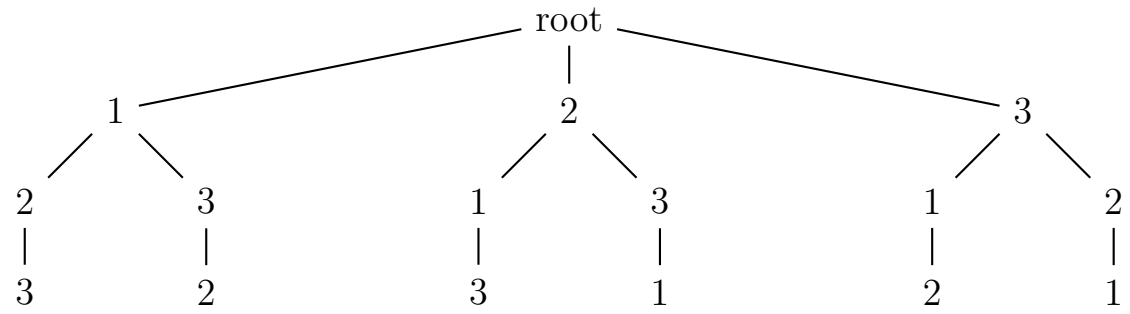
Lab07-Trees

VE281 - Data Structures and Algorithms, Xiaofeng Gao, TA: Qingmin Liu, Autumn 2019

∗ Please upload your assignment to website. Contact webmaster for any questions.

∗ Name:_____ Student ID:_____ Email: _____

Hint: You can use the package **tikz** to draw trees.



1. Red-black Tree

   (a) Suppose that we insert a sequence of keys 9, 3, 1 into an initially empty red-black tree. Draw the resulting red-black tree.

   (b) Suppose that we further insert key 6 into the red-black tree you get in Problem (1-a). Draw the resulting red-black tree.

   (c) Suppose that we further insert keys 2, 8 into the red-black tree you get in Problem (1-b). Draw the resulting red-black tree.

   (d) Suppose that we further insert key 7 into the red-black tree you get in Problem (1-c). Draw the resulting red-black tree.

   (e) Suppose that we further insert keys 4, 5 into the red-black tree you get in Problem (1-d). Draw the resulting red-black tree.

   When you draw the red-black tree, please indicate the color of each node in the tree. For example, you can color each node or put a letter **b/r** near each node.

2. Show the alphabet trie for the following collection of words: {chicken, goose, deer, horse, antelope, anteater, goldfish, ant, goat, duck}.

3. Show that any arbitrary n-node binary search tree can be transformed into any other arbitrary n-node binary search tree using $O(n)$ rotations.

   Hint: First show that at most $n-1$ right rotations suffice to transform the tree into a right-skewed binary search tree.

4. Suppose that an AVL tree insertion breaks the AVL balance condition. Suppose node P is the first node that has a balance condition violation in the insertion access path from the leaf. Assume the key is inserted into the left subtree of P and the left child of P is node A. Prove the following claims:

   (a) Before insertion, the balance factor of node P is 1. After insertion and before applying rotation to fix the violation, the balance factor of node P is 2.

   (b) Before insertion, the balance factor of node A is 0. After insertion and before applying rotation to fix the violation, the balance factor of node A cannot be 0.