendix) as we are using true class labels as ground truth and they are not one-hot encoded. The metric to be reported during training is accuracy (same as top-1 accuracy).

Optimiser

Adaptive Moment Estimation (ADAM) optimiser (equations in Appendix) was used as it is more effective than Stochastic Gradient Descent (SGD) (formula in Appendix). To update each learnable parameter (θ), instead of using gradient at current iteration step t (g_t), ADAM uses (bias-corrected) moving average of past g values ($\hat{m}_t$), which smooths noisy updates (u_t) due to fluctuating g, since now $u_t = -\alpha_t \hat{m}_t$. Moreover, instead of using a constant learning rate α, at each t, it divides α by square-root of (bias-corrected) moving average of past g^2 values to obtain dynamically adjusted learning rate α_t, so overall those θ with larger |g| (and thus larger $\hat{m}_t$) will generally have smaller α_t and vice-versa, so $|u_t| = |\alpha_t \hat{m}_t|$ will be balanced in magnitude across all θ and all t, which enables better performance while avoiding extensive hyper-parameter tuning.

Model Evaluation

Learning Curve



We used 60:20:20 train:validation:test split, with 10 epochs, and batch size = 32 to prevent memory overload. The learning curve shows a generally steady increase in accuracy and generally steady decrease in loss, with later epochs having less fluctuations, indicating that our model was able to learn effectively from the data. Also similar values for both the train and validation set mean that there is no overfitting on the train set.

Evaluation Metrics

Mean Reciprocal Rank (MRR)

We sort each row of the predicted probability matrix in descending order, then find the rank of the true class label, then calculate MRR (Hong et al., 2023) (formula in Appendix). Higher value means better, max score = 1

Top-k Accuracy

For k = 1, 5, 10, we also calculate the percentage of data inputs whose true next_location_id was among the k locations with highest predicted probabilities it

Evaluation Results

	Mean Reciprocal Rank	Top-1 Accuracy	Top-5 Accuracy	Top-10 Accuracy
Train	0.8514834	0.7491299	0.9815505	0.99698144
Validation	0.83921164	0.7421135	0.96038795	0.97703934
Test	0.8381379	0.74122274	0.95875496	0.9763466

Notably, across all the data including the completely unseen test set, our algorithm is able to achieve top-5 accuracy > 95% and MRR score > 0.83, meaning it has excellent performance in predicting the next location, i.e. taking only its top 5 predictions for next location of a user almost always contains the correct answer.

Limitations

Due to limited computational time and space, we demonstrated our algorithm's results using a subset (25%) of "kumamoto_challengedata_cityD.csv" data set only, but its results can be simply reproduced on any positionfixes data accompanied with appropriately formatted POI data (geometry, POI category, count). We were also not given mapping to real-world latitude-longitude coordinates so we have to train the model to fit each specific region, otherwise we would have been able to train the model simultaneously on multi-regional data

Conclusion

We successfully implemented a context-aware next-location prediction neural network algorithm. It is based on most effective MHSA transformer architecture and utilises Hong et al's innovative POI land-use context representation using LDA but is largely simplified to concisely use core input features, which optimises the balance between model complexity and accuracy. On unseen test sets, it is able to achieve excellent evaluation results, considering its simplicity. Its predictive performance is also higher than those reported in similar current literature, although this could be biased due to small regional data. For future extension, we can train and test our algorithm on global data using latitude-longitude coordinates to verify if it is truly effective.

