Design and Analysis of Algorithms

L42: Backtracking Algorithms Approach

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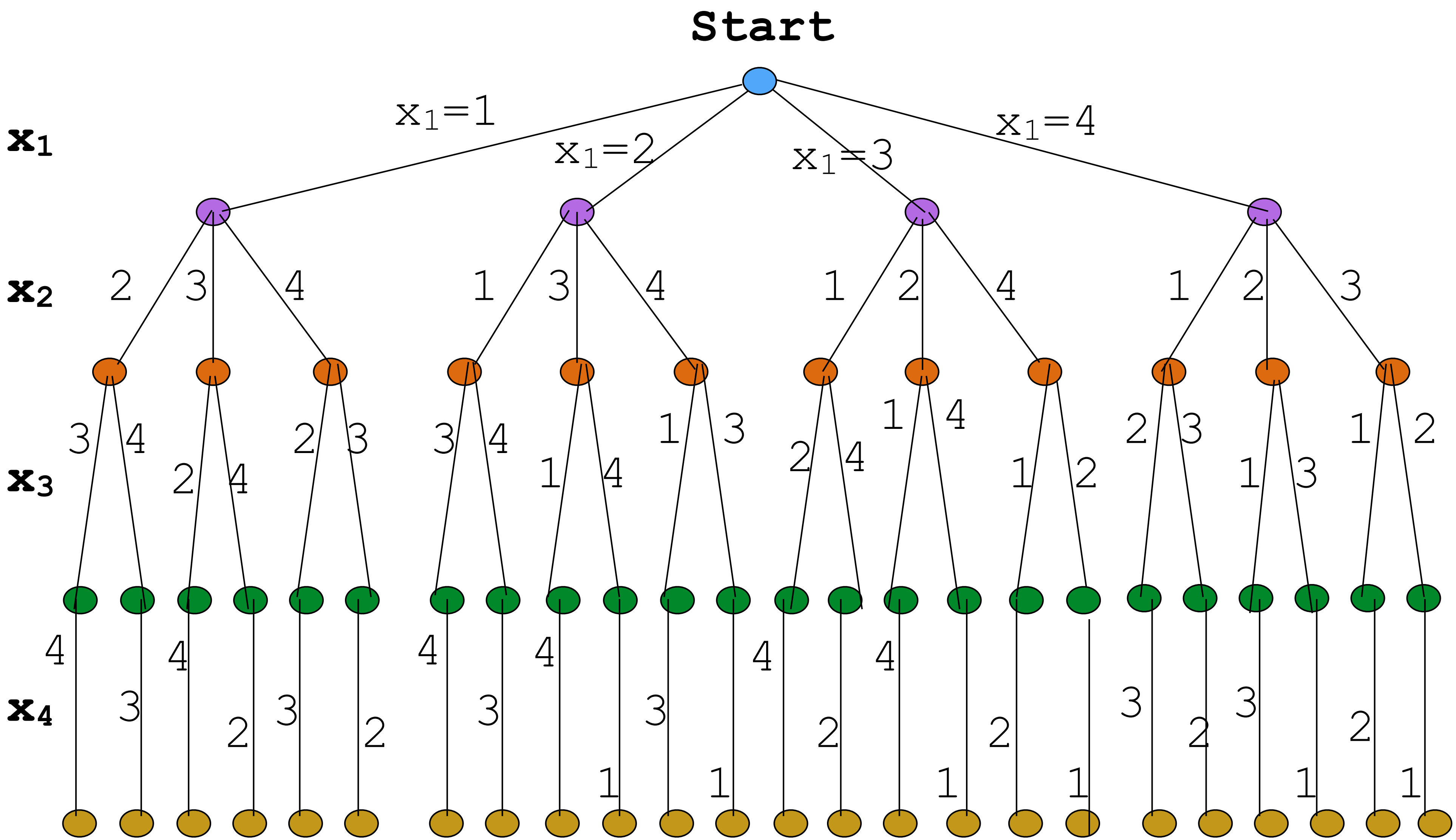
Resources

