Simulating Transmit and Receive Diversity Systems

or how I learned 3 Transmit is nontrivial

Jack Spencer "Danger" Langner
Dept. of Electrical Engineering
Cooper Union
Manhattan, NY
langner@cooper.edu

***Abstract*—The second project in ECE408: Wireless Communications was to recreate Fig. 4 in [1]. In order to do this both maximum receiver ratio and combining (MRRC) and Alamouti's transmit diversity schemes needed to implemented. Furthermore, since Alamouti developed the scheme for Rayleigh fading channels, we too had to simulate Rayleigh fading channels to ensure the diversity schemes worked as intended. In the end, I was successful in recreating Fig. 4 and I provide insight into my journey to get there.**

***Index Terms*—fading**

I. Introduction

INTRODUCTARY communications classes tend to deal exclusively with additive, white Gaussian noise (AWGN) as it provides a simple model for the channel that data is transmitted through. Additionally, the assumption of a purely AWGN channel allows analytical results to be derived easily and provide bounds on system performance or act as a reference when implementing a new system. However, the real world is not this nice and thus channel models have to be expanded to include additionally non-idealities. When there is no clear line of sight (LOS) between the transmitter and receiver along with scattering elements, it is a good idea to model the channel as fading. Furthermore, it has been shown that a Rayleigh distribution approximately describes the situation above. Thus, the signal is affected by both Rayleigh fading and AWGN, and life becomes harder. With the new channel model, new schemes have been proposed to overcome the degradation in the transmitted signal. In this paper, diversity is the scheme that will be described and implemented as a way to overcome the effects of the Rayleigh channel. Diversity describes a situation in which multiple transmitters and/or receivers are used to send and/or receive multiple copies of the same symbol in the hope of estimating the channel to undo the effect on the signal.

The paper is formatted as follows, in Section II a description of Rayleigh fading is provided along with how it was simulated. Following this, in Section III the technique of maximum receiver ratio combining is introduced along with its implementation. In Section IV the "new" technique that Alamouti developed in [1] is discussed along with its implementation. Then we get to the good stuff in Section V where we see if the goal was met. Finally, Section VI I briefly discuss potential future work to extend the simulations that were run for this paper.

II. Fading

A. Background on Fading

In a communication channel, fading refers to how a transmitted signal is affected by the path that it travels to the receiver. Here, fading is modeled as a multiplicative noise effecting the transmitted signal and AWGN is assumed to be added at the receiver. This is represented as:

$$r(t) = h(t)s(t) + n(t) \tag{1}$$

where $r(t)$ is the received signal, $h(t)$ is the channel, $s(t)$ is the transmitted signal, and $n(t)$ is the AWGN at the receiver. Additionally, all the terms are assumed to be complex valued and are allowed to vary with time.

In the simulations run, a slow, flat fading channel was assumed. This means that the channel does not change significantly over the period of a transmitted symbol (slow) and that all frequency components of the transmitted signal are affected in the same way (flat).

B. Rayleigh Fading

Since $h(t)$ is assumed to complex valued and changing in time, so it is reasonable to think that the components of $h(t)$ are randomly distributed. If a zero-mean Gaussian distribution is used to model the real and imaginary parts of $h(t)$, the envelope/magnitude of $h(t)$ will follow a Rayleigh distribution. The probability density function (pdf) of a Rayleigh distribution is :

$$p_r(x) = \left(\frac{x}{\sigma^2}\right) e^{-x^2/(2\sigma^2)}$$

where σ^2 is the variance of the independent Gaussian distributions used to generate the Rayleigh random variable. From the pdf, it can be shown that the mean of the distribution is

$$\bar{r} = \sigma\sqrt{\frac{\pi}{2}}$$

C. Generating Rayleigh Fading

Given that a Rayleigh random variable can be formed by taking the square root of the sum of squared Gaussian random variables with zero mean and the same variance, i.e. $R = \sqrt{X^2 + Y^2}$ for $X, Y \sim \mathcal{N}(0, \sigma^2)$. However, if we allow either the transmitter or receiver to move while the other is stationary, then a Doppler shift will be induced on the received signal. So if we set a maximum tolerable Doppler shift on the system, and using the algorithm in [2], it is possible to generate Rayleigh fading noise with the prescribed power spectrum. Additionally, the algorithm used in [2] was developed in [3]. A minor detail included in [3] that becomes helpful is that the generated data should be transformed to have a mean of 1 which is equivalent 0 dBm.

