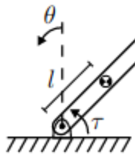


Figure 1: Sketch of the system with mass $m = 1$ kg, length to center of mass $l = 0.5$ m, and a friction given by $\tau_f = b\dot{\theta}$ with $b = 0.1$ Nm/(rad/s).

1. **System Modeling**
   (a) Setup a model (nth-order differential equation) of the system.
   (b) Reformulate the system model as a system of 1st order differential equations.
   (c) Derive a linearized model of the system at an angle of $\pi/3$ rad.

Equation: (Roterende Newtons 2. lov)

$$I\alpha = u - \tau_f + \sin(\theta)\cdot g\cdot l\cdot m$$

$I = ml^2$ (Punktmasse)

$$I\ddot{\theta} = u - b\dot{\theta} + \sin(\theta)\cdot m\cdot g\cdot l$$

$$\ddot{\theta} = \frac{u}{I} - \frac{b\dot{\theta}}{I} + \frac{\sin(\theta)\cdot m\cdot g\cdot l}{I}$$

As 2 first order equations:

$$x = \begin{bmatrix}\theta\\ \dot{\theta}\end{bmatrix}$$

$$\dot{x} = f(x,u) = \begin{bmatrix} x_2 \\ \frac{u}{I} - \frac{bx_2}{I} + \frac{\sin(x_1)\cdot mgl}{I}\end{bmatrix}$$

$$= \begin{bmatrix} x_2 \\ \frac{u}{I} - \frac{bx_2}{I} + \frac{\sin(x_1)\cdot g}{l}\end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

Linearize at $\theta = \frac{\pi}{3}$, $\dot{\theta} = 0$, $\ddot{\theta} = 0$:

$\bar{x}_1 = \frac{\pi}{3}$, $\bar{x}_2 = 0$, $\dot{x} = 0$

$\bar{u} = -mlg\sin(\frac{\pi}{3})$ (counteracts gravity at the point)

$\hat{x} = x - \bar{x}$, $\hat{u} = u - \bar{u}$

Linearizing by first order taylor expansion:

$$\dot{x} \approx \underbrace{f(\bar{x},\bar{u})}_{0} + \frac{\partial f}{\partial x}\bigg|_{\substack{x=\bar{x}\\ u=\bar{u}}} \hat{x} + \frac{\partial f}{\partial u}\bigg|_{\substack{x=\bar{x}\\ u=\bar{u}}} \hat{u}$$

$$\frac{\partial f}{\partial x} = \begin{bmatrix} 0 & 1 \\ \frac{\cos(x_1)\cdot g}{L} & -\frac{b}{I} \end{bmatrix}$$

$$\frac{\partial f}{\partial u} = \begin{bmatrix} 0 \\ \frac{1}{I} \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ \frac{\cos(x_1)\cdot g}{L} & -\frac{b}{I} \end{bmatrix}\Bigg|_{\substack{x=\bar{x}\\ u=\bar{u}}} \hat{x} + \begin{bmatrix} 0 \\ \frac{1}{I} \end{bmatrix}_{\substack{x=\bar{x}\\ u=\bar{u}}} \hat{u}$$

$$= \begin{bmatrix} 0 & 1 \\ \frac{\cos(\frac{\pi}{3})\cdot 9{,}82}{0{,}5} & -\frac{0{,}1}{0{,}25} \end{bmatrix} \cdot \hat{x} + \begin{bmatrix} 0 \\ \frac{1}{0{,}25} \end{bmatrix} \cdot \hat{u}$$

$$= \underbrace{\begin{bmatrix} 0 & 1 \\ 9{,}82 & -0{,}4 \end{bmatrix}}_{A} \cdot (x - \tfrac{\pi}{3}) + \underbrace{\begin{bmatrix} 0 \\ 4 \end{bmatrix}}_{B} \cdot (u - 4{,}2522)$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \underbrace{0}_{D} \cdot u$$

We measure position as output.