- Text book 2: Horowitz
  – Sec 5.1,5.2,5.4,5.8,5.9
- Text book 1: Levitin
  – Sec 8.2-8.4
- R1: Introduction to Algorithms
  - Cormen et al.
- https://en.wikipedia.org/wiki/Dynamic_programming
- https://www.codechef.com/wiki/tutorial-dynamic-programming
- https://www.hackerearth.com/practice/algorithms/dynamic-programming/introduction-to-dynamic-programming-1/tutorial/

Overview of Backtracking

- Basic approach of backtracking
  - Determine problem solutions by systematically searching the solution space
- Approach to search the solution space
  - Construct a tree of solution space
  - Node of this corresponds to a tuple variable assigned a possible feasible value
  - Edge of tree corresponds to tuple variables (x_i, x_{i+1}) where x_i is assigned a possible value
  - Leaf node satisfying the contraints (criterion function) represents a solution
- Two kind of trees
  - Static trees, Dynamic trees

State Space: 4 Queens



Size of 4-queens state space: $4! = 24$

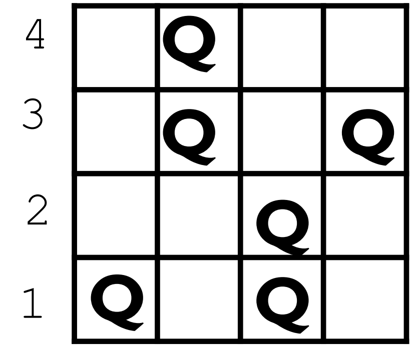
Solution: 4-Queens

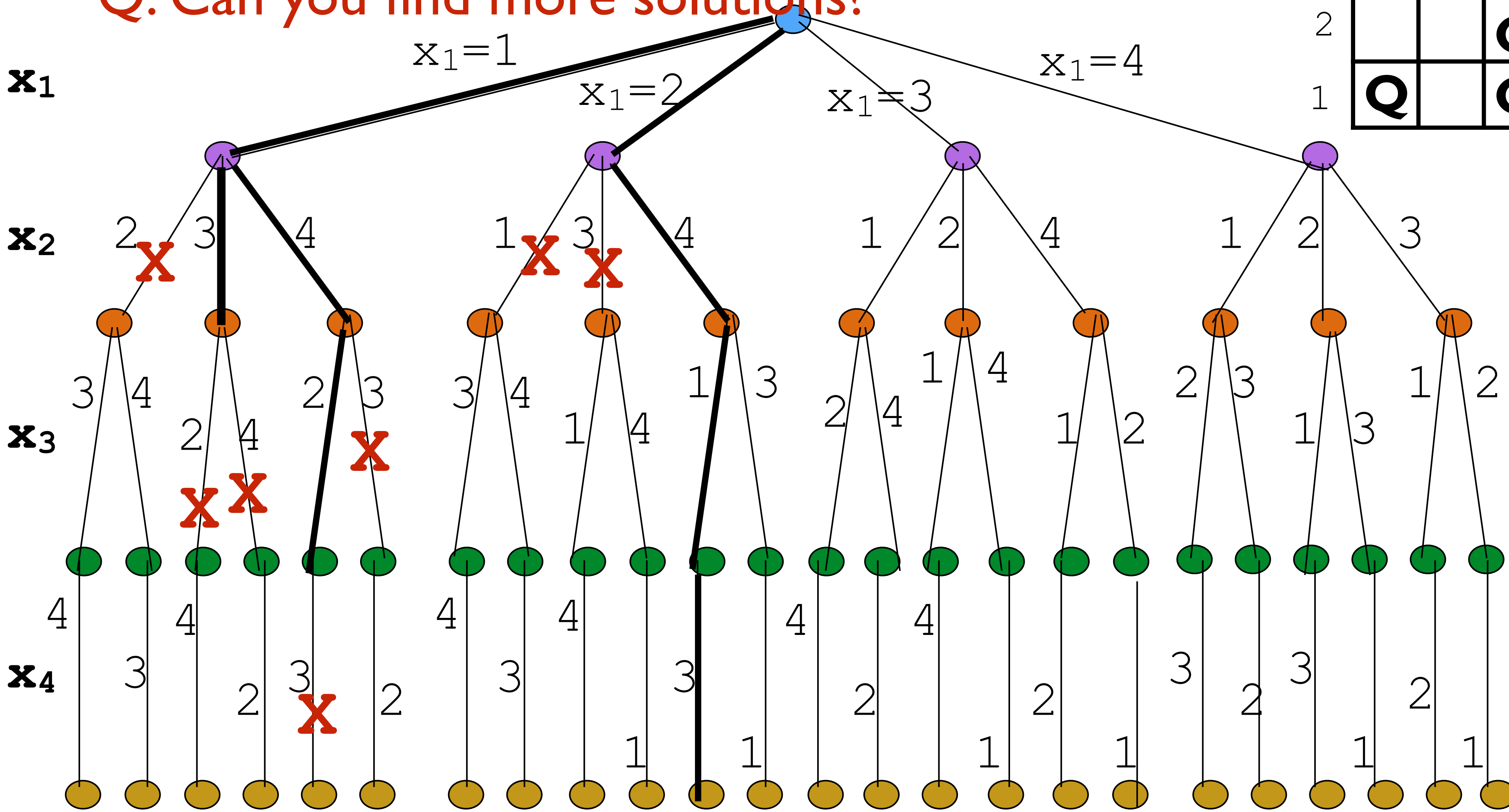
4		**Q**$_2$		
3				**Q**$_4$
2	**Q**$_1$			
1			**Q**$_3$	
	a	b	c	d