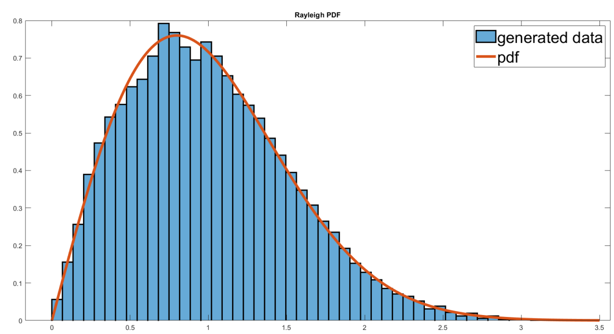

Fig. 1. Histogram of Generated Rayleigh data vs. PDF

The above, Fig. 1, shows the result of generating 16384 (2^{14}) independent Rayleigh variables with the algorithm described in [2] with a maximum Doppler shift of 1 Hz. A histogram of the magnitude of the samples which has been normalized to give a pdf is over layed with the ideal Rayleigh pdf with a unit mean.

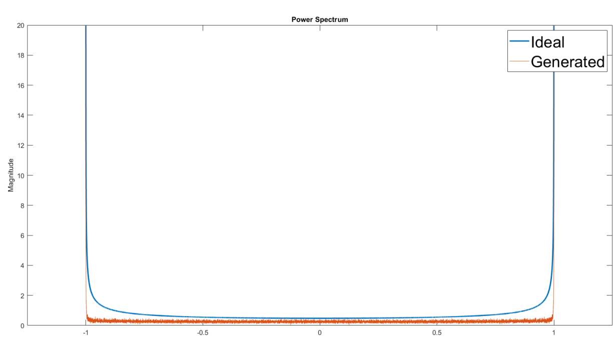

Fig. 2. Power Spectrum of Simulated Rayleigh variables

Fig. 2 shows a comparison between the ideal Doppler power spectrum and the average power spectrum of 5 independent iterations of the algorithm. Note, the the simulated power spectrum was scaled so that it comparable to the ideal. From this we can see that both the ideal and simulated power spectrums are mostly flat, which is good since the goal was to generate to a flat fading channel. Another note, the ideal spectrum is given as

$$S_{E_z}(f) = \frac{1.5}{\pi f_D \sqrt{1 - \left(\frac{f}{f_D}\right)^2}}$$

which happens to be the Fourier transform of the $0^t h$ order Bessel function of the first kind up to a scale factor.

III. Maximal-Ratio Receiver Combining

As alluded to in Section I, a fading channel presents an additional challenge that needs to be overcome in order to get reliable data transfer. To demonstrate how much harsher the fading channel is compared to an AWGN channel, the bit error rate (BER) for uncoded binary phase shift keying (BPSK) was determined for both channels.

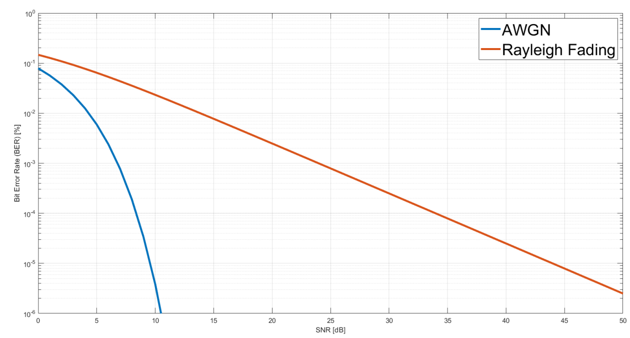

Fig. 3.