2. **Performance Specification**

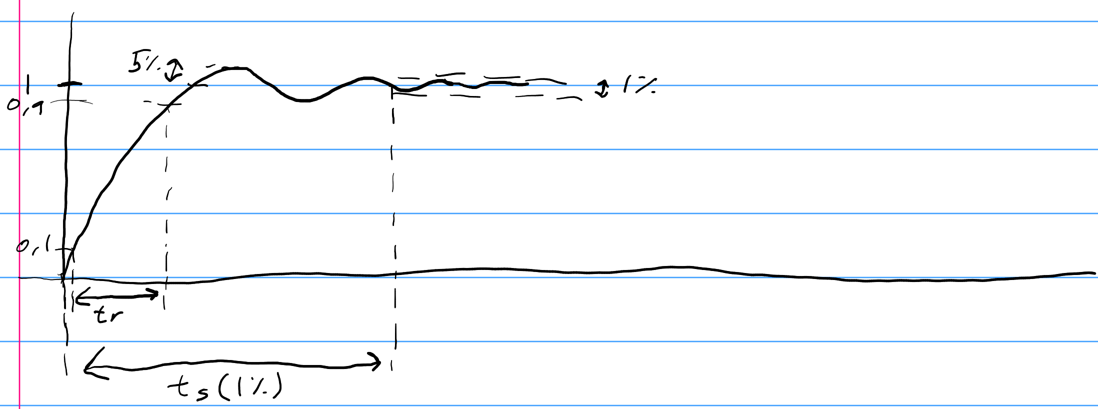
(a) Specify a desired performance of the system in time-domain.

(b) Set up a corresponding performance specification in frequency-domain.

a) $M_p = 5\%$ (overshoot)

$t_r = 1s$

$t_s(1\%) = 4s$



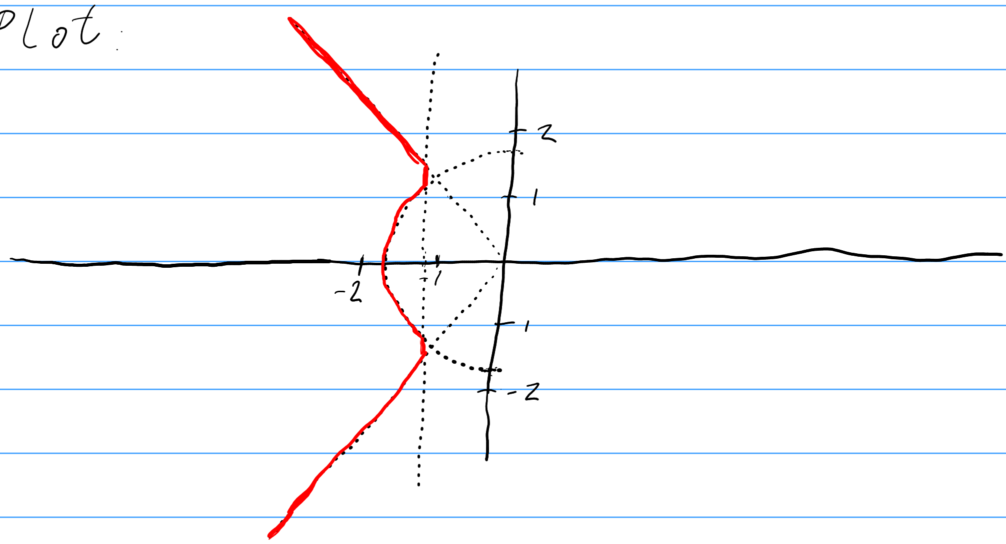
Frequency domain:

$$\omega_n = \frac{1{,}8}{t_r} = \frac{1{,}8}{1} = 1{,}8$$

$$\zeta = \sqrt{\frac{\left(\frac{\ln(M_p)}{-\pi}\right)^2}{1+\left(\frac{\ln(M_p)}{-\pi}\right)^2}} = \sqrt{\frac{\left(\frac{\ln(0{,}05)}{-\pi}\right)^2}{1+\left(\frac{\ln(0{,}05)}{-\pi}\right)^2}} = 0{,}6901$$

$$\sigma = \frac{-\ln(1/100)}{4} = 1{,}1513$$

Plot:



