Design and Analysis of Algorithms

L21: Greedy Algorithms

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Resources

- Text book 2: Sec 4.1, 4.3, 4.4
- Text book 1: Sec 9.1-5.4 - Levitin
- R1: Introduction to Algorithms
    - Cormen et al.

Overview: Greedy Algorithms

- Basis of greedy algorithm:
  - Make the choice that seems best at the moment.
- Basics of Greedy Algorithms
  - A paradigm that build solutions using one piece at a time
  - Chooses the next piece that is most obvious and provides immediate gain, i.e.
    - maximizes benefit or minimizes cost
  - Expecting such local optimal solutions may lead to global optimal solution,

Greedy Algorithms

- How to decide which choice is optimal
  - Define an objective function and optimize the same with the choice to be made
  - Repeat the process at each step.
  - There is no going back to reverse the decision
- Advantages:
  - Easy to design a greedy algo (there can be multiple)
  - Complexity time analysis is comparatively easier
    - For divide-n-conquer it may not be easy
      - Depends on number of sub-problems and size
- Disadvantages:
  - How to ensure that chosen algorithm is correct

Coin Change Problem

- Issue min number of coins for a given value
- Amount to be dispensed: Rs 43
  - Consider coin denomination: Rs 1, 2, 5, 10, 20
    - 2 coins of Rs 20
    - 1 coin of Rs 2, and Rs 1
      - » Total coins: 2+ 1 + 1 =4
  - Consider coin denomination as: Rs 1, 2, 5, 10, 20, 25
    - 1 coin of Rs 25, Rs 10, Rs 5, Rs 2, Rs 1
      - » Total coins : 5
    - Optimal case (4 coins)
      - » 2 coins of Rs 20,
      - » 1 coin of Rs 2, and Rs 1
- Q: What is the Greedy approach, Objective Function?

Writing Exams:

- Visiting friends to invite for a party:
- Greedy approach:?
  - – Start from visiting the nearest friend.
    - Criteria for nearest:
  - – Objective function?

Greedy Algorithm

- Subset paradigm: Approach
- Consider the input in an order
  - Determined by some selection procedure
- If the selected input leads to feasible solution
  - That input is added to the solution
- Else
  - do not consider that input
- Selection procedure is based on some optimization measure
  - The measure could be the objective function
  - There may several optimization measures possible

Greedy Algorithm

- Approach: subset paradigm

```
SolType Greedy(a[], n) {
  //a[1:n] contains the n inputs
  SolType solution = EMPTY // initialize
  for i=1 to n do
    Type x = Select(a)
    if Feasible(solution, x)
       solution= Union(solution, x)
  // end for
  return solution
} // end algo
```

Subset Paradigm: Analysis

- Function `Select`() selects an input from `a[]` and removes it.
- The selected input's value is assigned to `x`
- `Feasible` is a boolean valued function
  - Determines whether `x` can be included or not
- `Union()` combines `x` with the solution and updates the objective function
- Method `Greedy` describes how a typical greedy algorithm works

Coin Change Problem: Analysis

- Assumption: Enough number of coins for each denomination are available
- Initial solution: empty `Change`
- At each stage, 1 coin is selected and
  - added to `Change`
- Coin is selected using greedy criteria
  - It should increase total amount of `Change` as much as possible
- `Feasible` function:
  - `Change` given must equal the total amount
  - `Change` should not exceed the total amount

Max Water to Households

- A colony has number of houses and they get water from a water tanker of capacity `T`.
  - Each household H_i has a container having size C_i.
  - You are a politician and would like to oblige max number of households.
  - Each house gets a full container not partially filled.
  - At the same time would like to keep minimum water in tanker after filling the containers since that becomes wasted efforts. Can we solve this using greedy algorithm.
- Example:
  - $T=150L$, $C_1=50L$, $C_2=60L$, $C_3=90L$

Machine Scheduling Problem

- Given
  - N tasks and infinite supply of machines on which these tasks can be run.
  - Each task has a start time s_i and finish time $f_i > s_i$. $[s_i, f_i]$ is called processing interval.
  - Two tasks overlap if their intervals overlap at a point other than start time and end time.
- Problem: Find optimal number of machines on which these tasks can be assigned.
- Feasible solution:
  - Assign one task to each machine. It is feasible but not optimal since this would need N machines.

