第5章 常微分方程数值解

陈路 CHENLU.SCIEN@GMAIL.COM
201800301206

1. 上机实验题

问题 1
求

$$y' = 1 + y^2, \quad y(0) = 0 \tag{1}$$

的数值解(分别用欧拉显格式、梯形预估修正格式、4阶龙格库塔格式，并与解析解比较这三种格式的收敛性).

解:
方程1的精确解为$y(t) = tan(t)$，是一个周期为π的奇函数，我们不妨考察近似是其一个完整的最小周期的区间$[0, 3]$. 欧拉格式的代码实现

```
function E = euler(f, a, b, ya, M)
% Input  - f     is the function entered as a string 'f'
%        - a,b   are the left and right end points
%        - ya    is the initial condition y(a)
%        - M     is the number of steps
% Output - E=[T' Y'] where T is the vector of abscissas and
%                 Y is the vector of ordinates

h = (b-a)/M;
T = zeros(1, M+1);
Y = zeros(1, M+1);
T = a:h:b;
Y(1) = ya;
for j = 1:M
  Y(j+1) = Y(j) + h * feval(f,T(j),Y(j));
end
E=[T' Y'];
plot(T ,Y ,'r');
end
```

实验结果如下表1

将上表作图如下，其中红色曲线为方程1使用欧拉格式在步长$h = 1$时的计算结果，蓝色曲线为步长$h = 0.5$时的计算结果，绿色曲线为步长$h = 0.25$时的计算结果.

Table 1: 常微分方程1在欧拉格式下步长$h = 1, 0.5, 0.25$的输出结果

步长h	**迭代次数M**	**y(1)的逼近**	**y(2)的逼近**	**y(3)的逼近**
1	3	1	3	13
0.5	6	1.1250	5.306671142578125	2.181344382569777e+02
0.25	12	1.255186677910388	13.793965310277061	1.644599578587173e+10

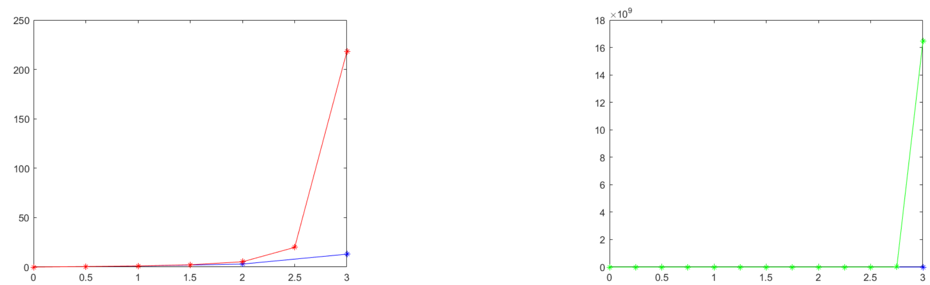

Figure 1: 常微分方程1在欧拉格式下步长$h = 1, 0.5, 0.25$的输出结果比较

梯形格式的代码实现

```
function H = heun(f, a, b, ya, M)
% Input - f     is the function entered as a string 'f'
%       - a,b   are the left and right end points
%       - ya    is the initial condition y(a)
%       - M     is the number of steps
% Output - H=[T' Y'] where T is the vector of abscissas and
%                    Y is the vector of ordinates

h = (b-a)/M;
T = zeros(1, M+1);
Y = zeros(1,M+1);
T = a:h:b;
Y(1) = ya;
for j = 1:M
   k1 = feval(f, T(j), Y(j));
   k2 = feval(f, T(j+1), Y(j) + h*k1);
   Y(j+1) = Y(j) + (h/2)*(k1+k2);
end
H=[T' Y'];
plot(T ,Y ,'r');
end
```

实验结果如下表2

将上表作图如下，其中红色曲线为方程1使用梯形格式在步长$h = 1$时的计算结果，蓝色曲线为步长$h = 0.5$时的计算结果，绿色曲线为步长$h = 0.25$时的计算结果.