3. Controller Design

(a) Design a PID controller for the linearized model of the system such that it attains the desired performance. The tuning procedure should be described (the template for the solution indicates that the controller type should be found by iterating over controller types starting with a P-controller).

```
sys =

        4
  ----------------
  s^2 + 0.4 s + 17
```

characteristic equation:

$$1 + K(s)\,G(s) = 0$$

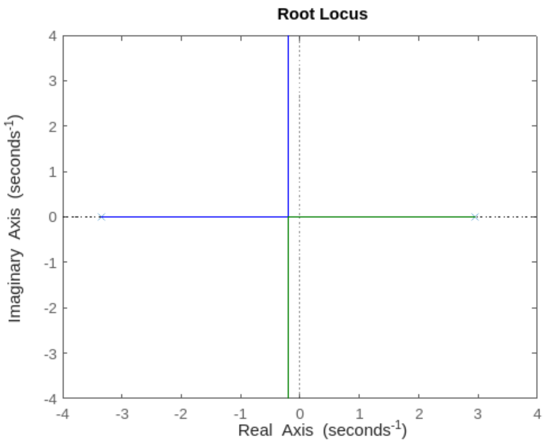
$$G(s) = \frac{4}{s^2 + 0{,}4s - 9{,}82}$$

P-controller:

$$K(s) = k_p \rightarrow 1 + K(s)\,G(s) = 0$$

$$\Downarrow$$

$$1 + k_p\,G(s) = 0$$



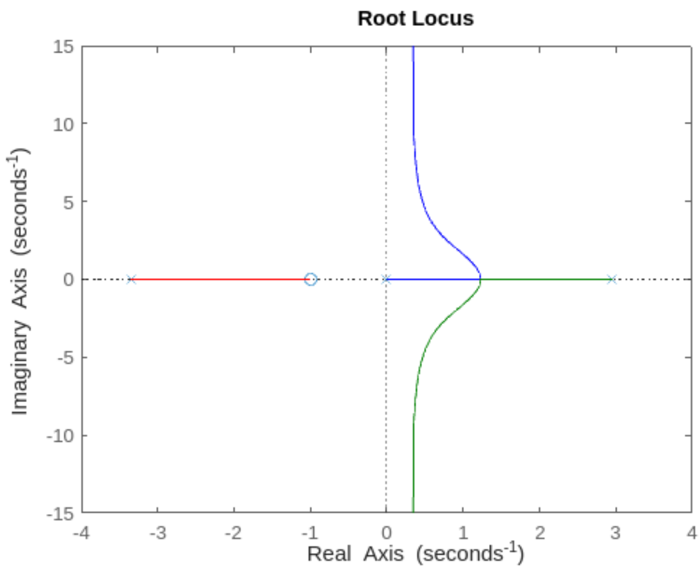
Not enough

PI-controller

(Automatic reset)

$$K(s) = k_p\left(1 + \frac{1}{T_i s}\right)$$

$$1 + K(s)\,G(s) = 0 \rightarrow 1 + k_p\left(1 + \frac{1}{T_i s}\right)\left(\frac{4}{s^2 + 0{,}4s + 17}\right)$$