References

Hong, Y., Zhang, Y., Schindler, K., Raubal, M., & The Author(s). (2023). Context-aware multi-head self-attentional neural network model for next location prediction. In *Transportation Research Part C* (Vol. 156, p. 104315) [Journal-article]. https://doi.org/10.1016/j.trc.2023.104315

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Contributions

Chong Geng Yang	Task 2.1	[illegible]
Ezra Koh See Hwa	Task 2.2	[illegible]
Jace Aung Kaung Kaung	Task 3	[illegible]
Jason Lim Jiasheng	Task 3	J

Appendix

Preliminary Definitions

A positionfix (track point) is a recorded position of a user, it records the raw time (tracked_at), spatial coordinates (geometry) and the user_id. A staypoint is a set of more than 1 positionfixes such that they are all within distance_threshold of the earliest positionfix and also the duration between earliest and latest positionfixes is more than time_threshold. Intuitively it is a region that the user stays around for some time. It has a user_id, geometry which is centroid of the geometries of its positionfixes, arrival time (started_at) and departure (finished_at) time. A location is a cluster of staypoints, its centre is centroid of the staypoints and its extent is the convex hull of the staypoints. Each positionfix also has staypoint_id indicating the staypoint it belongs to (null if it does not belong to any). Every positionfix belongs to at most 1 staypoint. Each staypoint also has a location_id indicating its cluster label

Model Details

Parameter settings

We set the embedding dimensions, as well as the following parameters

- sequence_length = 99
- Number of attention heads for MHSA module = 4
- key_dim for MHSA = 92/4 = 23
- total number of unique users considered = 3000
- Total number of unique locations = 22179

Inputs & Outputs

Each input to our model will have the following features: user_id_input (dimension 1 x 1), location_id_sequence_input, arrival_dayofweek_sequence_input, arrival_time_sequence_input, duration_sequence_input, x_coordinate_input, y_coordinate_input, convex_hull_area_input, POIcontext_sequence_input (dimension 99 x 20). Other than user_id_input and context_sequence_input, the rest are vectors with dimension 99 x 1. For a sample of inputs, our output will be a sample_size x 22179 probability matrix, where the (i,j)th element gives the probability that the next location of data i is location j. next_location_id is used as the ground truth class labels.

Dimensions of Embeddings & Intermediates

For a single data input,

- dimension of user_id embedding = 1 x 16
- dimension of location_id embedding = 99 x 32
- dimension for each location spatial coordinate embedding = 99 x 8
- dimension for staypoint arrival time embedding = 99 x 3
- dimension for staypoint arrival day embedding = 99 x 3
- dimension for staypoint duration embedding = 99 x 8
- dimension for a location's area embedding = 99 x 10
- dimension of POI context “embedding” = 99 x 20
- Dimension of P, E, E’, A, Z, M = 99 x 92
- Dimension of mean(M) = 1 x 92
- Dimension of U = 1 x 108

Embedding Row Vector For Time

$$\left[\sin\left(\frac{2\pi t}{T}\right),\ \cos\left(\frac{2\pi t}{T}\right),\ \frac{t}{T}\right]$$

where t = time, T = period

Sinusoidal Positional Encoding Formula

$$PE_{(pos,2i)} = sin(pos/10000^{2i/d_{\text{model}}})$$
$$PE_{(pos,2i+1)} = cos(pos/10000^{2i/d_{\text{model}}})$$

PE is the positional_encoding_matrix (P in our model).

MHSA Equations

$$Q = XW_Q, \quad K = XW_K, \quad V = XW_V \text{ , } Q_i = QW_i^Q, \quad K_i = KW_i^K, \quad V_i = VW_i^V$$

$$\text{MultiHead}(Q, K, V) = \text{Concat}(\text{head}_1, \ldots, \text{head}_h)W^O \text{ , } \text{head}_i = \text{Attention}(Q_i, K_i, V_i) = \text{softmax}\left(\frac{Q_i K_i^T}{\sqrt{d_k}}\right) V_i$$

$$\begin{aligned} W_Q &\in \mathbb{R}^{D_{\text{model}} \times D_{\text{key}}} \\ W_K &\in \mathbb{R}^{D_{\text{model}} \times D_{\text{key}}} \\ W_V &\in \mathbb{R}^{D_{\text{model}} \times D_{\text{value}}} \end{aligned} \text{ , } \begin{aligned} W_i^Q &\in \mathbb{R}^{D_{\text{key}} \times D_{\text{key}}} \\ W_i^K &\in \mathbb{R}^{D_{\text{key}} \times D_{\text{key}}} \\ W_i^V &\in \mathbb{R}^{D_{\text{value}} \times D_{\text{value}}} \end{aligned}$$

$$W^O \text{ has dimensions } \mathbb{R}^{(h \cdot D_{\text{value}}) \times D_{\text{model}}}$$