State Space: 4 Queens

Solution: (2,4,1,3)

Q: Can you find more solutions?





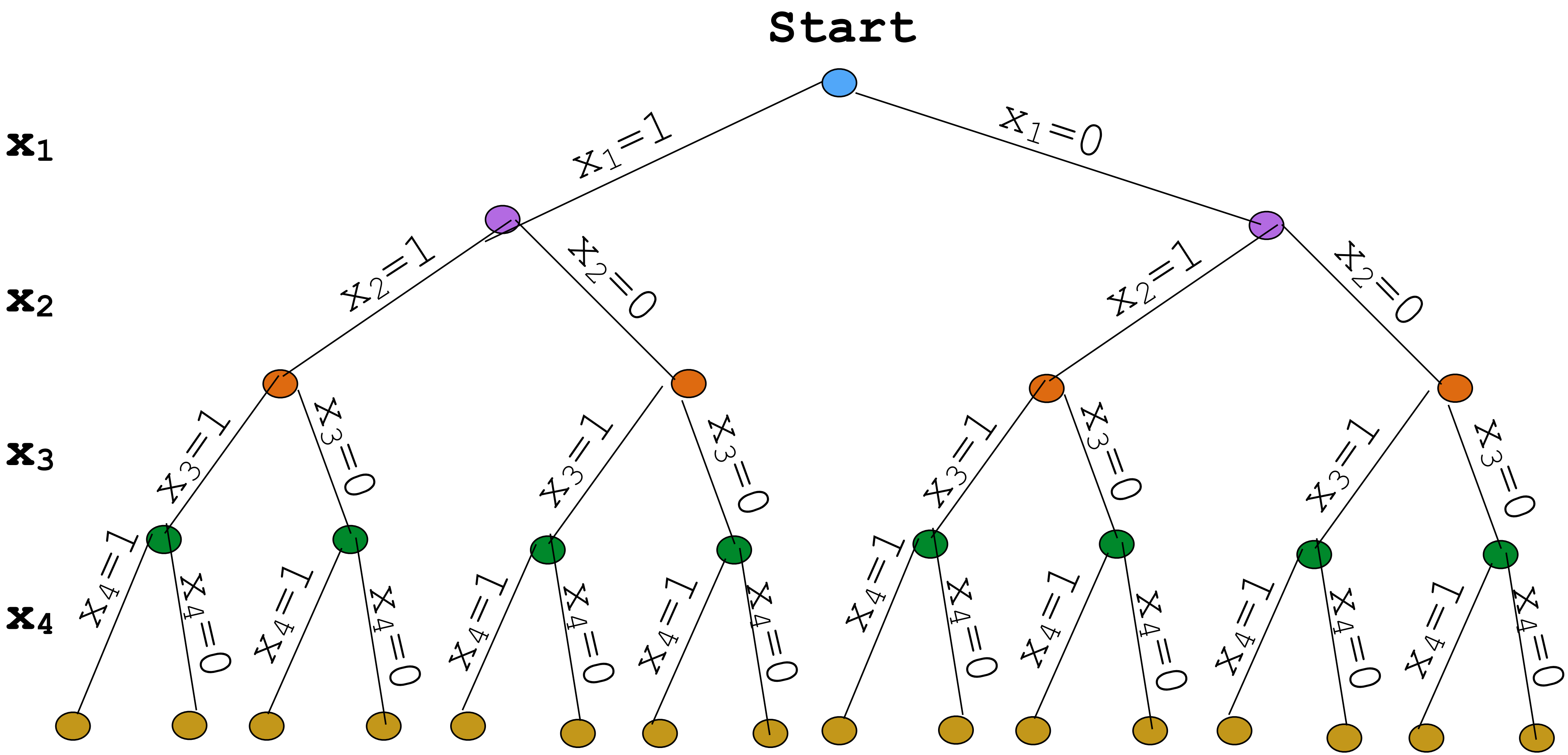
Size of 4-queens state space: $4! = 24$

State Space: 4 Queens

- Solving 4-Queens problems
  - – Build a complete possible tree
    - Called a static approach
  - – Explore (traverse) the tree for possible solutions.
  - – Prune the tree when come to a node where can not traverse further
  - – Backtrack to explore next path.
  - – When reach the leaf node, a solution is found.
- Note: Building a static tree is independent of problem instance.
  - – Tree is built for all possible solutions.