Machine Scheduling Problem

- Example: 7 tasks with their start and end times.

Task	A	B	C	D	E	F	G
Start time	0	3	4	9	7	1	6
Finish	2	7	7	11	10	5	8

- Q: What should be the Greedy approach for machine assignment
  - Define new machine: on which task is run 1st time.
  - Define old machine: on which some task is already completed
- Approach: Use old machine for next task if available, else use new machine.

Machine Assignment

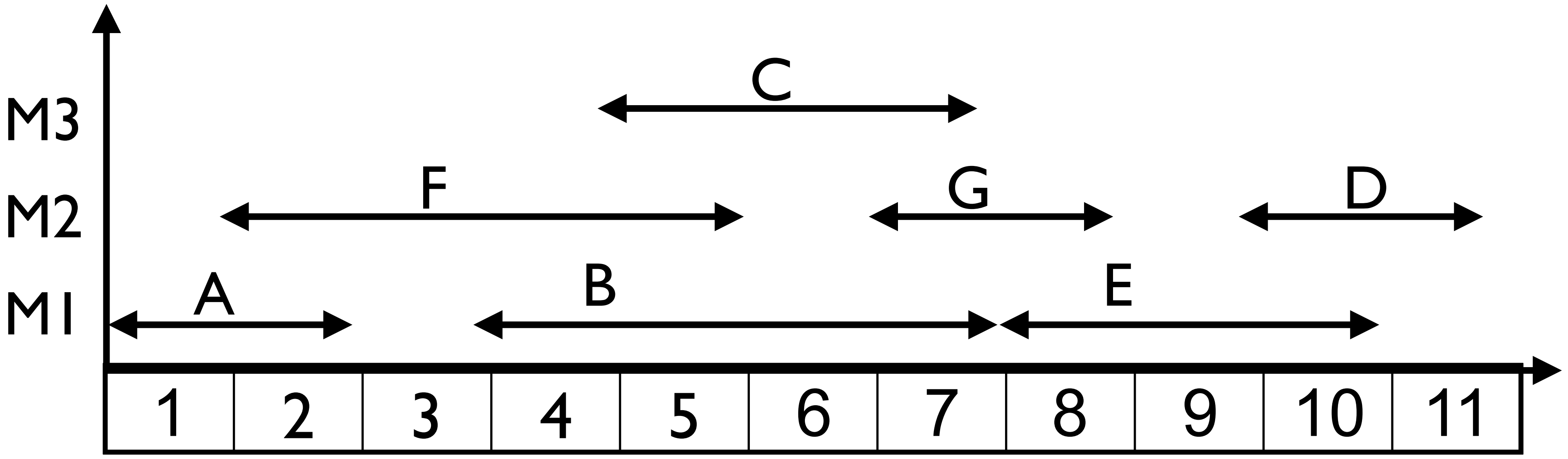
- Sort the tasks as per start time

Task	A	B	C	D	E	F	G
Start time	0	3	4	9	7	1	6
Finish	2	7	7	11	10	5	8

Task	A	F	B	C	G	E	D
Start time	0	1	3	4	6	7	9
Finish	2	5	7	7	8	10	11

Machine Assignment

Task	A	F	B	C	G	E	D
Start time	0	1	3	4	6	7	9
Finish	2	5	7	7	8	10	11



Q: If we sort the task as per finish time, will it work?
Which one would you prefer sort by start time or finish time?

Exercise

- Consider the machine assignment problem but with only one machine.
- Problem: Find the largest number of tasks that can be assigned to this machine.
- What should be the greedy approach?

Summary

- Greedy algorithm
- Coin Change problem
- Machine Task problem
- Water container fulfillment problem