Table 2: 常微分方程1在梯形格式下步长$h = 1, 0.5, 0.25$的输出结果

步长h	迭代次数M	y(1)的逼近	y(2)的逼近	y(3)的逼近
1	3	1.500000000000000	14.906250000000000	2.847341394853592e+04
0.5	6	1.514130592346191	97.698983198999300	7.746950417290489e+25
0.25	12	1.539404774465156	1.533032751706393e+05	Inf

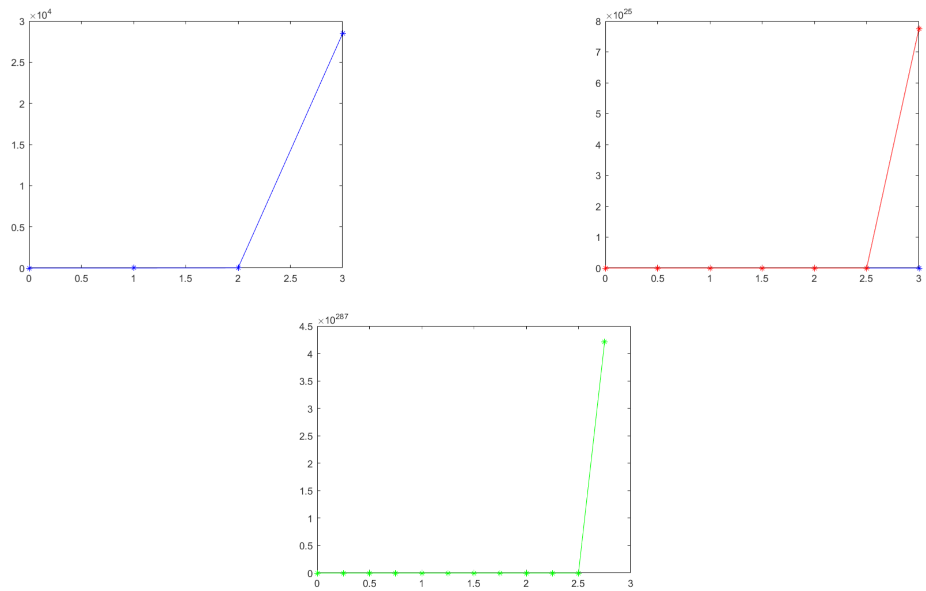

Figure 2: 常微分方程1在梯形格式下步长$h = 1, 0.5, 0.25$的输出结果比较

4阶龙格库塔格式的代码实现

```
function R = rk4(f, a, b, ya, M)
% Input - f   is the function entered as a string 'f'
%       - a,b are the left and right end points
%       - ya  is the initial condition y(a)
%       - M   is the number of steps
% Output - R=[T' Y'] where T is the vector of abscissas
%                 Y is the vector of ordinates

h = (b-a)/M;
T = zeros(1, M+1);
Y = zeros(1, M+1) ;
T = a:h:b;
Y(1) = ya;
for j = 1:M
   k1 = h*feval(f, T(j), Y(j));
   k2 = h*feval(f, T(j)+h/2, Y(j)+k1/2);
   k3 = h*feval(f, T(j)+h/2, Y(j)+k2/2);
   k4 = h*feval(f, T(j)+h, Y(j)+k3);
```

```
    Y(j+1) = Y(j) + (k1 + 2*k2 + 2*k3 + k4)/6;
  end
  R=[T' Y'];
  plot(T ,Y ,'r');
  end
```

实验结果如下表3

将上表作图如下，其中红色曲线为方程1使用4阶龙格库塔格式在步长$h=1$时的计算结果，蓝

Table 3: 常微分方程1在4阶龙格库塔格式下步长$h=1,0.5,0.25$的输出结果

步长h	**迭代次数M**	**y(1)的逼近**	**y(2)的逼近**	**y(3)的逼近**
1	3	1.535847981770833	5.177220263983291e+02	1.118005492535454e+39
0.5	6	1.554612104179646	1.112097692431962e+08	Inf
0.25	12	1.557247964295967	6.627407669161920e+80	Inf

色曲线为步长$h=0.5$时的计算结果.