As a point of reference in the AWGN channel, a BER of 10^{-3} is achieved at an SNR of 7 dB, for the Rayleigh fading channel the same BER is achieved at 18 dB.

Now that we have some idea of how difficult it is to transmit in a Rayleigh channel, the need for techniques that can reduce the effect of the Rayleigh fading channel is apparent. The first solution to be discussed here is using multiple receive antennas with one transmit antenna. This makes some sense because a single antenna can only send one symbol at time, so then this single symbol will be received multiple times and if there is prior knowledge of what the symbol is supposed to be, then the different channels (one unique channel per receive antenna) can be estimated and their effect accounted for at the receiver. This motivates the real world application of multiple receive antennas, however in [1] there is an assumption that each receive antenna has perfect knowledge of the channels. This is a little less real world, but allows for the developments of a straight forward decision rule. In the interest of time and my sanity , I leave the development of the decision rule to Section II. Classical Maximal-Ratio Receiver Combining (MRRC) Scheme in [1], however I will note that the decision rule turns out to be

$$d^2(\tilde{s}_0, s_i) \leq d^2(\tilde{s}_0, s_k)$$

where $d^2(\cdot,\cdot)$ is the Euclidean distance between two complex points and s_i and s_k are symbols that can be sent within the given modulation scheme. Furthermore, the rule is to pick the symbol that has the smallest distance from $\tilde{s}_0$. Since we are dealing with BPSK modulation there are only two symbols, namely 1 and -1, but this can be easily generalized. Furthermore, I implemented the decision rule by utilizing MATLAB's min function. By first calculating the distance from 1 or -1 (in that order) to $\tilde{s}_0$ and storing the result in an array, I then used min to return the index of, which would be a vector of either 1 or 2, by subtracting 1 from the result of the index, I was then able to recover the bit the MRRC decision rule would make. The order is important because with "normal" BPSK modulation you get $0 \longrightarrow 1$ and $1 \longrightarrow -1$, so by finding the difference from 1 first, you recover the correct bit in the substraction from the index. This methodology for decoding can be extended to higher order modulations by converting the bit pattern that would generate a given symbol to decimal and using the decimal to symbol relationship. Point is it can be extended.

IV. Alamouti's Method

This section is titled *Alamout's Method* becasue it describes the scheme that Alamouti put forward in his paper. He builds on the idea of using multiple receive antennas (receive diversity) by including an additional transmit antenna (transmit diversity). I again defer to [1] for a complete description of the methodoloy and highlight the important aspects.

Since two antennas can be used to transmit data, they can transmit symbols independently. However, if there is no correlation between the signals sent by the transmitters, then the receivers will see signals that are essentially noise. Take for example the situation where there are 2 transmitters and one receiver. If the two transmitters continually sent independent messages, then the receiver would never be able to properly decode either. For this reason, Alamouti describes an encoding scheme where the transmitters send their own symbol first and the next symbol they send is some variant of the symbol sent by the other transmitter. Here variant can conjugate and/or negative of the symbol sent. Additionally, $\tilde{s}_0$ and $\tilde{s}_1$ are constructed such that they be used with a decision rule that i essentially MRRC.

By including another transmit antenna, it becomes ver important to remember which symbol you are dealing with a each antenna and which channel the symbol travels through What helped me understand what was going on was equatio (14) in [1],

$$\begin{aligned} r_0 &= h_0 s_0 + h_1 s_1 + n_0 \\ r_1 &= -h_0 s_1^* + h_1 s_0^* + n_1 \\ r_2 &= h_2 s_0 + h_3 s_1 + n_2 \\ r_3 &= -h_2 s_1^* + h_3 s_0^* + n_3 \end{aligned}$$

This is helpful because it shows how to build each received signal. With Table 3 from [1] this provides the necessary information of constructing the received signal matrix on which the decision rule will be applied. The decision rule for transmit diversity is essentially the same, with the extension that each transmit antenna has to have the decision rule applied to what it sent.

