第8章　曲线拟合与函数逼近

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问题 1
已知观测数据如表1，求一个二次多项式拟合这组数据，试写出其最小二乘拟合模型，并给出其正则方程组及其解.

Table 1: 观测数据

x	-2	-1	0	1	2
f(x)	0	1	2	1	0

解: 对于N个观测点$(x_k, y_k)_{k=1}^N$，最小二乘抛物线的拟合模型为

$$y = f(x) = Ax^2 + Bx + C$$

求解A, B, C的线性方程组为

$$\begin{aligned}
\left(\sum_{k=1}^{N} x_k^4\right) A + \left(\sum_{k=1}^{N} x_k^3\right) B + \left(\sum_{k=1}^{N} x_k^2\right) C &= \sum_{k=1}^{N} y_k x_k^2 \\
\left(\sum_{k=1}^{N} x_k^3\right) A + \left(\sum_{k=1}^{N} x_k^2\right) B + \left(\sum_{k=1}^{N} x_k\right) C &= \sum_{k=1}^{N} y_k x_k \\
\left(\sum_{k=1}^{N} x_k^2\right) A + \left(\sum_{k=1}^{N} x_k\right) B + NC &= \sum_{k=1}^{N} y_k
\end{aligned} \tag{1}$$

在本题中，方程组1代入系数计算后表示为

$$\begin{cases}
34A + 10C = 2 \\
10B = 0 \\
10A + 5C = 4
\end{cases} \tag{2}$$

解向量为$[-3/7 \quad 0 \quad 58/35]$.
下面进行编程解决，最小二乘多项式拟合的代码实现如下：

```
function C = lspoly(X, Y, M)
% Least square polynomial approximation
```

```
% Input  -  X  is the 1xn abscissa vector
%        -  Y  is the 1xn ordinate vector
%        -  M  is the degree of least square polynomial
% Output -  C  is the coefficient list for the polynomial

n = length(X);
B = zeros(1:M+1);
F = zeros(n,M+1);
% Fill the columns of F with the powers of X
for k = 1:M+1
  F(:,k) = X'.^(k-1);
end

A = F'* F;
B = F'*Y';
C = A\B;
C = flipud(C);
end
```

解得 $C = [-0.428571428571429 \quad 0 \quad 1.657142857142857]^T$，故能够拟合表1中数据的二次多项式为 $y = -0.428571428571429x^2 + 1.657142857142857$.
作图1可直观感受.

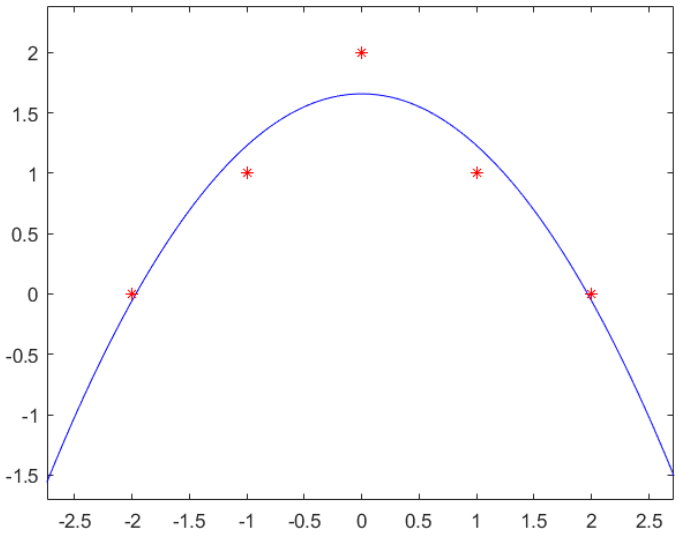

Figure 1: 观测数据及根据其拟合的二次多项式