Design and Analysis of Algorithms

L21: Greedy Algorithms

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Resources

- Text book 1: Sec 9.1-5.4 - Levitin
- Text book 2: Sec 4.1, 4.3, 4.4
- Introduction to Algorithms - A creative approach
    - Udi Manber
- R1: Introduction to Algorithms
    - Cormen et al.

Overview: Greedy Algorithms

- Basis of greedy algorithm:
  - Make the choice that seems best at the moment.
- Basics of Greedy Algorithms
  - A paradigm that build solutions using one piece at a time
  - Chooses the next piece that is most obvious and provides immediate gain, i.e.
    - maximizes benefit or minimizes cost
  - Expecting such local optimal solutions may lead to global optimal solution,

Greedy Algorithms

- How to decide which choice is optimal
  - Define an objective function and optimize the same with the choice to be made
  - Repeat the process at each step.
  - There is no going back to reverse the decision
- Advantages:
  - Easy to design a greedy algo (there can be multiple)
  - Complexity time analysis is comparatively easier
    - For divide-n-conquer it may not be easy
      - Depends on number of sub-problems and size
- Disadvantages:
  - How to ensure that chosen algorithm is correct

Coin Change Problem

- Issue min number of coins for a given value
- Amount to be dispensed: Rs 43
  - Consider coin denomination: Rs 1, 2, 5, 10, 20
    - 2 coins of Rs 20
    - 1 coin of Rs 2, and Rs 1
      - » Total coins: 2+ 1 + 1 =4
  - Consider coin denomination as: Rs 1, 2, 5, 10, 20, 25
    - 1 coin of Rs 25, Rs 10, Rs 5, Rs 2, Rs 1
      - » Total coins : 5
    - Optimal case
      - » 2 coins of Rs 20,
      - » 1 coin of Rs 2, and Rs 1
      - » Total coins: 4

Greedy Algorithm

- Subset paradigm: Approach
- Consider the input in an order
  – Determined by some selection procedure
- If the selected input leads to feasible solution
  – That input is added to the solution
- Else
  – do not consider that input
- Selection procedure is based on some optimization measure
  – The measure could be the objective function
  – There may several optimization measures possible

Greedy Algorithm

- Approach: subset paradigm

```
SolType Greedy(a[], n) {
   //a[1:n] contains the n inputs
   SolType solution = EMPTY // initialize
   for i=1 to n do
      Type x = Select(a)
      if Feasible(solution, x)
         solution= Union(solution, x)
   // end for
   return solution
} // end algop
```

Subset Paradigm: Analysis

- Function `Select()` selects an input from `a[]` and removes it.
- The selected input's value is assigned to `x`
- `Feasible` is a boolean valued function
  – Determines whether `x` can be included or not
- `Union()` combines `x` with the solution and updates the objective function
- Metod `Greedy` describes how a typical greedy algorithm works

Coin Change Problem: Analysis

- Assumption: Enough number of coins for each denomination are available
- Initial solution: empty `Change`
- At each stage, 1 coin is selected and
  - added to `Change`
- Coin is selected using greedy criteria
  - It should increase total amount of `Change` as much as possible
- `Feasible` function:
  - `Change` given must equal to total amount
  - `Change` should not exceed the total amount

Max Water to Households

- A colony has number of houses and they get water from a water tanker of capacity T. Each household H_i has a container having size C_i. You are a politician and would like to oblige max number of households. Each house only gets a full container not partially filled container. At the same time would like to keep minimum water in tanker after filling the container since that becomes wasted efforts. Can we solve this using greedy algorithm.
- Example:
  - $T=150L$, $C_1=50L$, $C_2=60L$, $C_3=90L$

Machine Scheduling Problem

- Given
  - N tasks and infinite supply of machines on which these tasks can be run.
  - Each task has a start time s_i and finish time $f_i > s_i$. $[s_i, f_i]$ is called processing interval.
  - Two tasks overlap if there intervals overlap at a point other than start time and end time.
- Problem: Find optimal number of machines on which these tasks can be assigned.
- Feasible solution:
  - Assign one task to each machine. It is feasible but no optimal since this would need N machines.

Machine Scheduling Problem

- Example: 7 tasks with their start and end times.

Task	A	B	C	D	E	F	G
Start time	0	3	4	9	7	1	6
Finish	2	7	7	11	10	5	8

- Q: What should be the Greedy approach for machine assignment
  - Define new machine: on which task is run 1st time.
  - Define old machine: on which some task is already completed
- Approach: Use old machine for next task if available, else use new machine.

Machine Assignment

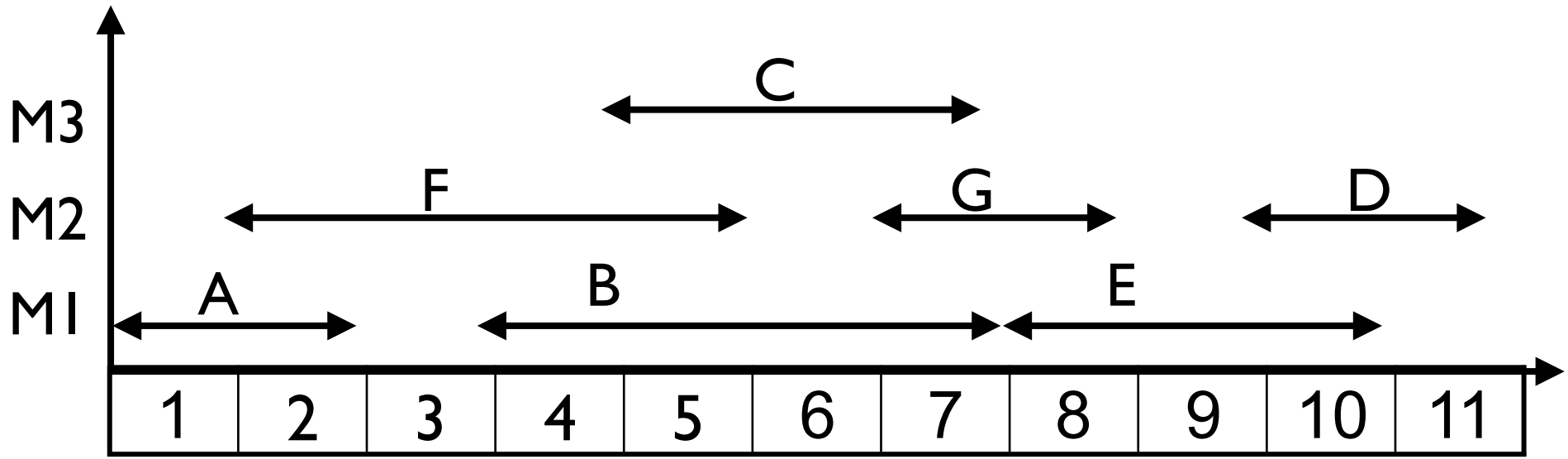
- Sort the tasks as per start time

Task	A	B	C	D	E	F	G
Start time	0	3	4	9	7	1	6
Finish	2	7	7	11	10	5	8

Task	A	F	B	C	G	E	D
Start time	0	1	3	4	6	7	9
Finish	2	5	7	7	8	10	11

Machine Assignment

Task	A	F	B	C	G	E	D
Start time	0	1	3	4	6	7	9
Finish	2	5	7	7	8	10	11



Exercise

- Consider the machine assignment problem but with only one machine.
- Problem: Find the largest number of tasks that can be assigned to this machine.
- What should be the greedy approach?

Summary

- Greedy algorithm
- Coin Change problem
- Machine Task problem
- Water container fulfillment problem

Summary