Where h is number of attention heads (4 in our model), X is the MHSA input (E' in our model), D_{model} = 92 in our model, D_{key} = d_k = D_{value} = key_dim = 23 in our model. All the W matrices are learnable parameters.

Layer Normalisation Formula

1. **Calculate the mean** for each row:

$$\mu_i = \frac{1}{d} \sum_{j=1}^{d} Z_{i,j}$$

2. **Calculate the standard deviation** for each row:

$$\sigma_i = \sqrt{\frac{1}{d} \sum_{j=1}^{d} (Z_{i,j} - \mu_i)^2}$$

3. **Apply learnable scaling and shifting**:

$$M_{i,j} = \gamma_j \cdot \frac{Z_{i,j} - \mu_i}{\sigma_i} + \beta_j$$

SCCE Formula

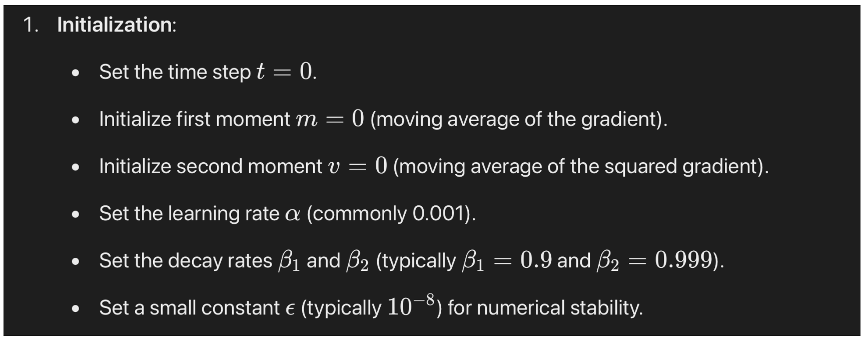
$$\text{Loss} = -\frac{1}{N} \sum_{i=1}^{N} \log(p_{i,y_i})$$

Where:

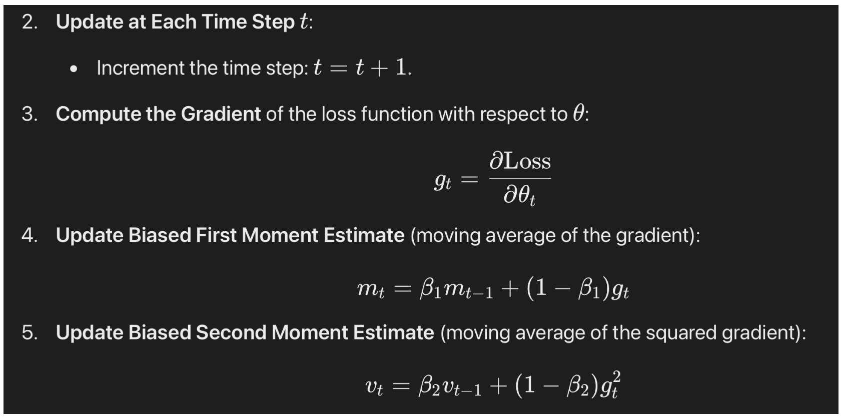
- N is the number of samples in the batch.
- y_i is the true class label (integer) for sample i.
- p_{i,y_i} is the predicted probability for the true class y_i of sample i.

