0.1 HWT

0.1.1 Raw Problem

- Set V' of points in the Euclidean space $\mathbb{R}^2$,
- Bound h_{max},
- Cost per meter c,
- Cost d_i for $i \in \{1, 8, 16\}$, where d_i is the cost of using a degree at most i.

We want to connect all points in V' with V'

0.1.2 Transformed problem

- $V := V \cup \{s, t\}$ where s represents the tower and t an artificial final node.
- Graph (V, E) with $E = V^2 \setminus V$.
- Edge costs c_e for each $e \in E$ defined by

  $$c_{(v,w)} := l_{(v,w)} \cdot c,$$

  where $l_{(v,w)}$ is the Euclidean distance between v and w.
- Cost d_i stays the same.

0.1.3 Variables

- x_e for all $e \in E$, where $x_e \equiv 1$ if and only if the edge e is used in the solution
- f_e for all $e \in E$, where f_e represents the remaining capacity of the component of the edge e
- $y_{v,i}$ for all $v \in V$, where $y_{v,i} \equiv 1$ if the degree of v in the solution is at most i, otherwise $y_{v,i}$ is arbitrarily 0 or 1.

0.1.4 Cost Function

Given $x = (x_e)_{e \in E}, f = (f_e)_{e \in E}$,

$$\texttt{cost}(x, f) := \sum_{e \in E} x_e c_e + \sum_{v \in E, i \in \{1,8,16\}} y_{v,i} d_i$$

0.1.5 Constraints

- *Flow Conservation:* For all $v \in V$,

  $$\sum_{w \in V} x_{vw} f_{vw} = \sum_{u \in V} x_{uv} f_{uv}$$

- *Part Size Bound (1):* For all $v \in V, v \neq s$,

  $$f_{vt} >= 1,$$

- *Part Size Bound (2):* For all $w \in W, w \neq T$,

$$f_{T,v} = h_{max}$$

- *Degree constraints:* For all $v \in V$,

$$\sum_{w \in V} x_{vw} \leq 1y_{v,1} + 8y_{v,8} + 16y_{v,16}$$

- *Integrity:* For all $v, w \in V, i \in \{1, 8, 16\}$,

$$x_{vw}, y_{v,i} \in \{0, 1\}$$