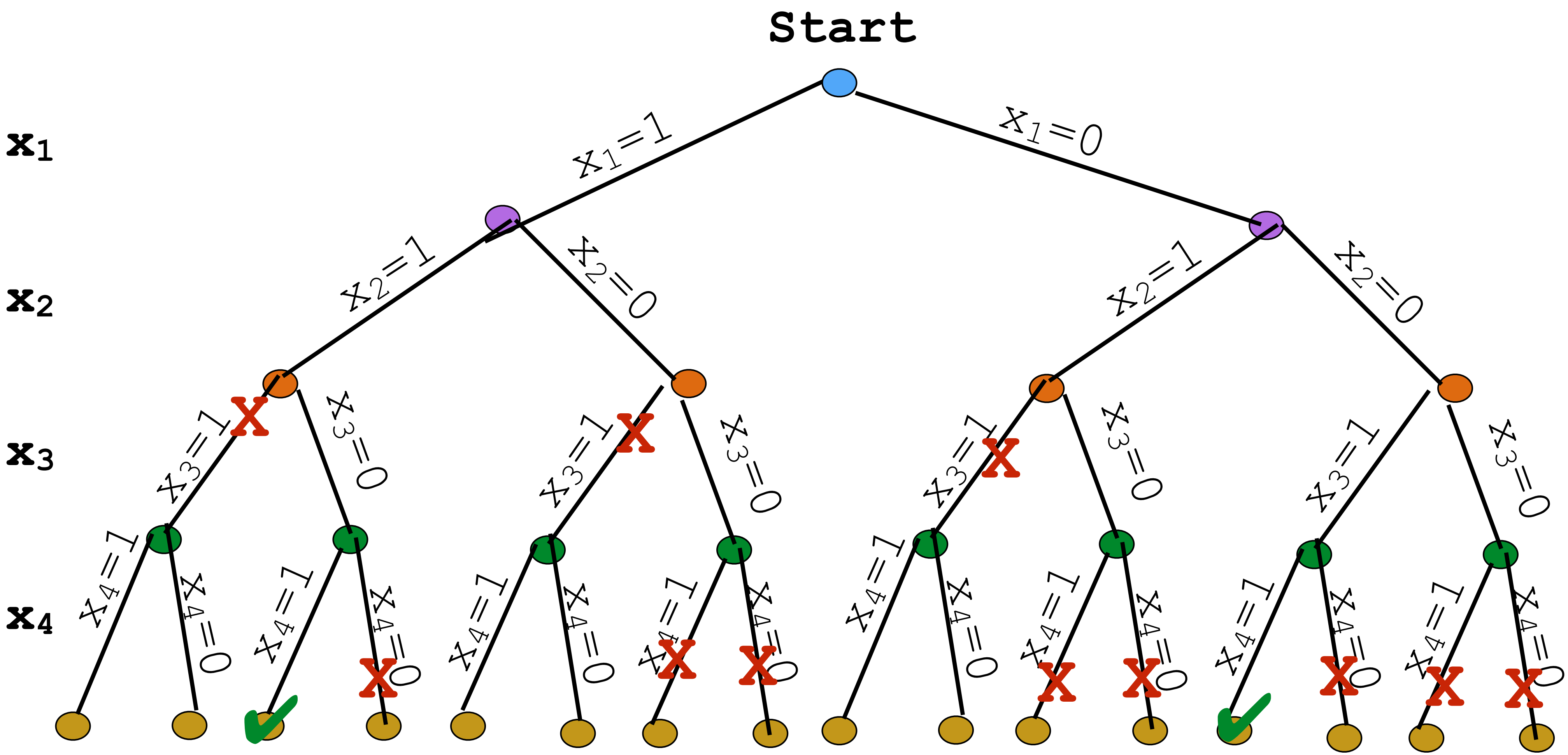
State Space: Sub of Subset problem

- **Ex:** `S={11,13,24,7}`, **and** `m=31`



Solution Space: Subset sum problem

- **Ex:** `S={11,13,24,7}`, **and** `m=31`



Sol^n 1: {1,1,0,1}   Sol^n 2={0,0,1.1}

State Space: Subset Sum

- Solving sum of subset problems
  - Build a complete possible tree
    - Again a static approach
  - Explore (traverse) the tree for possible solutions.
  - Prune the tree when come to a node where can not traverse further
  - Backtrack to explore next path.
  - When reach the leaf node, it is not necessary that a solution is found.
- Note: Building a static tree is independent of problem instance.
  - Tree is built for all possible solutions.