ADAM Optimizer For Single Learnable Parameter θ

Equations



1. **Initialization**:
   - Set the time step $t = 0$.
   - Initialize first moment $m = 0$ (moving average of the gradient).
   - Initialize second moment $v = 0$ (moving average of the squared gradient).
   - Set the learning rate α (commonly 0.001).
   - Set the decay rates β_1 and β_2 (typically $\beta_1 = 0.9$ and $\beta_2 = 0.999$).
   - Set a small constant ϵ (typically 10^{-8}) for numerical stability.



2. **Update at Each Time Step t**:
   - Increment the time step: $t = t + 1$.
3. **Compute the Gradient** of the loss function with respect to θ:

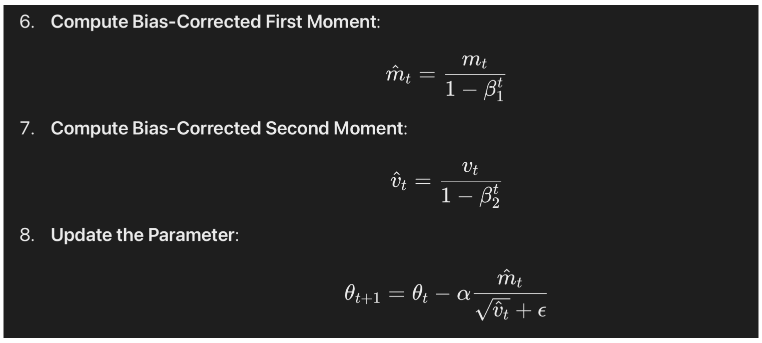
$$g_t = \frac{\partial \text{Loss}}{\partial \theta_t}$$

4. **Update Biased First Moment Estimate** (moving average of the gradient):

$$m_t = \beta_1 m_{t-1} + (1 - \beta_1) g_t$$

5. **Update Biased Second Moment Estimate** (moving average of the squared gradient):

$$v_t = \beta_2 v_{t-1} + (1 - \beta_2) g_t^2$$



6. **Compute Bias-Corrected First Moment**:

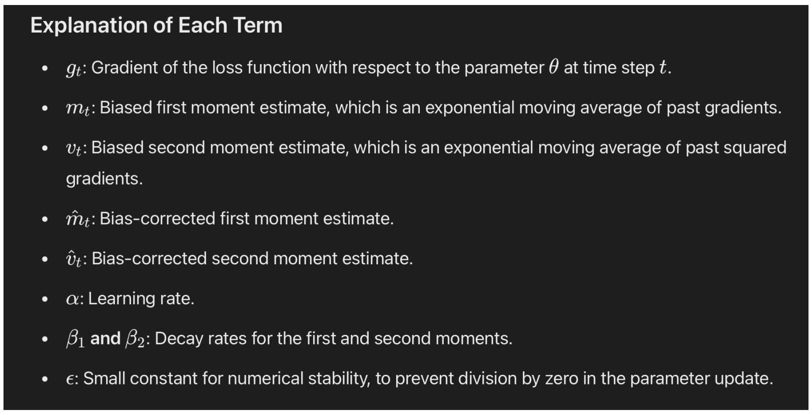
$$\hat{m}_t = \frac{m_t}{1 - \beta_1^t}$$

7. **Compute Bias-Corrected Second Moment**:

$$\hat{v}_t = \frac{v_t}{1 - \beta_2^t}$$

8. **Update the Parameter**:

$$\theta_{t+1} = \theta_t - \alpha \frac{\hat{m}_t}{\sqrt{\hat{v}_t} + \epsilon}$$