Also wrong

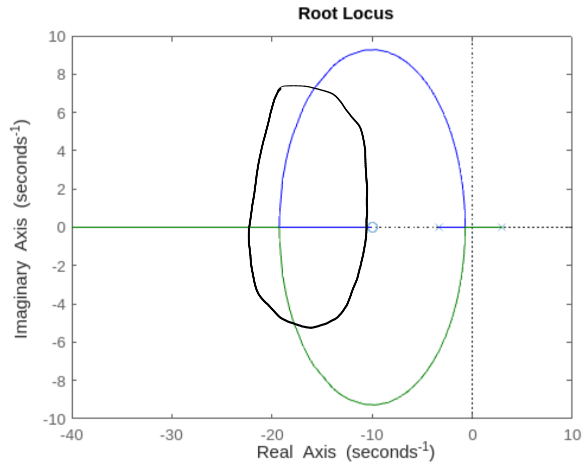
PD-controller

$$K(s) = k_p(1 + T_d s)$$

$$1 + K(s)\,G(s) = 0$$

$$\Updownarrow$$

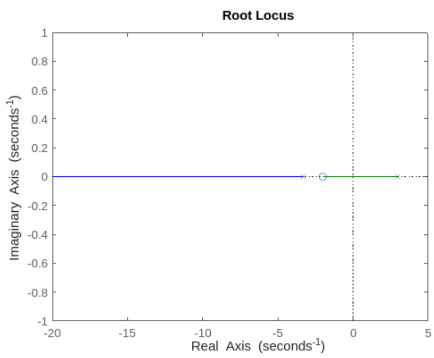
$$1 + k_p(1+T_d s)\left(\frac{4}{s^2+0{,}4s-9{,}82}\right) = 0$$

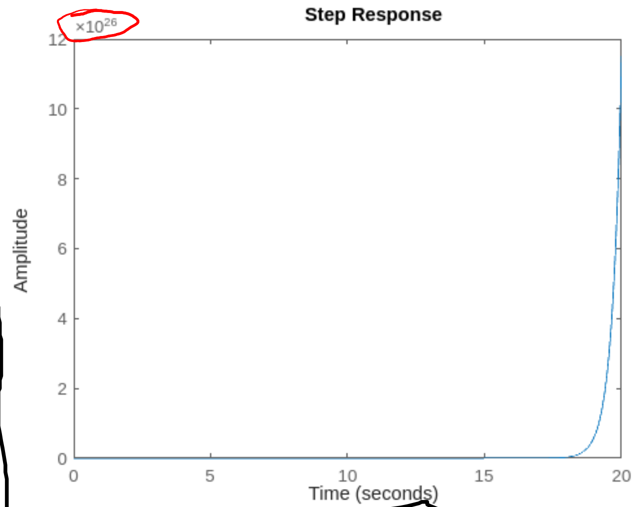


$$T_d = \frac{1}{10}$$

Hvis man vælger et polpar indenfor den sorte cirkel, troede jeg det ville virke, men stepresponset ser forkert ud.

T_d = 1/2






```
index = 52;
poles(:,index)
```

```
ans = 2×1
   -700.4095
     -1.9905
```

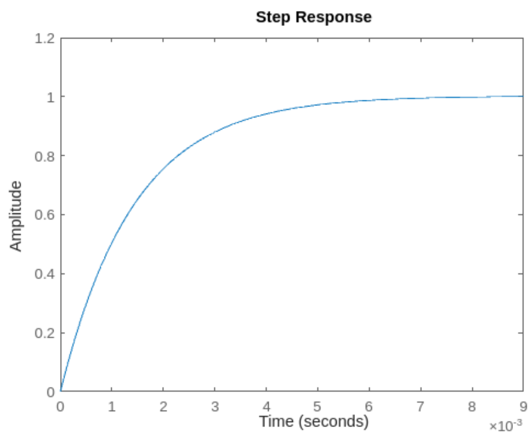
```
gains(1,index)
```

```
ans = 351.0000
```

```
G = gains(1,index)*(1+T_d*s);
sys1 = G*sys / (1+G*sys)
```

```
sys1 =

         702 s^3 + 1685 s^2 - 6332 s - 1.379e04
  ---------------------------------------------------
  s^4 + 702.8 s^3 + 1665 s^2 - 6340 s - 1.369e04
```




```
stepinfo(sys1)
```

```
ans = struct with fields:
         RiseTime: 0.0032
    TransientTime: 0.0060
     SettlingTime: 0.0060
      SettlingMin: 0.9066
      SettlingMax: 1.0026
        Overshoot: 0
       Undershoot: 0
             Peak: 1.0026
         PeakTime: 0.0400
```