Mathematical Foundations of Computer Science

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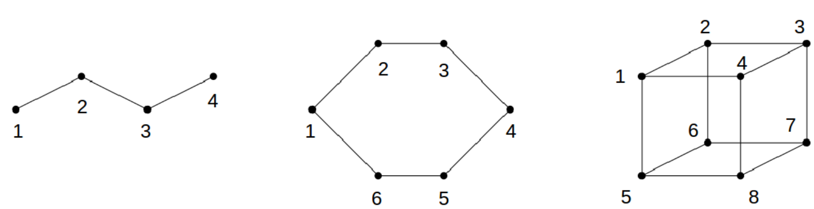
- Monday, 2018-04-02, homework handed out
- Sunday, 2018-04-08, 12:00: submit questions and first submissions. You'll get feedback until Wednesday.
- 2018-04-15: submit final solution.

6 Graph

Exercise 6.1. ex 6-1

Answer: 2, 12, 48

Proof. First we give each vertex of graphs an ID for convenience.



In the first graph, there are two vertex $1, 4$ with only one degree, which means their corresponding vertices in automorphism have only one degree.

Therefore we have

$$f(1) = 1, f(4) = 4$$

or

$$f(1) = 4, f(4) = 1$$

Either case the automorphism can be determined. There are 2 automorphic graphs. The functions are

$$f_1 = \{\{1, 1\}, \{2, 2\}, \{3, 3\}, \{4, 4\}\}$$

$$f_2 = \{\{1, 4\}, \{2, 3\}, \{3, 2\}, \{4, 1\}\}$$

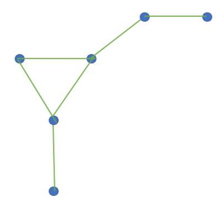
In the second graph, we take vertex 1 and 2 and the edge between them e_{12}. The corresponding edge in the automorphism can be $e_{12}, e_{21}, e_{23}, e_{32}, ..., e_{61}, e_{16}$. Once the corresponding edge of e_{12} is determined, the automorphism is determined. So there are 12 automorphisms.

In the third graph, we will illustrate our methods by an example first. If we take e_{12} and choose e_{43} as its mapping in automorphism, there are 4 choices left for e_{14}, as e_{14} can be $e_{14}, e_{23}, e_{48}, e_{37}$. Once the mappings of e_{12} and e_{14} are determined, the automorphism is determined. We have 12 choices for e_{12} and 4 choices for e_{14}, so there are 48 automorphisms.

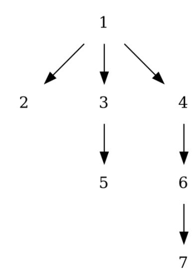
Exercise 6.2. ex 6-2

Proof. First we choose a vertex in G to be corresponding to our first vertex, say vertex 1. There is 2^n ways. Note that $n - 1$ adjacent vertexes to it can uniquely form a hyperplane and they are all symmetrical. So we arrange them to the adjacent vertexes and there is only $n!$ ways since all the hyperplanes are unique. As a result, the number of automorphism is $2^n n!$. □

Exercise 6.3. ex 6-3



Exercise 6.4. ex 6-4



Exercise 6.5. ex6-5

Proof. Since the edges of a graph and its automorphism are same, total edges of the complete graph be composed of both of them must be even.However, a graph consists of 999 vertices has $C_{999}^2 = 498501$ edges, which is odd. So there is no self-complementary graph on 999 vertices. $\square$

Exercise 6.6. ex6-6

Answer: $n = 4k \text{ } or \text{ } 4k + 1$

Proof. Maybe we can prove it by adjacent matrix. $\square$