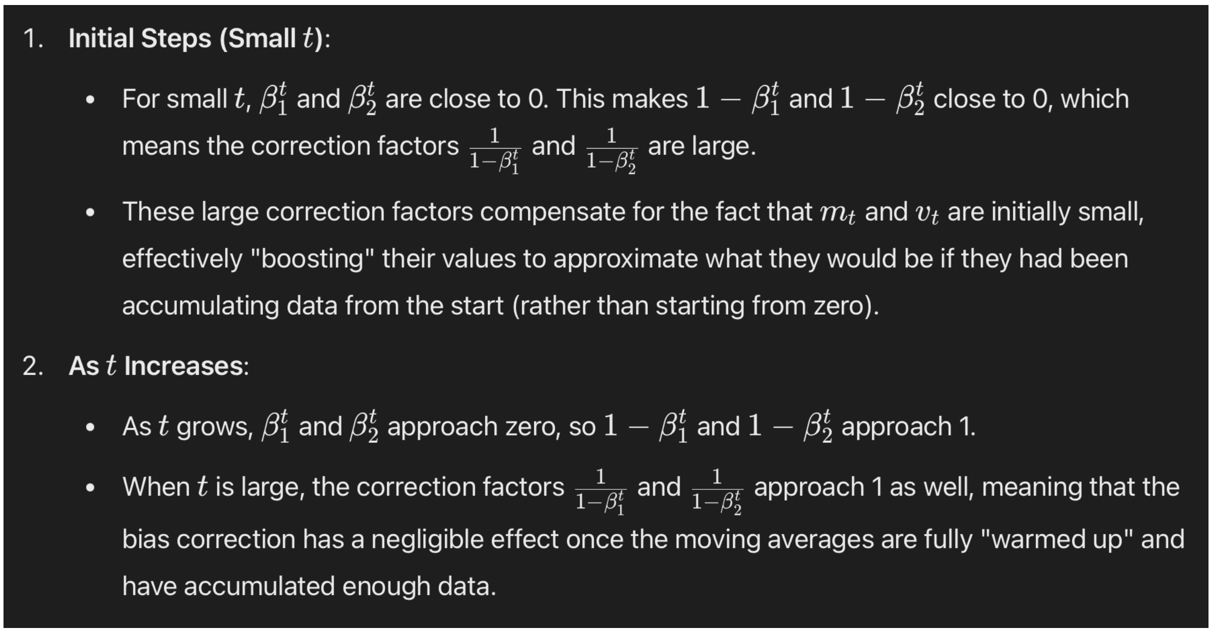


Explanation of Each Term

- g_t: Gradient of the loss function with respect to the parameter θ at time step t.
- m_t: Biased first moment estimate, which is an exponential moving average of past gradients.
- v_t: Biased second moment estimate, which is an exponential moving average of past squared gradients.
- $\hat{m}_t$: Bias-corrected first moment estimate.
- $\hat{v}_t$: Bias-corrected second moment estimate.
- α: Learning rate.
- β_1 **and** β_2: Decay rates for the first and second moments.
- ϵ: Small constant for numerical stability, to prevent division by zero in the parameter update.

Rationale for Bias Correction

Since m and v are initialised to 0, with β meant to be set close to 1, they will be biased towards 0.



1. **Initial Steps (Small t)**:
   - For small t, β_1^t and β_2^t are close to 0. This makes $1 - \beta_1^t$ and $1 - \beta_2^t$ close to 0, which means the correction factors $\frac{1}{1-\beta_1^t}$ and $\frac{1}{1-\beta_2^t}$ are large.
   - These large correction factors compensate for the fact that m_t and v_t are initially small, effectively "boosting" their values to approximate what they would be if they had been accumulating data from the start (rather than starting from zero).
2. **As t Increases**:
   - As t grows, β_1^t and β_2^t approach zero, so $1 - \beta_1^t$ and $1 - \beta_2^t$ approach 1.
   - When t is large, the correction factors $\frac{1}{1-\beta_1^t}$ and $\frac{1}{1-\beta_2^t}$ approach 1 as well, meaning that the bias correction has a negligible effect once the moving averages are fully "warmed up" and have accumulated enough data.

SGD Formula For Single Learnable Parameter θ

$$\theta_{t+1} = \theta_t - \alpha \cdot g_t$$

MRR Formula

$$\text{MRR} = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{\text{rank}_i}$$