Design and Analysis of Algorithms

L18: Applications of Decrease/Divide & Conquer

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Resources

- Text book 1: Sec 5.1-5.3 - Levitin
- Introduction to Algorithms - A creative approach
    - Udi Manber
- R1: Introduction to Algorithms
    - Cormen et al.

Celebrity Problem

- Q10 (Levitin):
  - A celebrity among a group of N people is a person who knows nobody but is known to everybody else. Identify the celebrity by only asking the questions to the people of the form: "Do you know him/her?" Design an efficient algorithm to identify a celebrity or determine that the group has no such person. How many questions does your algorithm need in the worst case?

Celebrity Problem

- Approach 1 :
  - Build an adjacency matrix A
  - Ask each person if he knows all other persons
    - Total num of Qs: $n(n-1) = O(n^2)$
    - $A[i,j]=1$ if i^{th} person knows person j
      - 0 otherwise
  - Find a colum k, such that $\forall i$
    - $\Sigma A(i,k)=n-1$, and
    - $\Sigma A(k,i)=0$
- person k is celebrity

Celebrity Problem

- Approach 2 :
  - Build a graph
  - Ask each person if he knows all other persons
    - Total num of Qs: `n(n-1) =O(n2)`
    - Draw an edge `(i,j)` if person `i` knows person `j` .
  - Find a node `k` such that its
    - indegree is `(n-1)`, and
    - outdegree is `0`.
- person `k` is celebrity

Celebrity Problem

- Approach 3: Using Decrease and conquer.
- Design function `celebrity(N)` which returns `k`
  - if `k` is non-zero, then `k` is celebrity
  - if `k` is zero, there there is no celebrity.
- `celebrity(N)` Using Decrease and conquer.
  - Invoke `k=celebrity(N-1)`
    - if `k=N`, and `N` does not anyone, `N` is celebrity
      - Complexity: O(N)
    - if `k≠N`, and `N` knows `k`, `k` is celebrity, complexity `O(1)`
    - Else no celebrity
- Time Complexity:
  $T(n) = T(n-1)+O(n) = O(n^2)$

Celebrity Problem

- Approach 4: Using stacks
- Push all persons(elements) on the stack
  – stack size is `N`
- Repeat until stack size becomes `1`
  – pop two persons `A`, `B` from stacks
  – If `A` knows `B`, then `A` is not a celebrity
    - Push `B` on the stack
  – If `A` doesn't know `B`, then `B` is not a celebrity
    - Push `A` on the stack.
- The last person on the stack is celebrity
- Complexity: `3N-1 = O(N)`
  – `2N` pop operations, `N` push operations

Summary

- Celebrity problem