V. Results

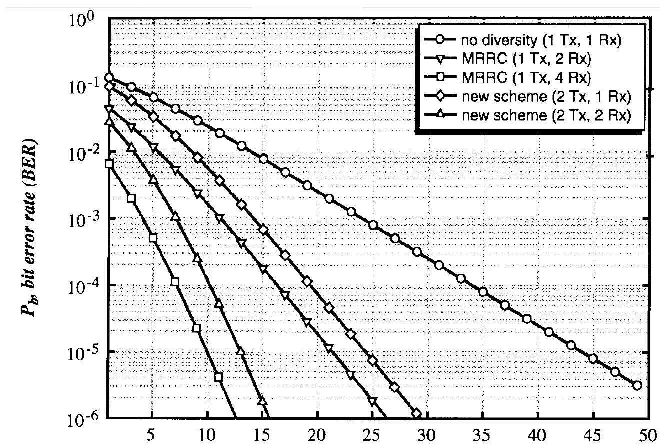

Fig. 4. Fig. 4 from [1]

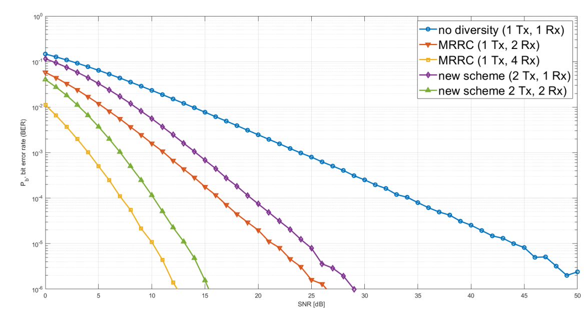

Fig. 5. My recreation of Fig. 4 in [1]

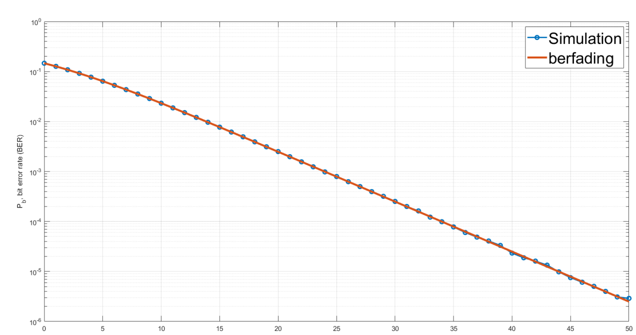

Fig. 6. Comparison of my Rayleigh fading with Analytical result

The first thing that is worth checking is the validity of the Rayleigh fading channel that was implemented as recreating

Fig.4 in [1] is the main goal of this project. This is seen in Fig. 6. The fact that it appears to be one curve is very reassuring as that means my simulated Rayleigh fading channel matches with the function provided by MATLAB in the Communications toolbox, berfading. With this out of the way, we get to the true purpose of the assignment, which was to recreate Fig. 4 in [1].

In Fig. 4 we see the famed Fig. 4 from [1]. We see how the new scheme (Alamouti's scheme) has 3 dB worse performance compared to MRRC, but as Alamouti points out this is because there is an assumption that the total radiated power is held constant, so for two transmitters, they both have to operate at half power, and $10 \log_{10}\left(\frac{1}{2}\right) \approx -3$, thus explaining the difference in performance. So following Alamouti's advice, I also ran the simulation the SNR doubled for the new scheme which can be in Fig. 7.

