Exercise 2.1

Figure 2 illustrates two masses connected via a lossless spring, moving at a surface with no friction.

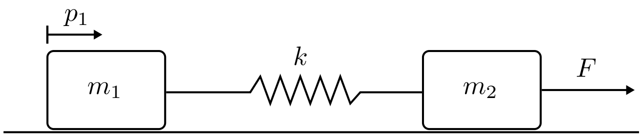

Figure 2: Two masses connected via a lossless spring, where a controllable force F is applied to m_2.

1. Derive a state space model for the system, where the force is considered to be an input, and the velocity $v_1 = \dot{p}_1$ is considered to be the output.
2. Compute the eigenvalues of the system matrix. Where in the complex plane are they located and why?
3. Does the system have any zeros? If yes, where are they located?
4. How can the transfer function for the system be computed based on the state space model?
5. Derive a transfer function from the state space model for $m_1 = m_2 = 1$ kg and $k = 1$ N/m, and determine its poles and zeros?

Exercise 2.2

It is important to know the relation between the pole locations of a system and its time-behavior, to shape the behavior of a dynamical system. Consider a system with the following specification

- Overshoot smaller than 5 %
- 1 % settling time lower than 10 s
- Rise time lower than 5 s

Analyze the system as follows

1. Sketch the step response of a 2nd order system having the above properties.
2. Where in the complex plane can the poles of a system having the above properties be located?
3. Add one zero to the 2nd order system

$$G(s) = \frac{1}{s^2 + 2s + 1}$$

   How is the step response changed, when the zero is moved between the values $s = -8$, $s = -4$, $s = -2$, $s = -1$, $s = -0.5$, $s = 1$ (ensure that the DC gain is one for all systems)?