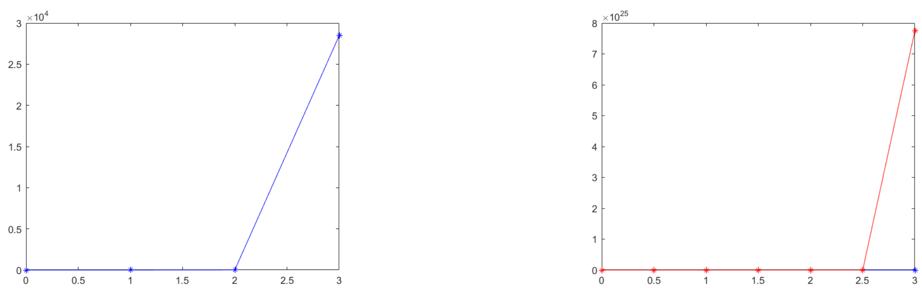

Figure 3: 常微分方程1在4阶龙格库塔格式下步长$h=1,0.5$的输出结果比较

问题 2

用龙格库塔4阶方法求解描述振荡器的经典的$van \quad der \quad Pol$微分方程

$$\left\{\begin{array}{l}\frac{\mathrm{d}^2 y}{\mathrm{d}t^2}-\mu\left(1-y^2\right)\frac{\mathrm{d}y}{\mathrm{d}t}+y=0\\ y(0)=1, y'(0)=0\end{array}\right.$$

分别取 $=0.01,0.1,1$，作图比较计算结果.

解:

首先把二阶常微分方程化为常微分方程组. 设$y_1=y, y_2=y'$，则有：

$$\left\{\begin{array}{l}y_1'=y_2\\ y_2'=\mu(1-y_1^2y_2-y_1)\\ y(0)=1, y'(0)=0\end{array}\right. \tag{2}$$

描述该微分方程组的函数如下：

```
  function dy = vdp(t,y)
    dy = zeros(2,1);
    dy(1) = y(2);
    dy(2) = 0.01 * (1-y(1)^2)*y(2) - y(1); % change mu
  end
```

分别取$\mu = 0.01, 0.1, 1$，输出图形如下图4，其中蓝色曲线为$\mu = 0.01$时微分方程2的数值解，红色曲线为$\mu = 0.1$时的数值解，绿色曲线为$\mu = 1$时的数值解.

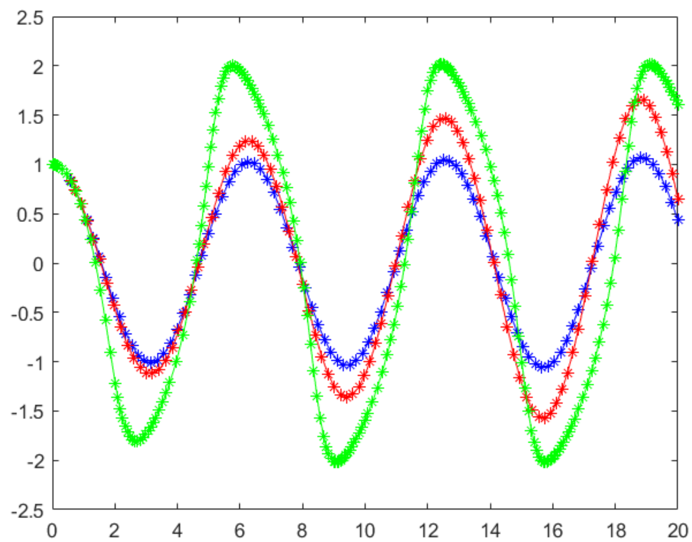

Figure 4: 微分方程2在$\mu = 0.01, 0.1, 1$时的数值解

方程组在区间$[a, b]$上的四阶龙格库塔算法代码实现如下：

```
function [T, Z] = rks4(F, a, b, Za, M)
% Input - F     is the system input as a string 'F'
%       - a,b   are the end points of the interval
%       - Za    =[x(a) y(a)] are the initial conditions
%       - M     is the number of steps
% Output - T    is the vector of steps
%        - Z    =[x1(t). . .xn(t)]; where xk(t) is the approximation
%                    to the kth dependent variable

h = (b-a)/M;
T = zeros(1, M+1) ;
Z = zeros(M+1, length(Za));
T = a;h:b;
Z(1,:) = Za;
for j = 1:M
k1 = h * feval(F,T(j), Z(j,:));
```

```
k2 = h * feval(F, T(j)+h/2, Z(j,:)+k1/2);
k3 = h * feval(F, T(j)+h/2, Z(j,:)+k2/2);
k4 = h * feval(F, T(j)+h, Z(j,:)+k3);
Z(j+1,:) = Z(j,:) + (k1+2*k2+2*k3+k4)/6;
end
```