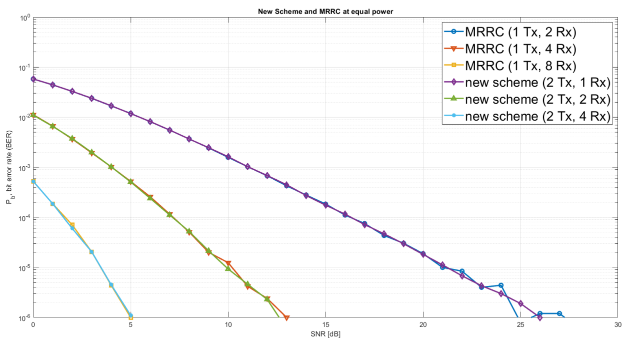

Fig. 7. New Method with +3 dB for SNR

As promised, by doubling the SNR, the new scheme matches the MRRC. This is helpful because this serves as a proof that Alamouti's scheme has the same diversity order as the MRRC. This is helpful because it either allows for the signal to be transmitted at a lower power or the same level of diversity gain can be be achieved with exponentially less receivers. This is is highlighted in Fig. 7 because for 8 receive antennas there are 9 antenna elements, whereas using Alamouti's method only 6 antennas are necessary. So if space at the receiver becomes a constraint, using transmit diversity as proposed in [1] be help attain the same performance with fewer receivers.

VI. Future Work

I am going to be brief here, if given more time I would want to extend Alamouti's transmit diversity scheme to support N transmit antennas. When I set down this path for this project, I realized that Alamouti carefully set up his transmit scheme so that when constructing the $\tilde{s}$ terms they would correspond to exactly one symbol scaled by the channel magnitude plus noise. This was done by carefully selecting which terms to conjugate and/or negate so that there was cancellation among the symbols not of interest. So in order to extend Alamouti's scheme while taking advantage of the MRRC rule, the correct operators need to be applied so that there is a map that describes the relationship between s_0 and $\tilde{s}_0$ without any of the other s_i's involved in the expression. This is an interesting problem and I would like to solve it, I would also be open to learning about how more sophisticated transmitted diversity schemes are implemented.

I would also like to take a moment to point out that I figured out how to index the transmitted symbols for T transmit antennas. If n symbols are to be sent from T transmit antennas, then each antenna will send nT total symbols, keeping with Alamouti's scheme. In order to keep the order of transmitted symbols, it can be set by using a loop variable t which ranges from 1 to T, the indices correspond to $t : T : nT - (-t) \bmod T$.

Acknowledgment

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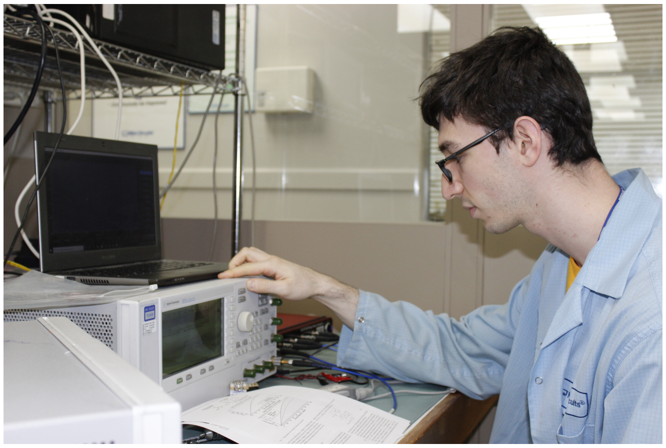
About the Author



Jack Spencer "Danger" Langner received his Bachelors from the Cooper Union for the Advancement of Science and Art in May 2019 and began working on his masters in the Autumn of 2019 at the same institution.

He currently works at Mini-Circuits, an RF and mircowave component manufacturer, which is located in Brooklyn, where he is a pipeline engineer in the engineering test department. Currently, he is in the process of developing new procedures that relate to measuring residual phase noise (pictured above). Additionally, he is in the midst of a "software revolution" in the hopes of improving the lives of the other test engineers. Jack's hobbies include learning about RF instrumentation and differential equations, he's an odd one for sure.