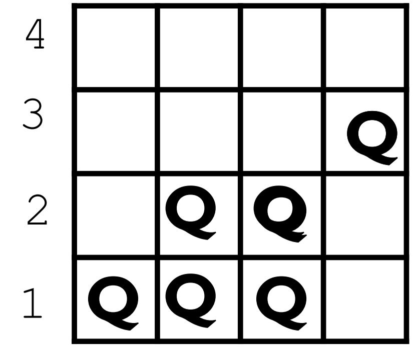
State Space Trees: Terminology

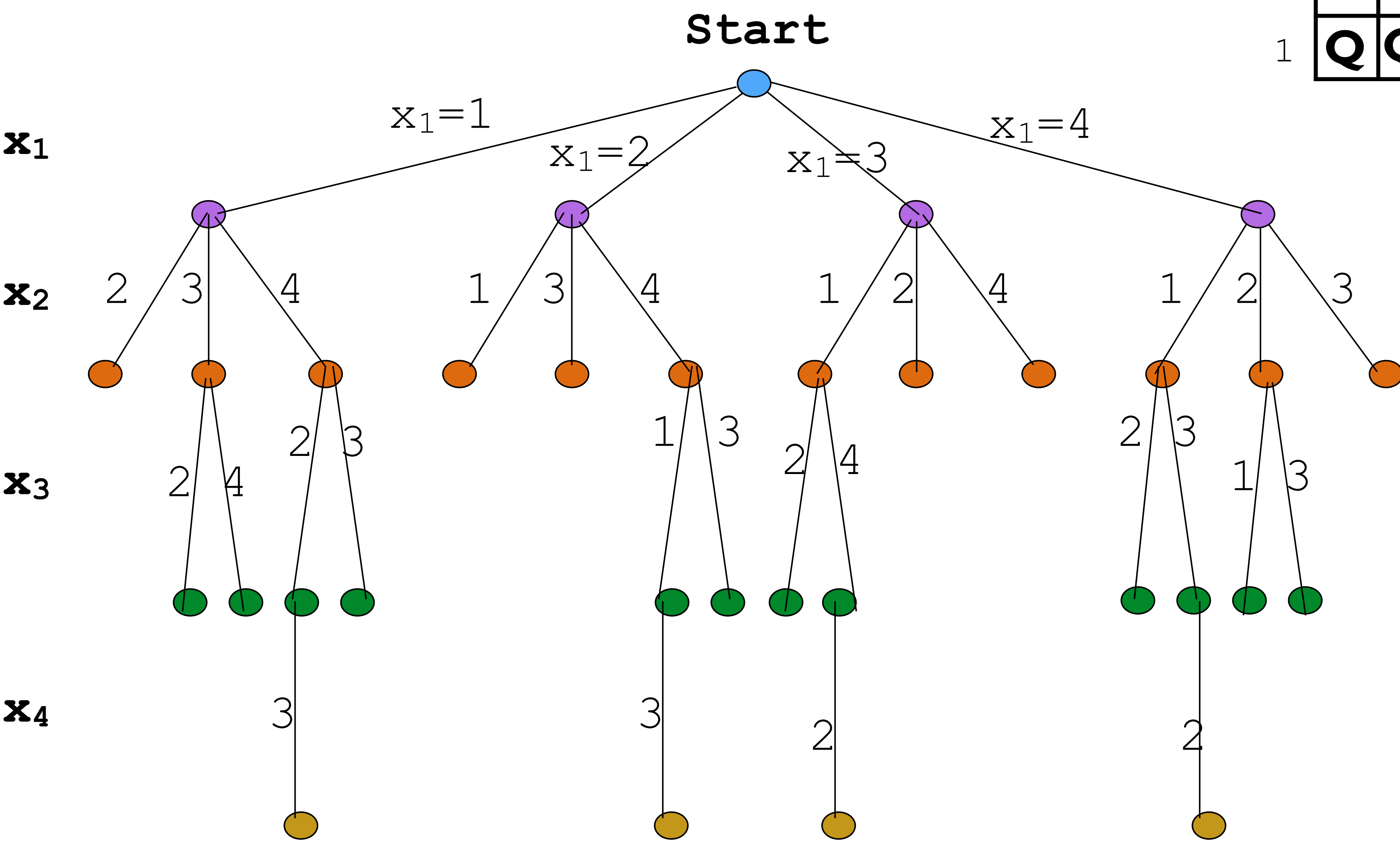
- Terminology
  - Each node in the tree defines a problem state
  - All paths from root to other nodes define *state space* of problem
  - *Solution states* are those problem states `s` for which the path from root to `s` defines a tuple in the solution space.
  - *Answer states* are are those solution states s for which the path from root to s defines a tuple that is a member of the set of solutions.
  - Tree organization is referred to *Space Tree*.

State Space Trees

- Dynamic Trees:
  - Tree organization is determined dynamically as the state space is being searched.
  - Tree organizations that are dependent on problem instance are called dynamic trees.
- Two ways to generate problem states
  - Backtracking, and Branch-n-Bound
  - Both begin with the root and generate other nodes
  - A node whose all children have not been generated is called a *live* node
  - A *dead* node is a generated node which is not to be expanded further, or whose all children are generated.

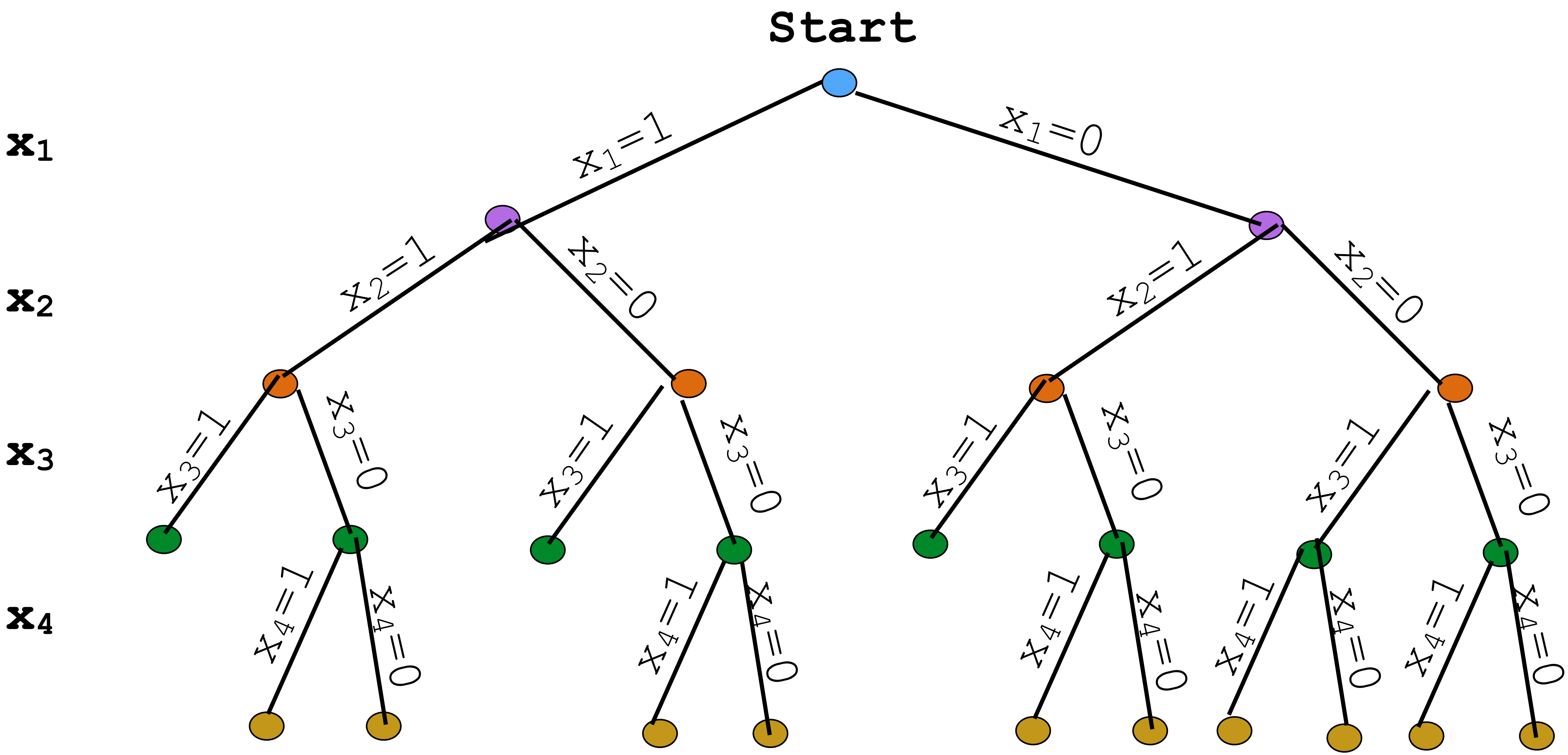
Backtracking Tree: 4 Queens



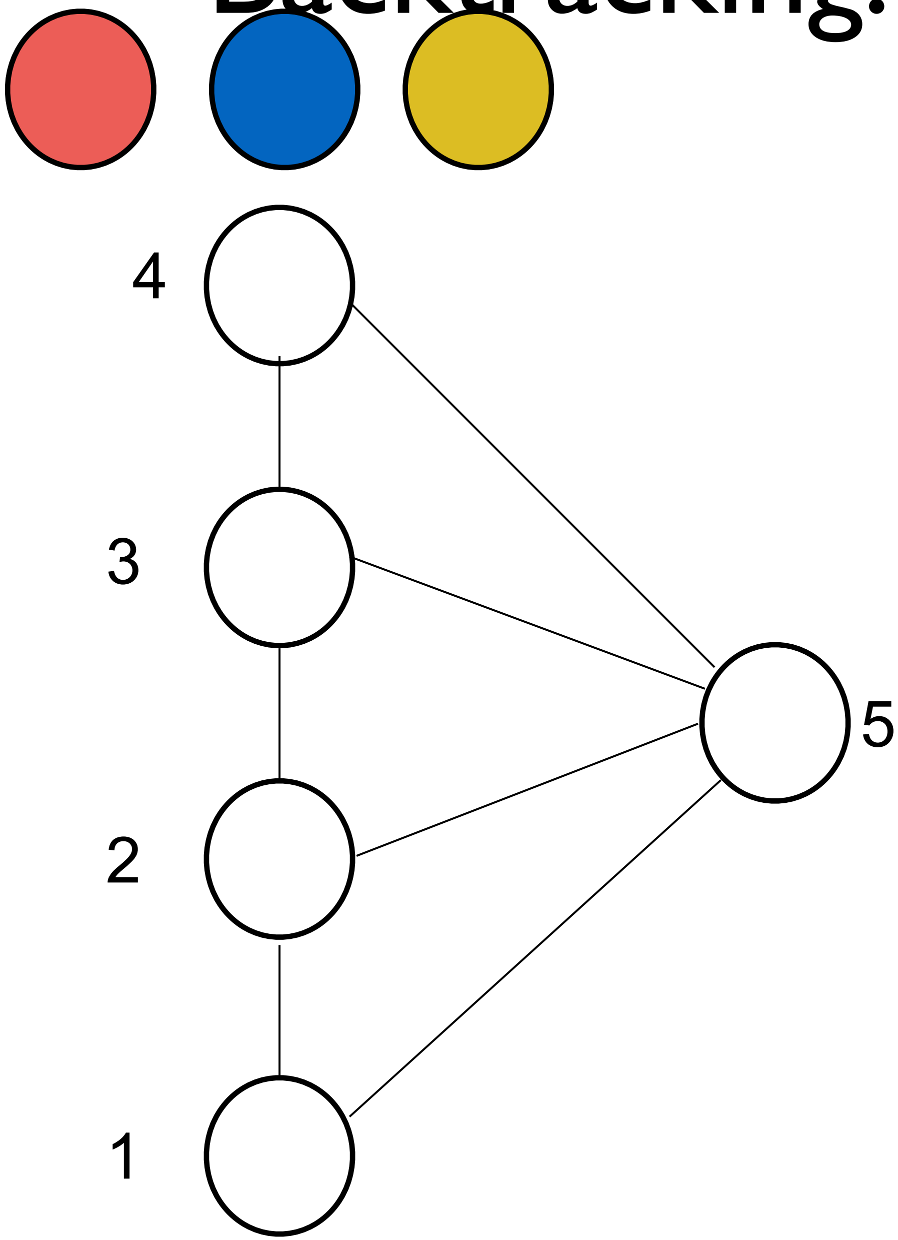


Backtracking Tree: Subset Sum

- **Ex:** `S={11,13,24,7}`, **and** `m=31`



Backtracking: 3-color problem



- Exercise:
  - Construct the state space tree for the graph shown for 3-color problem.

Algo: Backtracking

- Notations:
  - $(x_1, x_2, \ldots, x_i)$ be the path from root to a node in state space tree.
  - $T(x_1, x_2, \ldots, x_i)$ be the set of all possible values for x_{i+1} such that $(x_1, x_2, \ldots, x_{i+1})$ is also a path to problem state
  - B_{i+1} is boundary function predicate i.e. if $B_{i+1}(x_1, x_2, \ldots, x_{i+1})$ is false for path $(x_1, \ldots, x_{i+1})$, then path can't be extended to reach an answer state
  - $T(x_1, x_2, \ldots, x_n) = \varnothing$
  - Candidates for position $i+1$ of solution vector $(x_1, \ldots, x_n)$
    - Those values generated by T and satisfies B_{i+1}

Algo: Backtracking

```
Algo Backtrack(int n)
    int k=1
    while (k) {
        if there remains an untried x[k] such that
        x[k] is in T(x[1],…,x[k-1]), and
        B(x[1],…,x[k]) is true, then {
            if (x[1],…[x[k]) is a path to an answer
            node,
                output x[1:k]
            k++
    }
    k--
```

Summary

- Basic approach of backtracking
- Static tree
- Dynamic tree